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Computer Algebra in Scientific Computing

23rd International Workshop, CASC 2021 Sochi, Russia, September 13–17, 2021 Proceedings

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Preface

The International Workshop on Computer Algebra in Scientific Computing (CASC) provides the opportunity both for researchers in theoretical computer algebra (CA) and engineers, as well as other allied professionals applying CA tools for solving problems in industry and in various branches of scientific computing, to present their results annually. CASC is the forum of excellence for the exploration of the frontiers in the field of computer algebra and its applications in scientific computing. It brings together scholars, engineers, and scientists from various disciplines that include computer algebra. This workshop provides a platform for the delegates to exchange new ideas and application experiences, share research results, and discuss existing issues and challenges.

Sirius Mathematics Center (SMC), located in the city of Sochi, Russian Federation, was established in 2019 by the "Talent and Success" Educational Foundation. This is an international institution for research and postgraduate training in mathematical sciences. Currently, the center uses the facilities of the Omega Sirius Hotel located between Sochi Olympic Park and the former Olympic Village near the Black Sea coast. The mission of the center is to support mathematical research in Russia as well as to promote personal and scientific contacts between mathematicians. The center strives to be a meeting point for scientists working in mathematical sciences, enabling them to exchange ideas, initiate new projects, meet, and train students and young scientists.

The SMC Scientific Board is responsible for establishing selection criteria for proposals of activities at the SMC, evaluating the proposals, and developing the scientific program of the center. The current members are Maria J. Esteban (CEREMADE, CNRS, and Université Paris-Dauphine, Paris), Sergey Lando (Higher School of Economics, Moscow and Skolkovo Institute of Science and Technology, Moscow), Ari Laptev (Imperial College, London), Alexey Shchuplev (SMC, Director), and August Tsikh (Siberian Federal University, Krasnoyarsk). In the autumn of 2020, the SMC administration offered the CASC workshop organizers significant financial support for arranging the CASC 2021 workshop on the SMC platform.

Therefore, it was decided, in the autumn of 2020, that the 23rd CASC International Workshop would be held at the Sirius Mathematics Center, Sochi, on September 13–17, 2021.

The organizing committee of the CASC 2021 International Workshop has been monitoring the developing COVID-19 pandemic. The safety and well-being of all conference participants have been our priority. Due to the current international situation, CASC 2021 was exceptionally held in the hybrid format: those able to travel to Sochi have attended in person while those prevented from coming by the restrictions on international travel were offered the opportunity to present their work remotely.

This year, the CASC International Workshop had two categories of participation: (1) talks with accompanying papers to appear in these proceedings, and (2) talks with accompanying extended abstracts for distribution locally at the conference only. The latter was for work either already published, or not yet ready for publication, but in either case still new and of interest to the CASC audience. The former was strictly for new and original research results, ready for publication.

All papers submitted for the LNCS proceedings received a minimum of three reviews. In addition, the whole Program Committee was invited to comment and debate on all papers. In total, this volume contains 23 papers and two invited talks. The paper by Ioannis Emiris presents an invited talk but went through the regular review process.

The invited talk by Alicia Dickenstein is devoted to the motivation and description of several algebraic-geometric computational techniques used for the study of families of polynomials that arise in the realm of biochemical reaction networks. The standard modelling of biochemical reaction networks gives rise to systems of ordinary polynomial differential equations depending on parameters. One is thus led to study families of polynomial ordinary differential equations, with a combinatorial structure that comes from the digraph of reactions. Attempts to explore the parameter space, in order to predict properties of the associated systems, challenge the standard current computational tools because, even for moderately small networks, there are many variables and many parameters. It is shown that different techniques can be strengthened and applied for systems with special structure even if the number of variables and parameters is arbitrarily large; in particular, for the systems defined by Alicia Dickenstein and Pérez Millán termed MESSI (Modifications of type Enzyme-Substrate or Swap with Intermediates), which are abundant among the enzymatic mechanisms.

The invited talk presented by Ioannis Emiris addresses one of the main problems in distance geometry: given a set of distances for some pairs of points, one must determine the unspecified distances. This is highly motivated by applications in molecular biology, robotics, civil engineering, sensor networks, and data science. A new method is proposed that introduces a combinatorial process in terms of directed graphs with constrained orientations, and manages to improve in all dimensions the existing bounds for roots count; this is achieved by employing the m-Bézout bound, thus arriving at tighter results than using the classic Bézout bound. The method readily leads to bounds on the m-Bézout number of a polynomial system, provided that the given system can be modelled by a graph whose vertices correspond to the variable subsets and whose edges correspond to the given equations.

Polynomial algebra, which is at the core of CA, is represented by contributions devoted to the use of comprehensive Gröbner bases for testing binomiality of chemical reaction networks, the parallel factorization of polynomials with multivariate power series coefficients, a new version of the root radii algorithm for finding the roots of a univariate polynomial, the use of subresultant chains for the solution of polynomial systems, the extension of Fulton's algorithm for determining the intersection multiplicity of two plane curves to the higher-dimensional case, the use of the resultants and of the computer algebra system (CAS) Maple in the investigation of the geometric properties of Fermat–Torricelli points on a sphere, and the derivation with the aid of Gröbner bases of new optimal symplectic fourth-order Runge–Kutta–Nyström methods for the numerical solution of molecular dynamics problems.

Four papers deal with ordinary and partial differential equations: the use of Weil algebras for the symbolic computation of univariate and multivariate higher-order partial derivatives, establishing the relationship between differential algebra and tropical differential algebra, applications of primitive recursive ordered fields to the computability and complexity of solution operators for some partial differential equations (PDEs), and the solution with guaranteed precision of the Cauchy problem for linear evolutionary systems of PDEs in the case of real analytic initial data.

Two papers are devoted to the applications of symbolic-numerical computations for computing orthonormal bases of the SU(3) group for orbital angular momentum implemented in the CAS MATHEMATICA and symbolic and numeric computations of the Frobenius norm real stability radius for some classes of matrices.

Applications of computer algebra systems in mechanics, physics, and chemistry are represented by the following themes: the derivation of first integrals and invariant manifolds in the generalized problem of the motion of a rigid body in a magnetic field with the aid of Gröbner bases and the CAS MATHEMATICA, and the detection of toricity of steady state varieties of chemical reaction networks with the aid of the CAS REDUCE.

The remaining topics include a new algorithm for decoupling multivariate fractions with the aid of trees, the simplification of nested real radicals in the CASs of a general kind, improved algorithms for approximate GCD in terms of robustness and distance, rational solutions of pseudo-linear systems, a new algorithm for testing the supersingularity of elliptic curves by using the Legendre form of elliptic curves, a new deterministic method for computing the Milnor number of an isolated complete intersection singularity, a new algorithm for computing the integer hull of a rational polyhedral set, and the construction of 8958 new nonisomorphic parallelisms of the three-dimensional projective space over the finite field $\mathbb{F}_5$.

Sadly, Vladimir P. Gerdt, who was one of the two co-founders (along with Prof. Dr. Ernst W. Mayr, Technical University of Munich) of the CASC International Workshops, passed away on January 5, 2021. In honor and memory of V.P. Gerdt, this volume contains an obituary which describes his contributions to different branches of computer algebra and to quantum computing. A special session dedicated to Gerdt's memory was held during this workshop.

The CASC 2021 workshop was supported financially by a generous grant from the Sirius Mathematics Center headed by Dr. Alexey Shchuplev. We appreciate that the SMC provided free accommodation for a number of participants. We also gratefully acknowledge support by the Ministry of Science and Higher Education of the Russian Federation, grant No. FSSW-2020-0008.

The local organizing committee of CASC 2021 at the Sirius Mathematics Center in Sochi provided excellent conference facilities, which enabled foreign participants to present their talks remotely.

Our particular thanks are due to the members of the CASC 2021 local organizing committee and staff at the SMC, i.e., Vitaly Krasikov (Chair), Alexey Shchuplev, Natalia Tokareva, Irina Klevtsova, Peter Karpov, Sergey Tikhomirov, and Timur Zhukov who ably handled all the local arrangements in Sochi.

Furthermore, we want to thank all the members of the Program Committee for their thorough work. We also thank the external referees who provided reviews.

We are grateful to the members of the group headed by Timur Sadykov for their technical help in the preparation of the camera-ready files for this volume. We are grateful to Dmitry Lyakhov (King Abdullah University of Science and Technology, Kingdom

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July 2021

François Boulier
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Memories on Vladimir Gerdt

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Prof. Vladimir P. Gerdt

It is our deepest regret to inform you that Vladimir Petrovich Gerdt, Professor, Head of the Algebraic and Quantum Computing Group of the Scientific Department of Computational Physics of the Laboratory of Information Technologies (LIT) at the Joint Institute of Nuclear Research (JINR) in Dubna, Oblast Moscow, Russia, died on January 5th, 2021 at the age of 73, following complications caused by COVID-19. Vladimir Gerdt was born on January 21, 1947 in the town of Engels, Saratov region of the USSR. He began his scientific career at JINR in November 1971, after graduating from the Physics Department of Saratov State University, first in the Department of Radiation Safety, and from February 1977 on in the Laboratory of Computer Technology and Automation, which, in the year 2000, was renamed to Laboratory of Information Technologies, where he was engaged in the deployment of analytical computing software systems on the computers of the JINR Central Research Center, as well as their development and application for solving physical problems. In 1983, he became the head of the Computer Algebra Research Group (renamed in 2007 to Algebraic and Quantum Computing Group) at LIT. In 1976, Vladimir Gerdt successfully defended his Ph.D. thesis (for *Kandidat nauk*) in the field *Theoretical and Mathematical Physics*, and in 1992, his doctoral dissertation (for *Doktor nauk*, D.Sc.) in the field *Application of Computer Technology, Mathematical Modeling, and Mathematical Methods for Scientific Research.* In 1997, he was awarded the academic title of Professor. In his long and distinguished research career, Vladimir Gerdt worked on many different topics. Even when he started to work on something new, he never forgot the old topics. Often, he also looked for, in a creative form, possible relationships between his various research questions exhibiting numerous interesting connections. In the following, we try to organize his research works into seven fields in which he was active and which we list roughly in chronological order according to his first publication in the respective field (and we also apologize for any omissions or errors due to our bias and the requirement to be succinct):

1. Physics: high energy physics, gauge theory and constrained dynamics
2. Differential equations: integrable systems, symmetry theory and completion questions
3. Lie algebra: representations and classifications
4. Commutative algebra: Gröbner and involutive bases, polynomial system solving
5. Differential and difference algebra: differential/difference ideal theory and non-commutative Gröbner and involutive bases
6. Quantum computing: quantum circuits and related algebraic problems, simulation of quantum algorithms, quantum error correction, mixed states
7. Numerical analysis: algebraic construction of finite difference methods, symbolic-numerical solution of quantum mechanical problems

In the sequel, we try to trace the main steps in Vladimir's scientific activities over his whole career spanning a period of almost 50 years. Of course, it is not possible to present everything he did, and our selection is certainly subjective and biased by our own research interest and, possibly, lack of knowledge. Nevertheless, we believe that this account is able to convey how broad his research interests were and how many important contributions he made. As, over the years, Vladimir collaborated with so many different people, we here omit the names of his cooperation partners in the various fields.

Vladimir began his career like many of the pioneers in computer algebra as a physicist. His first publications in the mid 1970s were concerned with phenomenological computations in high energy physics aiming at predicting the results of accelerator experiments. As such computations tend to be very demanding and time consuming, it was a natural thought to try to automatize them at least partially using computer algebra. Thus, his first publication with the words "computer algebra" in the title appeared in 1978 and was concerned with the computation of Feynman integrals (essentially the same problem that inspired a bit over a decade earlier Tony Hearn to develop Reduce, together with Macsyma the first general purpose computer algebra systems). At this time, for most physicists or mathematicians, computer algebra was still something rather exotic and a comprehensive list of articles describing such applications of computer algebra was rather short.

As many problems in physics boil down to the analysis of differential equations, it is not surprising that from the early 1980s on Vladimir got more and more involved in their theory. In the beginning, he was mainly interested in two topics: the explicit solution of ordinary differential equations and the theory of (completely) integrable systems. He developed for example a method to solve certain linear ordinary differential equations in terms of elliptic functions. Following ideas developed in the school of A.B. Shabat, he worked on computer algebra methods for the algorithmic classification of integrable systems in the form of evolution equations using symmetry methods (mainly generalized symmetries, often incorrectly called Lie-Bäcklund symmetries, although neither Lie nor Bäcklund ever worked on them). Again, in most cases, a symmetry analysis requires extensive computations and thus represents a natural application field for computer algebra. In fact, Vladimir never ceased to be interested in symmetry methods for differential equations. It was probably through these works that for the first time Vladimir also attracted the attention of a larger audience in the western computer algebra world, when he published no less than four articles in the proceedings of the EUROCAL '87 conference in Leipzig. His first paper in the Journal of Symbolic Computation, published in 1990, was also devoted to integrable systems.

The integrability analysis of evolution equations raises many interesting problems. In intermediate steps, one often has to solve large overdetermined systems of linear differential equations or one has to deal with polynomial systems. Symmetry reductions typically lead to ordinary differential equations, which one would like to solve analytically. The theory of Lie groups and algebras also features here prominently. Hence, in the early 1990s Vladimir started to work on these topics, independently from their direct application in the context of integrability analysis. He co-authored a computer algebra package for the analysis of polynomial systems using Gröbner basis techniques. In parallel, he began with the investigation of (super) Lie algebras — partially again using Gröbner bases. In the beginning, he was interested in automatically recognizing isomorphic Lie algebras. Later, he was more concerned with finitely presented Lie algebras and superalgebras. Here he developed in particular an algorithm for the construction of such (super)algebras out of a finite set of generators and relations.

The late 1990s represent a key phase in Vladimir's scientific oeuvre. From his research in Lie symmetry theory, he was familiar with the Janet-Riquier theory of differential equations, as it provides a popular approach to analyzing the large determining systems arising in the construction of Lie symmetry algebras. And, as just mentioned, he also was familiar with Gröbner bases from commutative algebra. From Janet's work on differential equations, he abstracted a general notion of what he called an involutive division and introduced, by combining it with concepts like normal forms and term orders, the notion of an involutive basis of a polynomial ideal as a Gröbner basis with additional combinatorial properties. For the rest of his life, involutive bases played a dominant role in Vladimir's research.

He was particularly interested in their algorithmic aspects. The basic involutive algorithm —rooted in Janet's work— can be seen as an optimization of the basic form of Buchberger's algorithm for the construction of Gröbner bases. Vladimir developed further optimizations specific to the involutive algorithm and adapted optimizations for the Buchberger algorithm to make them applicable also in the involutive setting. His group at JINR wrote the GINV package in C/C++ as a standalone program for (mainly) computing Janet bases and he participated in a Maple implementation of involutive bases.

Being a physicist, Vladimir recognized the possibilities offered by Gröbner or involutive bases in the context of mechanical systems with constraints. The famous Dirac procedure is essentially a differential completion procedure for the special case of Hamiltonian systems with constraints followed by a separation of the constraints into two different classes: first, constraints generating gauge symmetries, and second, constraints reducing the dimension of the phase space. While, in principle, the procedure is quite straightforward, it involves a notorious number of subtleties and pitfalls when applied to concrete systems. Vladimir showed that in the case of a polynomial Lagrangian most of these can be handled using Gröbner bases and provided a corresponding Maple package. Later, he co-authored a number of papers where these ideas were used to extend the classical Dirac procedure to light-cone Yang-Mills mechanics.

Rings of linear differential or difference operators may be considered as simple examples of non-commutative polynomial rings, and it is rather straightforward to adapt Gröbner or involutive bases to them. All implementations of involutive bases co-authored by Vladimir cover these two cases as well. For systems of linear differential or difference equations, such algorithms for instance allow for an effective completion to involutive or passive form, i.e., for the construction of all hidden integrability conditions, a fact relevant for analytic as well as numerical studies of the systems. In particular, it is crucial for determining the size of the solution space or consistent initial value problems.

The situation becomes much more complicated for non-linear systems.Around 2000, Vladimir started to look more deeply into differential algebra, in particular into differential ideal theory, and a bit later also into difference algebra. His key achievement here was the revival of the Thomas decomposition, an almost forgotten approach to both algebraic and differential ideal theory based on triangular sets and — in the differential case — Janet-Riquier theory. In a Thomas decomposition, an arbitrary system composed of equations and inequations is split into a disjoint union of so-called simple systems which are comparatively easy to analyze, because of their special properties. The disjointness of the resulting simple systems represents a specific feature of the Thomas decomposition, setting it apart from most other decompositions. Together with a group at RWTH in Aachen, Vladimir developed a fully algorithmic version of both the algebraic and the differential Thomas decomposition and co-authored implementations of them in Maple.

In effect, the Thomas decomposition was the second research topic which Vladimir studied intensively right until his death. He applied it in many different fields, ranging from the integrability analysis of fully non-linear systems of (partial) differential equations to an extension of the Dirac procedure to cases where the ranks of certain Jacobians are not constant (a case about which one can find nothing in the classical literature, but which is not uncommon in applications). His last significant and unfortunately unfinished project consisted of developing a difference version of it.

One reason for Vladimir's interest in difference algebra was the analysis and construction of numerical methods. So-called mimetic methods aim at setting up difference equations that have qualitative properties similar to the original differential equations. Such qualitative properties can be conserved quantities or more generally symmetries, but also certain structural features, in particular for equations which are not in Cauchy-Kovalevskaya form. Starting in the mid 2000s, Vladimir became interested in the effective construction of finite difference and finite volume methods preserving certain algebraic structures of the differential ideal generated by the given differential equations. A rigorous formulation of these ideas required parallel theories of differential and difference algebra. For linear differential equations, classical techniques from Gröbner and involutive bases were sufficient to effectively realize his approach; for a fully algorithmic treatment in the case of non-linear equations a difference Thomas decomposition would have been necessary. Vladimir treated a number of concrete non-linear examples, but the algebraic computations had to be done partially by hand. The numerical methods arising from this approach are quite non-standard, differing significantly from the usually applied methods, and the numerical experiments presented so far appear to indicate good performance. For the analysis of these methods, he introduced new notions of consistency and developed computer algebra methods for verifying the corresponding conditions.

In another line of work combining many of his research interests, Vladimir participated in projects for the symbolic-numerical solution of quantum mechanical problems, in particular in atomic physics, ranging from solving time-dependent Schrödinger equations to eigenvalue problems and on to the computation of matrix elements and boundary value problems for elliptic systems. The emphasis was on finite-dimensional quantum systems like atoms in external fields or quantum dots.

Also since the mid 2000s, Vladimir and his group was quite active in the field of quantum computing (in fact, to such an extent that his group at JINR was renamed to better reflect this additional research focus). He concentrated on related algebraic problems to which he applied e.g. involutive methods. In the beginning, the emphasis was on the circuit model of quantum computing. Vladimir developed algorithms for the construction of polynomial systems or unitary matrices describing such circuits and co-authored corresponding Mathematica and C# packages. He was also concerned with the simulation of quantum computations on classical computers and co-authored a Mathematica package for this task. After a brief study of quantum error correction, he moved on to investigating mixed states, mainly by group-theoretic means. Here the emphasis was on the effective construction of local invariants, since these facilitate checking whether a state is entangled or uncoupled. For this purpose, he showed how involutive bases can be used within computational invariant theory.

During his last years, Vladimir returned to the topic of Lie symmetry theory. He was interested in the algorithmic linearization of ordinary differential equations, i.e., in the construction of a point transformation reducing the given equation to a linear one. Lie already had shown for certain cases that one can decide whether a given non-linear ordinary differential equation can be linearized, based on its Lie symmetry group. Later, this topic was studied extensively by Bluman and his group. Vladimir derived fully algorithmic criteria for linearizability (in part based on the differential Thomas decomposition), a result for which he and his co-authors received the distinguished paper award at the ISSAC conference in 2017. He continued to work on improvements of this result, putting more emphasis on the symmetry algebra instead of the symmetry group, but, unfortunately, he died before this project was finished.

Altogether, Vladimir was the author or co-author of more than 240 scientific papers (a listing is available at his CV at JINR, and he was a leading expert in the field of symbolic and algebraic computation. He devoted a lot of effort and energy to train young researchers in these modern scientific areas. He was a professor at the Department of Distributed Information Computing Systems of Dubna State University, where, under his supervision, seven students successfully defended their Ph.D. thesis.

Vladimir also was the organizer of many international conferences on computer algebra. He was the (co-)chair of 29 conferences, a member of the organizing committee of 11 conferences, a member of the Program Committee for 27 conferences, and a member of the Scientific and Advisory Committee of 7 conferences: 74 conferences in total during the period from 1979 to 2020. Thus, Vladimir had, on average, organizational roles in almost two conferences each year, showing his inexhaustible energy.

In the context of this CASC conference (of 2021 in Sochi), it may be an opportunity (and even appropriate) to enlarge a bit on the history of CASC, the international workshop series *Computer Algebra in Scientific Computing*, in particular the events before its birth in St. Petersburg on April 20, 1998. The other co-founder of CASC (one of the present authors, referred to EWM in the text below), first became aware of Vladimir's scientific work in October of 1996 when he (EWM) was working together with his Ph.D. student Klaus Kühnle on an optimal worst-case space bound for algorithms to compute Gröbner bases. Since this bound (exponential space) is independent of the algorithm used, the news about involutive bases were very interesting. The year after, on June 5, EWM invited Vladimir to give a seminar talk about Involutive Gröbner Bases at TUM, which was very well received. During the after-session-get-together at the Parkcafe in Munich, the question was raised about the share of theoretical talks vs. the talks devoted to the numerous applications of the methods and algorithms of computer algebra in the natural sciences. EWM said that "I am a theoretician and trying to connect to applications", and Vladimir said "I am more applied but don't mind theory". The two of them also agreed that there were excellent scientists in the computer algebra field in Russia as well as in Germany. EWM also said that he admired the science that had been going on in certain parts of what was then the Commonwealth of Independent States (CIS) (like Tashkent) since his early study years, that he always had wanted to go there but never managed (since, among other things, he went to Stanford and the US for almost ten years). And suddenly the idea was "Why don't we have a joint (between Russia and Germany, or CIS and Germany) scientific workshop (with the title CASC, that was discussed there already)". Vladimir then right away persuaded Ph.D. Nikolay Vasiliev in St. Petersburg to organize the first instantiation), so this went very fast. For the following fifteen years, the team at EWM's chair at TUM could always rely on Vladimir and his excellent connections in Russia and CIS to persuade very competent colleagues at a number of very interesting places to locally organize CASC.

1990 (ISSAC)

2019 (CASC)

It also turned out that the Deutsche Forschungsgemeinschaft (DFG) was willing to support the CASC workshop in St. Petersburg as well as those in the series for about the following ten years. This support was very helpful in the beginning of CASC, since whenever CASC took place outside of CIS, the funds were used solely for supporting participants from CIS; for CASC workshops in CIS, the method was a bit more difficult and indirect, but with basically the same result. It is clear that in the beginning of CASC, when the financial situation was much more restricted than now, this support from DFG was invaluable. Of course, there was also some organizational work for the conference (in addition to the local organization; like designing and putting out the call for papers (including the conference poster), running the PC, organizing travel, –, putting together the proceedings, ...). As everybody handling the nitty-gritty of conferences knows this was considerable work, at times quite stressful (the less money you have the more stress), and performed by just a few people in EWM's group (in particular, his secretary A. Schmidt, his research assistant Dr. W. Meixner, his programmer Ernst Bayer, and his Russian-Bavarian coordinator Dr. Victor Ganzha). They also deserve a lot of thanks for their efforts and contributions.

Since 1998, the CASC workshops have been held annually (with one gap in 2008, because of political unrest in Georgia), alternating in principle between Russia and Germany, but also including other countries of CIS, in Western and Central Europe, and even in Japan and China. Giving evidence to its widespread attractiveness, the sequence of locations was: St. Petersburg, Herrsching (Munich), Samarkand, Konstanz, Big Yalta, Passau, St. Petersburg/Ladoga, Kalamata, Chisinau, Bonn, – , Kobe, Tsakhkadzor, Kassel, Maribor, Berlin, Warsaw, Aachen, Bucharest, Beijing, Lille, Moscow, Linz, Sochi (also see the CASC bibliography).

Vladimir was the co-chair of the CASC series from its foundation in 1998 onward until 2019. He also was very active in the *Applications of Computer Algebra* (ACA) conference series where he regularly organized sessions, in particular on differential and difference algebra.

He was a member of the editorial board of the *Journal of Symbolic Computation* (JSC), from its foundation on until 2020. Since 1991, he was a member of the largest international scientific and educational computing society *Association for Computing Machinery* (ACM) and the German special interest group for Computer Algebra.

During the period from 1981 to 2013, Vladimir presented 34 lecture courses for students and young scientists in various universities of the USSR/Russian Federation as well as in China, France, Sweden, and especially in Germany. As his family was partially of German origin, he felt very attached to Germany, where he had a number of relatives, whom he frequently visited. Since the late 1990s, he came to Germany almost every year. As a guest lecturer or visiting professor, he spent in total more than five years at German universities and applied universities in Greifswald, Ravensburg-Weingarten, Aachen and Kassel, teaching a wide variety of courses.

Vladimir was the winner of the first prize of JINR in 1986, the second prize of JINR in 2015 in the competition of scientific and methodological works. He was awarded the medal "In memory of the 850th anniversary of Moscow", the departmental badge of distinction in the field "Veteran of Nuclear Energy and Industry", and the "Certificate of Honor" of JINR. He was the founder of and a scientific leader at the School of Computer Algebra and Quantum Computing of JINR. As such, he largely defined the public perception of the Laboratory.

Optimism, openness, goodwill, and sincere interest in science always characterized Vladimir. He will be sadly missed by all who had the pleasure to collaborate and interact with him, and we would like to extend our sincere condolences to his colleagues and friends and, above all, his wife Evgeniya Almazova and his two sons, Anton and Peter.

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