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Reasoning and Language at Work

A Critical Essay



Springer

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*Dedicated to the memory
of Professors Luis A. Santaló (1911–2001)
and Abe Mamdani (1942–2010).*

Series Editor's Foreword

This insightful book—maybe short in size but big in ideas, and deep explanations and inspirations concerning relevant aspects and relations—is concerned with many issues that are crucial for the human cognition, thinking, and acting, and also related issues that are also crucial for some artificial systems mimicking the humans exemplified by multi-agent systems, and also artificial intelligence-based (AI) systems that take science and technology by storm.

Briefly speaking, the authors deal with fundamental aspects of, first, reasoning which is the key element of all kinds of systems, both human-centric and artificial, that are meant for broadly perceived problem solving, notably decision making. In such systems, we have some premises, for instance some evidence and judgments, and we have to find conclusions which can then possibly be employed for some purposeful activities like to find some best option from available or feasible ones. Second, since natural language is the only fully natural means of articulation and communication for the human being, the authors consider natural language, in particular methods, to represent and then handle its syntax and semantics.

What differentiates this book from similar treatises on similar topics is that the authors, first, consider these above-mentioned topics in a broadly perceived logical framework. Second, to adequately represent an inherent imprecision in the meaning of linguistic terms and relations, expressed by the humans in the form natural language, they explicitly refer to the pioneering works by the late Lotfi A. Zadeh on the concept of a fuzzy set, then fuzzy logic, and finally computing with words, sometimes called computing with words and perceptions. Moreover, the book contains extremely valuable references to many concepts and problems considered in various classical and extended logics, often using different languages and motivations.

More specifically, the authors provide an extremely valuable and insightful exposition of, first, some basic more general types of reasoning, that is, deductive, inductive, and abductive. However, they also refer the reader to various non-standard types of reasoning, notably those that have recently appeared, for instance in relations to multivalued, uncertain, temporal, etc., logics. As examples, one can cite here defeasible, paraconsistent, probabilistic, or statistical reasoning, to just name a few. Particular emphasis is put on the broadly perceived approximate reasoning,

and the role of fuzzy sets and fuzzy logic has been underlined. The authors go even deeper and discuss commonsense reasoning, notably expressed by using elements of the fuzzy logic-based paradigm of computing with words, the inclusion of which can be decisive for the development and implementation of all kinds of artificial intelligence-based (AI) systems.

To summarize, the book is a remarkable source of information, explanations, and inspirations which may be a basic reference for all readers, both novice and advanced, interested in an insightful and inspiring exposition of the topics covered, notably logics, reasoning, fuzzy logic, natural language, computing with words, commonsense language, and related topics. It is highly recommendable!

Warsaw, Poland
May 2021

Janusz Kacprzyk

Preface

Wege nicht Werke
(Martin Heidegger)

What follows is not a proper text on fuzzy logic, the basic field of research which has seen the now retired first two authors active for around 50 years, and the third for over twenty. Instead, it is but a booklet containing a collection of reflections that fly further from fuzzy logic by continuing where some previous papers by the three authors have left.

Such reflections are a tentative way to show that fuzzy logic is not only fertile due to its being relevant in many technologic fields. Indeed, it is also as a facilitator for building reflections on thinking, language, reasoning, and its mechanization, in a way that intermingles both ‘scientific’ and ‘philosophical’ aspects. It is something that, consequently, can be seen as able to generate a new ‘Humanistic Culture’ for the twenty-first century, not too far from what prevailed along the seventeenth-century century’s European Enlightening, and including science, as today nobody can doubt it is a relevant part of culture. A possible, innovative field of debate this booklet presents, especially, to the young scientists and philosophers.

A general consensus in the community exists that the good scientists begin reasoning on a delimited subject, of which some previous knowledge exists, looking at first for questions that are new as well as good, and subsequently find adequate answers. Such answers are even more satisfactory when their fertility expands to fields different from the one in which the problem was initially posed.

The goodness of a question and its fertility are obviously linked. One could also say that, perhaps, the evaluation of a satisfactory judgment cannot but be retrospective: A question ‘was’ a good one if the obtained answers are subsequently demonstrated to be fertile. Just to exemplify, let us recall the questions asked by Einstein on motion, those of Kekulé on the Benzene’s molecule, and Cajal’s ones on the nervous cell. Their answers did all have a noteworthy import in other fields: For instance, Kekulé’s discover is one of the bases on which the German Chemical Industry of Colorants developed.

Most of the current technology of information and communication, as well as many results of the pharmaceutical industry (but those two cases are not exhaustive), come from fertile answers to science's good questions. Currently, the percentage of GNP devoted to R&D is in direct proportion with the true power of a nation and all developed countries have to do with what is called 'Politics for science,' or 'Scientific Politics' which, frankly, sounds horrible.

Such more or less obvious considerations are stated here—at the beginning of these pages—keeping in mind some remarks done by Isaac Rabi, Nobel Prize for Physics, and the great geometer and thinker Karl Menger. They are also tuned with the spirit informing the book 'Combining Experimentation and Theory' which articulates and develops an homage to the late Abe Mamdani, just starting from the relationships existing between the two concepts present in the title [1].

We are tempted to state that fruitful new ideas—be they in definitive shape or still in an informal state—can provoke the asking of unusual, vitalizing questions which, when answered, can allow us to see things from a new, different and, in some cases, enlarged perspective. 'Good thinking,' then, means that it is not enough to 'think' and 'reason' correctly against an untouchable background of general presuppositions which cannot be questioned, but it also requires, *à la* Nietzsche, systematically, meticulously doubting of what is considered already well known and definitely assessed. Submitting thinking to a rigorous control, pushing it outside the borders of the 'received view,' must be considered, then, an issue of intellectual hygiene.

1

It is just in such sense that this small book is presented as a 'Critical Essay', the choice of terms signaling, respectively, that it raises some doubts even if these do not always turn into an explicit criticism, and that this is a surface level 'survey', with the aim of focusing the attention on some issues more than treating them in detail, and in a relatively contained number of pages. It tries to rethink already known topics by looking at them from a point of view that is new as well as *naïve*, the term used in the same sense in which it characterizes 'Naïve Set Theory.' In somewhat a kind of joke, the authors try to 'shake before drinking' what they previously believed as well known for what concerns reasoning.

Notwithstanding, such 'critical' approach is not only outward directed against what others express, but as well and mainly inward, against what the authors believe is an acquired knowledge. Years and years of debate among the authors have not produced any certainty, but in a serious twist for practitioners of fuzziness, a number of uncertainties. It is, partly, due to such uncertainties that the critical approach is not always explicit in this essay.

Einstein once observed that 'Science comes from refining the usual thinking,' underlining both the important continuity between the known and unknown, and where differences reside. The same idea underlies the efforts done by deep authors when writing ponderous volumes on known subjects with the didactical intention of

showing the multi-facets of the evolution of scientific concepts. It is with regret we note the current trend followed by young (not exclusively young, indeed) researchers, when affording a problem, to concentrate their activity on the scrutiny of a multitude of very recent papers, while the reading of such kind of comprehensive and reflexive books very seldom is part of their engagement.

Almost all new findings have deep roots in what past scientists or thinkers wrote; almost nothing, in a sense, is actually completely original since also the most innovative ideas spring from a revision of old paradigms. Let us also add that although what appears as totally new seems to be reserved to few authors, experience shows that often in the ‘antecedents’ one can eventually find some suggestive perspective also when doing a traditional, normal, daily research activity on specific questions.

Such trend is so widespread that cannot be exclusively attributed to the behavior and choices of single scientists; it seems fairer to also pin it on a radical change that has occurred in the organization of scientific research and to the surrendering to the high velocity and great pressure the current ‘publish or perish’ paradigm demands. It is up to some extent surprising that many if not most principal researchers have never published a standalone book on what it is supposed they are leading specialists of. Instead, there is a proliferation of books edited by several of them and collecting chapters around subjects not always concordant.

This essay aims not to stop at what seems to be well known; it tries to show the possibility of embarking into an intellectual journey that can allow to see new things zigzagging, simultaneously forwards and backwards, from some questions to what is considered known in the same way in which one discovers new things in a town by walking back and forth with curious eyes. The essay aims to help people, and mainly the younger scientists, to speculatively look for further new horizons; a task which, we want to underline, can actually be carried on only through personal effort and conviction, which can be only triggered and stimulated by others.

This small book tries to rethink what is known and following, again, Einstein in his belief that posing questions is essential for the scientific progress: even in an implicit form, like in this essay, that often leaves to the reader to make such questions explicit using their own insight. This is a reason for reading it slowly.

The text does not search to widening the ‘work’ of its authors. Benefiting from the decades of combined research experience of them—that surpasses 50 years for each of the elder two—it aims to open a new path toward the clarification of some basic concepts in thinking and reasoning, as well as that in their possible mechanization. This is a book that, ideally, should be read as if ‘doing nothing,’ quietly and in a kind of working suspension; turning each paragraph again, on and on. It is meant for a Sunday afternoon activity!

Just concerning the previously mentioned aspect, let us recall the words of the late Ebrahim (Abe) Mamdani (1942–2010), in the already cited book that is dedicated to him:

My interest stems from my fascination with the digital computer and how it can be used for simulating behavior.

Such approach persuaded the third, and younger, author that speculations, the special kind of conjectures introduced and described by the first author, should be included as a first-class member in the toolbox of logic instruments used to make a crack at mechanizing commonsense reasoning.

While the start of such inclusion has pondered the role of speculations when the structure dealt with is that of a finite Boolean Algebra, we have also laid out some work on expanded structures, such as the Borel Algebra. In the general context of the construction of algorithmic models that mimic and reconstruct some of the inner mechanisms of a working mind, a deductive approach is limiting and does exclude a lot of different interpretations and nuances: by implementing, in addition to classical deductions, speculations, seen as a back-and-forth process of deducing and abducting, aiming at such integration, and unlocking the key to an important aspect of what is commonly defined as creative thinking.

2

In the last 100 years, the mathematical analysis of reasoning (mathematical logic) was conducted by means of symbolic and discrete models, mainly using algebraic resources and sometimes even (abstract) topological ones. This actually implies a limitation John von Neumann regretted at the half of twentieth century, and at which even George Boole did not offer an answer, as for instance he freely employed, with his ‘symbols’, Taylor’s developments in his logical work on ‘Thought.’

We think it could be worthwhile to clarify that von Neumann remarks are done in the specific context of his work at a particular time. He introduces the role of error in the Theory of Automata; thus,

The subject matter is the role of error in logics, or in the physical implementation of logics—in automata synthesis. Error is viewed, therefore, not as an extraneous and misdirected or misdirecting accident, but as an essential part of the process under consideration—its importance in the synthesis of automata being fully comparable to that of the factor which is normally considered, the intended and correct logical structure [2].

However, already in 1949 (the Proceedings will be published 2 years later), he had spoken specifically of mathematical logic maintaining that:

There exists today a very elaborate system of formal logic, and, specifically, of logic as applied to mathematics. This is a discipline with many good sides, but also with certain serious weaknesses ... About the inadequacies ... this may be said: Everybody who has worked in formal logic will confirm that it is one of the most refractory parts of mathematics. The reason for this is that it deals with rigid, all-or-none concepts, and has very little contact with the continuous concept of the real or of the complex number, that is, with mathematical analysis. Yet analysis is the technically most successful and best-elaborated part of mathematics. Thus, formal logic is, by the nature of its approach, cut off from the best cultivated portions of mathematics, and forced onto the most difficult part of the mathematical terrain, into Combinatorics [3].

We must add that von Neumann did not contribute to what is considered 'Logic' from an academic and disciplinary point of view in periods after his remarks quoted above, although he was certainly able to do so and at the utmost level. Instead, he indicated a different path to be followed to come at terms with approximation and error.

A very meaningful choice, we deem. He could have forged and provided new tools by using 'the technically most successful and best-elaborated part of mathematics' (his words) along the lines followed by the disciplinary tradition of mathematical logic. In the midst of a furious and creative intellectual climate, we must add, and under the pressure of the urgencies dictated by the realizations of projects like the building up of electronic computers, he chose to follow another road: showing that crucial logical issues can be fruitfully pursued in those more general contexts emerging from the questions asked by new fields of investigation like information processing or cybernetics [4, 5].

It can be also added that an important intellectual certification of the reliability of this line of thought can be attributed to Popper's position—although no direct influence can be traced on the developments in which we are interested—when he assert that 'it is always undesirable to make an effort to increase precision for its own sake—especially linguistic precision—since this usually leads to loss of clarity ... : one should never try to be more precise than the problem situation demands ... an increase in precision or exactness has only a pragmatic value as a mean to some definite end' [6].

The previous passage comes after he has reassured the reader: 'I do not suggest, of course, that an increase in the precision of, say, a prediction, or even of a formulation, may not sometimes be highly desirable.' For more on this, see [7]. Popper's warning teams with a similar one due to Aristotle in *Nicomachean Ethics* (1094b), when he states that the treatment of a discipline 'will be adequate, if it achieves that amount of precision which belongs to its subject matter' and adds that 'the same exactness must not be expected' in all arguments or activities.

So, the choice of accepting error 'as an essential part of the process under consideration,' as well as the warning against the 'effort to increase precision for its own sake' does not suggest relaxing the canons of scientific rigor but instead that a prejudicially wrong attitude toward the treatment of errors and precision could contribute to lower them.

Actually, such limitations have a number of reasons: the lack of an approach that looks inside natural language statements or, usually, to the classes of statements with the same meaning that logicians call *propositions*; the idea of not considering its own linguistic form and components, whose meaning can be drastically modified by slightly varying the linguistic terms in it; the attitude of considering propositions as 'units' without parts. Those modifications are especially visible when the linguistic terms in a statement are not precise, rigid, bi-valuate, or crisp, but imprecise, vague, flexible, or fuzzy, which is a permanent fixture of natural languages more than an exception.

It was fuzzy set theory, introduced by Lotfi A. Zadeh after 1965 that, allowing to look at linguistic terms and connectives as ‘functions,’ permitted to include the above-mentioned flexibility through continuity, a basic concept of mathematical analysis. Such viewing of words as functions meant a real, tangible progress, akin to what was obtained by exploring the heart’s inner workings using an electrocardiogram instead of only a stethoscope. Zadeh opened a new path toward using mathematical analysis in the study of commonsense reasoning expressed in a natural language.

Among other topics, this functional approach allowed to analyze the validity of classical logic laws with fuzzy sets determining, by solving functional equations and inequalities, which operations of conjunction, disjunction, and negation can or cannot hold.

All that is in the backstage of this booklet and did facilitate a direct work with commonsense reasoning that, involved as it is with natural language, needs to take into account what is *inside* the statements, its components, and their variations.

The connection between language and reasoning is so strong that the second can be seen as ‘Language in Action,’ or ‘Language at Work.’

Sometimes, the close relationship between language and reasoning creeps up directly in language itself: It is in this sense that it can be noted that Catalan seems to be the only Latin language in which ‘parlar’ (talking) enjoys the synonym ‘enraonar,’ with ‘raonar’ meaning ‘to reason,’ and ‘enraonar’ to enchain reasons [8].

3

Another topic of some relevance, also derived from fuzzy set theory, is the consideration of ‘measurable’ linguistic statements, that is, those consisting of words whose meaning can be represented by means of scalar magnitudes. In this context, the ‘measure’ of their meaning is nothing else than the membership function of the fuzzy set whose linguistic label is just the corresponding word. It is the concept of a measure that instills science in the study of meaning since, resuming the words of Lord Kelvin, ‘If you can’t measure it, you are not doing Science.’

Now, and thanks to all that, semantics can start to be a scientific subject, in the sense specified above: and fuzzy logic, or Zadeh’s ‘Computing with Words and Perceptions,’ can become an experimental and theoretic discipline. It seems that a kind of physics of language and reasoning could be finally approached.

It is of some relevance to notice that transforming a statement into a function requires to ‘design’ all the measures of meaning concerning the linguistic terms of which the statement consists of. This process of design, being these measures not necessarily unique, should be done with the help of all the contextual information available to the designer, and is a mark distinguishing what is presented here as ‘Language in Action,’ or commonsense reasoning (reasoning as everybody does it), from the previous classical logic approach.

Even if it is not considered in this booklet, an important notion in the multifaceted field in which the presence of a lack of crispness has been studied is the philosophical concept of ‘vagueness’ [9, 10].

This general notion when seen from the perspective of measurable words converts in that of ‘fuzziness,’ [11–14] even allowing to compute *how fuzzy*, or *how crisp*, is a fuzzy set, and thanks to what the second author did introduce already in 1972, jointly with the late Aldo de Luca, under the name of ‘fuzzy entropies’ [15, 16]. Among important developments on the subject, we limit here to remember some early contributions [17, 18], and interesting related concepts in the setting of MV Algebras pointed out by Di Nola [19]. It is a study allowing to say, perhaps paradoxically, that fuzzy logic can be seen as the scientific study of fuzziness.

4

All this opens a door toward the series of reflections of which this essay consists of and, in the first place, to the introduction of the so called Formal Skeleton of commonsense reasoning from which classical logic, or the calculus of rigid statements, quantum logic, multiple-valued logics, and fuzzy logic, follow. All of them, with their many interesting applications, show the same minimal skeleton of laws.

A skeleton that not only allows to define refuting, conjecturing and classifying conjectures as consequences, hypotheses, and speculations (with these last appearing by the first time in the logics literature), but also to recognize reasoning as an inferential zigzag in search of speculations, similar to a kind of Brownian movement around the premise, perhaps shaking it thanks to what is known of it, and consisting of sequentially deducing and abducting. A zigzag that sometimes can be algorithmically simulated as it is shown in the booklet, and placing speculation in the center of reasoning, as well as showing it is in the frontier with directed thinking, and that reasoning consists in the end in a sequence of trials and errors.

Additionally, but important, is that the skeleton shows that most of the known logical laws are not always valid in commonsense reasoning, are not universal in it but are no more than local laws; this ‘locality’ is something that can also be considered as a new view in the formal study of reasoning.

The skeleton permits to note the importance of the transitive law. A specific instance of a law that is not universal in commonsense reasoning but just local. Using transitivity in concurrence with the basic laws of the skeleton permits the formal simplification of reasoning as just the zigzag of inference conducting to Refuting and Conjecturing, as well as to analyze the behavior of conclusions when the initial information grows, and seeing that only speculations are neither monotonic, nor anti-monotonic, but non-monotonic. Speculation seems to be equivalent to non-monotonic logic. The skeleton is a minimal support for the formal study of commonsense reasoning.

After all that, the worrying problem of the breaking of deductive chains is dealt with. A problem that is analyzed starting from the idea of ‘meaning indistinguishability,’ and modeling it by means of the indistinguishability operators, introduced and studied previously in fuzzy logic. Such approach seems to model well enough a linguistic phenomenon that is often due to the limitation of our senses perception, such as the chain of synonyms.

5

Finally, the book goes back in time and looks for some historical roots on the ideas hidden in it and appearing, perhaps surprisingly, in the Middle Ages and in the writings of Dante Alighieri, and the English Franciscans Roger Bacon and William of Ockham. Another Franciscan Tertiary Friar, the Majorcan Ramon Llull, or Raimundus Lullus, with his famous ‘Ars Magna’ tried, for the first time after the Greek’s Antikithera Machine in the second century BC, and as soon as the fourteenth century BC, to mechanize reasoning, that is, in the flux of ideas of not making things more complicated than they are, and trying to approach them with the greater than possible intellectual courtesy, the clarity.

Perhaps those roots are not actually so surprising: It is shown in the famous novel ‘The Name of the Rose’ by Umberto Eco that in such time a lot of new ideas that further were successful in Europe were advanced. Actually, it was ‘modern thinking’ that come into being in that time; in some sense, it can be said that some thinkers of that time did shape the future thought in modern Europe.

Some scholars have put forward, in fact, the thesis that the ‘turn’ toward *modernity* was just happening in that period and was interrupted by a catastrophic event (the plague of fourteenth century) which postponed the full realization of this ‘research program,’ we could say, for a few centuries. This process has been described, in a both suggestive and very clear way, by Pietro Greco in his monumental work on Science and Europe [20] to which we refer the reader.¹

6

The essay’s contents consist in three parts and a final conclusion. The first part, influenced by fuzzy logic, reflects on reasoning, and the second, devoted to the breaking of deductive chains, is not only influenced by fuzzy logic but also by Poincaré’s continua. The third part starts with an outlook of fuzzy calculus, but focalizes in a reflection on the mentioned roots of the essay’s spirit. What follows is not a direct lift from fuzzy logic, but is partially inspired by its study.

¹ The particular argument referred above is treated in volumes 1 (2014) and 2 (2015).

For what concerns the final conclusions, they are but a closing reflection on all that are presented and discussed in this booklet. The main intention being to suggest the readers some possible and innovative line of research that concerns language at work, and reasoning, both in itself and in view of its mechanization. Something that should encourage the research on computers' expansion from deducing to 'thinking' in, at least, a directed form.

What can be done better, today—being research on neuroscience so far away from our own intellectual and practical capacities—than trying, at least, to help computers to simulate directed thinking and reasoning as a possible 'practical use/experimental verification' of what is beyond this booklet? This is, in part, a reason for which some hints on the computing with conjectures, and especially with speculations, are included.

It is in such direction that the Afterword Rudolf Seising kindly wrote for this booklet outlines the historical path that refers, in its background, to the development of artificial intelligence. In it, language and reasoning acquired the relevance that the necessity of its computer simulation gives them, step by step with the progress of digital computers.

In the authors view, this essay reinforces the relevance of fuzzy logic that, in addition to its fertility with applications in so many technology fields, can also facilitate a determinant contribution toward foreseeing a new science of language and reasoning, of 'Language at Work.'

Slightly modifying the words in the Pray of Saint Francis of Assisi, this essay aims at helping to accomplish 'Where there is darkness let us show light' for what concerns the study and mechanization of commonsense reasoning.

At the same time and from another point of view, we must declare (due the intellectual 'Customs of Reliability' grounded on Wittgenstein's words 'on what we cannot speak we must be silent') that some 'sins' against clarity and the intellectual courtesy can appear at some points in this booklet, when ideas that are not yet clear and perfectly formed seemed to the authors a nonetheless valuable contribution that can be suggestive for the readers. With Horace, *Nihil est ab omni parte beatum*.

Oviedo, Spain
Palermo, Italy
Palermo, Italy
May 2021

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About the Authors



Enric Trillas born in 1940 in Barcelona, earned his Ph.D. in Sciences from the University of Barcelona, and was Full Professor at the Technical Universities of Catalonia (of which he was Vice-Rector), and Madrid. From 1983 to 1996, he held several positions concerning the scientific and technological activities in the Spanish Government. Notably, he was President of the High Council for Scientific Research (CSIC), Director General of the National Institute for Aerospace Technology (INTA), and Secretary General of the National Plan for R&D. From 2006 to 2018, he was Researcher at the (now closed) European Centre for Soft Computing (ECSC) in Mieres, Asturias, Spain, and then Emeritus Professor at the University of Oviedo, Asturias, Spain. He is also Distinguished Visiting Professor at the Universidad Nacional de Córdoba in Argentina, and holds the honorary doctorate (doctor honoris causa) of two Spanish universities, the Universidad Pública de Navarra, and Universidad de Santiago de Compostela. He is the author and the editor of more than a dozen of books and volumes in Catalan, Spanish, and English and published more than 400 papers in journals, conference proceedings, and volumes. He supervised 23 Ph.D. dissertations and delivered courses and lectures at universities and research centers in Germany, Italy, France, Argentina, Chile, and the USA. He translated three books into Spanish. He is recognized as the initiator in Spain, and a pioneer in Europe, of—first—Karl Menger’s probabilistic metric spaces, and then of Lotfi A. Zadeh’s fuzzy Sets and fuzzy logic. He had

served for years in many editorial boards of international journals and international program committees of conferences. He received a dozen of official distinctions from Spain, Catalonia, Italy, and Peru, and was awarded medals and honors exemplified by the Kampé de Fériet Medal, the European Fuzzy Pioneer Award, the IEEE Fuzzy Pioneer Award, the Outstanding Contributions Award of the International Fuzzy Systems Association (IFSA), and the Honorary Membership at the European Society of Fuzzy Logic and Technology. He is Member of the Accademia Nazionale di Scienze, Lettere e Arti di Palermo (Italy). He retired in August, 2018, and intensively continued research activities until May, 2021. This essay somehow subsumes his life as a scientific researcher, as he finally moves to different intellectual activities.



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Codice, 2007 (with Pietro Greco); *Memoria e Progetto*, GEM 2010 (edited with Pietro Greco) have been the most influential.



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