

# Combining Event Calculus and Description Logic Reasoning via Logic Programming

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**Abstract.** The paper introduces a knowledge representation language that combines the event calculus with description logic in a logic programming framework. The purpose is to provide the user with an expressive language for modelling and analysing systems that evolve over time. The approach is exemplified with the logic programming language as implemented in the Fusemate system. The paper extends Fusemate’s rule language with a weakly DL-safe interface to the description logic  $\mathcal{ALCIF}$  and adapts the event calculus to this extended language. This way, time-stamped ABoxes can be manipulated as fluents in the event calculus. All that is done in the frame of Fusemate’s concept of stratification by time. The paper provides conditions for soundness and completeness where appropriate. Using an elaborated example it demonstrates the interplay of the event calculus, description logic and logic programming rules for computing possible models as plausible explanations of the current state of the modelled system.

**This is a corrected version of the published paper. It adds a missing case in the definition of the semantics of body literals (Section 2.1), and it fixes a flaw in the definition of possible models (See Note 3).**

## 1 Introduction

This paper presents an expressive logical language for modelling systems that evolve over time. The language is intended for model computation: given a history of events until “now”, what are the system states at these times, in particularly “now”, expressed as logical models. This is a useful reasoning service in application areas with only partially observed events or incomplete domain knowledge. By making informed guesses and including its consequences, the models are meant to provide plausible explanations for helping understand the current issues, if any, as a basis for further decision making.

For example, transport companies usually do not keep detailed records of what goods went on what vehicle for a transport on a particular day. Speculating the whereabouts of a missing item can be informed by taking known locations of other goods of the same batch on that day into account; problems observed with goods on delivery site, e.g., low quality of fresh goods, may or may not be related to the transport conditions, and playing through different scenarios may lead to plausible explanations while eliminating others (truck cooling problems? tampering?).

There are numerous approaches for modelling and analysing systems that evolve over time. They are often subsumed under the terms of stream processing, complex event recognition, and situational awareness, temporal verification among others, see [1,14,2,3,4] for some logic-based methods. Symbolic event recognition, for

instance, accepts as input a stream of time-stamped low-level events and identifies high-level events — collections of events that satisfy some pattern [1]. See [36] for a recent sophisticated event calculus. Other approaches utilize description logics in a temporalized setting of ontology-based data access (OBDA) [30]. For instance, [29] describes a method for streaming data into a sequence of ABoxes, which can be queried in an SQL-like language with respect to a given ontology.

The knowledge representation language put forward in this paper combines Kowalski’s event calculus (EC) with description logics (DL) in a logic programming framework. The rationale is, DLs have a long history of developments for representing structured domain knowledge and for offering reliable (decidable) reasoning services. The EC provides a structured way of representing actions and their effects, represented as fluents that may change their truth value over time. For the intended model computation applications mentioned above, the EC makes it easy to take snapshots of the fluents at any time. The full system state at a chosen time then is derived from the fluent snapshot and DL reasoning. The logic programming rules orchestrate their integration and serve other purposes, such as diagnosis.

This paper uses the Fusemate logic programming language and system [11,12]. Fusemate computes possible models of stratified disjunctive logic programs [33,34], see Section 2 for details. Fusemate was introduced in [11] with the same motivation as here. In [12] it was extended with novel language operators improved with a weaker form of stratification. Their usefulness in combination was demonstrated by application to description logic reasoning. In [12] it was shown how to transform an  $\mathcal{ALCIF}$ <sup>1</sup> knowledge base into a set of Fusemate rules and facts that is satisfiable if and only if the knowledge base is  $\mathcal{ALCIF}$ -satisfiable. All of that is used in this paper.

*Paper contributions.* This paper builds on the Fusemate developments summarized above and extends them in the following ways:

1. Integration of the description logic reasoner of [12] as a subroutine callable from Fusemate rules. Section 3 details the semantics of the combination and conditions for its soundness and completeness. This is an original contribution in its own right which exploits advantages of a stratified setting.
2. A version of the event calculus [20] that fits Fusemate’s model computation and notion of stratification. Details are in Section 4.
3. Integrating DL and EC by means of rules, and utilizing rules for KR aspects not covered by either. Details in particular in Section 5
4. Providing an elaborated example the integrated EC/DL/rules language. It is included in the Fusemate distribution which is available at <https://bitbucket.csiro.au/users/bau050/repos/fusemate/>.

To the best of my knowledge, a combination of DL with EC has not been considered before. Given the long history of applying DL reasoning (also) for time evolving systems, I find this surprising. From that perspective, a main contribution of this paper is to fill the gap and to argue that the proposed combination makes sense.

<sup>1</sup>  $\mathcal{ALCIF}$  is the well-known description logic  $\mathcal{ALC}$  extended with inverse roles and functional roles. See [5] for background on description logics.

There is work on integrating DLs into the situation calculus (SitCalc) and similar methods [15,9,8,10]. SitCalc [23] is a first-order logic formalism for specifying state transitions in terms of pre- and post-conditions of actions. It is mostly used for planning and related applications that require reachability reasoning for state transitions. Indeed, the papers [9] and [15] investigate reasoning tasks (executability and projection, ABox updates) that are relevant in that context. Both approaches are restricted to acyclic TBoxes. In [10], actions are specified as sets of conditional effects, where conditions are based on epistemic queries over the knowledge base (TBox and ABox), and effects are expressed in terms of new ABoxes. The paper investigates verification of temporal properties. As a difference to the EC, none of these approaches supports a *quantitative* notion of time.

## 2 Stratified Logic Programs and Model Computation

This paper uses the extended “Fusemate” rule language introduced in [12] without the earlier belief revision operator introduced in [11]. This section complements the earlier paper [12] with a rigorous definition of the semantics of the extended language. It also provides soundness and completeness arguments, under certain conditions, wrt. abstract fixpoint iteration and wrt. Fusemate’s procedure more concretely.

Terms and atoms of a given first-order signature with “free” *ordinary* function and predicate symbols are defined as usual. Let  $\mathbb{T}$  be a countably infinite discrete set of *time points* equipped with a well-founded total strict ordering  $<$  (strictly earlier), e.g., the natural numbers. Assume that the time points, comparison operators  $=, \leq$  (earlier), and a next time function  $+1$  are also part of the signature and interpreted in the obvious way. A *time term* is a (possibly non-ground) term over the sub-signature  $\mathbb{T} \cup \{+1\}$ . The signature may contain other “built-in” interpreted predicate and function symbols for predefined types such as strings, arithmetic data types, sets, etc. We only informally assume that all terms are built in a well-sorted way, and that interpreted operators over ground terms can be evaluated effectively to a value represented by a term.

Let  $\text{var}(e)$  denote the set of variables occurring in a term or atom  $e$ . We say that  $e$  is *ground* if  $\text{var}(e) = \emptyset$ . We write  $e\sigma$  for applying a substitution  $\sigma$  to  $e$ . The domain of  $\sigma$  is denoted by  $\text{dom}(\sigma)$ . A substitution  $\gamma$  is a *grounding substitution for a finite set of variables  $X$*  iff  $\text{dom}(\gamma) = X$ . In the following, the letters  $x, y, z$  stand for variables, *time* for a time term variable,  $s, t$  for terms, and  $tt$  for a time term, possibly indexed. Lists of terms or other expressions are written as vectors, e.g.,  $\vec{t}$  is a list of terms  $t_1, \dots, t_n$  for some  $n \geq 0$ . A (*Fusemate*) *rule* is an implication written in Prolog-like syntax as

$$H :- b_1, \dots, b_k, \text{not } \vec{b}_1, \dots, \text{not } \vec{b}_n . \quad (1)$$

In (1), the rule *head*  $H$  is either (a) *ordinary*, a disjunction  $h_1 \vee \dots \vee h_m$  of ordinary atoms, for some  $m \geq 1$ , or (b) the expression **fail**.<sup>2</sup> In case (a) the rule is *ordinary* and in case (b) it is a *fail rule*. The list to the right of  $:-$  is the rule *body*. Bodies are defined by recursion as follows, along with associated sets  $\text{fvar}$  (free variables).

<sup>2</sup> This definition of head is actually simplified as Fusemate offers an additional head operator for belief revision, see[11]. This is ignored here.

Name	Form	$fvar$	Comment
Ordinary atom	$p(tt, \vec{t})$	$var(tt, \vec{t})$	$tt$ time term, $p$ free predicate
Comprehension with time term $x$	$p(x \circ tt, \vec{t}) \text{ sth } B$	$\{x\} \cup var(tt, \vec{t})$	$\circ \in \{<, \leq, >, \geq\}$ , $B$ is a body
Built-in call	$p(\vec{t})$	$var(\vec{t})$	$p$ is built-in predicate
Time comparison	$s \circ t$	$var(s, t)$	$s, t$ time terms, $\circ \in \{<, \leq, >, \geq\}$
Let special form	$\text{let}(x, t)$	$\{x\} \cup var(t)$	
Choose special form	$\text{choose}(x, ts)$	$\{x\} \cup var(ts)$	$ts$ is a set of terms
Collect special form	$\text{collect}(x, t \text{ sth } B)$	$\{x\}$	
Positive body $\vec{b}$	$b_1, \dots, b_k$	$\cup_{i=1..k} fvar(b_i)$	$k \geq 0$ , $b_i$ is one of above
Negative body literal	$\text{not } \vec{b}$	$\emptyset$	$\vec{b}$ is non-empty positive body
Body $B$	$\vec{b}, \text{not } \vec{b}_1, \dots, \text{not } \vec{b}_n$	$fvar(\vec{b})$	$n \geq 0$ , and $\vec{b}, \vec{b}_j$ positive bodies

A *positive body literal* is of one of the forms up to **collect**. Examples are below.

*Note 1 (Implicit quantification).* In a body  $B$ , the variables  $fvar(B)$  are implicitly existentially quantified in front of that  $B$ .<sup>3</sup> Rules may contain extra variables in negative body literals. An example is the rule  $p(\text{time}, x) :- q(\text{time}, x, y), \text{not}(z < \text{time}, r(x, y, z))$  which corresponds to the (universal quantification of the) formula  $q(\text{time}, x, y) \wedge \neg \exists z. (z < \text{time} \wedge r(x, y, z)) \rightarrow p(\text{time}, x)$ . The extra variable  $z$  will be picked up for existential quantification after ground instantiating the rule body's  $fvars$   $\{\text{time}, x, y\}$ . If  $\gamma$  is such a grounding substitution then indeed  $fvar((z < \text{time}, r(x, y, z))\gamma) = \{z\}$  as desired. The formal definition of the possible model semantics below will make this precise.  $\square$

A *normal rule* is an ordinary rule with one head literal ( $m = 1$  in (1)). A *Horn* rule is a normal rule or a fail rule. A *fact* is an ordinary rule with empty body ( $k, n = 0$  in (1)) and is simply written as  $H$ . A rule  $H :- B$  is *range-restricted* iff  $var(H) \subseteq fvar(B)$ . A (*Fusemate*) *program* is a finite set of range-restricted and *stratified* rules.

*Stratification.* The standard notion of stratification (“by predicates”) means that the call graph of a program has no cycles going through negative body literals [31]. Every strongly connected component of the call graph is called a stratum and contains the predicates that are defined (in rule heads) mutually recursive with each other. All head predicates of the same rule are put into the same stratum. Fusemate employs a weaker *stratification by time and by predicates (SBTP)* [12]. With SBTP, every ordinary non-fact rule (1) must have an ordinary body literal  $b_i$ , for some  $1 \leq i \leq k$ , with a *pivot* variable  $\text{time}$ , such that every other time term in the head (body) is syntactically constrained to  $\geq$  ( $\leq$ , respectively) than  $\text{time}$ , and the literals within negative body literals are syntactically constrained to be (a)  $<$  than  $\text{time}$  or (b)  $\leq$  than  $\text{time}$  and must be in a stratum strictly lower than the head stratum. For example, the rule  $p(\text{time}, x) :- q(\text{time}, x), \text{not}(r(t, y), t \leq \text{time})$  is SBTP if  $r$  is in a strictly lower stratum than  $p$ , and  $p(\text{time}, x) :- q(\text{time}, x), \text{not}(r(t, y), t < \text{time})$  is SBTP even if  $r$  is in the same stratum as  $p$ . This has the effect that model computation can be done in time/stratum layers in increasing (lexicographic) order using only already derived atoms.

<sup>3</sup> The variables  $var(t)$  in the **collect** special form have to be excluded from that because they are quantified within their “sth  $B$ ” body scope. To avoid name conflicts, we assume that  $var(t) \cap fvar(B') = \emptyset$  for all bodies  $B'$  such that  $B = B'$  or  $B$  occurs in  $B'$ .

Comprehension and **collect** must be stratified for the same reason. For the purpose of SBTP, a comprehension  $p(x \circ tt, \vec{t}) \text{ sth } B$  is taken as if  $p(x, \vec{t})$  and  $B$  were negative body literals, and **collect**( $x, t \text{ sth } B$ ) is taken as if  $B$  were a negative body literal.

## 2.1 Possible Models

We need some preliminaries pertaining to the semantics of rules before formally defining “possible models”. A *(rule) closure* is a pair  $(H :- B, \beta)$  such that  $\beta$  is a grounding substitution for  $fvar(B)$  called *body matcher* in this context. For a program  $P$ , its *full closure*  $cl(P)$  is the set of all closures of all rules in  $P$ .

Full closures supplant the usual full ground instantiation of programs. They make it easy to define rule semantics in presence of the special forms, comprehension operators, and implicit existential quantification without full grounding. This works as follows.

An *interpretation*  $I$  is a (possibly infinite) set of ordinary atoms. Let  $I$  be an interpretation and  $\beta$  a grounding substitution for some set of variables. Let  $B$  be a body as in (1). If  $fvar(B\beta) = \emptyset$  define  $I, \beta \models B$  iff  $I, \beta \models b_1, \dots, b_k$  and  $I, \beta \models \text{not } \vec{b}_j$  for all  $j = 1..n$ , where the following table provides the definitions for body literals:

Name	Form	Def
Ordinary atom	$I, \beta \models p(tt, \vec{t})$	iff $p(tt, \vec{t})\beta \in I$
Comprehension with time term $x$	$I, \beta \models p(x < tt, \vec{t}) \text{ sth } B$	iff $x\beta$ is the maximal (latest) time point s.th. $x\beta < tt\beta$ , $I, \beta \models p(x, \vec{t})$ and $I, \beta \models \exists B$ . Accordingly for $\geq, <, \leq$
Built-in call	$I, \beta \models p(\vec{t})$	iff $p(\vec{t})\beta$ evaluates to true
Time comparison	$I, \beta \models s \circ t$	iff $s\beta \circ t\beta$
Let special form	$I, \beta \models \text{let}(x, t)$	iff $x\beta = t\beta$
Choose special form	$I, \beta \models \text{choose}(x, ts)$	iff $x\beta \in ts\beta$
Collect special form	$I, \beta \models \text{collect}(x, t \text{ sth } B)$	iff $x\beta = \{t\gamma \mid I, \beta\gamma \models B \text{ for some grounding substitution } \gamma \text{ for } fvar(B\beta)\}$
Positive body $\vec{b}$	$I, \beta \models b_1, \dots, b_k$	iff $I, \beta \models b_i$ for all $i = 1..k$
Negative body literal	$I, \beta \models \text{not } \vec{b}$	iff $I, \beta \not\models \exists \vec{b}$

In the table above, define  $I, \beta \models \exists B$  iff there is a grounding substitution  $\gamma$  for  $fvar(B\beta)$  such that  $I, \beta\gamma \models B$  ( $\beta\gamma$  is  $\beta$  extended with bindings for the implicitly existentially quantified variables in  $B\beta$ ). For closures define  $I \models (H :- B, \beta)$  iff  $I, \beta \not\models B$  or else  $H$  is an ordinary head  $h_1 \vee \dots \vee h_m$  and  $h_i\beta \in I$  for some  $1 \leq i \leq m$ . In this case we say that  $I$  *satisfies*  $(H :- B, \beta)$ . An interpretation  $I$  is a *model of a set  $C$  of closures*, written as  $I \models C$  iff  $I$  satisfies every closure in  $C$ . It is *minimal* iff  $J \not\models C$  for every  $J \subsetneq I$ . It is *supported* iff for every  $a \in I$  there is a  $(h :- B, \beta) \in C$  such that  $a = h\beta$  and  $I, \beta \models B$ .

*Note 2 (Fixpoint iteration for DLPs [33]).* The possible model semantics [33,34] assigns to a disjunctive logic program sets of Horn programs and takes their intended models as the possible models of the disjunctive program. The Horn programs represent all possible ways of making one or more head literals true, for every disjunctive rule. As a propositional example, the disjunctive program  $\{a :- b, a \vee c :- b, b :- \text{not } d\}$  is split into the Horn programs  $\{a :- b, b :- \text{not } d\}$  and  $\{a :- b, c :- b, b :- \text{not } d\}$ .

The possible models are  $\{a, b\}$  and  $\{a, b, c\}$ . Non-ground programs have to be fully ground-instantiated using the program's (possibly infinite) Herbrand base first.

As explained in [33], the possible models of such ground-instantiated *stratified* programs can be constructed by iterated fixpoint computation along the program's stratification. For each stratum, in ascending order, the rules with a head predicate from that stratum are evaluated in the model so far, up to that stratum, and, only if necessary, made true by adding the head to the model, until fixpoint. In general this construction requires transfinite induction with a limit ordinal at each stratum.  $\square$

From a practical (Fusemate) perspective we are mostly interested in finite fixpoints for making model computation effective. We start with a definition for the possible models splitting operator in terms of closures.

**Definition 1 (Split program closure).** *Let  $P$  be a program and  $cl(P)$  its full closure. A *split program closure* of  $P$  is obtained from  $cl(P)$  by replacing every closure  $(h_1 \vee \dots \vee h_m \leftarrow B, \beta)$  in  $cl(P)$  by the *split closures*  $(h \leftarrow B, \beta)$ , for every  $h \in S$ , where  $S$  is some non-empty subset of  $\{h_1, \dots, h_m\}$ .*

*Note 3 (Flawed Definition of Possible Models).* The original paper [33] defines, in our words, an interpretation  $I$  as a *possible model* of a program  $P$  iff  $I$  is a minimal supported model of some split program of  $P$ . Unfortunately, there is a flaw in this definition. To explain, by way of example, take the program  $P = \{a :- a, b :- \text{not } a\}$ . It has two minimal supported models,  $I_{\text{good}} = \{b\}$  and  $I_{\text{bad}} = \{a\}$  which are exactly the possible models of  $P$  according to this definition. However, while  $I_{\text{good}}$  will be computed by fixpoint iteration,  $I_{\text{bad}}$  will be not. Clearly,  $I_{\text{bad}}$  is not intended as a possible model in [33]. The example, thus, disproves the completeness claim for fixpoint iteration in [33] (Theorem 3.1).

The flaw stems from requiring minimality of models *as a whole*. A fixed definition needs to match the iterated fixpoint construction, which computes minimal (and supported) models on a *per stratum* basis. In the example, only  $\emptyset$  is a minimal model of the first stratum  $\{a :- a\}$  which is extended to the minimal model  $\{b\}$  of  $P$ . The *perfect model semantics* of [31] achieves that and will be used below as a more suitable basis for defining possible models. With that fix, a Theorem 3.1 in [33] will hold.  $\square$

**Definition 2 (Possible models, adapted from [33] and corrected).** *An interpretation  $I$  as a *possible model* of  $P$  iff  $I$  is a perfect model of some split program closure of  $P$ .*

## 2.2 Fusemate Soundness and Completeness

We wish to apply the fixpoint model construction (Note 2) to Fusemate programs. For this to work, rules must be *monotonic* and *compact*.

**Definition 3.** *Let  $(H :- B, \beta)$  be an ordinary rule closure. It is *monotonic* iff for all  $I$  and  $J \supseteq I$  such that every atom in  $J \setminus I$  is in the same stratum as  $H\beta$ , if  $I, \beta \models B$  then  $J, \beta \models B$ . It is *compact* iff for all  $I$ , if  $I, \beta \models B$  then  $J, \beta \models B$  for some finite  $J \subseteq I$ .*

In general, monotonicity of an operator guarantees the existence of a least fixpoint, and compactness guarantees that it can be found by fixpoint iteration. For satisfiable Horn programs, monotonicity entails the “model intersection property” which entails



the existence of a unique minimal model. These are all well-known standard results [24], and the above definitions are formulated in a way to make these results applicable.

Fusemate rules are always monotonic. In particular for comprehension and **collect** this is due to stratification. However **collect** is not always compact. Given a body literal **collect**( $x, t \text{ sth } B$ ), there could be infinitely many substitutions  $\gamma$  in the comprehension  $\{t\gamma \mid I, \beta\gamma \models B \text{ for some grounding substitution } \gamma \text{ for } fvar(B\beta)\}$ . Because infinite sets have no term representation, such a **collect** literal renders its rule body always unsatisfied, resulting in incompleteness. One possible way out is to make sure that the variables in  $t$  range only over finite domains, e.g., sets of constants. With this fix, it follows that fixpoint iteration (Note 2) wrt. SBTP is sound and complete for possible models of Fusemate programs (Def. 2). The proof is an adaptation of the corresponding one in [33].

Soundness and completeness of fixpoint iteration holds in particular for *finite* models. This suggests another “fix”: thanks to stratification, the mentioned incompleteness can occur only when  $I$  itself is infinite at a limit step in the fixpoint iteration. Because computing (rather, finitely representing) infinite models is out of scope anyway, it is safe to ignore the compactness problem for finite model computation.

*Fusemate.* Fusemate implements a bottom-up model computation procedure in the style of hyper tableaux [13] in a stratified way (SBTP). The Fusemate main loop computes body matchers  $\beta$  of bodies  $B$  of program rules  $H :- B$  against a current branch (a model candidate) and closes it or branches out according to possible models splitting. Each new branch is for a set  $S$  in Def. 1 and receives all  $h\beta$  for  $h \in S$ .<sup>4</sup> This constructs tableau in a depth-first left-to-right order. Body matcher computation is made more practical by guaranteed left-to-right evaluation of bodies. This helps to avoid unexpected undefinedness of comprehensions and special forms. For example, in the body of  $r(time, xs) :- q(time, y), \text{collect}(xs, x \text{ sth } (p(time, x), x > y))$  the **collect** special form binds the variable  $xs$  to the list of all  $x$  such that  $p(time, x)$  and  $x > y$  hold, where  $y$  has *already* been bound by the preceding  $q(time, y)$ . See [12] for a formal definition of left-to-right body matcher computation.

Other than that, Fusemate model computation follows the abstract fixpoint computation procedure (see Note 2) for finite interpretations. This entails *finite model soundness*: if Fusemate terminates on a program  $P$  with an open exhausted branch then this branch contains a finite possible model of  $P$ . It also entails *finite model completeness*: if every possible model of  $P$  is finite then Fusemate will compute each of them in its open exhausted branches. A formal theorem for these results could be given but is not stated here because it would require more formalization.

Fusemate’s termination behavior could be improved with a breadth-first strategy, however at the expense of one-branch-at-a-time space efficiency. In the programs below this is not a problem.

### 3 Description Logic Interface

Fusemate can be used as a description logic (DL) reasoner by mapping a DL knowledge base into a logic program and running that program for satisfiability [12]. This section

<sup>4</sup> Body matcher are represented internally in the Scala runtime system without explicit grounding.

makes that reasoner callable from rules, but other DL reasoners could be coupled, too. It describes the syntax, semantics, and soundness and completeness properties of the coupling, and it discusses related work.

The DL terminology follows [5]. To summarize, a DL knowledge base KB consists of a TBox and an ABox. A TBox  $T$  is a set of GCIs (general concept inclusions), each of the form  $C \sqsubseteq D$  where  $C$  and  $D$  are DL *concept expressions*, or just *concepts*. An ABox  $A$  is a set of *ABox assertions*, i.e., concept assertions and role assertions of the forms  $a : C$  and  $(a, b) : r$ , respectively, where  $a$  and  $b$  are individuals and  $r$  is a role. Fusemate currently implements  $\mathcal{ALCIF}$ , which is  $\mathcal{ALC}$  extended with inverse roles and functional roles. A *role*, hence, is either a role name  $n$  or an inverse role name  $n^{-1}$ . Roles can be declared as functional (right-unique). As usual, KB-satisfiability is assumed to be decidable and concept formation must be closed under negation, so that query entailment can be reduced to KB unsatisfiability as follows. Given a KB  $(A, T)$  and an ABox  $Q$ , the *(ground) query*, define  $(A, T) \models_{\text{DL}} Q$  iff the KB entails  $Q$  wrt. the usual first-order logic semantics of description logics, or, equivalently: for all  $a : C \in Q$  the KB  $(A \cup \{a : \neg C\}, T)$  is unsatisfiable and for all  $(a, b) : r \in Q$  the KB  $(A \cup \{a : \forall r. \neg B, b : B\}, T)$  is unsatisfiable, where  $B$  is a fresh concept name.

The coupling between the rules and the DL reasoner is *two-way* and *dynamic*: it is two-way in the sense that rules can not only *call* the DL reasoner wrt. a fixed ABox and a TBox, the rules can also *construct* ABoxes during model computation, individually in each possible model. It is *dynamic* in the sense that ABox assertions are time-stamped, like ordinary atoms, and also all earlier ABoxes are accessible by the rules.

*Syntax.* Concepts and roles are treated as constants by the rule language while any free ground term can be a DL individual. More precisely, assume a DL signature whose concept and role names are disjoint with the signature of the rule language. Let  $t, t_1, t_2$  be free possibly non-ground terms,  $C$  a concept,  $r$  a role and  $tt$  a time term. An *untimed DL-atom* is of the form  $t : C$  or  $(t_1, t_2) : r$ . Let  $\text{IsAAt}/3$  and  $\text{HasAAt}/4$  be distinguished ordinary predicate symbols. A *timed DL-atom* is an ordinary atom  $\text{IsAAt}(t, C, tt)$  or  $\text{HasAAt}(t_1, r, t_2, tt)$ , usually written as  $t : C @ tt$  or  $(t_1, t_2) : r @ tt$ , respectively. Timed DL-atoms can appear in heads (and bodies) of ordinary rules. This allows to create time-stamped ABoxes initially as sets of facts and dynamically during program execution. For calling the DL reasoner, the rule language is extended by the following *DL-call* special forms, where  $T$  is a TBox,  $A$  is an ABox, and  $\vec{q}$  (“query”) is a list of untimed DL-atoms.

$T \models \vec{q}$	$\text{DLISSAT}(T)$	$\text{DLISUNSAT}(T)$
$(A, T) \models \vec{q}$	$\text{DLISSAT}(A, T)$	$\text{DLISUNSAT}(A, T)$

The free variables are  $\text{fvar}(\vec{q})$  in the left column cases, otherwise empty.

*Semantics.* Logic programming considers syntactically different terms as unequal. This is not enforced in DLs. Indeed, e.g., if  $A = \{(a, c) : r, (a, b) : r\}$  and  $r$  is a functional role then  $A$  is satisfiable by making  $b$  and  $c$  *equal*. To avoid such discrepancies, DL individuals are explicitly equipped with a unique name assumption, as follows.

Given an ABox  $A$ , let  $K(A) = \{a_1, \dots, a_n\}$  be the set of all (“known”) individuals mentioned in  $A$  and define  $\text{UNA}(A) = \{a_i : N_{(a_i, a_j)}, a_j : \neg N_{(a_i, a_j)} \mid a_i, a_j \in$



$K(A)$  and  $1 \leq i < j \leq n\}$ . In that,  $N_{(a_i, a_j)}$  are fresh concept names. The set  $UNA(A)$  specifies that all individuals in  $A$  must be pairwise unequal (a, b and c in the example).

The definition of rule semantics in Section 2 is extended by DL-calls as follows:  $I, \beta \models ((A, T) \models \vec{q})$  iff  $(A \cup UNA(A) \cup UNA(\vec{q}\beta), T) \models_{DL} \vec{q}\beta$  ( $\vec{q}\beta$  as a set);  $I, \beta \models DLISSAT(A, T)$  iff  $(A \cup UNA(A), T)$  is satisfiable;  $I, \beta \models DLISUNSAT(A, T)$  iff  $(A \cup UNA(A), T)$  is unsatisfiable.

For the DL-calls on the first line, let *time* be the pivot variable of the rule containing the DL-call and take  $A = abox(I, time \beta)$  for the corresponding definition with explicit  $A$ , where  $abox(I, d) = \{t : C \mid t : C @ d \in I\} \cup \{(t_1, t_2) : r \mid (t_1, t_2) : r @ d \in I\}$  is the *induced ABox from I at time d*. Intuitively, such a DL-call gets its ABox from the current interpretation by projection from its timed DL-atoms at the current time.

Notice the implicit dependency of an induced ABox on timed DL-atoms at pivot time. This is why for the purpose of stratification every line one DL-call stands for the two subgoals  $IsAAt(\_, \_, time)$  and  $HasAAt(\_, \_, time)$ . For constant ABoxes on the second line stratification is not an issue. (As such they are not very useful - but see Example 2 and the example in Section 5 below.)

With all that in place, the possibly model semantics for stratified programs defined in Section 2.1 carries over to the DL coupling without change. Notice that the semantics of the coupling is agnostic of the notion of (un)satisfiability and entailment in the DL part. This way, the coupling respects the usual open world semantics of DL reasoning. Notice also that it is possible that a program has a possible model  $I$  whose induced ABox is unsatisfiable with some TBox  $T$ . If this is not desirable it is easy to reject such a model with a **fail** rule utilising a  $DLISUNSAT(T)$  call.

*Soundness and completeness.* Soundness and completeness carries over from Section 2.2 with some caveats. Incompleteness can arise due to potentially infinite ABoxes induced at limit ordinals. With an interest in finite models only, this issue can safely be ignored, as before. A more relevant issue is monotonicity (Def. 3).  $DLISSAT$  calls can be non-monotonic because first-order logic satisfiability is, of course, not always preserved when a KB grows. This can lead to both incompleteness/unsoundness, depending on a positive/negative call context. The other two forms are based on unsatisfiability, hence monotonic, and cause no problem. With those only, iterated fixpoint computation and Fusemate model computation are both sound and complete for finite possible models.

*Related work.* According to the classification in [16], ours is a hybrid approach with a loose coupling between the description logic and the rule reasoner. The coupling is done in a DL-safe way [27], in fact, essentially, in a *weakly* DL-safe [32] way as in DL+Log. DL+log [32] is among the most expressive languages that combines rules with ontologies. DL+log rules can query a DL reasoner by taking concept/role names as unary/binary predicates and using (in our terms) extra existentially quantified variables in queries. With Fusemate rules one would equivalently use existential role restrictions. Unlike DL+log, Fusemate allows DL-calls within default negation, cf. Example 2. Most other hybrid languages, like the one in [27] and dl+Programs [17] do not allow DL atoms in the head. Unlike as in the other approaches, concepts and roles are *terms* here and, hence, can be quantified over in rules. This is advantageous for writing domain independent rules involving concepts and roles, such as the event calculus in Section 4.

### 3.1 Example

As a running example we consider a highly simplified transport scenario. Boxes containing goods are loaded onto a truck, moved to a destination, and unloaded again. The boxes can contain perishable goods that require cooling, fruits, or non-perishable goods, toys. Boxes of the former kind (and only those) can be equipped with temperature sensors and provide a temperature value, which is classified as low (unproblematic) or high (problematic). A part of this domain is modelled in the description logic  $\mathcal{ALC}$  extended with functional roles. The following KB has a TBox on box properties (left), and an ABox on temperature classes (middle) and box properties (right):

$\text{Box} \sqsubseteq \forall \text{Temp}.\text{TempClass}$	$\text{Low} : \text{TempClass}$	$\text{Box}_0 : \text{FruitBox}$
$\text{FruitBox} \sqsubseteq \exists \text{Temp}.\text{TempClass}$	$\text{High} : \text{TempClass}$	$\text{Box}_1 : \text{FruitBox}$
$\text{ToyBox} \sqsubseteq \neg \exists \text{Temp}.\text{TempClass}$		$\text{Box}_2 : \text{Box}$
$\text{FruitBox} \sqsubseteq \text{Box}$		$\text{Box}_3 : \text{ToyBox}$
$\text{ToyBox} \sqsubseteq \text{Box}$		$\text{Box}_4 : \text{Box} \sqcap \forall \text{Temp}.\neg \text{TempClass}$
$\text{Temp}$ is a functional role		$\text{Box}_5 : \text{Box} \sqcap \exists \text{Temp}.\text{TempClass}$

*Example 1.* The ABox assertions can be represented as a program with facts timed at, say, 0 (“beginning of time”), e.g.,  $\text{Box}(5) : \text{Box} \sqcap \exists \text{Temp}.\text{TempClass} @ 0$ .<sup>5</sup> Let  $\text{tbox}$  denote the TBox above. Some example rules with DL-calls are

- 1  $x : \text{Box} @ \text{time} :- x : \_ @ \text{time}, \text{tbox} \models x : \text{Box}$
- 2  $\text{TempBox}(\text{time}, \text{box}) :- \text{box} : \text{Box} @ \text{time}, \text{tbox} \models \text{box} : (\exists \text{Temp}.\text{TempClass})$
- 3  $\text{KnownTempBox}(\text{time}, \text{box}) :-$
- 4  $\quad \text{box} : \text{Box} @ \text{time}, \text{choose}(\text{temp}, \text{List}(\text{Low}, \text{High})), \text{tbox} \models (\text{box}, \text{temp}) : \text{Temp}$

The first rule materializes the Box concept. Any known individual at a given time that is provable a Box will explicitly become a Box individual at time. While this is redundant for DL-reasoning, it comes in handy for rules. For example, the second rule applies to explicitly given Boxes at time that provably have a Temp attribute. Thanks to the first rule,  $\text{TempBox}(0, \text{Box}(i))$  is derivable for  $i \in \{0, 1, 5\}$ . (Recall that the ABox in the DL-call is formed from the timed DL-atoms at pivot time.) The third rule is a variation of the second rule and tests if a box has a *concrete* Temp attribute Low or High instead of *some*.  $\square$

*Example 2.* This is an example for a stratified DL-call within default negation and explicit ABox:

- 1  $\text{ColdBox}(\text{time}, \text{box}) :-$
- 2  $\quad \text{box} : \text{Box} @ \text{time},$
- 3  $\quad \text{not} (\text{t} < \text{time}, (\text{l.aboxAt}(\text{t}), \text{tbox}) \models \text{box} : \text{Box}, (\text{box}, \text{High}) : \text{Temp})$

<sup>5</sup> The concrete Fusemate syntax is  $\text{IsAAt}(\text{Box}(5), \text{And}(\text{Box}, \text{Exists}(\text{Temp}, \text{TempClass})), 0)$  but we stick with the better readable “:-”-syntax. TBoxes have similar syntax and are typically bound to (Scala) variables like  $\text{tbox}$  in the example. In concrete syntax, free constant, function and predicate symbols start with a capital letter, variables with a lower case letter. An underscore  $\_$  is an anonymous variable.

According to this rule, a box is a ColdBox at a given time if it never provably was a Box in the past with a High temperature. The (Scala) expression `I.aboxAt(t)` can be used in Fusemate to retrieve the induced abox at time  $t$  from the current interpretation  $I$ .<sup>6</sup> Notice that  $t$  is strictly earlier than time which renders the DL-call stratified.

An example for the DLISUNSAT DL-call is in the rule **fail** :– Now(time), DLISUNSAT(tbox). This rule abandons a current model candidate if its induced abox at the current time “Now” is inconsistent with tbox.  $\square$

## 4 Event Calculus Embedding

The event calculus (EC) is a logical language for representing and reasoning about actions and their effects [20,35]. At its core, effects are fluents, i.e., statements whose truth value can change over time, and the event calculus provides a framework for specifying the effects of actions in terms of initiating or terminating fluents.

Many versions of the EC exists, see [26] for a start. The approach below makes do with a basic version that is inspired by the discrete event calculus in [28] with integer time. The event calculus of [28] is operationalized by translation to propositional SAT. Its implementation in the “decreasoner” is geared for efficiency and can be used to solve planning and diagnosis tasks, among others. The version below is tailored for the model computation tasks mentioned in the introduction, where a fixed sequence of events at given timepoints can be supposed.<sup>7</sup> It rests on minimal model semantics and stratified default negation. Most of it is not overly specific to Fusemate, and answer set programming encodings of the event calculus, e.g. [21], should be applicable as well.

The rest of this section explains the EC/DL integration grouped into “axiom sets”:

- Domain independent EC axioms: principles of actions initiating/terminating fluents
- Domain independent EC/DL integration axioms: ABox assertions as fluents
- Domain dependent axioms: initial situation and concrete actions effects
- Concrete actions: events driving the model computation
- Fusemate specific rules

*Domain independent axioms.* The EC main syntactic categories are *Fluents* and *Actions*, both given via designated sub-signatures of the term signature. They are used with *EC-predicates* in intended sorting as follows:

Initiates : $\mathbb{T} \times \text{Action} \times \text{Fluent}$	Initiated : $\mathbb{T} \times \text{Fluent}$
Terminates : $\mathbb{T} \times \text{Action} \times \text{Fluent}$	Terminated : $\mathbb{T} \times \text{Fluent}$
StronglyTerminates : $\mathbb{T} \times \text{Action} \times \text{Fluent}$	StronglyTerminated : $\mathbb{T} \times \text{Fluent}$
HoldsAt : $\mathbb{T} \times \text{Fluent}$	Happens : $\mathbb{T} \times \text{Action}$

The EC was originally introduced as a Prolog logic program. The following *domain independent rules* are similar but modified for stratified bottom-up model computation.

<sup>6</sup> Access to  $I$  is unusual for logic programming systems. See [12] for a discussion of this features.

<sup>7</sup> Actually, events can be inserted in retrospect using Fusemate’s revision operator, restarting the model computation from there. The paper [11] already has a “supply-chain” example for that.

Some rules use a “strong negation” operator **neg** which can be applied to ordinary atoms in the body or the head. Fusemate implements the usual semantic [18] which amounts to adding the rules **fail** :-  $p(\text{time}, \vec{x})$ , **neg**  $p(\text{time}, \vec{x})$  for every ordinary predicate  $p$ .

- <sup>1</sup> Initiated(time+1, f) :- Happens(time, a), Initiates(time, a, f) // H1
- <sup>2</sup> Terminated(time+1, f) :- Happens(time, a), Terminates(time, a, f) // H2
- <sup>3</sup> StronglyTerminated(time+1, f) :- Happens(time, a), StronglyTerminates(time, a, f) // H3
- <sup>4</sup> Terminated(time, f) :- StronglyTerminated(time, f) // H4
  
- <sup>6</sup> HoldsAt(time, f) :- Initiated(time, f), **not** Terminated(time, f) // EC3
- <sup>7</sup> **neg**(HoldsAt(time, f)) :- StronglyTerminated(time, f), **not** Initiated(time, f) // EC4
  
- <sup>9</sup> HoldsAt(time, f) :- Step(time, prev), HoldsAt(prev, f), **not** Terminated(time, f) // EC5
- <sup>10</sup> **neg**(HoldsAt(time, f)) :- Step(time, prev), **neg**(HoldsAt(prev, f)), **not** Initiated(time, f) // EC6

In the rules above, the variable  $f$  stands for fluents and  $a$  for actions. The axioms H1 – H3 specify the dependencies between fluents and actions in general. The distinction between Initiates and Initiated was made for being able to distinguish between initiation by actions (“loading a box on a truck *initiates* the box being on the truck”) and initiation as a matter of circumstances or their consequences (“smoke *initiated* alarm bell ringing”).

The core relation is HoldsAt(time,  $f$ ) which can hold true at time because  $f$  is Initiated at time (EC3), or was true at the previous time step but not terminated (EC5, frame axiom). Similarly for the negated case. Notice the difference between Terminated and StronglyTerminated. The former removes HoldsAt(time,  $f$ ) from the model, the latter inserts **neg**(HoldsAt(time,  $f$ )) into it. That is, this is a three-valued logic. With default negation one can distinguish the three cases.

Notice that fluents are initiated or terminated in H1 – H3 with a delay of one time step. This was done so that the Initiates and Terminates predicates can be defined in a stratified way in terms of HoldsAt at the current time. Without the delay SBTP would be violated in such cases. The increase in time will not cause non-termination of model computation because H1 – H3 are conditioned on events happening (as long as there are only finitely many events).

#### 4.1 Linking Description Logic with the Event Calculus

Section 3 introduced timed DL-atoms for specifying (timestamped) ABoxes. Typically, ABox assertions should be preserved over time unless there is reason for change. Examples are the initial ABox assertions in Example 1 and role assertions in Example 4 below. This immediately suggests to utilize the event calculus for treating ABox assertions as fluents. The following explains this in more detail.

*Domain independent axioms.* From now on, *untimed* DL-atoms are allowed in fluent positions. Untimed DL-atoms are enough because fluents occur within EC-predicate atoms which by themselves provide the time. The following axioms are added as domain independent axioms to restore the timed DL-atom versions of the fluents:

- <sup>1</sup>  $x : c @ \text{time} :- \text{HoldsAt}(\text{time}, x : c) // DL1$

<sub>2</sub>  $x : \text{Neg}(c) @ \text{time} :- \text{neg}(\text{HoldsAt}(\text{time}, x : c)) // \text{DL2}$   
<sub>3</sub>  $(x, y) : r @ \text{time} :- \text{HoldsAt}(\text{time}, (x, y) : r) // \text{DL3}$

Notice the use of variables  $c$  and  $r$  in concept and role positions, which makes it possible to formulate these axioms independent of a concrete DL KB. The DL2 axiom expresses strongly negated concept membership equivalently by membership in the negated concept.

The axioms DL1 – DL3 are obviously reasonable in any domain. Their converse is not, however. Not everything holding true at a point in time should by default extend into the future, e.g., a person’s birthday.

*Domain dependent axioms.* Domain dependent axioms comprise fluents that hold initially and specifications of action effects in terms of initiation and termination of fluents. An example for the former is the fact for Box(5) in Example 1, which could be rewritten as  $\text{HoldsAt}(0, \text{Box}(5) : \text{Box} \sqcap \exists \text{Temp}.\text{TempClass})$ .

*Example 3.* The following rules specify the effects of Load and Unload actions of boxes in terms of these boxes being OnTruck.

- <sub>1</sub>  $\text{Initiates}(\text{time}, \text{Load}(\text{box}), \text{OnTruck}(\text{box})) :- \text{box} : \text{Box} @ \text{time}$
- <sub>2</sub>  $\text{StronglyTerminates}(\text{time}, \text{Unload}, \text{OnTruck}(\text{box})) :- \text{HoldsAt}(\text{time}, \text{OnTruck}(\text{box}))$

The first rule makes sure in its body that only boxes that exist at a time can be loaded. The second rule concludes that all boxes loaded will definitely not be not on the truck after unload. All other boxes are untouched. Notice that the OnTruck fluent is not a DL concept (it doesn’t have to be).  $\square$

*Concrete actions.* What is still missing are concrete actions happening for triggering the model computation in the combined Rules/DL/EC domain model. In the running example we consider the following scenario unfolding:

Time	10	20	30	40	50
Action	Load Box <sub>0</sub> Load Box <sub>1</sub>	Load Box <sub>2</sub>	Load Box <sub>3</sub> Load Box <sub>4</sub>		Unload
Sensor	Box <sub>0</sub> : −10°	Box <sub>2</sub> : 10°	Box <sub>0</sub> : 2°	Box <sub>0</sub> : 20°	

These actions are easily represented as facts, e.g.,  $\text{Happens}(10, \text{Load}(\text{Box}(0)))$ . The temperature measurement at time 20 for Box(2) becomes  $\text{Happens}(20, \text{SensorEvent}(\text{Box}(2), 10))$ .

*Concrete domains.* Real-world applications require reasoning with concrete domains (numeric types, strings, etc). Extending DLs with concrete domains while preserving satisfiability is possible only under tight expressivity bounds. See [25] for a survey. One way to mitigate this problem is to use rules and built-ins for concrete domains and to pass symbolic abstractions to the DL reasoner.

*Example 4.* This rule demonstrates abstracting a concrete box temperature sensor reading as a Temp attribute.

- <sub>1</sub>  $\text{Initiates}(\text{time}, \text{SensorEvent}(\text{box}, \text{temp}), (\text{box}, \text{High}) : \text{Temp}) \text{ and}$
- <sub>2</sub>  $\text{Terminates}(\text{time}, \text{SensorEvent}(\text{box}, \text{temp}), (\text{box}, \text{Low}) : \text{Temp}) :-$
- <sub>3</sub>  $\text{Happens}(\text{time}, \text{SensorEvent}(\text{box}, \text{temp})), \text{temp} > 0$

The given action `Happens(20, SensorEvent(Box(2), 10))` with the rule above and rules H1 and H6 will derive `HoldsAt(21, (Box(2), High) : Temp)`. From that, with DL1 and the rules in Example 1, `Box(2)` will become a `TempBox` and even a `KnownTempBox` from time 21 onwards.  $\square$

*Fusemate specific rules.* Fusemate provides the user with a number of non-standard operators, see [12]. One of them is the aggregation operator `COLLECT`.

*Example 5.* Consider the rule

```

1 Unloaded(time+1, boxes) :-
2   Happens(time, Unload),
3   COLLECT(boxes, box STH HoldsAt(time, OnTruck(box)))

```

This rule aggregates all unloaded boxes into one set, `boxes`, one tick after `Unload` time. It is not formulated as a fluent to make it a *timepoint* property. In the example, the `Unload` happens at time 50, which leads to `Unloaded(51, Set(Box(0), Box(1), Box(2), Box(3), Box(4)))`. Notice that these are exactly the boxes loaded over time, at timepoints 10, 20, and 30.  $\square$

## 4.2 Ramification Problem

The ramification problem is concerned with indirect consequences of an action. Such consequences could be in conflict with facts holding at the time of the action or other consequences. This problem is particularly prominent in the combination with DL, where effects (i.e., fluents) can be entailed implicitly by the DL KB, and possibly in an opaque way. Trying to terminate such a fluent can be futile, as it could be re-instated implicitly or explicitly by materialization.

A good example is the entailment of `TempBox(0, Box(0))` as discussed in Example 1. Suppose we wish to re-purpose `Box(0)` and no longer use it for temperature sensitive transport. In terms of the modelling, `Box(0)` shall no longer belong to the (entailed) concept  $\exists \text{Temp}.\text{TempClass}$ .

The ramification problem has been extensively researched in the EC literature, see [35]. For instance, one could impose state constraints, if-and-only if conditions, so that terminating an entailed fluent propagates down; or one could use effect constraints that propagate termination of actions to other actions. A first attempt in this direction is a rule that terminates a fluent that entails the property to be removed:

```

1 Terminated(time+1, (box, temp) : Temp) :-
2   RemoveTemp(time, box), // Some condition for removing box Temp
3   (box, temp) : Temp @ time // Attribute to be removed

```

This rule works as expected for `Box2` after explicitly having received a `Temp`-attribute at time 20, cf. Example 4. It does not work, however, for, e.g., `Box0`. As a `FruitBox`, `Box0` has a `Temp` attribute implied by the `TBox`.

One way to fix this problem *in the running example* is to terminate *all* concept assertions for the box as any of them might entail a `Temp` attribute, and only retain that it is a `Box`:



```

1 (Terminated(time, box : concept) and Initiated(time, box : Box)):-
2     RemoveTemp(time, box), // Some condition for removing box Temp
3     box : concept @ time, concept != Box // Concept to be removed
4 // Similar rule for removing role assertions omitted

```

While this measure achieves the desired effect, it may also remove box properties that could be retained, e.g., the size of the box (if it were part of the example, that is).

The KB revision problem has been studied extensively in database and AI settings. For DLs, there are algorithms for *instance level updates* of an ABox, where, in first-order logic terms, the ABox is a set of ground atoms over known individuals, see [19]. Very recently, Baader et al [7,6] devised algorithms for semantically optimally revising ABoxes that may contain quantifiers (e.g. Box<sub>5</sub> in the running example). All these result are for lightweight description logics, though.

## 5 Putting it all Together

This section completes the running example with rules for diagnostic reasoning. Suppose a given subset of the boxes {Box<sub>0</sub>, . . . , Box<sub>5</sub>} is unloaded at the destination. We are interested in determining the status of the delivery and computing possible models as explanations under these constraints:

1. If there is no unloaded box with known high temperature then the status is OK.
2. If some unloaded box has a known high temperature then this box has been tampered with or the truck cooling is broken.
3. If some unloaded box has a known low temperature then the truck cooling is not broken (because a broken cooling would affect all boxes).
4. If all unloaded boxes with a temperature sensor can consistently be assumed to have high temperature then box tampering can be excluded (because broken cooling is the more likely explanation).

The following rules determine the status of the delivery as “ok” or “anomalous”. There are two cases of anomalies, (a) the truck cooling is broken or (b) some box has been tampered with. The rules feature disjunctive heads, strong negation, DL-calls, Scala builtin calls and the set datatype.

```

1 OK(time) :- Unloaded(time, boxes), not Anomaly(time, _)

3 Anomaly(time, TamperedBox(box)) or Anomaly(time, BrokenCooling) :-
4     Unloaded(time, boxes),
5     (box, High) : Temp @ time,
6     boxes ∋ box

8 neg(Anomaly(time, BrokenCooling)) and neg(Anomaly(time, TamperedBox(box))) :-
9     Unloaded(time, boxes),
10    (box, Low) : Temp @ time,
11    boxes ∋ box

```

```

13 fail :- Anomaly(time, TamperedBox(box)),
14      Unloaded(time, unloadedBoxes),
15      COLLECT(boxes, box STH (TempBox(time, box), unloadedBoxes  $\ni$  box)),
16      LET(assertions, boxes map { (_, High) : Temp }, // unloaded boxes ascribed High Temp
17      DLISSAT(I.aboxAt(time) ++ assertions, tbox)

```

The first rule makes the delivery ok in absence of any anomaly. The second rule observes an anomaly if some unloaded box has a High temperature. The anomaly could be either type, or both, this rule makes a guess. The third and the fourth rule are eliminating guesses. The third rule says that the truck cooling is not broken if evidenced by the existence of a Low temperature box. Moreover, each of these boxes has not been tampered with. The fourth rule is the most interesting one. It eliminates a tampered-box anomaly by considering all unloaded boxes that are known to be equipped with temperature sensors. The rationale is that if *all* these boxes can consistently be assumed to have High temperature then box tampering is unlikely (broken cooling is more likely).

This reasoning is achieved by collecting in line 15 in the boxes variable the mentioned boxes (TempBox was defined in Example 1). Line 16 assigns to a variable assertions the value of the stated Scala expression for constructing High temperature role assertions for boxes. Finally, the DL-call on line 17 checks the satisfiability of the KB consisting of the current abox temporarily extended with assertions and the static TBox. It is important to know that fail rules are always tried last for a fixed current time, after all ordinary rules. This way, the usages of COLLECT and DLISSAT in the last rule *are* stratified.

The correct diagnosis is Anomaly(51, BrokenCooling). In the course of events, the TempBoxes are Box<sub>0</sub>, Box<sub>1</sub>, Box<sub>2</sub>, and Box<sub>5</sub> (Box<sub>2</sub> becomes one only at time 20.) The unloaded boxes at time 50 are Box<sub>0</sub>, Box<sub>1</sub>, Box<sub>2</sub>, and Box<sub>4</sub>. In their intersection, Box<sub>0</sub> and Box<sub>2</sub> have High Temp values, which gives rise to an anomaly. Only the box Box<sub>1</sub> has an unknown Temp value, which is consistent with High and, hence, excludes a TamperedBox anomaly. Moreover, for every box, neither a TamperedBox anomaly nor a negated TamperedBox anomaly is derived.

If the Box<sub>0</sub> sensor reading at time 40 is changed from 10 to -10 then the diagnosis is

```

1 Anomaly(51, TamperedBox(Box(2)))
2 neg(Anomaly(51, TamperedBox(Box(0))))
3 neg(Anomaly(51, BrokenCooling))

```

Both diagnosis are the only possible models in each case and nothing is known about Box<sub>1</sub>. The Fusemate runtime is approx. 4 seconds in each case on a modern PC. The main bottleneck is lack of performance of the coupled DL-reasoner, which is a proof-of-concept implementation only.

## 6 Conclusions

This paper introduced a knowledge representation language that, for the first time, combines the event calculus with description logic in a logic programming framework for model computation. The paper demonstrated the interplay of these three components by means of an elaborated example.

Results are in parts at an abstract level. They include conditions for finite-model soundness and completeness of the rules/DL reasoner coupling that are re-usable in other systems that support stratification in a similar way ([37], e.g.).

The diagnosis rules in Section 5, among others, utilized Fusemate’s specific set comprehension operator (COLLECT) and might be hard to emulate in other systems. It might be possible to run the example in this paper with an expressive system like DLV [22] without too many changes.

The modelling in the example emphasised the possibility to distinguish between absent, unknown or known attribute values, which was enabled by the description logic-/rules integration. One might want to go a step further and add “dynamic existentials” to the picture. These are unknown or implicit actions that must have existed to cause observed effects. Recovered or speculating such actions can be expressed already with the (implemented) belief revision framework of [11]. Experimenting with that within the framework here is future work.

The perhaps most pressing open issue is the EC ramification problem (Section 4.2), which is particularly pronounced with the DL integration into the EC. Recent advances on ABox updates might help [7,6].

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