Distributed static state feedback control for DC Microgrids

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Abstract. This paper addresses the problem of Current Sharing (CS) and Average Voltage Regulation (AVR) in Direct Current (DC) microgrids composed of several interconnected Distributed Generation Units (DGUs), power lines and loads. To achieve the control objectives (CS and AVR), the system is augmented with distributed integral actions. A distributed-based static state feedback control architecture is proposed. This latter guarantees the global asymptotic convergence of the system state to the set of all equilibrium points for which the control objectives are achieved, thanks to the passivity property of the DGU with local controller. Simulation results are provided to illustrate the effectiveness of the proposed methodology.

Keywords: Distributed Control · Multi-Agent Systems · DC Microgrids.

1 Introduction

Microgrids (MG) are a novel concept of distributed electrical network that can be composed of several interconnected power supplies and loads. This concept represents an efficient key component to simplify the integration of renewable energy sources. Moreover, Direct Current (DC) MGs have received an increasing interest in power system control engineering community. This growing interest is due to its efficiency, simplicity and wide range of applicability [2] [4].
Effective control strategies are needed to achieve high performance operation and ensure the stability of the MG. These objectives require not only local management but also cooperation between the interconnected Distributed Generation Units (DGUs) and loads [4].

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Common problems in the control of DC MG are Current Sharing (CS) and Voltage Regulation (VR). CS or, equivalently, load sharing aims to share proportionally the current demand between the different DGUs taking into consideration their power capacity. VR aims to guarantee a certain voltage level for the loads since this latter must be supplied by a nominal value of voltage under any perturbation [9]. Achieving these goals is an arduous and generally impossible task as the CS requires a voltage deviation from its reference values. Therefore, an alternative is to provide an average voltage regulation (AVR), i.e., the average values of voltages at the Points of Common Coupling (PCC) is equal to the average of its references [1].
Many works are presented in the literature to control DC MGs. Generally, the main difficulty is to guarantee global stability when CS and AVR objectives are simultaneously considered. In [7] and [9], only voltage stabilization is considered. Moreover, in [10] and [5], the aforementioned objectives are considered but without proof of global convergence.

In this paper, a new distributed methodology to control DC-MG is proposed where each element of the MG has its controller and exchanges information with its neighbors over a communication network. The novelty of this work is the use of two distributed integral actions to achieve both AVR and CS objectives. In addition, the proposed control approach is LMI-based which makes it attractive numerically. Finally, the use of passivity of interconnected systems to prove the global asymptotic convergence allows to extend the result to more general MG-problems, e.g., MG with Storage Units, etc.

The paper is organized as follows: in Section 2, some notation and preliminaries are given. In Section 3, the general framework of the studied DC-MG model is presented. The control objectives are detailed in Section 4. In Section 5, integral actions are considered to deal with the control objectives, the design of the proposed distributed control is presented. In Section 6, the simulation results are presented. Finally, Section 7 concludes the paper.

2 Notation and Preliminaries

Notation: The symbols, $\mathbb{R}$ and $\mathbb{R}_{>0}$ stand respectively for the set of real and positive real numbers. To simplify notation we denote a column vector as an n-tuple $(x_1, x_2, \cdots, x_n)$ whose entries x_i can be also column vectors or equivalently $(x_1, x_2, \cdots, x_n) = \left[x_1^T \; x_2^T \; \cdots \; x_n^T\right]^T$. The notation $\mathbf{I}_n$ is used to denote the identity matrix of the size $(n \times n)$. The transpose of a matrix A is denoted by A^T. The vector of dimension n with all components equal 1 is denoted $\mathbb{1}_n$. $\mathbb{0}_{m \times p}$ stands for the zero matrix of the size $(m \times p)$. The empty set is represented by $\emptyset$. The symbol $\otimes$ represents the Kronecker product. The notation $diag(A_1, ..., A_n)$ denotes the block diagonal matrix having the matrices A_1 to A_n on the diagonal and 0 every where else.
Convergence to a Set: If $d(x, y)$ denotes a distance in a metric space, the distance of a point x to a set S is defined by: $d(x, S) = \inf_{y \in S} d(x, y)$. A trajectory $x(\cdot)$ is said to converge asymptotically to a set S if $\lim\limits_{t \to +\infty} d(x(t), S) = 0$.

Passivity theory: A linear system (A, B, C) is strictly passive [3] if there exists a matrix $P = P^T > 0$ and a scalar $\epsilon > 0$ s.t.: $A^T P + PA < -\epsilon P$, $PB = C^T$.

3 DC Microgrid Model

In this work we consider, a DC-MG composed of N distributed Generation Units (DGUs) connected through q resistive power lines. A simple electrical scheme example of the considered model is shown in Fig. 1. The generic energy source of

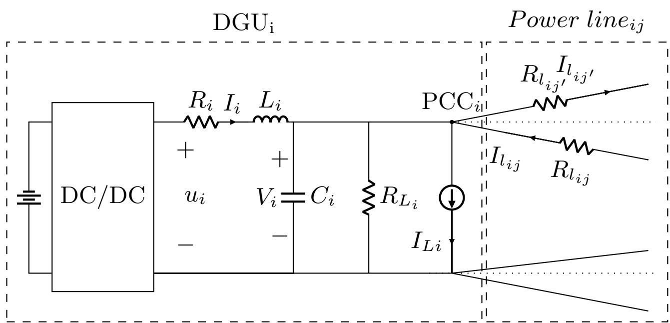

Fig. 1. The considered electrical scheme of the DC Microgird with DGUs and power lines.

each DGU is modeled as a DC voltage source that supplies a local load through a DC-DC converter. The local load is connected to the Point of Common Coupling (PCC) through an RLC (low-pass) filter. Furthermore, two types of local load are considered, Resistive load R_{Li} and unknown constant current source I_{Li}. The model of the DGU$_\mathrm{i}$ is described by the following dynamic equations:

$$\mathrm{DGU_i}\begin{cases} L_i \dot{I}_i = -R_i I_i - V_i + u_i, \\ C_i \dot{V}_i = I_i - I_{Li} - \frac{V_i}{R_{Li}} - \sum\limits_{j \in \mathcal{N}_i^{pow}} \frac{1}{R_{l_{ij}}}(V_i - V_j). \end{cases} \tag{1}$$

where I_i is the generated current, V_i is the voltage at the PCC near the DGU$_\mathrm{i}$, $I_{l_{ij}}$ is the power line current, L_i and R_i are, respectively, the output filter inductance and resistance, C_i is the output shunt capacitor, R_{Li} is the local resistive load, $R_{l_{ij}}$ is the power line resistance and $\mathcal{N}_i^{pow}$ denotes the set of nodes connected, respectively, by power lines to the i-th DGU.

The DC power network is represented by a connected and undirected graph $\mathcal{G}^{pow} = (\mathcal{V}^{pow}, \mathcal{L}^{pow})$ (see [6] for more details about graph theory). The nodes, $\mathcal{V}^{pow} = \{1, ..., N\}$, represent the DGUs. The topology of the power network is represented by a weighted Laplacian matrix $\mathcal{L}^{pow} \in \mathbb{R}^{N \times N}$ whose elements are related to the coupling term $\sum_{j \in \mathcal{N}_i^{pow}} \frac{1}{R_{l_{ij}}}(V_i - V_j)$ in (1).

The overall microgrid system for all the DGUs can be written, compactly, as:

$$\text{MG}\begin{cases} L\dot{I} = -RI - V + u, \\ C\dot{V} = I - R_L^{-1}V - \mathcal{L}^{pow}V - I_L, \end{cases} \tag{2}$$

where $I,\ V,\ I_L,\ u \in \mathbb{R}^N$. As well, $C,\ R_L,\ R,\ L \in \mathbb{R}^{N\times N}$ are positive definite diagonal matrices, e.g, $L = \text{diag}(L_1, \cdots, L_N)$.

4 Motivation and Problem Formulation

First, we present the considered control objectives. When sharing current between several supplies, the current demand should be shared proportionally, but not necessarily equally.

Objective 1. ***(Current Sharing)*** *At steady state, currents need to fulfill the following requirement*

$$\lim_{t\to\infty} \omega_i I_i = \omega_i I_i^e = \omega_j I_j^e \quad \forall i, j \in \mathcal{V}^{pow},$$

where the weight ω_i, $i = 1, \cdots, N$ are given parameters.

In fact, ω_i^{-1} can be chosen as the corresponding DGU_i rated current. Hence, a relatively small value of ω_i corresponds to a relatively large generation capacity of DGU_i.

Generally, achieving Objective 1 does not permit to attend an equilibrium voltage $V^e = V^{ref}$ at the same time. Hence, as in [8] an average voltage regulation is considered, where the aim of the controller is to have the weighted average value of V^e equal to the weighted average value of the desired reference voltages V^{ref}. Assuming that there exists a reference voltage V_i^{ref} at the PCC, for all DGU_i, the second control objective can be stated as

Objective 2. ***(Average Voltage Regulation)***

$$\lim_{t\to\infty} \mathbb{1}_N^T W^{-1} V(t) = \mathbb{1}_N^T W^{-1} V^e = \mathbb{1}_N^T W^{-1} V^{ref},$$

where $W = \text{diag}(\omega_1, \cdots, \omega_N)$, $\omega_i > 0$, for all DGU_i.

The choice of the weights for voltages as ω_i^{-1} is motivated by the fact that the DGU_i with the highest capacity should impose the voltage of the MG [8].

Now, we are able to state the control problem as :

Control Problem: *For a given reference voltage V^{ref} and an unknown load current I_L, design a distributed-based control scheme s.t. the state of system (2) in closed-loop converges globally and asymptotically to a set of equilibrium points S^e whose elements satisfy Objectives 1-2.*

5 Distributed Controller Design

In this section, a solution to the control problem defined in Section 4 is provided. First, we assume the following:

Assumption 1. ***(communication network)*** *A communication network modeled as a connected and undirected graph $\mathcal{G}^{com} = (\mathcal{V}^{com}, \mathcal{L}^{com})$ where $\mathcal{V}^{pow} = \{1, ..., N\}$ represent the DGUs and $\mathcal{L}^{com} \in \mathbb{R}^{N\times N}$ is a symmetric positive semidefinite Laplacian matrix, allows to exchange voltage V_i and current I_i measured at each* $\mathrm{DGU_i}$*, $i = 1, ..., N$.*

Assumption 2. ***(Nominal Model)*** *All the DGUs have the same nominal values of parameters, i.e., $L_i = L^*$, $C_i = C^*$, $R_i = R^*$ and $R_{Li} = R_L^*$ $\forall i = 1, \cdots, N$ with L^*, C^*, R^*, $R_L^* \in \mathbb{R}_{>0}$ represent the nominal values. Thus, $L = L^*\boldsymbol{I}_N$, $C = C^*\boldsymbol{I}_N$, $R = R^*\boldsymbol{I}_N$ and $R_L = R_L^*\boldsymbol{I}_N$.*

Our aim is to determine a controller including N integral actions in order to achieve Objectives 1-2. Consider system (2), let us introduce an augmented state $\mathcal{X} = (I, V, \phi, \gamma)$ whose dynamics is given by the following equations:

$$\Sigma \begin{cases} L\dot{I} = -RI - V + u, & (3a) \\ C\dot{V} = I - (R_L^{-1} + \mathcal{L}^{pow})V - I_L, & (3b) \\ \tau_\phi\dot{\phi} = W^T\mathcal{L}^{com}WI, & (3c) \\ \tau_\gamma\dot{\gamma} = -W^T\mathcal{L}^{com}W\gamma + (V - V^{ref}), & (3d) \end{cases}$$

where τ_ϕ, $\tau_\gamma \in \mathbb{R}_{>0}$ and where $\mathcal{L}^{com}$ is defined in Assumption 1.

Definition 1. ***(Set of Equilibrium Points)*** *For a given reference voltage V^{ref} and an unknown load current I_L, the set of all the equilibrium points is defined by $S^e(I_L, V^{ref}) = \{\mathcal{X}^e = (I^e,\ V^e,\ \phi^e,\ \gamma^e) \in \mathbb{R}^{4N}$* and *$u^e \in \mathbb{R}^N$ s.t.: $0 = -RI^e - V^e + u^e$, $0 = I^e - (R_L^{-1} + \mathcal{L}^{pow})V^e - I_L$, $0 = W^T\mathcal{L}^{com}WI^e$, and $0 = -W^T\mathcal{L}^{com}W\gamma^e + (V^e - V^{ref}).\}$*

For a given reference voltage V^{ref} and an unknown load current I_L, one can easily prove that the set $S^e(I_L, V^{ref})$ is not empty and that Objectives 1-2 are always achieved in this set. The next part concerns the design of a state feedback controller of the form

$$u = -K(I, V, \phi, \gamma), \tag{4}$$

with $K \in \mathbb{R}^{N\times 4N}$ and s.t. the state $\mathcal{X} = (I, V, \phi, \gamma)$ converges asymptotically to the set S^e.

5.1 Local controllers design

Since the controller u should be distributed, the local controllers u_i $i = 1, ..., N$ should depend only on local variables $x_i = (I_i, V_i, \phi_i, \gamma_i)$. Hence, the gain matrix K (see (4)) should be restricted to the form:

$$K = (K_I, K_V, K_\phi, K_\gamma), \tag{5}$$

where $K_I,\ K_V,\ K_\phi,\ K_\gamma \in \mathbb{R}^{N\times N}$ are diagonal matrices. The main difficulty to find a gain matrix of this form for system (3) is the existence of physical ($\mathcal{L}^{pow}$) and communication ($\mathcal{L}^{com}$) coupling terms. Hence, to simplify the design let us introduce the following change of coordinates:

$$(\tilde{I}, \tilde{V}, \tilde{\phi}, \tilde{\gamma}) = (\mathbf{I}_4 \otimes U^T)(I, V, \phi, \gamma), \tag{6}$$

where $U \in \mathbb{R}^{N\times N}$ is a unitary matrix s.t.:

$$\tilde{\mathcal{L}}^{com} = U^T W^T \mathcal{L}^{com} W U = \mathrm{diag}(0, \lambda_2, \dots, \lambda_N)$$

where $\lambda_i < \lambda_j\ \forall\ i < j$. The matrix U exists because $W^T \mathcal{L}^{com} W$ is a symmetric matrix and $\lambda_1 = 0$ since the graph $\mathcal{G}^{com}$ is connected. In this new basis, system (3) can be rewritten as follows:

$$\tilde{\Sigma}\begin{cases} L\dot{\tilde{I}} = -R\tilde{I} - \tilde{V} + \tilde{u}, & \text{(7a)}\\ C\dot{\tilde{V}} = \tilde{I} - (R_L^{-1} + \tilde{\mathcal{L}}^{pow})\tilde{V} - \tilde{I}_L, & \text{(7b)}\\ \tau_\phi \dot{\tilde{\phi}} = \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0\\ 0 & \lambda_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_N \end{bmatrix}}_{\tilde{\mathcal{L}}^{com}} \tilde{I}, & \text{(7c)}\\ \tau_\gamma \dot{\tilde{\gamma}} = -\tilde{\mathcal{L}}^{com}\tilde{\gamma} + (\tilde{V} - \tilde{V}^{ref}). & \text{(7d)} \end{cases}$$

where $\tilde{\mathcal{L}}^{pow} = U^T \mathcal{L}^{pow} U$ and $(\tilde{u}, \tilde{I}_L, \tilde{V}^{ref}) = (\mathbf{I}_3 \otimes U^T)(u, I_L, V^{ref})$. Note that the matrices $L,\ C,\ R$ and R_L remain unchanged by Assumption 2. Consider a controller $\tilde{u}$ of the form:

$$\tilde{u} = -\tilde{K}(\tilde{I}, \tilde{V}, \tilde{\phi}, \tilde{\gamma}), \tag{8}$$

where $\tilde{K} = (\mathcal{K} \otimes \mathbf{I}_N)$ and $\mathcal{K} = \begin{bmatrix} k_I & k_V & k_\phi & k_\gamma \end{bmatrix} \in \mathbb{R}^{1\times 4}$. Let us remark that system $\tilde{\Sigma}$ with the control law (8) is composed of N interconnected subsystems which can be written using a permutation matrix as follows, for $i = 1, \cdots, N$:

$$\tilde{\Sigma}_i\ : \dot{\tilde{x}}_i = Acl_i \tilde{x}_i + d_i - B_p \sum_{j \in \mathcal{N}_i^{pow}} l_{i,j} C_p (\tilde{x}_i - \tilde{x}_j), \tag{9}$$

where $\tilde{x}_i = (\tilde{I}_i, \tilde{V}_i, \tilde{\phi}_i, \tilde{\gamma}_i)$, $d_i = -(0, C^{*-1}\tilde{I}_{Li}, 0, \frac{1}{\tau_\gamma}\tilde{V}_i^{ref})$, $B_p = (0, C^{*-1}, 0, 0)$, $C_p = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$, $l_{i,j}$ for $1 \le\ i, j \le N$ denotes the elements of $\tilde{\mathcal{L}}^{pow}$ and

$$Acl_i = \begin{bmatrix} A & & \mathbb{0}_{2\times 2} & \\ \frac{\lambda_i}{\tau_\phi} & 0 & 0 & 0\\ 0 & \frac{1}{\tau_\gamma} & 0 & -\frac{\lambda_i}{\tau_\gamma} \end{bmatrix} - \begin{bmatrix} L_t^{*-1}\\ \mathbb{0}_{3\times 1} \end{bmatrix} \mathcal{K}, \quad \text{and where } A = \begin{bmatrix} -R^* L^{*-1} & -L^{*-1}\\ C^{*-1} & -(C^* R_L^*)^{-1} \end{bmatrix}. \tag{10}$$

In this basis, it can be noticed that the local variables $\tilde{x}_i = (\tilde{I}_i, \tilde{V}_i, \tilde{\phi}_i, \tilde{\gamma}_i)$ $i = 1, ..., N$ are only coupled by the term $B_p \sum_{j \in \mathcal{N}_i^{pow}} l_{i,j} C_p(\tilde{x}_i - \tilde{x}_j)$ (related to the matrix $\tilde{\mathcal{L}}^{pow}$). The next theorem shows how it is possible to determine the unique gain matrix $\mathcal{K}$ in (10) for all the subsystems by removing the last coupling terms in the right member of (9) and using some passivity arguments.

Theorem 1. *(**Main result**) If there exists a static state feedback $\mathcal{K} = \begin{bmatrix} k_I \ k_V \ k_\phi \ k_\gamma \end{bmatrix}$ s.t. the triples (Acl_i, B_p, C_p) for $i = 2, ..., N$ and $(\breve{A}cl_1, \breve{B}_1, \breve{C}_1)$ are strictly passive where:*

$$\begin{aligned} \breve{A}cl_1 &= \begin{bmatrix} A & \mathbb{0}_{2\times1} \\ 0 \ \frac{1}{\tau_\gamma} & 0 \end{bmatrix} - \begin{bmatrix} {L^*}^{-1} \\ \mathbb{0}_{2\times1} \end{bmatrix} \begin{bmatrix} k_I \ k_V \ k_\gamma \end{bmatrix}, \\ \breve{B}_1 &= (0, {C^*}^{-1}, 0), \quad \breve{C}_1 = \begin{bmatrix} 0 \ 1 \ 0 \end{bmatrix}, \end{aligned} \tag{11}$$

and where Acl_i, B_p, C_p and A are given with subsystems (9), then the state of the augmented system (3) in closed-loop with

$$u = -(\mathcal{K} \otimes \boldsymbol{I}_N)(I, V, \phi, \gamma)$$

converges asymptotically to an equilibrium $\mathcal{X}^e \in S^e(V^{ref}, I_L)$ for which the control objectives 1-2 are satisfied.

6 Simulation

In this section we aim to validate the proposed controller by simulation. We consider a MG composed of 4 DGUs with non-identical electrical parameters and communication links (see Fig. 2). The controller was designed using the

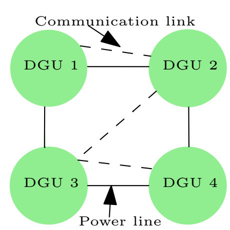

Fig. 2. MG with 4 DGUs, power lines, and communication links.

nominal parameter of the MG and then applied on the MG model with the real parameters. The system is initially at a steady state with load current $I_L(0) = [5\ 10\ 30\ 20]$ A. Then, at the time instant $t = t_1$ the load current is stepped up with $\Delta I_L = [10\ 15\ 20\ 30]$ A. As we can see in Fig. 3-4, the weighted average voltage converges to the weighted average value of the reference voltages (see Objective 2). Furthermore, the voltages at the PCC converge, without oscillations, to a steady state near to the reference voltage $V^{ref} = 380V$. Moreover,

Fig. 4 shows clearly that the weighted currents converge to the same consensus value achieving Objective 1 and the generated currents converge asymptotically to the desired steady state, asymptotically.
The results illustrate the robust performance of the proposed controllers under the change in the load current and the presence of parametric discrepancies from the nominal values.

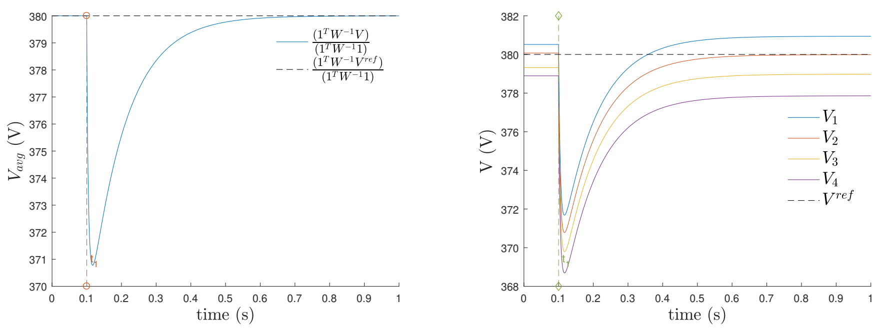

Fig. 3. From the left: weighted average voltage at the PCC and the weighted average reference voltage value (dashed line); voltage at the PCC of each DGU together with the reference value (dashed line).

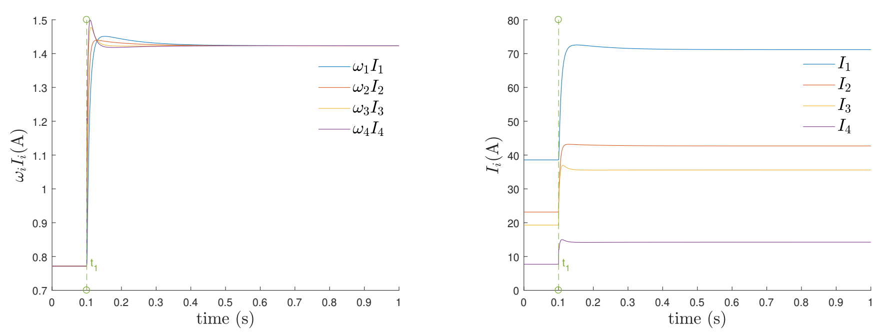

Fig. 4. From the left: the weighted generated currents of the DGUs; generated currents.

7 Conclusion

A distributed-based Static-State-feedback control scheme, including integral actions to achieve both proportional Current Sharing and Average Voltage Regulation in DC power-networks has been proposed. Distributed integral actions have been used to achieve the control objectives by exploiting a communication network. The simulation results clearly show that the control objectives are achieved with unknown load and even with significant discrepancies between nominal and real parameters of the DGUs.

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