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Interactive multiobjective optimization in lot sizing with safety stock and safety lead time

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Abstract. In this paper, we integrate a lot sizing problem with the problem of determining optimal values of safety stock and safety lead time. We propose a probability of product availability formula to assess the quality of safety lead time and a multiobjective optimization model as an integrated lot sizing problem. In the proposed model, we optimize six objectives simultaneously: minimizing purchasing cost, ordering cost, holding cost and, at the same time, maximizing cycle service level, probability of product availability and inventory turnover. To present the applicability of the proposed model, we consider a real case study with data from a manufacturing company and apply the interactive NAUTILUS Navigator method to support the decision maker from the company to find his most preferred solution. In this way, we demonstrate how the decision maker navigates without having to trade-off among the conflicting objectives and could find a solution that reflects his preference well.

Keywords: Inventory management · Uncertain demand · Uncertain lead time · Interactive decision making · NAUTILUS Navigator.

1 Introduction

Lot sizing has emerged as one of the key factors for the effective supply chain management. The purpose of lot sizing is to determine an optimal order quantity that minimizes costs while satisfying demand. After Harris's economic order quantity concept for solving a simple lot sizing problem [10], there has been a dramatic increase in interest over the last century in developing lot sizing models to adapt to more complex situations [1,8].

Uncertainties complicate lot sizing problems. In fact, predicting the exact demand for future needs is challenging. Commonly, many companies hold a certain amount of stock, known as a safety stock (SS), as a buffer to cope when demand exceeds the prediction [31]. Another source of uncertainty is the delivery lead time [24]. Companies usually have an agreement with their suppliers for the delivery time, but for many reasons, there can be delays. To overcome this issue, an additional time period, known as a safety lead time (SLT), is defined [31]. During the SLT period, companies keep their stocks available to satisfy the demand.

The problem of determining an optimal SS value has been studied by many researchers [9]. Various methods have been developed [26] to find an optimal value of SS that should be small enough to reduce costs while satisfying demand and guaranteeing a high service level. Most studies expand the cycle service level (CSL) formula [23] to adapt to various conditions. When lead time is uncertain, the CSL formula takes into account the average and standard deviation of the lead time [28]. On the other hand, the problem of finding an optimal SLT value is not as popular as the previous one [7]. In [12], inventory costs are minimized subject to a service level constraint to find an optimal SLT, and an optimization model based on Markov Chain is proposed in [6]. However, there is a lack of formula to control the quality of SLT.

The relationship between lot sizing problems with SS and SLT has been studied in [22]. Keeping stock for SS and SLT increases order quantity, which also increases the costs. Some researchers have studied lot sizing problems with uncertainty on demand and lead time [7]. However, they mostly use statistical tools to handle uncertainty in lot sizing models, but not simultaneously find SS or SLT. Some of them use simulation to find an optimal SS and SLT. There is a lack of integration of a lot sizing problem and problems of determining SS and SLT values in the literature. The problem of integrating lot sizing and SS determination is proposed in [18], but they consider SLT as the input value.

Lot sizing problems naturally include a conflict between minimizing costs and satisfying demand simultaneously. Additional problems of determining SS and SLT increase the conflict because holding more stock for SS and SLT makes the costs higher. For this reason, multiobjective optimization [19] is a good tool to solve lot sizing problems [2]. A multiobjective optimization problem has several mathematically incomparable solutions, called Pareto optimal solutions. Solving a multiobjective optimization problem can be understood as finding the most preferred solution for a decision maker (DM), who has expertise in the problem domain. Interactive methods [20] are regarded as promising because the solution process is iterative and they allow the DM to gain insight into the problem and change his/her preferences during the solution process, thanks to learning. So far, however, there have been a few studies applying interactive multiobjective optimization in lot sizing problems [29].

In this research, we consider a single item multi period lot sizing problem with uncertainty on demand and lead time. The main contributions of this paper are threefold. First, we propose a novel formula, named probability of product availability (PPA), for measuring the quality of SLT to handle unpredicted lead time. Second, we develop a multiobjective optimization model that determines the optimal lot sizes for each period and simultaneously finds the optimal values of SS and SLT. Last but not least, we support a DM to find the most preferred solution for the optimization model by applying an interactive NAUTILUS Navigator method [25].

The proposed multiobjective optimization model has six objectives to optimize simultaneously. Three of them are minimizing cost functions, i.e. purchasing cost (PC), ordering cost (OC), and holding cost (HC). We consider them separately to see trade-offs between objectives. The CSL is maximized to improve safety against demand uncertainty. We propose a PPA formula to assess the quality of SLT to buffer lead time uncertainty, which is maximized in the model. Lastly, inventory turnover (ITO) as the primary performance indicator for inventory management [27] is maximized to measure the effectiveness of this model in handling the inventory system.

Most lot sizing problems are difficult to solve because of their complexity [14]. In this paper, we use the interactive NAUTILUS Navigator method [25]. The strength of this method in handling computationally expensive problems meets the need of this kind of problem. Another strength of this method is allowing the DM to find his/her most preferred solution without sacrifices, which meets the needs of the DM. In this, the strategy is starting from a bad point and improving all objectives simultaneously. We use real data from a manufacturing company and a real DM to prove the validity of our proposed model. Finally, we support the DM to find the most satisfying solution for him by using this method.

The remainder of the paper is organized as follows. Section 2 reviews the basic concepts of multiobjective optimization and the NAUTILUS Navigator method. Then, the proposed multiobjective optimization model is presented in Section 3. In Section 4, the case study together with the real data from a manufacturing company is described, following by results and analysis of the decision making process using NAUTILUS Navigator. Finally, conclusions and discussions of possible extensions are presented.

2 Background on Multiobjective Optimization

In this section, we briefly review the basic concepts and definitions related to multiobjective optimization, followed by the NAUTILUS Navigator method.

2.1 Basic Concepts and Definitions

A multiobjective optimization problem can be formulated in the following form:

$$\begin{aligned} &\text{minimize} \quad && \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), ..., f_k(\boldsymbol{x}))^T \\ &\text{subject to} \quad && \boldsymbol{x} \in S, \end{aligned} \tag{1}$$

where $k \geq 2$ objective functions, $f_i : S \to \mathbb{R}$ for $1 \leq i \leq k$, are simultaneously optimized. The vector of decision variables $\boldsymbol{x} = (x_1, ..., x_n)^T$ belongs to the feasible region $S \subset \mathbb{R}^n$, which is formed by constraints. The image of the feasible region $Z = \boldsymbol{f}(S), Z \subset \mathbb{R}^k$ is called a feasible objective region, which is formed by the vectors of objective values $\boldsymbol{z} = \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), ..., f_k(\boldsymbol{x}))^T$, $\boldsymbol{z} \in Z$, $\boldsymbol{x} \in S$.

Because of the conflicting objectives, a multiobjective optimization problem (1) has several different solutions, called Pareto optimal solutions, which reflect the trade-offs among the conflicting objectives. A solution $\boldsymbol{z}^1 \in Z$ is said to dominate another solution $\boldsymbol{z}^2 \in Z$ if $z_i^1 \leq z_i^2$ for all $i = 1, ..., k$ and $z_j^1 < z_j^2$ for at least one $j = 1, ..., k$. A solution $\boldsymbol{z} \in Z$ is called a Pareto optimal solution if $\boldsymbol{z}$ is not dominated by any other solution. The lower and upper bounds of the Pareto optimal solutions are called an ideal point $\boldsymbol{z}^*$ and a nadir point $\boldsymbol{z}^{nad}$, respectively, which reflect the best and the worst values that each objective function in the Pareto optimal solutions can achieve.

Pareto optimal solutions are incomparable mathematically. Additional preference information from a DM is needed to identify the most preferred solution as the final solution. A DM is an expert who has a responsibility to make a decision in the problem domain, who is usually a supply chain manager in lot sizing. The preference information from the DM can be incorporated before the optimization process (a priori methods), after having generated a representative set of Pareto optimal solutions (a posteriori methods), or during an iterative optimization process (interactive methods) [19]. The advantages of interactive methods, which allow the DM to learn different aspects of the problem during the solution process and change their preferences during the solution process if desired, are the main reasons we chose this type of methods. Many interactive methods have been developed [20]. In this paper, we apply the NAUTILUS Navigator method [25] because of its ability in handling computationally expensive problems and the possibility to find the most preferred solution without trading-off. This is important since DMs sometimes get anchored around the initial solution and a trade-off free method avoids anchoring.

2.2 NAUTILUS Navigator

The NAUTILUS Navigator method combines the idea of NAUTILUS methods [21] to avoid trading-off and navigation ideas elaborated in [11]. Due to the fact that people do not respond similarly to losses and gains [15], trading-off among Pareto optimal solutions causes some decisional stress to the DM [17]. Motivated by this fact, NAUTILUS methods start from the worst possible objective function values and iteratively gain in all objectives without sacrificing any of the current values. Methods in the NAUTILUS family [21] differ in the way used to interact with the DM to find the final solution. NAUTILUS Navigator uses navigation to direct the movement from the worst starting point, which is the nadir point or any undesirable point provided by the DM, to a Pareto optimal solution as the final solution. In this process, the DM specifies a desirable value for each objective function, which are the components of a reference point, as a search direction to direct the movement towards desired Pareto optimal solutions. During the navigation process, the DM can change the reference point, the movement speed, or even go backwards if he/she wishes so.

To handle computationally expensive problems, a set of Pareto optimal solutions is generated before the interactive process starts. The generation may take time because of expensive functions, but it is done without involving the

DM. Any a posteriori methods can be used to generate a set of Pareto optimal solutions or a set that approximates Pareto optimal solutions. When involving the DM, the navigation process takes place using this set without solving the original computationally expensive problem. This allows showing real-time movement without waiting times to the DM. The detailed algorithm can be seen in [25].

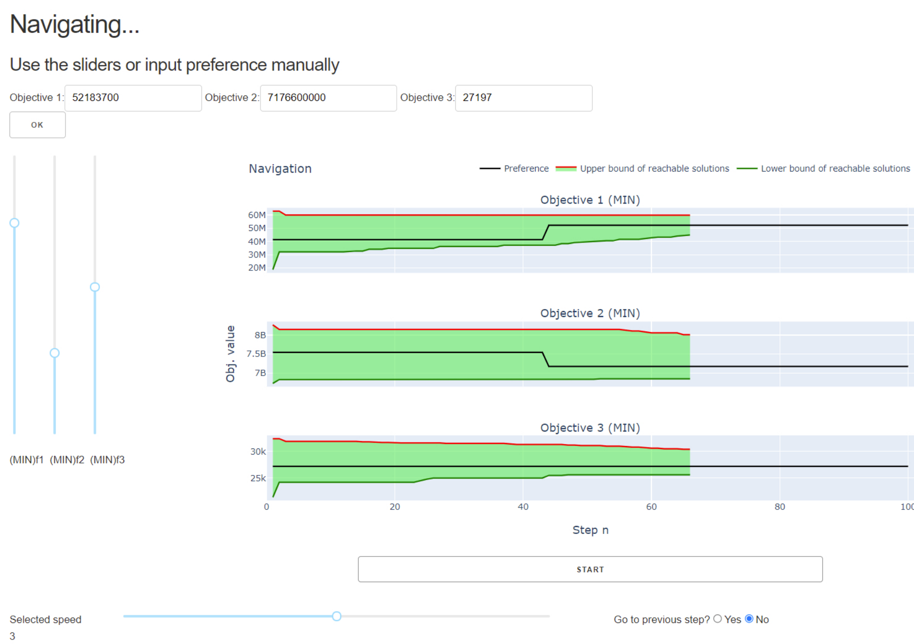

Fig. 1. GUI of the NAUTILUS Navigator method

A graphical user interface (GUI) is important for NAUTILUS Navigator to visualize the navigation process. Figure 1 shows the available GUI that can be freely downloaded from `https://desdeo.it.jyu.fi`. The DM provides his/her preferences using the sliders on the left side or inputs values in text boxes at the top. The green area in the graph shows the reachable ranges, which are the best and the worst objective function values, that each objective can reach from the current step without sacrifices in any objectives. Thus, the reachable ranges shrink when approaching Pareto optimal solutions. Whenever the DM wants to change his/her preference, he/she can stop the process and change the reference point. The black lines in the middle of the graphs show the positions of the components of the reference point. The DM is allowed to jump to any previous step using the radio button in the bottom right. He/she then needs to provide which step to go to and re-specify his/her preferences in order to define a new direction. The DM can navigate until he/she finds his/her most preferred

Pareto optimal solution at the end of the solution process. In that case, the ranges shrink to a single point.

3 Problem Formulation

We study a lot sizing problem for a single item with a single supplier and in multiple time periods. We follow a periodic review policy, where orders are reviewed over discrete time periods $t = 1, ..., T$. The order quantity $(Q(t))$ is reviewed at the beginning of period t, and the order arrives after a stochastic lead time. The following assumptions are made throughout this paper.

1. The predicted demand during period t $(D(t))$ follows a normal distribution with a mean μ and a standard deviation σ. The demand in each period is independent of other periods.
2. The lead time follows a normal distribution with a mean L and a standard deviation s.
3. The price for purchasing one unit of item (p) is constant in all time periods and does not depend on the order quantity.
4. The cost for a single order is c without any capacity limit.
5. The cost of holding one unit of item (h) is constant throughout all time periods.
6. There is no backorder cost involved.
7. There is an agreement between the company and the supplier that the company must order with a minimum order quantity moq and it rounds up by a rounding value r. Therefore, the order can only be placed by following the formula $moq + a\,r$ for any integer $a \geq 0$.

3.1 Safety Stock and Safety Lead Time Formulation

As said, we focus on the lot sizing problem with uncertainty in demand and lead time. Many researchers have utilized a SS to protect against demand uncertainty and a SLT to handle lead time uncertainty [16]. A SS means keeping more stocks as a buffer against demand fluctuations. To control the amount of SS, the cycle service level (CSL) formula is applied [4]. CSL is the probability of not hitting a stockout in a replenishment time (RT). A RT is a time needed to refill the stock, that is from the arrival of one order to the arrival of the next one. We set $RT = 1 + SLT$ since we order in each period and prepare for late delivery in the SLT period. To prevent stockout during a RT, the difference between an actual demand (D^*_{RT}) and a predicted demand (D_{RT}) must be less than SS. We adopt the CSL formula for demand and lead time uncertainty [28] with our definition of RT, which can be written as follows:

$$\begin{aligned} CSL &= P(D^*_{RT} \leq D_{RT} + SS) \\ &= F(D_{RT} + SS, D_{RT}, \sigma_{RT}) = F\left(\frac{SS}{\sigma_{RT}}\right), \end{aligned} \tag{2}$$

where F is the standard normal distribution function and σ_{RT} is the standard deviation of demand during a RT, which can be formulated as $\sigma_{RT} = \sqrt{\sigma^2(1+SLT)+\mu^2 s^2}$.

A SLT is assigned to handle unpredicted lead time. During the SLT period, the availability of stock to cover predicted and unpredicted demand must be guaranteed. Therefore, we consider an additional SLT period in the fill rate (FR) constraint to secure the availability of the stock during SLT to cover the predicted demand. A SLT period is also considered in CSL to buffer unpredicted demand during SLT. However, if the order arrives after the SLT period, the stockout may occur. Therefore, it is important to decide an optimal SLT value with a low possibility of having stockout. In this paper, we propose the probability of product availability (PPA) formula to measure the quality of SLT. PPA is defined as the probability of not having stockout because of the late delivery, which occurs when the actual order arrives during the period $L+SLT$. The PPA formula can be written as follows:

$$\begin{aligned} PPA &= P(\text{actual delivery time} \leq L+SLT) \\ &= F(L+SLT, L, s) = F\left(\frac{SLT}{s}\right). \end{aligned} \tag{3}$$

This formula can be used to find the SLT value by defining an appropriate PPA level.

3.2 Multiobjective Optimization Model

As said, we propose a multiobjective optimization model with six objectives, three to minimize and three to maximize. The main goal of this model is to find the order quantity of each period $(Q(t), t=1,...,t_n)$ together with SS and SLT values with the best balance between the objective functions. We define $I(t)$ as the inventory level at the end of period t where $I(t)=I(t-1)+Q(t-\lfloor L \rfloor)-D(t)$, and $Y(t)$ as the order indicator where $Y(t)=1$ if the order is placed $(Q(t)>0)$, otherwise $Y(t)=0$. The proposed optimization model can be written as follows.

$$\begin{aligned}
\text{min} \quad & PC = \sum_t Q(t)\,p,\ OC = \sum_t Y(t)\,c,\ HC = \sum_t \frac{I(t-1)+I(t)}{2}\,h, \\
\text{max} \quad & CSL\ (2),\ PPA\ (3),\ ITO = \sum_t \frac{D(t)}{(I(t-1)+I(t))/2}, \\
\text{s.t.} \quad & \frac{I(t-1)+\sum_{i=t-\lfloor L \rfloor}^{t} Q(i) - SS}{\sum_{j=t}^{t+\lfloor P \rfloor} D(j) + (P-\lfloor P \rfloor) D(\lceil P \rceil).} \geq 1 \text{ , for } t=1,...,T, && (4) \\
& Q(t) = Y(t)\,(moq + a\,r) \text{ , for any integer } a \geq 0 \text{ and } t=1,...,T, && (5) \\
& SS \geq 0 \text{ and } SOT \geq 0. && (6)
\end{aligned}$$

Following the dynamic lot sizing problem [14,30], three types of cost are considered: PC, OC, and HC. We consider them separately to see the trade-offs. Minimizing PC implies minimizing HC, but OC has a trade-off with HC because ordering the same amounts of items many times makes OC higher and HC lower. For inventory management purposes, it is important to understand both HC and OC. In order to prevent partial optimization, which could be the case if only total costs were measured, it is important to separate them. When targeting at low HC only, one can be misled, as then there could be a temptation to order more often, resulting in higher OC.

We maximize CSL to prevent stockout because of the demand uncertainty and maximize PPA to avoid stockout due to late delivery. Keeping a high value of SS raises the CSL but PC and HC increase, which is a conflict as we need to maximize CSL but minimize OC and HC. Having a long SLT increases the PPA but decreases CSL with the same SS value. Then PPA has a trade-off with CSL, PC and HC. Maximizing ITO is our last objective function. To have a high ITO, the order must be as close to the demand as possible in order to hold less stock, which has a trade-off with OC. Furthermore, ITO has a trade-off with CSL and PPA as less stock is needed to have a high ITO, but CSL and PPA need more stock to have better safety in handling uncertainties.

FR represents customer service for an inventory control system. It is defined as the fraction of orders that are filled from stock [13]. It is an important indicator in daily operations. In the proposed model, FR is the first constraint (4) to fulfill the predicted demand. In each period, we guarantee that our stock (excluding SS) can satisfy the predicted demand. The consideration period for one order (P) in the periodic review policy is $1 + L$ [4], but an additional SLT period is also considered to ensure the stock availability during SLT. Thus, we set $P = 1 + L + SLT$. FR is a fraction between available stock without SS and the predicted demand during P. When FR is at least one, the stock availability to handle the predicted demand is guaranteed. Furthermore, we ensure that all orders follow the agreement of minimum order quantity and rounding value in constraint (5), while constraint (6) is defined to confine the lower bounds of SS and SLT.

4 Computational Results

We consider a case study from a manufacturing company to demonstrate the applicability of the proposed model. We apply the interactive NAUTILUS Navigator method to support the supply chain manager of the said company, acting as the DM, to find his most preferred solution without trading-off.

4.1 Information about the Case

We review a weekly single item lot sizing problem for 41 weeks. Thus, the optimization model has 43 integer decision variables, including weekly order quantities, SS and SLT. We received data of an item, which is a component of the company's product. The data is generated from the company's planning system. The data contains current inventory information for the item as well as a consumption projection according to the company's production plan. Based on the data, the price to purchase one unit of the item is €91.18, the cost for a single order is €200, and the cost of holding one unit of item is ten percent of the price annually. The lead time for this item is 6 weeks, with a standard deviation $s = 0.93$ days. The company has made a prediction for the weekly demand data based on its historical data, which varies with a mean $\mu = 116.22$ and a standard deviation $\sigma = 29.04$. The opening inventory is 312 units and the company has made previous orders for the next six weeks, which are $(48, 119, 120, 120, 48, 96)$. Based on the agreement between the company and the supplier, the company must place an order with a minimum of 48 units and round by 48 units.

As a request from the DM, bounds for SS and SLT were defined as additional constraints. The DM was only interested in SS values lower than μ and SLT values below four days. He also requested to see at least one day SLT or one day's worth of demand for SS, which is $\mu/5$. Furthermore, low ITO values below ten were not interesting for the DM.

As said, a GUI plays an important role in NAUTILUS Navigator. A few modifications of the available GUI were done in this research to make the GUI more useful for the DM in this case. The DM preferred to see the probability of product unavailability (PPU) rather than PPA. Thus, we switched to minimize $PPU = 1 - PPA$ in the fifth objective. Furthermore, the DM wanted to see the information of days of stock (DoS). DoS is an inventory performance indicator describing the number of days needed to sell an item. DoS is calculated as the number of days in one year (we use 254 working days) divided by ITO.

4.2 Computational results

As described in Section 2.2, the starting point of the NAUTILUS Navigator method is a set of pre-generated solutions. As said, lot sizing problems are computationally expensive problems. Because of their complexity, many researchers use metaheuristic methods, like evolutionary algorithms, to solve various problems of lot sizing [14]. In this paper, we applied NSGA-III [5] by using the pymoo framework [3] because of its ability to solve constrained multiobjective optimization problems with integer variables. Evolutionary algorithms cannot guarantee Pareto optimality but can generate sets of solutions where no solution dominates the others.

Some strategies were needed to generate a large amount of nondominated solutions. Because a single run of NSGA-III was not able to generate enough solutions, we ran the algorithm several times with different initial populations. Furthermore, to get more solutions, various parameters of evolutionary operators were used that were available in the framework. Finally, all solutions were combined, dominated solutions were deleted, and 1503 nondominated solutions were obtained that approximate Pareto optimal solutions.

The DM started the navigation process by investigating the reachable ranges for the first step, which were represented by the ideal point and the nadir point initially derived from the set. With the bounds defined by the DM, the ideal point was $\boldsymbol{z}^{\star} = (358\ 884.48,\ 1\ 000,\ 674.73,\ 0.9945,\ 0,\ 97.45)$ and the nadir point was $\boldsymbol{z}^{nad} = (367\ 637.76,\ 6\ 800,\ 4\ 782.04,\ 0.5,\ 0.5,\ 10.19)$ (remember that the fourth and sixth objectives are to be maximized and the others are to be minimized). Initially, the DM wanted to set the ideal point as the reference point to investigate how the navigation ran and which Pareto optimal solutions can be found if he wanted all the objectives to navigate towards their best values.

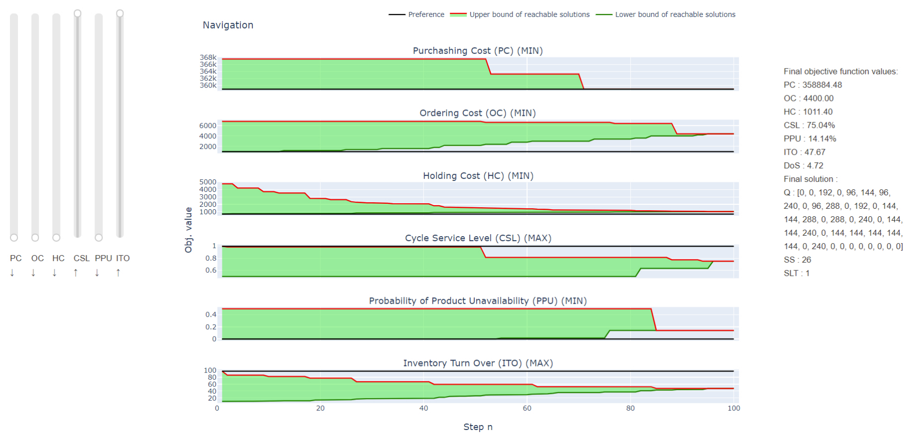

Fig. 2. A Pareto optimal solution for the ideal point as the reference point

Because of the trade-offs among the objectives, getting the best possible values for all objectives is naturally impossible, but, the DM navigated till the Pareto optimal solution $\boldsymbol{z} = (358\ 884.48,\ 4\ 400,\ 1\ 011.40,\ 0.7504,\ 0.1414,\ 47.67)$ was reached. Thus, the reachable range was finally a single point. Figure 2 shows this navigation. The DM analyzed that, in step 52, there was a significant decrease of the upper bound for the reachable CSL values to 0.8116, and the ITO reachable range shrunk with the upper bound 59.53. Because of this, the DM decided to go backwards to step 50 and provided new preferences.

The DM wanted to keep the ITO in the best value at this step, which was 59.53. He then set the components of the reference point for PC and OC to their worst values, and keep the other components as their best reachable values at this step. Therefore, the new reference point was (367 637.76, 6 800, 901.98, 0.9835, 0, 59.53). He let the navigation continue until the end to check the Pareto optimal solution that could be reached. The Pareto optimal solution obtained was $\boldsymbol{z} = (363\ 261.12,\ 6\ 000,\ 1\ 108.19,\ 0.8437,\ 0.0159,\ 39.25)$. He found the CSL value better but it was not satisfactory enough for him. He learned that the upper bound of the CSL's reachable values started to decrease at step 80. He then decided to return to this step to set a new reference point.

The DM navigated with different desired values of ITO to observe how much he needed to sacrifice in ITO to get better values for CSL. He returned to step 80 a few times with different desired values for ITO, but he only got 0.9041 as the best value for CSL. He decided to go further backwards to step 16 because the upper bound of ITO and HC in reachable values had a significant decrease after this step. He set all cost objectives in their worst reachable values, CSL and PPU in their best reachable values, and ITO=48. He let the reachable ranges shrink till the Pareto optimal solution $\boldsymbol{z} = (363\,261.12, 6\,400, 1\,183.94, 0.9366, 0.0159, 35.68)$. The DM found that the CSL value was not satisfactory enough.

The DM realized that CSL had a trade-off with PPU, and he needed to relax PPU to get better CSL. He decided to return to step 75 when the CSL decreased. He then relaxed the ITO value to the worst reachable value, and got the Pareto optimal solution $\boldsymbol{z} = (363\,261.12, 5\,800, 1\,066.10, 0.9272, 0.1414, 42.69)$. He was happy with the improvement of ITO but was still curious to find a better CSL value.

The DM wanted to investigate how much he needed to sacrifice in ITO when he desired to improve CSL. He then decided to go to the very first step and set his preferences at the best reachable value for CSL and the worst reachable values for costs and PPU. For ITO, he set 40 as the desired level. He let the navigation converge to a single solution. He got the best CSL value and the Pareto optimal solution was $\boldsymbol{z} = (367\,637.76, 5\,800, 1\,061.90, 0.9945, 0.5, 42.94)$. He was very happy with this solution. He thought that the CSL value was very good and the other objective values were acceptable. He decided to accept this solution as the final one.

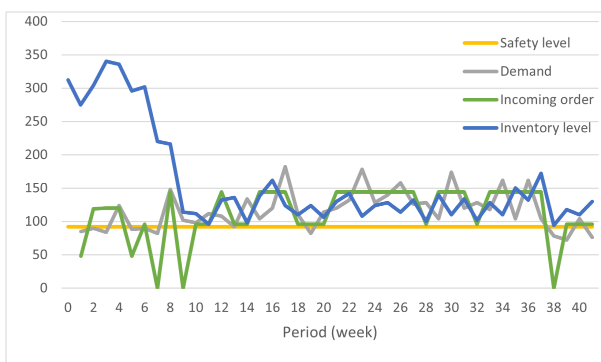

Fig. 3. The decision variables corresponding to the final solution

The decision variables corresponding to the final solution for order quantities can be seen in Figure 3. The other decision variables were $SS = 92$ and $SLT = 0$. The green line in Figure 3 shows the incoming order quantities for each week, which are the previously set order data for $t = 1, ..., 6$ and the optimized order quantities $Q(t-L)$ for $t = 7, ..., 41$. The inventory level in the blue line shows that during the first six weeks, which cannot be controlled by the model due to the lead time, the company had excess inventory. The inventory level then decreased and followed the demand quantity to have a higher ITO, which is a useful indicator for inventory management and planning purposes.

By deepening his understanding of the interdependencies between conflicting objectives, the DM learned a lot from his own area of responsibility as a supply chain manager and also gained the confidence to modify his original preferences. At first, he was not willing to sacrifice on any objectives, but during the decision making process, there was a growing awareness that not everything can be achieved, but sacrifices have to be made. These included, among other things, the CSL and ITO. However, in his day-to-day operations, ITO is a goal set by the company's top management. Therefore, deviating from this objective must be strongly justified to the management.

As a result of the learning process, the DM gained confidence in setting his preferences, and thus multiobjective optimization and NAUTILUS Navigator supported his understanding and ability to justify his decisions. The DM greatly appreciated the fact that as the decision making process progressed, he constantly saw the navigator's results and understanding of achieving objectives, which guided him in setting his preferences. The possibility to stop the process at any time and the feature to go backwards in the navigator, were, in his view, excellent opportunities to make decisions easily. The GUI of the navigation and the real-time updating of the results also supported his decision making. The navigator graphs and the sliders for setting the reference point were, in the DM's view, a clear advantage in support of decision making. The whole process was so instructive and professionally useful.

As can be seen in Figure 3, the inventory level was significantly reduced from its original level. The DM commented that this is a typical example of decisions being made in the past "for the sake of certainty", where typically stock levels tend to rise. NAUTILUS Navigator as a method responded precisely to the need for decisions to be based on calculations rather than assumptions. The DM was pleased with the result of the objective function values, as well as the corresponding decision variables. Overall, the DM was satisfied with the results and operation of NAUTILUS Navigator and found an interactive method very suitable for learning. He is willing to adopt the method more widely for inventory planning and control, especially for critical items.

5 Conclusions

In this paper, we considered a single item multi period lot sizing problem in a periodic review policy under a stochastic environment on demand and lead time. We used a SS to handle uncertainty on demand and CSL to measure the quality of SS. To handle uncertainty on lead time, a SLT was used and we proposed the PPA formula to measure the quality of SLT. The aim of this paper was to integrate the lot sizing problem with the problem of determining the optimal values of SS and SLT. We developed a multiobjective optimization model to solve the integrated lot sizing problem. Six objectives were optimized simultaneously to find the optimal order quantity in each period and at the same time determine the optimal values of SS and SLT.

Real data from a manufacturing company was used to demonstrate the applicability and usefulness of the proposed model. A supply chain manager from the said company acted as the DM to draw managerial insights into the decision making process. The interactive NAUTILUS Navigator method was successfully applied to solve our integrated computationally expensive lot sizing problem. The DM appreciated the navigation process that allowed him to learn during the decision making process and find the most satisfying solution for him. He confirmed the validity of the solution and found it useful for his daily operation.

For future research, considering many items would present more computational challenges but meet the needs of real industrial problems. A company may have thousands of items that are impossible to consider separately. Another possible future research topic is to address the variation of price based on the order quantity, or integrating the model with the problem of determining minimum order quantity and rounding value.

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