# Termination Analysis for the $\pi$-Calculus by Reduction to Sequential Program Termination 

 Ryosuke Sato ${ }^{1}$ © , and Takeshi Tsukada ${ }^{2}$ ©<br>${ }^{1}$ The University of Tokyo, Japan<br>${ }^{2}$ Chiba University, Japan


#### Abstract

We propose an automated method for proving termination of $\pi$-calculus processes, based on a reduction to termination of sequential programs: we translate a $\pi$-calculus process to a sequential program, so that the termination of the latter implies that of the former. We can then use an off-the-shelf termination verification tool to check termination of the sequential program. Our approach has been partially inspired by Deng and Sangiorgi's termination analysis for the $\pi$-calculus, and checks that there is no infinite chain of communications on replicated input channels, by converting such a chain of communications to a chain of recursive function calls in the target sequential program. We have implemented an automated tool based on the proposed method and confirmed its effectiveness.


## 1 Introduction

We propose a fully automated method for proving termination of $\pi$-calculus processes. Although there have been a lot of studies on termination analysis for the $\pi$-calculus and related calculi $[13,11,26,19,29,12,28]$, most of them have been rather theoretical, and there have been surprisingly little efforts in developing fully automated termination verification methods and tools based on them. To our knowledge, Kobayashi's TyPiCal [18,19] is the only exception that can prove termination of $\pi$-calculus processes (extended with natural numbers) fully automatically, but its termination analysis is quite limited (see Section 6).

Our method is based on a reduction to termination analysis for sequential programs: we translate a $\pi$-calculus process $P$ to a sequential program $S_{P}$, so that if $S_{P}$ is terminating, so is $P$. The reduction allows us to use powerful, mature methods and tools for termination analysis of sequential programs [17,14,24,21,7].

The idea of the translation is to convert a chain of communications on replicated input channels to a chain of recursive function calls of the target sequential program. Let us consider the following Fibonacci process:

```
* fib? \((n, r)\).if \(n<2\) then \(r!(1)\)
    else \(\left(\nu s_{1}\right)\left(\nu s_{2}\right)\left(f i b!\left(n-1, s_{1}\right)\left|f i b!\left(n-2, s_{2}\right)\right| s_{1} ?(x) \cdot s_{2} ?(y) . r!(x+y)\right)\)
| \(f i b!(m, r)\)
```

Here, the process $* f i b ?(n, r) \ldots$ is a function server that computes the $n$-th Fibonacci number in parallel and returns the result to $r$, and $f i b!(m ; r)$ sends a request for computing the $m$-th Fibonacci number; those who are not familiar with the syntax of the $\pi$-calculus may wish to consult Section 2 first. To prove that the process above is terminating for any integer $m$, it suffices to show that there is no infinite chain of communications on $f i b$ :

$$
f i b(m, r) \rightarrow f i b\left(m_{1}, r_{1}\right) \rightarrow f i b\left(m_{2}, r_{2}\right) \rightarrow \cdots
$$

We convert the process above to the following program: ${ }^{3}$

```
let rec fib(n) = if n<2 then () else (fib(n-1) [] fib(n-2)) in
fib(m)
```

Here, [] represents the non-deterministic choice. Note that, although the calculation of Fibonacci numbers is not preserved, for each chain of communications on fib , there is a corresponding sequence of recursive calls:

$$
\mathrm{fib}(m) \rightarrow \mathrm{fib}\left(m_{1}\right) \rightarrow \mathrm{fib}\left(m_{2}\right) \rightarrow \cdots
$$

Thus, the termination of the sequential program above implies the termination of the original process. As shown in the example above, (i) each communication on a replicated input channel is converted to a function call, (ii) each communication on a non-replicated input channel is just removed (or, in the actual translation, replaced by a call of a trivial function defined by $f(\tilde{x})=())$, and (iii) parallel composition is replaced by a non-deterministic choice. We formalize the translation outlined above and prove its correctness.

The basic translation sketched above sometimes loses too much information. For example, consider the following process:

$$
\begin{aligned}
& * \operatorname{pred} ?(n, r) . r!(n-1) \\
& \mid * f ?(n, r) . \text { if } n<0 \text { then } r!(1) \text { else }(\nu s)(\operatorname{pred}!(n, s) \mid s ?(x) . f!(x, r)) \\
& \mid f!(m, r)
\end{aligned}
$$

The translation sketched above would yield:

```
let pred(n) = n-1 in
let rec f(n) = if n<0 then () else (pred(n) [] f(*)) in
f(m)
```

Here, * represents a non-deterministic integer: since we have removed the input $s ?(x)$, we do not have information about the value of $x$. As a result, the sequential program above is non-terminating, although the original process is terminating. To remedy this problem, we also refine the basic translation above by using a refinement type system for the $\pi$-calculus. Using the refinement type system, we can infer that the value of $x$ in the original process is less than $n$, so that we can refine the definition of $f$ to:

[^0]```
let rec f(n) = ... else (pred(n) [] let x=* in assume(x<n);f(x))
```

The target program is now terminating, from which we can deduce that the original process is also terminating. We have implemented an automated tool based on the refined translation above.

The contributions of this paper are summarized as follows.

- The formalization of the basic translation from the $\pi$-calculus (extended with integers) to sequential programs, and a proof of its correctness.
- The formalization of a refined translation based on a refinement type system.
- An implementation of the refined translation, including automated refinement type inference based on CHC solving, and experiments to evaluate the effectiveness of our method.

The rest of this paper is structured as follows. Section 2 introduces the source and target languages of our translation. Section 3 formalizes the basic translation, and proves its correctness. Section 4 refines the basic translation by using a refinement type system. Section 5 reports an implementation and experiments. Section 6 discusses related work, and Section 7 concludes the paper.

## 2 Source and Target Languages

This section introduces the source and target languages for our reduction. The source language is the polyadic $\pi$-calculus [22] extended with integers and conditional expressions, and the target language is a first-order functional language with non-determinism.

## $2.1 \pi$-Calculus

Syntax Below we assume a countable set of variables ranged over by $x, y, z, w, \ldots$ and write $\mathbb{Z}$ for the set of integers, ranged over by $i$. We write ? for (possibly empty) finite sequences; for example, $\tilde{x}$ abbreviates a sequence $x_{1}, \ldots, x_{n}$. We write len $(\tilde{x})$ for the length of $\tilde{x}$ and $\epsilon$ for the empty sequence.

The sets of processes and simple expressions, ranged over by $P$ and $v$ respectively, are defined inductively by:

$$
\begin{aligned}
& P \text { (processes) }::=\mathbf{0}|x!(\tilde{v} ; \tilde{w}) . P| x ?(\tilde{y} ; \tilde{z}) . P|* x ?(\tilde{y} ; \tilde{z}) \cdot P|\left(P_{1} \mid P_{2}\right) \mid(\nu x: \kappa) P \\
& \quad \mid \text { if } v \text { then } P_{1} \text { else } P_{2} \mid \text { let } \tilde{x}=\tilde{\star} \text { in } P \\
& v \text { (simple expressions) }::=x|i| \text { op }(\tilde{v})
\end{aligned}
$$

The syntax of processes on the first line is fairly standard, except that the values sent along each channel consist of two parts: $\tilde{v}$ for integers, and $\tilde{w}$ for channels; this is for the sake of technical convenience in presenting the translation to sequential programs. The process $\mathbf{0}$ denotes an inaction, $x!(\tilde{v} ; \tilde{w}) . P$ sends a tuple $(\tilde{v}, \tilde{w})$ along the channel $x$ and behaves like $P$, and the process $x ?(\tilde{y} ; \tilde{z}) . P$ receives a tuple $(\tilde{v}, \tilde{w})$ along the channel $x$, and behaves like $[\tilde{v} / \tilde{y}, \tilde{w} / \tilde{z}] P$. We often just
write $\tilde{v}$ for $\tilde{v} ; \epsilon$ or $\epsilon ; \tilde{v}$. The process $* x ?(\tilde{y} ; \tilde{z}) . P$ represents infinitely many copies of $x ?(\tilde{y} ; \tilde{z}) . P$ running in parallel. The process $P_{1} \mid P_{2}$ runs $P_{1}$ and $P_{2}$ in parallel, and $(\nu x: \kappa) P$ creates a fresh channel $x$ of type $\kappa$ (where types will be introduced shortly) and behaves like $P$. The process if $v$ then $P_{1}$ else $P_{2}$ executes $P_{1}$ if the value of $v$ is non-zero, and $P_{2}$ otherwise. The process let $\tilde{x}=\tilde{\star}$ in $P$ instantiates the variables $\tilde{x}$ to some integer values in a non-deterministic manner, and then behaves like $P$. The meta-variable op ranges over integer operations such as + or $\leq$.

The free and bound variables are defined as usual. The only binders are $(\nu x: \kappa)($ which binds $x)$, let $\tilde{x}=\tilde{\star}$ in (which binds $\tilde{x}), x ?(\tilde{y} ; \tilde{z})$. and $* x ?(\tilde{y} ; \tilde{z})$. (which bind $\tilde{y}$ and $\tilde{z}$ ). Processes are identified up to renaming of bound variables, and we implicitly apply $\alpha$-conversions as necessary.

We write $P \rightarrow Q$ for the standard one-step reduction relation on processes. The base cases of the communication are given by:

$$
\begin{aligned}
x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\left|x!(\tilde{v} ; \tilde{w}) \cdot P_{2} \rightarrow[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1}\right| P_{2} \\
* x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\left|x!(\tilde{v} ; \tilde{w}) \cdot P_{2} \rightarrow * x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\right|[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1} \mid P_{2}
\end{aligned}
$$

provided that $\tilde{v}$ evaluates to $\tilde{i}$. The full definition is given in Appendix A. We say that a process $P$ is terminating if there is no infinite reduction sequence $P \rightarrow P_{1} \rightarrow P_{2} \rightarrow \cdots$.

In the rest of the paper, we consider only well-typed processes. We write $\iota$ for the type of integers. The set of channel types, ranged over by $\kappa$, is given by:

$$
\kappa::=\operatorname{ch}_{\rho}(\tilde{\imath} ; \tilde{\kappa})
$$

The type $\boldsymbol{\operatorname { c h }}_{\rho}(\tilde{\imath} ; \tilde{\kappa})$ describes channels used for transmitting a tuple $(\tilde{v} ; \tilde{w})$ of integers $\tilde{v}$ and channels $\tilde{w}$ of types $\tilde{\kappa}$. Below we will just write $\tilde{\iota}$ for $\tilde{\iota} ; \epsilon$ and $\tilde{\kappa}$ for $\epsilon ; \tilde{\kappa}$. The subscript $\rho$, called a region, is a symbol that abstracts channels; it is used in the translation to sequential programs. For example, $\boldsymbol{c h}_{\rho_{1}}\left(\iota ; \mathbf{c h}_{\rho_{2}}(\iota)\right)$ is the type of channels that belong to the region $\rho_{1}$ and are used for transmitting a pair $(i, r)$ where $r$ is a channel of region $\rho_{2}$ used for transmitting integers. We use a meta-variable $\sigma$ for an integer or channel type.

Type judgments for processes and simple expressions are of the form $\Gamma ; \Delta \vdash P$ and $\Gamma ; \Delta \vdash v: \sigma$, where $\Gamma$ and $\Delta$ are sequences of bindings of the form $x: \iota$ and $x: \kappa$, respectively. The typing rules are shown in Figure 1. Here $\Gamma ; \Delta \vdash \tilde{v}: \tilde{\sigma}$ means $\Gamma ; \Delta \vdash v_{i}: \sigma_{i}$ holds for each $i \in\{1, \ldots$, len $(\tilde{v})\}$. We omit the explanation of the typing rules as they are standard.

### 2.2 Sequential Language

We define the target language of our translation, which is a first-order functional language with non-determinism.

$$
\begin{aligned}
& \frac{\Gamma ; \Delta \vdash \mathbf{0}}{\Gamma ; \Delta \vdash v: \iota \quad \Gamma ; \Delta \vdash P_{1} \quad \Gamma ; \Delta \vdash P_{2}} \begin{array}{r:|}
\Gamma ; \text { if } v \text { then } P_{1} \text { else } P_{2}
\end{array} \\
& \frac{\Gamma ; \Delta \vdash P_{1} \quad \Gamma ; \Delta \vdash P_{2}}{\Gamma ; \Delta \vdash P_{1} \mid P_{2}} \quad \frac{\Gamma ; \Delta, x: \kappa \vdash P}{\Gamma ; \Delta \vdash(\nu x: \kappa) P} \quad \frac{\Gamma, \tilde{x}: \tilde{c} ; \Delta \vdash P}{\Gamma ; \Delta \vdash \operatorname{let} \tilde{x}=\tilde{\star} \text { in } P} \\
& \frac{\Gamma ; \Delta \vdash x: \mathbf{c h}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P}{\Gamma ; \Delta \vdash x ?(\tilde{y} ; \tilde{z}) . P} \\
& \frac{\Gamma ; \Delta \vdash x: \boldsymbol{\operatorname { c h }}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma ; \Delta \vdash \tilde{v}: \tilde{\iota} \quad \Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa} \quad \Gamma ; \Delta \vdash P}{\Gamma ; \Delta \vdash x!(\tilde{v} ; \tilde{w}) . P} \\
& \frac{\Gamma ; \Delta \vdash x: \boldsymbol{c h}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P}{\Gamma ; \Delta \vdash * x ?(\tilde{y} ; \tilde{z}) . P} \\
& \frac{x: \iota \in \Gamma}{\Gamma ; \Delta \vdash x: \iota} \quad \frac{x: \kappa \in \Delta}{\Gamma ; \Delta \vdash x: \kappa} \quad \overline{\Gamma ; \Delta \vdash i: \iota} \quad \frac{\Gamma ; \Delta \vdash \tilde{v}: \tilde{\iota}}{\Gamma ; \Delta \vdash o p(\tilde{v}): \iota}
\end{aligned}
$$

Fig. 1. The typing rules of the simple type system for the $\pi$-calculus

A program is a pair $(\mathcal{D}, E)$ consisting of (a set of) function definitions $\mathcal{D}$ and an expression $E$, defined by:

$$
\begin{aligned}
& \mathcal{D} \text { (function definitions) }::=\left\{f_{1}\left(\tilde{x}_{1}\right)=E_{1}, \ldots, f_{n}\left(\tilde{x}_{n}\right)=E_{n}\right\} \\
& E \text { (expression) }::=() \mid \text { let } \tilde{x}=\tilde{\star} \text { in } E|f(\tilde{v})| E_{1} \oplus E_{2} \\
& \mid \text { if } v \text { then } E_{1} \text { else } E_{2} \mid \operatorname{Assume}(v) ; E \\
& v \text { (simple expressions) }::= x|i| \text { op }(\tilde{v})
\end{aligned}
$$

In a function definition $f_{i}\left(x_{1}, \ldots, x_{k_{i}}\right)=E_{i}$, the variables $x_{1}, \ldots, x_{k_{i}}$ are bound in $E_{i}$; we identify function definitions up to renaming of bound variables, and implicitly apply $\alpha$-conversions. The function names $f_{1}, \ldots, f_{n}$ need not be distinct from each other. If there are more than one definition for $f$, then one of the definitions will be non-deterministically used when $f$ is called. We explain the informal meanings of the nonstandard expressions. The expression let $\tilde{x}=\tilde{\star}$ in $E$ instantiates $\tilde{x}$ to some integers in a non-deterministic manner. The expression $E_{1} \oplus E_{2}$ non-deterministically evaluates to $E_{1}$ or $E_{2}$. The expression Assume $(v) ; E$ evaluates to $E$ if $v$ is non-zero; otherwise the whole program is aborted. The other expressions are standard and their meanings should be clear.

We write $(\mathcal{D}, E) \rightsquigarrow\left(\mathcal{D}, E^{\prime}\right)$ for the one-step reduction relation, whose definition is given in Appendix A. We say that a program is terminating if there is no infinite reduction sequence.

## 3 Basic Transformation

This section presents our transformation from a $\pi$-calculus process to a sequential program, so that if the transformed program is terminating then the original process is terminating.

Table 1. Correspondence between processes and sequential programs

| processes | sequential programs |
| :--- | :--- |
| replicated input $(* x ?(\tilde{y} ; \tilde{z}) . \cdots)$ | function definition $f_{\rho_{x}}(\tilde{y})=\cdots$ |
| non-replicated input $(x ?(\tilde{y} ; \tilde{z}) . \cdots)$ | non-deterministic instantiation (let $\tilde{y}=\tilde{\star}$ in $\cdots)$ |
| output $(x!(\tilde{v} ; \tilde{w}) . \cdots)$ | function call $\left(f_{\rho_{x}}(\tilde{v}) \oplus \cdots\right)$ |
| parallel composition $(\cdots \mid \cdots)$ | non-deterministic choice $(\cdots \oplus \cdots)$ |

As explained in Section 1, the idea is to transform an infinite chain of message passing on replicated input channels to an infinite chain of recursive function calls. Table 1 summarizes the correspondence between processes and sequential programs. As shown in the table, a replicated input process is transformed to a function definition, whereas a non-replicated input process is just ignored, and integer bound variables are non-deterministically instantiated. Note that channel arguments $\tilde{z}$ are ignored in both cases. Instead, we prepare a global function name $f_{\rho}$ for each region $\rho ; \rho_{x}$ in the table indicates the region assigned to the channel type of $x .^{4}$

We define the transformation relation $\Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$, which means that the $\pi$-calculus process $P$ well-typed under $\Gamma ; \Delta$ is transformed to the sequential program $(\mathcal{D}, E)$. The relation is defined by the rules in Figure 2.

We explain some key rules. In SX-NiL, $\mathbf{0}$ is translated to $(\mathcal{D},())$, where $\mathcal{D}$ is the set of trivial function definitions. In SX-In, a (non-replicated) input is just removed, and the bound variables are instantiated to non-deterministic integers; this is because we have no information about $\tilde{y}$; this will be refined in Section 4. In contrast, in SX-RIn, a replicated input is converted to a function definition. Since $\mathcal{D}$ generated from $P$ may contain $\tilde{y}$, they are bound to non-deterministic integers and merged with the new definition for $f_{\rho}$. In SX-Out, an output is replaced by a function call. In SX-PAR, parallel composition is replaced by nondeterministic choice.

Example 1. Let us revisit the Fibonacci example used in the introduction to explain the actual translation. Using the syntax we introduced, the Fibonacci process $P_{\text {fib }}$ can now be defined as:

$$
\begin{aligned}
& \left(\nu f i b: \operatorname{ch}_{\rho_{1}}\left(\iota ; \boldsymbol{\operatorname { c h }}_{\rho_{2}}(\iota)\right)\right) * f i b ?(n ; r) . \\
& \text { if } n<2 \text { then } r!(1) \text { else }\left(\nu r_{1}: \operatorname{ch}_{\rho_{2}}(\iota)\right)\left(\nu r_{2}: \operatorname{ch}_{\rho_{2}}(\iota)\right) \\
& \left(f i b!\left(n-1 ; r_{1}\right)\left|f i b!\left(n-2 ; r_{2}\right)\right| r_{1} ?(x) \cdot r_{2} ?(y) \cdot r!(x+y)\right) \\
& \mid \text { let } m=\star \text { in }\left(\nu r: \operatorname{ch}_{\rho_{2}}(\iota)\right) f i b!(m ; r)
\end{aligned}
$$

[^1]\[

$$
\begin{align*}
& \overline{\Gamma ; \Delta \vdash \mathbf{0} \Rightarrow\left(\left\{f_{\rho}(\tilde{y})=() \mid x: \mathbf{c h}_{\rho}(\tilde{\imath} ; \tilde{\kappa}) \in \Delta, \operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{\iota})\right\},()\right)} \\
& \frac{\Gamma ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P \Rightarrow(\mathcal{D}, E)}{\Gamma ; \Delta \vdash x ?(\tilde{y} ; \tilde{z}) . P \Rightarrow(\operatorname{let} \tilde{y}=\tilde{\star} \text { in } \mathcal{D}, \text { let } \tilde{y}=\tilde{\star} \operatorname{in} E)}  \tag{SX-In}\\
& \frac{\Gamma ; \Delta \vdash x: \boldsymbol{c h}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P \Rightarrow(\mathcal{D}, E)}{\Gamma ; \Delta \vdash * x ?(\tilde{y} ; \tilde{z}) \cdot P \Rightarrow\left(\left\{f_{\rho}(\tilde{y})=E\right\} \cup(\operatorname{let} \tilde{y}=\tilde{\star} \operatorname{in} \mathcal{D}),()\right)}  \tag{SX-RIn}\\
& \frac{\Gamma ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma ; \Delta \vdash \tilde{v}: \tilde{\imath} \quad \Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa} \quad \Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)}{\Gamma ; \Delta \vdash x!(\tilde{v} ; \tilde{w}) \cdot P \Rightarrow\left(\mathcal{D}, f_{\rho}(\tilde{v}) \oplus E\right)} \\
& \frac{\Gamma ; \Delta \vdash P_{1} \Rightarrow\left(\mathcal{D}_{1}, E_{1}\right) \quad \Gamma ; \Delta \vdash P_{2} \Rightarrow\left(\mathcal{D}_{2}, E_{2}\right)}{\Gamma ; \Delta \vdash P_{1} \mid P_{2} \Rightarrow\left(\mathcal{D}_{1} \cup \mathcal{D}_{2}, E_{1} \oplus E_{2}\right)}  \tag{SX-Out}\\
& \frac{\Gamma ; \Delta, x: \kappa \vdash P \Rightarrow(\mathcal{D}, E)}{\Gamma ; \Delta \vdash(\nu x: \kappa) P \Rightarrow(\mathcal{D}, E)}  \tag{SX-Nu}\\
& \frac{\Gamma ; \Delta \vdash v: \iota \quad \Gamma ; \Delta \vdash P_{1} \Rightarrow\left(\mathcal{D}_{1}, E_{1}\right) \quad \Gamma ; \Delta \vdash P_{2} \Rightarrow\left(\mathcal{D}_{2}, E_{2}\right)}{\Gamma ; \Delta \vdash \text { if } v \text { then } P_{1} \text { else } P_{2} \Rightarrow\left(\mathcal{D}_{1} \cup \mathcal{D}_{2}, \text { if } v \text { then } E_{1} \text { else } E_{2}\right)}  \tag{SX-IF}\\
& \frac{\Gamma, \tilde{x}: \tilde{\iota} ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)}{\Gamma ; \Delta \vdash \text { let } \tilde{x}=\tilde{\star} \text { in } P \Rightarrow(\text { let } \tilde{x}=\tilde{\star} \text { in } \mathcal{D} \text {, let } \tilde{x}=\tilde{\star} \text { in } E)}  \tag{SX-LetND}\\
& \text { let } \tilde{x}=\tilde{\star} \text { in } \mathcal{D}:=\{f(\tilde{y})=(\text { let } \tilde{x}=\tilde{\star} \text { in } E) \mid f(\tilde{y})=E \in \mathcal{D}\}
\end{align*}
$$
\]

Fig. 2. The rules of simple type-based program transformation

Note that $(\nu f i b)$ and let $m=\star$ in have been added to close the process. We can derive $\emptyset ; \emptyset \vdash P_{\text {fib }} \Rightarrow(\mathcal{D}, E)$, where $\mathcal{D}$ and $E$ are given as follows: ${ }^{5}$

$$
\begin{aligned}
\mathcal{D}= & \left\{f _ { \rho _ { 1 } } ( z ) = \text { if } z < 2 \text { then } f _ { \rho _ { 2 } } ( 1 ) \text { else } \left(f_{\rho_{1}}(z-1) \oplus f_{\rho_{1}}(z-2)\right.\right. \\
& \left.\oplus \text { let } x=\star \text { in let } y=\star \text { in } f_{\rho_{2}}(x+y)\right) \\
& \left.f_{\rho_{2}}(z)=()\right\} \\
E= & \text { let } m=\star \text { in } f_{\rho_{1}}(m)
\end{aligned}
$$

Here $f_{\rho_{1}}$ is the "Fibonacci function" because $\rho_{1}$ is the region assigned to the channel $f i b$ in $P_{\text {fib }}$. The function call $f_{\rho_{2}}(x+y)$ corresponds to the output $r!(x+y)$; the argument of the function call is actually a nondeterministic integer because $r ?(x)$ and $r ?(y)$ are translated to non-deterministic instantiations. Since the program $(\mathcal{D}, E)$ is terminating, we can verify that $P_{\text {fib }}$ is also terminating.

[^2]Example 2. To help readers understand the rule SX-RIn, we consider the following process, which contains a nested input:

$$
* f ?(x ; r) \cdot * g ?(y, z) .(\text { if } y \leq 0 \text { then } r!(z) \text { else } g!(y-1, x+z)) \mid f!(2 ; r) \cdot g!(3,0)
$$

where $f: \boldsymbol{c h}_{\rho_{1}}\left(\iota ; \boldsymbol{c h}_{\rho_{2}}(\iota)\right)$ and $g: \boldsymbol{\operatorname { c h }}_{\rho_{3}}(\iota, \iota)$. This process computes $x * y+z$ (which is 6 in this case) and returns that value using $r$. This program is translated to:

$$
\begin{aligned}
& f_{\rho_{1}}(x)=() \quad f_{\rho_{2}}(z)=() \\
& f_{\rho_{3}}(y, z)=\text { let } x=\star \text { in if } y \leq 0 \text { then } f_{\rho_{2}}(z) \text { else } f_{\rho_{3}}(y-1, x+z)
\end{aligned}
$$

with the main expression $f_{\rho_{1}}(2) \oplus f_{\rho_{3}}(3,0)$. Note that the body of $f_{\rho_{1}}$, which is the function corresponding to $f$, is (). This is because when the rule SX-RIN is applied to $* g ?(y, z) \ldots$, the main expression of the translated program becomes (). Observe that the function definition for $f_{\rho_{3}}$ still contains a free variable $x$ at this moment. Then $f_{\rho_{3}}$ is closed by let $x=\star$ in when we apply the rule SX-RIn to $* f ?(x ; r) \ldots$. We can check that the above program is terminating, and thus we can verify that the original process is terminating. Note that some precision is lost in the application of SX-RIn above since we cannot track the relation between the argument of $f_{\rho_{1}}$ and the value of $x$ used inside $f_{\rho_{3}}$. This loss causes a problem if, for example, the condition $y \leq 0$ in the process above is replaced with $y \leq x$. The body of $f_{\rho_{3}}$ would then become let $x=\star$ in if $y \leq x \cdots$, hence the sequential program would be non-terminating.

Remark 1. A reader may wonder why a non-replicated input is removed in SX-In, rather than translated to a function definition as done for a replicated input. It is actually possible to obtain a sound transformation even if we treat nonreplicated inputs in the same manner as replicated inputs, but we expect that our approach of removing non-replicated inputs often works better. For example, consider $x ?(y) \cdot x!(y) \mid x!(0)$. Our translation generates $\left(\left\{f_{\rho_{x}}(z)=()\right\},(\right.$ let $y=$ $\star$ in $\left.\left.f_{\rho_{x}}(y)\right) \oplus f_{\rho_{x}}(0)\right)$ which is terminating, whereas if we treat the input in the same way as a replicated input, we would obtain $\left(\left\{f_{\rho_{x}}(z)=f_{\rho_{x}}(z)\right\}, f_{\rho_{x}}(0)\right)$ which is not terminating. Our approach also has some defect. For example, consider $x!(0) \mid x ?(y)$.if $y=0$ then $\mathbf{0}$ else $\Omega$ where $\Omega$ is a diverging process. Our translation yields $\left(\left\{f_{\rho_{x}}(z)=()\right\}, f_{\rho_{x}}(0) \oplus\right.$ let $y=\star$ in if $y=0$ then () else $\Omega^{\prime}$ ) which is non-terminating. On the other hand, if we treat the input like a replicated input, we would obtain $\left(\left\{f_{\rho_{x}}(z)=\right.\right.$ if $z=0$ then () else $\left.\left.\Omega^{\prime}\right\}, f_{\rho_{x}}(0)\right)$ which is terminating. This issue can, however, be mitigated by the extension with refinement types in Section 4. Our choice of removing non-replicated inputs is also consistent with Deng and Sangiorgi's type system [13], which prevents an infinite chain of communications on replicated input channels by using types and ignores non-replicated inputs.

The following theorem states the soundness of our transformation.
Theorem 1 (soundness). Suppose $\emptyset ; \emptyset \vdash P \Rightarrow(\mathcal{D}, E)$. If $(\mathcal{D}, E)$ is terminating, then so is $P$.

We briefly explain the proof strategy; see Appendix B for the actual proof. Basically, our idea is to show that the translated program simulates the original process. Then we can conclude that if the original process is non-terminating then so is the sequential program. However, there is a slight mismatch between the reduction of a process and that of the sequential program that we need to overcome. Recall that $* f ?(x) . P|f!(1)| f!(2)$ is translated to $f_{\rho_{f}}(1) \oplus f_{\rho_{f}}(2)$ with a function definition for $f_{\rho_{f}}$. In the sequential program, we need to make a "choice", e.g. if $f_{\rho_{f}}(1)$ is called, we cannot call $f_{\rho_{f}}(2)$ anymore. On the other hand, the output $f!(2)$ can be used even if $f!(1)$ has been used before. To fill this gap, we introduce a non-standard reduction relation, which does not discard branches of non-deterministic choices and show the simulation relation using that non-standard semantics. Then we show that if there is an infinite non-standard reduction sequence, then there is an infinite subsequence that corresponds to a reduction along a certain choice of non-deterministic branches. This step is essentially a corollary of the König's Lemma. This is because the infinite nonstandard reduction sequence can be reformulated as an infinite tree in which branches correspond to non-deterministic choices $\oplus$ (thus the tree is finitely branching) and paths correspond to reduction sequences.

The following example indicates that the basic transformation is sometimes too conservative.

Example 3. Let us consider the following process $P_{\text {dec }}$ :

$$
\begin{aligned}
& * \operatorname{pred} ?(n ; r) . r!(n-1) \\
& \mid * f ?(n ; r) . \text { if } n<0 \text { then } r!(1) \text { else }\left(\nu s: \operatorname{ch}_{\rho_{2}}(\iota)\right)(\text { pred! }(n ; s) \mid s ?(x) . f!(x ; r)) \\
& \mid f!(m ; r)
\end{aligned}
$$

where pred: $\boldsymbol{c h}_{\rho_{1}}\left(\iota ; \mathbf{c h}_{\rho_{2}}(\iota)\right), f: \boldsymbol{\operatorname { c h }}_{\rho_{3}}\left(\iota ; \mathbf{c h}_{\rho_{4}}(\iota)\right)$ and $r: \mathbf{c h}_{\rho_{4}}(\iota)$. This process, which also appeared in the introduction, keeps on decrementing the integer $m$ until it gets negative and then returns 1 via $r$. We can turn this process into a closed process $P_{\text {dec }}^{\prime}$ by restricting the names pred, $f, r$ and adding let $m=\star$ in in front of the process. Note that $P_{\mathrm{dec}}^{\prime}$ is terminating.

The process $P_{\mathrm{dec}}^{\prime}$ is translated to:

$$
\begin{aligned}
f_{\rho_{1}}(n)= & f_{\rho_{2}}(n-1), \quad f_{\rho_{2}}(x)=(), \quad f_{\rho_{4}}(x)=(), \\
f_{\rho_{3}}(n)= & \text { if } n<0 \text { then } f_{\rho_{4}}(1) \\
& \quad \text { else }\left(f_{\rho_{1}}(n) \oplus \text { let } x=\star \text { in } f_{\rho_{3}}(x)\right)
\end{aligned}
$$

with the main expression let $m=\star$ in $f_{\rho_{3}}(m)$. Observe that the function $f_{\rho_{3}}$ is applied to a non-deterministic integer, not $n-1$. Thus, this program is not terminating, meaning that we fail to verify that the original process is terminating. This is due to the shortcoming of our transformation that all the integer values received by non-replicated inputs are replaced by non-deterministic integers. This problem is addressed in the next section.

## 4 Improving Transformation Using Refinement Types

In this section, we refine the basic transformation in the previous section by using a refinement type system.

Recall that in Example 3, the problem was that information about values received by non-replicated inputs was completely lost. By using a refinement type system for the $\pi$-calculus, we can statically infer that $x<n$ holds between $x$ and $n$ in the process in Example 3. Using that information, we can transform the process in Example 3 and obtain

$$
f_{\rho_{3}}(n)=\text { if } n<0 \text { then } \cdots \text { else }\left(f_{\rho_{1}}(n) \oplus \text { let } x=\star \text { in } \operatorname{Assume}(x<n) ; f_{\rho_{3}}(x)\right)
$$

for the definition of $f_{\rho_{3}}$. This is sufficient to conclude that the resulting program is terminating.

In the rest of this section, we first introduce a refinement type system in Section 4.1 and explain the refined transformation in Section 4.2. We then discuss how to automatically infer refinement types and achieve the refined transformation in Section 4.3.

### 4.1 Refinement Type System

The set of refinement channel types, ranged over by $\kappa$, is given by:

$$
\kappa::=\operatorname{ch}_{\rho}(\tilde{x} ; \phi ; \tilde{\kappa})
$$

Here, $\phi$ is a formula of integer arithmetic. We sometimes write just $\boldsymbol{c h}_{\rho}(\tilde{x} ; \phi)$ for $\boldsymbol{c h}_{\rho}(\tilde{x} ; \phi ; \epsilon)$. Intuitively, $\boldsymbol{\operatorname { c h }}_{\rho}(\tilde{x} ; \phi ; \tilde{\kappa})$ describes channels that are used for transmitting a tuple $(\tilde{x} ; \tilde{y})$ such that (i) $\tilde{x}$ are integers that satisfy $\phi$, and (ii) $\tilde{y}$ are channels of types $\tilde{\kappa}$. For example, the type $\boldsymbol{c h}_{\rho_{1}}\left(x ; \boldsymbol{\operatorname { t r u e }} ; \boldsymbol{c h}_{\rho_{2}}(z ; z<x)\right)$ describes channels used for transmitting a pair $(x, y)$, where $x$ may be any integer, and $y$ must be a channel of type $\mathbf{c h}_{\rho_{2}}(z ; z<x)$, i.e., a channel used for passing an integer $z$ smaller than $x$.Thus, if $u$ has type $\boldsymbol{c h}_{\rho_{1}}\left(x ;\right.$ true; $\left.\boldsymbol{c h}_{\rho_{2}}(z ; z<x)\right)$, then the process $u ?(n ; r) . r!(n-1)$ is allowed but $u ?(n ; r) . r!(n)$ is not.

Type judgments for processes and expressions are now of the form $\Gamma ; \Phi ; \Delta \vdash$ $P$ and $\Gamma ; \Phi ; \Delta \vdash v: \sigma$, where $\Phi$ is a sequence of formulas. Intuitively, $\Gamma ; \Phi ; \Delta \vdash P$ means that $P$ is well-typed under the environments $\Gamma$ and $\Delta$ assuming that all the formulas in $\Phi$ holds.

The selected typing rules are shown in Figure 3. The rules for the other constructs are identical to that of the simple type system; the complete list of typing rules appears in Appendix C. The rules shown in Figure 3 are fairly standard rules for refinement type systems. In RT-OUT, the notation $\Phi \vDash \phi$ means that $\phi$ is a logical consequence of $\Phi$; for example, $x<y, y<z \vDash x<z$ holds. In the typing rules, we implicitly require that all the type judgments are well-formed, in the sense that all the integer variables occurring in a formula is properly declared in $\Gamma$ or bound by a channel type constructor; see Appendix C for the well-formedness condition.

$$
\begin{aligned}
& \frac{\Gamma ; \Phi ; \Delta \vdash x: \boldsymbol{c h}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\imath} ; \Phi, \phi ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P}{\Gamma ; \Phi ; \Delta \vdash x ?(\tilde{y} ; \tilde{z}) . P}(\mathrm{RT}-\mathrm{IN}) \\
& \Gamma ; \Phi ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma ; \Phi ; \Delta \vdash \tilde{v}: \tilde{\imath} \quad \Phi \vDash[\tilde{v} / \tilde{y}] \phi \\
& \frac{\Gamma ; \Phi ; \Delta \vdash \tilde{w}:[\tilde{v} / \tilde{y}] \tilde{\kappa} \sqrt{\Gamma ; \Phi ; \Delta \vdash P}}{\Gamma ; \Phi ; \Delta \vdash x!(\tilde{v} ; \tilde{w}) . P} \text { (RT-OUT) } \\
& \frac{\Gamma ; \Phi ; \Delta \vdash x: \boldsymbol{\operatorname { c h }}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{c} ; \Phi, \phi ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P}{\Gamma ; \Phi ; \Delta \vdash * x ?(\tilde{y} ; \tilde{z}) . P}(\text { RT-RIn }) \\
& \frac{\Gamma ; \Phi ; \Delta \vdash v: \iota \quad \Gamma ; \Phi, v \neq 0 ; \Delta \vdash P_{1} \quad \Gamma ; \Phi, v=0 ; \Delta \vdash P_{2}}{\Gamma ; \Phi ; \Delta \vdash \text { if } v \text { then } P_{1} \text { else } P_{2}} \text { (RT-IF) } \\
& \frac{x: \kappa \in \Delta}{\Gamma ; \Phi ; \Delta \vdash x: \kappa}(\text { RT-VAR-CH })
\end{aligned}
$$

Fig. 3. Selected typing rules of the refinement type system for the $\pi$-calculus

### 4.2 Program Transformation

Based on the refinement type system above, we refine the transformation relation to $\Gamma ; \Phi ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$. The only change is in the following rule for nonreplicated inputs. ${ }^{6}$

$$
\begin{align*}
& \frac{\Gamma ; \Phi ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Phi, \phi ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P \Rightarrow(\mathcal{D}, E)}{\Gamma ; \Phi ; \Delta \vdash x ?(\tilde{y} ; \tilde{z}) \cdot P} \\
& \Rightarrow(\operatorname{let} \tilde{y}=\tilde{\star} \operatorname{in} \operatorname{Assume}(\phi) ; \mathcal{D}, \text { let } \tilde{y}=\tilde{\star} \text { in } \operatorname{Assume}(\phi) ; E) \tag{RX-In}
\end{align*}
$$

Here, we insert Assume $(\phi)$, based on the refinement type of $x$. The expression let $\tilde{y}=\tilde{\star}$ in Assume $(\phi) ; E$ first instantiates $\tilde{y}$ to some integers in a nondeterministic manner, but proceeds to evaluate $E$ only if the values of $\tilde{y}$ satisfy $\phi$. Thus, the termination analysis for the target sequential program may assume that $\tilde{y}$ satisfies $\phi$ in $E$.

Example 4. Let us explain how the process $P_{\text {dec }}$ introduced in Example 3 is translated by the refined translation. Recall that the following simple types were assigned to the channels:

$$
\text { pred }: \boldsymbol{\operatorname { c h }}_{\rho_{1}}\left(\iota ; \boldsymbol{\operatorname { c h }}_{\rho_{2}}(\iota)\right), \quad f: \boldsymbol{\operatorname { c h }}_{\rho_{3}}\left(\iota ; \boldsymbol{\operatorname { c h }}_{\rho_{4}}(\iota)\right), \quad r: \boldsymbol{\operatorname { c h }}_{\rho_{4}}(\iota), \quad s: \boldsymbol{\operatorname { c h }}_{\rho_{2}}(\iota) .
$$

By the refinement type system, the above types can be refined as:

$$
\begin{aligned}
& \text { pred }: \boldsymbol{\operatorname { c h }}_{\rho_{1}}\left(n ; \boldsymbol{\operatorname { t r u e }} ; \boldsymbol{\operatorname { c h }}_{\rho_{2}}(x ; x<n)\right), \quad f: \boldsymbol{\operatorname { c h }}_{\rho_{3}}\left(n ; \boldsymbol{\operatorname { t r u e }} ; \boldsymbol{\operatorname { c h }}_{\rho_{4}}(x ; \text { true })\right), \\
& r: \operatorname{ch}_{\rho_{4}}(x ; \text { true }), \quad s: \operatorname{ch}_{\rho_{2}}(x ; x<n) .
\end{aligned}
$$

For example, one can check that the output $r!(n-1)$ on the first line of $P_{\text {dec }}$ is well-typed because $\models[n-1 / x] x<n$ holds. Note that this $r$ is the variable bound by pred? $(n ; r)$ and thus has the type $\boldsymbol{c h}_{\rho_{2}}(x ; x<n)$.

[^3]Therefore, by the rule RX-In, the input $s ?(x) . f!(x ; r)$ is now translated as follows:

$$
\begin{aligned}
& \Gamma ; \Phi ; \Delta \vdash s: \operatorname{ch}_{\rho_{2}}(x ; x<n) \quad \Gamma, x: \iota ; \Phi, x<n ; \Delta \vdash f!(x ; r) \Rightarrow\left(\mathcal{D}, f_{\rho_{3}}(x)\right) \\
& \Gamma ; \Phi ; \Delta \vdash s ?(x) \cdot f!(x ; r) \\
& \Rightarrow\left((\text { let } x=\star \operatorname{in} \operatorname{Assume}(x<n) ; \mathcal{D}),\left(\text { let } x=\star \operatorname{in} \operatorname{Assume}(x<n) ; f_{\rho_{3}}(x)\right)\right)
\end{aligned}
$$

with suitable $\Gamma, \Phi$ and $\Delta$. By translating the whole process, we obtain

$$
\begin{aligned}
f_{\rho_{3}}(n)= & \text { if } n<0 \text { then } f_{\rho_{4}}(1) \\
& \text { else }\left(f_{\rho_{1}}(n) \oplus \text { let } x=\star \text { in Assume }(x<n) ; f_{\rho_{3}}(x)\right)
\end{aligned}
$$

as desired. The other function definitions are given as in the case of Example 3 (except for the fact that some redundant assertions let $x=\star$ in $\operatorname{Assume}(x<$ $n)$ are added).

The soundness of the refined translation is obtained from the following argument. We first extend the $\pi$-calculus with the Assume statement. Then the refined translation can be decomposed into the following two steps: (a) given a $\pi$-calculus process $P$, insert Assume statements based on refinement types and obtain a process $P^{\prime}$; and (b) apply the translation of Section 3 to $P^{\prime}$ (where Assume is just mapped to itself) and obtain a sequential program $S$. The soundness of step (b) follows by an easy modification of the proof in Appendix B for the basic transformation (just add the case for Assume). So, the termination of $S$ would imply that of $P^{\prime}$. Now, from the soundness of the refinement type system (which follows from a standard argument on type preservation and progress), it follows that the Assume statements inserted in step (a) always succeed. Thus, the termination of $P^{\prime}$ would imply that of $P$. We can, therefore, conclude that if $S$ is terminating, so is $P$.

### 4.3 Type Inference

This section discusses how to infer refinement types automatically to automatically achieve the transformation. As in refinement type inference for functional programs $[25,27,5]$, we can reduce refinement type inference for the $\pi$-calculus to the problem of CHC (Constrained Horn Clauses) solving [4].

We explain the procedure through an example. Once again, we use the process $P_{\text {dec }}$ introduced in Example 3. We first perform type inference for the simple type system in Section 2, and (as we have seen) obtain the following simple types for pred and $f$ :

$$
\text { pred }: \boldsymbol{\operatorname { c h }}_{\rho_{1}}\left(\iota ; \boldsymbol{\operatorname { c h }}_{\rho_{2}}(\iota)\right), \quad f: \boldsymbol{\operatorname { c h }}_{\rho_{3}}\left(\iota ; \boldsymbol{\operatorname { c h }}_{\rho_{4}}(\iota)\right)
$$

Here, we have omitted the types for other (bound) channels $r, s, y$, as they can be determined based on those of pred and $f$. Based on the simple types, we prepare the following templates for refinement types.

$$
\text { pred : } \boldsymbol{c h}_{\rho_{1}}\left(n ; P_{1}(n) ; \boldsymbol{c h}_{\rho_{2}}\left(x ; P_{2}(n, x)\right)\right), \quad f: \boldsymbol{\operatorname { c h }}_{\rho_{3}}\left(n ; P_{3}(n) ; \boldsymbol{c h}_{\rho_{4}}\left(x ; P_{4}(n, x)\right)\right) .
$$

Here, $P_{i}(i \in\{1, \ldots, 4\})$ is a predicate variable that represents unknown conditions.

Based on the refinement type system, we can generate the following constraints on the predicate variables.

$$
\begin{aligned}
& \forall n \cdot\left(P_{1}(n) \Longrightarrow P_{2}(n, n-1)\right) \quad \forall n \cdot\left(P_{3}(n) \wedge n<0 \Longrightarrow P_{4}(n, 1)\right) \\
& \forall n \cdot\left(P_{3}(n) \wedge n \geq 0 \Longrightarrow P_{1}(n-1)\right) \\
& \forall n, x \cdot\left(P_{3}(n) \wedge n \geq 0 \wedge P_{2}(n-1, x) \Longrightarrow P_{3}(x)\right) \\
& \forall m \cdot\left(\text { true } \Longrightarrow P_{3}(m)\right)
\end{aligned}
$$

Here, the first constraint comes from the first line of the process, and the second constraint (the third and fourth constraints, resp.) comes from the then-part (the else-part, resp.) of the second line of the process. The last constraint comes from $f!(m ; r)$.

The generated constraints are in general a set of Constrained Horn Clauses (CHCs) [4] of the form $\forall \tilde{x} .\left(P_{1}\left(\tilde{v}_{1}\right) \wedge \cdots \wedge P_{k}\left(\tilde{v}_{k}\right) \wedge \phi \Longrightarrow H\right)$, where $P_{1}, \ldots, P_{k}$ are predicate variables, $\phi$ is a formula of integer arithmetic (without predicate variables), and $H$ is either of the form $P(\tilde{v})$ or $\phi^{\prime}$. The problem of finding a solution (i.e. an assignment of predicates to predicate variables) of a set of CHCs is undecidable in general, but there are various automated tools (called CHC solvers) for solving the problem [20,5]. Thus, by using such a CHC solver, we can solve the constraints on predicate variables, and obtain refinement types by substituting the solution for the templates of refinement types.

For the example above, the following is a solution.

$$
P_{1}(n) \equiv \text { true } \quad P_{2}(n, x) \equiv x<n \quad P_{3}(x) \equiv \text { true } \quad P_{4}(n, x) \equiv \text { true } .
$$

This is exactly the predicates we used in Example 4 to translate $P_{\text {dec }}$ using the refined approach.

Adding extra CHCs. Actually, a further twist is necessary in the step of CHC solving. As in the example above, all the CHCs generated based on the refinement typing rules are of the form $\cdots \Longrightarrow P_{i}(\tilde{v})$ (i.e., the head of every CHC is an atomic formula on a predicate variable). Thus, there always exists a trivial solution for the CHCs , which instantiates all the predicate variables to true. For the example above,

$$
P_{1}(n) \equiv \text { true } \quad P_{2}(n, x) \equiv \text { true } \quad P_{3}(n) \equiv \text { true } \quad P_{4}(n, x) \equiv \text { true }
$$

is also a solution, but using the trivial solution, our transformation yields the non-terminating program. This program is essentially the same as the one in Example 3 since let $x=\star$ in Assume(true); $E$ is equivalent to let $x=\star$ in $E$. Typical CHC solvers indeed tend to find the trivial solution.

To remedy the problem above, in addition to the CHCs generated from the typing rules, we add extra constraints that prevent infinite loops. For the example above, the definition of $f_{\rho_{3}}$ (which corresponds to the channel $f$ ) in the translated program is of the form
$f_{\rho_{3}}(n)=$ if $n<0$ then () else $f_{\rho_{1}}(n) \oplus\left(\right.$ let $x=\star$ in $\left.\operatorname{Assume}\left(P_{2}(n, x)\right) ; f_{\rho_{3}}(x)\right)$.

Thus we add the clause:

$$
P_{2}(n, x) \Longrightarrow n \neq x
$$

to prevent an infinite loop $f_{\rho_{3}}(m) \rightarrow f_{\rho_{3}}(m) \rightarrow \cdots$. With the added clause, a CHC solver HoIce [5] indeed returns $n<x$ as the solution for $P_{2}(n, x)$.

In general, we can add the extra CHCs in the following, counter-exampleguided manner.

1. $\mathcal{C}:=$ the CHCs generated from the typing rules
2. $\theta:=\operatorname{callCHCsolver}(\mathcal{C})$
3. $S:=$ the sequential program generated based on the solution $\theta$
4. if $S$ is terminating then return OK; otherwise, analyze $S$ to find an infinite reduction sequence, add an extra clause to $\mathcal{C}$ to disable the infinite sequence, and go back to 2 .

More precisely, in the last step, the backend termination analysis tool generates a lasso as a certificate of non-termination. We extract a chain $f(\tilde{x}) \rightarrow \cdots \rightarrow$ $f\left(\tilde{x}^{\prime}\right)$ of recursive calls from the lasso, and add an extra clause requiring $\tilde{x} \neq \tilde{x}^{\prime}$ to $\mathcal{C}$. This is naive and insufficient for excluding out an infinite sequence like $f(1) \rightarrow f(2) \rightarrow f(3) \rightarrow \cdots$. We plan to refine the method by incorporating more sophisticated techniques developed for sequential programs [16].

## 5 Implementation and Preliminary Experiments

### 5.1 Implementation

We have implemented a termination analysis tool for the $\pi$-calculus based on the method described in Sections 3 and 4. This tool was written in OCaml. We chose C language as the actual target of our translation, and used Ultimate Automizer [17] (version 0.2.1) as a termination analysis tool for C.

For the refinement type inference described in Section 4.3, we have used HoIce [5] (version 1.8.3) and Z3 [10] (version 4.8.10) as backend CHC solvers. Since a stronger solution for CHCs is preferable as discussed at the end of Section 4.3, if HoICE and Z3 return different solutions $\left\{P_{1} \mapsto \phi_{1}, \ldots, P_{n} \mapsto \phi_{n}\right\}$ and $\left\{P_{1} \mapsto \phi_{1}^{\prime}, \ldots, P_{n} \mapsto \phi_{n}^{\prime}\right\}$, then we used the solution $\left\{P_{1} \mapsto \phi_{1} \wedge \phi_{1}^{\prime}, \ldots, P_{n} \mapsto\right.$ $\left.\phi_{n} \wedge \phi_{n}^{\prime}\right\}$ for inserting Assume commands.

To make the analysis precise, the implementation is actually based on an extension of the refinement type system in Section 4.1 with subtyping; see Appendix D.

### 5.2 Preliminary Experiments

We prepared a collection of $\pi$-calculus processes, and ran our tool on them. Our experiment was conducted on Intel Core i7-10850H CPU with 32GB memory. For comparison, we have also run the termination analysis mode of TyPiCaL $[18,19]$ on the same instances.

Table 2. Results of the experiments

| Test case | Basic | Refined | TYPICAL |
| :---: | :---: | :---: | :---: |
| client-server | 2.5 | 2.7 | 0.006 |
| stateful-server-client | FAIL | FAIL | 0.006 |
| parallel-or | 2.4 | 2.9 | 0.006 |
| broadcast | 3.6 | 3.3 | 0.004 |
| btree | FAIL | FAIL | 0.011 |
| stable | FAIL | FAIL | 0.003 |
| ds-ex5-1 | FAIL | FAIL | 0.002 |
| factorial | 3.9 | 4.4 | 0.002 |
| ackermann | 22.4 | 26.0 | 0.003 |
| fibonacci | 4.8 | 4.4 | 0.003 |
| even/odd | 7.0 | 7.6 | 0.002 |
| factorial-pred | FAIL | 28.2 | FAIL |
| fibonacci-pred | FAIL | 28.2 | FAIL |
| even/odd-pred | FAIL | 10.1 | FAIL |
| sum-neg | 7.6 | 13.1 | FAIL |
| upperbound | 3.8 | 3.9 | FAIL |
| nested-replicated-input1 | 2.3 | 2.4 | FAIL |
| nested-replicated-input2 | FAIL | FAIL | FAIL |
| nested-replicated-input3 | 3.7 | 4.0 | 0.010 |
| deadlock | FAIL | 2.9 | FAIL |

The experimental results are summarized in Table 2. The columns "Basic" and "Refined" show the results for the basic method in Section 3 and the refined method in Section 4 respectively. The numbers show the running times measured in seconds, and "FAIL" means that the verification failed due to the incompleteness of the reduction; non-terminating sequential programs were obtained in those cases. The column "TyPiCal" shows the analogous result for TyPiCal. The termination analysis of TyPiCal roughly depends on Deng and Sangiorgi's method [13]. "FAIL" in the column means that the process does not satisfy the (sufficient) conditions for termination [13]. The termination analysis of TYPICAL treats numbers as natural numbers, and is actually unsound in the presence of arbitrary integers (for example, $f!(m ; r) \mid$ $* f ?(x ; r)$.if $x=0$ then $r!(1)$ else $f!(x-1 ; r)$ is judged to be terminating for any $m$ ).

The test cases consist of two categories. The first one, shown above the horizontal line, has been taken from the sample programs of TyPiCal. Among them, we have excluded out those that are not related to termination analysis (note that TyPiCal can perform deadlock/lock-freedom analysis and information flow analysis besides termination analysis). The second category, shown below the horizontal line, consists of those prepared by us, ${ }^{7}$ including the samples discussed in the paper. All the processes in the test cases are terminating.

[^4]For "stateful-server-client", "btree", "stable", and "ds-ex5-1" in the first category, and "nested-replicated-input2" in the second category, our analysis fails for essentially the same reason. The following is a simplified version of "ds-ex51":

$$
a!()|b!()| * a ?() \cdot b ?() \cdot a!()
$$

The process above is terminating because each run of the third process consumes a message on $b$. Our reduction however ignores communications on $b$ and produces the following non-terminating program:

$$
\left(\left\{f_{\rho_{a}}()=f_{\rho_{a}}(), f_{\rho_{b}}()=()\right\}, f_{\rho_{a}}() \oplus f_{\rho_{b}}()\right)
$$

For the second category, our refined method clearly outperforms the basic method and TyPiCal. We explain some of the test cases in the second category. The test cases "fibonacci" and "nested-replicated-input3" are from Example 1 and 2 respectively, and "even/odd" is a mutually recursive process that judges whether a given number is even or odd. The process "deadlock" is the following one:

$$
* \text { loop?().loop!()|r?().loop!(). }
$$

This process is terminating, because the subprocess $r$ ?().loop!() is blocked forever, without ever sending a message to loop. With the refinement type system, the channel $r$ is given type: $\boldsymbol{c h}_{\rho}(\epsilon ;$ false $)$, and $r ?()$.loop!() is translated to:

$$
\text { let } \epsilon=\epsilon \text { in Assume }(\text { false }) ; f_{\rho_{\text {loop }}}(),
$$

which is terminating by Assume(false). The process "upperbound" is the following process:

$$
f!(0) \mid * f ?(x) \text {.if } x>10 \text { then } 0 \text { else } f!(x+1)
$$

It is terminating because the argument of $f$ monotonically increases, and is bounded above by 10. TyPiCal cannot make such reasoning.

## 6 Related Work

As mentioned in Section 1, there have been a number of studies on termination of the $\pi$-calculus [ $13,11,26,19,29,12,28]$, but most of them have been rather theoretical, and few tools have been developed. Our technique has been partially inspired by Deng and Sangiorgi's work [13], especially by their observation that a process is terminating just if there is no infinite chain of communications on replicated input processes. Deng and Sangiorgi ensured the lack of infinite chains by using a type system. They actually proposed four system, a core system and three kinds of extensions. Our approach roughly corresponds to the first extension of their system ([13], Section 4), which requires that, in every chain of communications, the values of messages monotonically decrease. An advantage of our approach is that we can use mature tools for sequential programs to reason
about how the values of messages change. Our approach does not subsume the second and third extensions of Deng and Sangiorgi's system, which take into account synchronizations over multiple channels; it is left for future work to study whether such extensions can be incorporated in our approach.

To our knowledge, TyPiCal [18,19] is the only automated termination analysis tool. TyPiCal's termination analysis is based on Deng and Sangiorgi's method [13], but is quite limited in reasoning about the values sent along channels; it only considers natural numbers, and the ordering on them is limited to the standard order on natural numbers. Thus, for example, TyPiCal cannot prove the termination of the process "upperbound" as described in Section 5.

Recently, there have been studies on type systems for estimating the (time) complexity of processes for the $\pi$-calculus [1,2] and related session calculi $[9,8]$. Since the existence of a finite upper-bound implies termination, those analyses can, in principle, be used also for reasoning about termination, but the resulting termination analysis would be too conservative. It would be interesting to investigate whether our approach of reduction to sequential programs can be extended to achieve complexity analysis for the $\pi$-calculus. Refinement types for variants of the $\pi$-calculus have been studied before [15,3]. Our contribution in this regard is the application to termination analysis.

Cook et al. [6] proposed a method for proving termination of multi-threaded programs. Their technique also makes use of a termination tool for sequential programs. As their language model is quite different from ours (they deal with imperative programs with shared memory and locks, rather than messagepassing programs), however, their method is quite different from ours.

## 7 Conclusion

We have proposed a method for reducing termination verification for the $\pi$ calculus to that for sequential programs and implemented an automated termination analysis tool based on the method. Our approach allows us to reuse powerful termination analysis tools developed for sequential programs.

Future work includes (i) a further refinement of our reduction and (ii) applications of our method to other message-passing-style concurrent programming languages. As for the first point, there are a few known limitations in the current reduction. Besides the issues mentioned at the end of Example 2 and Section 5, there is a limitation that channels of the same region are merged to the same function, which leads to the loss of precision. For example, consider:

$$
\begin{aligned}
& * c ?(x) \text {.if } x<0 \text { then } 0 \text { else } c!(x-1) \\
& \mid * d ?(x) \text {.if } x>0 \text { then } \mathbf{0} \text { else } d!(x+1) \\
& |e!(c)| e!(d) \mid c!(0)
\end{aligned}
$$

The process is terminating, but our approach fails to prove it. Since the same region is assigned to $c$ and $d$ (because both are sent along $e$ ), the replicated input
processes are translated to non-deterministic function definitions:

$$
\begin{aligned}
& f_{\rho}(x)=\text { if } x<0 \text { then }() \text { else } f_{\rho}(x-1) \\
& f_{\rho}(x)=\text { if } x>0 \text { then }() \text { else } f_{\rho}(x+1)
\end{aligned}
$$

which cause an infinite reduction $f_{\rho}(0) \rightarrow f_{\rho}(-1) \rightarrow f_{\rho}(0) \rightarrow \cdots$. One remedy to this problem would be to introduce region polymorphism and translate processes to higher-order functional programs.

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$$
\begin{aligned}
& \begin{array}{c|c|c}
\operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{v}) \quad \operatorname{len}(\tilde{z})=\operatorname{len}(\tilde{w}) & \tilde{v} \Downarrow \tilde{i} \\
\hline x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\left|x!(\tilde{v} ; \tilde{w}) \cdot P_{2} \rightarrow[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1}\right| P_{2}
\end{array}(\text { R-Comm }) \\
& \frac{P_{1} \rightarrow P_{1}^{\prime}}{P_{1}\left|P_{2} \rightarrow P_{1}^{\prime}\right| P_{2}}(\mathrm{R}-\mathrm{PAR}) \quad \frac{P \rightarrow P^{\prime}}{(\nu x: \kappa) P \rightarrow(\nu x: \kappa) P^{\prime}} \text { (R-Nu) } \\
& \frac{\operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{v}) \quad \operatorname{len}(\tilde{z})=\operatorname{len}(\tilde{w}) \quad \tilde{v} \Downarrow \tilde{i}}{* x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\left|x!(\tilde{v} ; \tilde{w}) \cdot P_{2} \rightarrow * x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\right|[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1} \mid P_{2}} \text { (R-RComm) } \\
& \frac{v \Downarrow i \neq 0}{\text { if } v \text { then } P_{1} \text { else } P_{2} \rightarrow P_{1}}(\mathrm{R}-\mathrm{IF}-\mathrm{T}) \\
& \frac{v \Downarrow 0}{\text { if } v \text { then } P_{1} \text { else } P_{2} \rightarrow P_{2}} \text { (R-IF-F) } \\
& \frac{\operatorname{len}(\tilde{x})=\operatorname{len}(\tilde{i})}{\text { let } \tilde{x}=\tilde{\star} \text { in } P \rightarrow[\tilde{i} / \tilde{x}] P} \text { (R-LETND) } \\
& \frac{P \equiv_{\pi} P_{1} \rightarrow P_{1}^{\prime} \equiv_{\pi} P^{\prime}}{P \rightarrow P^{\prime}}(\text { R-Cong }) \\
& \frac{\tilde{v} \Downarrow i v i}{o p}(\mathrm{R}-\mathrm{INT}) \quad(\mathrm{R}-\mathrm{Op})
\end{aligned}
$$

Fig. 4. The reduction rules of the $\pi$-calculus. Here $\llbracket o p \rrbracket: \mathbb{Z}^{n} \rightarrow \mathbb{Z}$ represents the interpretation of the operation op whose arity is $n$.

## A Operational Semantics

## A. 1 Reduction Semantics of the $\pi$-Calculus

We define a reduction relation [22] as the operational semantics of the $\pi$-calculus.
As usual, we first define the structural congruence relation $\equiv_{\pi}$ on the set of processes.

Definition 1 (structural congruence for processes). The structural congruence relation $\equiv_{\pi}$ on $\pi$-calculus processes is defined as the least congruence relation that satisfies the following rules.

$$
\begin{gathered}
P_{1}\left|P_{2} \equiv_{\pi} P_{2}\right| P_{1} \quad\left(P_{1} \mid P_{2}\right)\left|P_{3} \equiv_{\pi} P_{1}\right|\left(P_{2} \mid P_{3}\right) \\
P \mid \boldsymbol{O} \equiv_{\pi} P \quad(\nu x) \boldsymbol{0} \equiv_{\pi} \boldsymbol{O} \quad(\nu x)(\nu y) P \equiv_{\pi}(\nu y)(\nu x) P \\
(\nu x)\left(P_{1} \mid P_{2}\right) \equiv_{\pi} P_{1} \mid(\nu x) P_{2} \quad \text { if } x \text { does not freely occur in } P_{1}
\end{gathered}
$$

Next, we define the reduction relation on processes.
Definition 2. The reduction relation $\rightarrow$ on processes is defined by the set of rules in Figure 4. We write $\rightarrow^{*}$ and $\rightarrow^{+}$for the reflexive transitive closure and the transitive closure of the reduction relation $\rightarrow$, respectively.

$$
\begin{gathered}
\frac{\operatorname{len}(\tilde{x})=\operatorname{len}(\tilde{i})}{(\mathcal{D}, \text { let } \tilde{x}=\tilde{\star} \text { in } E) \rightsquigarrow(\mathcal{D}, \tilde{i} / \tilde{x}] E)}(\text { SR-LETND }) \\
\frac{(\lambda \tilde{y} . E) \in \mathcal{D}(f) \quad \operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{v}) \quad \tilde{v} \Downarrow \tilde{i}}{(\mathcal{D}, f(\tilde{v})) \rightsquigarrow(\mathcal{D},[\tilde{i} / \tilde{y}] E)}(\text { SR-APP }) \\
\frac{v \Downarrow i \quad i \neq 0}{\left(\mathcal{D}, \text { if } v \text { then } E_{1} \text { else } E_{2}\right) \rightsquigarrow\left(\mathcal{D}, E_{1}\right)}(\text { SR-IF-T) } \\
\frac{v \Downarrow 0}{\left(\mathcal{D}, \text { if } v \text { then } E_{1} \text { else } E_{2}\right) \rightsquigarrow\left(\mathcal{D}, E_{2}\right)}(\text { SR-IF-F) } \\
\frac{(\text { SR-CHO-L })}{\left(\mathcal{D}, E_{1} \oplus E_{2}\right) \rightsquigarrow\left(\mathcal{D}, E_{1}\right)}(\text { SR-CHO-R }) \\
\frac{\left(\mathcal{D}, E_{1} \oplus E_{2}\right) \rightsquigarrow\left(\mathcal{D}, E_{2}\right)}{i \neq 0} \\
\frac{v \Downarrow i}{(\mathcal{D}, \text { Assume }(v) ; E) \rightsquigarrow(\mathcal{D}, E)}(\text { SR-Ass-T }) \\
\frac{v \Downarrow 0}{(\mathcal{D}, \operatorname{Assume}(v) ; E) \rightsquigarrow(\mathcal{D},())}(\text { SR-Ass-F) }
\end{gathered}
$$

Fig. 5. Reduction rules of the sequential language

## A. 2 Reduction Semantics of the Sequential Language

Here, we define the reduction semantics for the sequential language. We actually define two kinds of semantics: one is a standard reduction relation $(\mathcal{D}, E) \rightsquigarrow$ ( $\mathcal{D}^{\prime}, E^{\prime}$ ), which evaluates $E_{1} \oplus E_{2}$ to either $E_{1}$ or $E_{2}$; the other is a non-standard reduction relation $(\mathcal{D}, E) \longrightarrow\left(\mathcal{D}^{\prime}, E^{\prime}\right)$, which does not discard branches of nondeterministic choices.

Definition 3. The reduction relation $\rightsquigarrow$ on sequential programs is defined by the set of rules in Figure 5. In the rule SR -App we are considering $\mathcal{D}$ as a map that maps $f$ to $\mathcal{D}(f)=\{\lambda \tilde{x} . E \mid f(\tilde{x})=E \in \mathcal{D}\}$.

We now define a non-standard reduction relation that keeps all the nondeterministic branches during the reduction. This non-standard reduction relation has a better match with the reduction of processes. Since processes have structural rules, we also introduce structural rules on expressions.
Definition 4 (structural congruence for sequential expressions). The structural congruence relation for expressions, written $E_{1} \equiv_{E} E_{2}$, is defined as the least congruence relation that satisfies the following rules.
$E_{1} \oplus E_{2} \equiv_{E} E_{2} \oplus E_{1} \quad\left(E_{1} \oplus E_{2}\right) \oplus E_{3} \equiv_{E} E_{1} \oplus\left(E_{2} \oplus E_{3}\right) \quad E \oplus() \equiv_{E} E$
Definition 5. The non-standard reduction relation $\rightarrow$ on the set of sequential programs is defined by the set of rules in Figure 6 together with all the rules in Figure 5 (with $\rightsquigarrow$ replaced by $\rightarrow$ ), except for SR-Chо-L and SR-Cho-R. To simplify the notation, we may write $E \rightarrow \rightarrow_{\mathcal{D}} E^{\prime}$ for $(\mathcal{D}, E) \rightarrow\left(\mathcal{D}, E^{\prime}\right)$ or even $E \rightarrow E^{\prime}$ if $\mathcal{D}$ is clear from the context.

$$
\begin{gathered}
E \equiv_{\mathrm{E}} E_{1}\left(\mathcal{D}, E_{1}\right) \rightarrow\left(\mathcal{D}, E_{1}^{\prime}\right) \quad E_{1}^{\prime} \equiv_{\mathrm{E}} E^{\prime} \\
\hline(\mathcal{D}, E) \rightarrow\left(\mathcal{D}, E^{\prime}\right) \\
\frac{(\text { SR-ConG })}{\left(\mathcal{D}, E_{1}\right) \cdots\left(\mathcal{D}, E_{1}^{\prime}\right)} \\
\frac{\left(\mathcal{D}, E_{2}\right) \longrightarrow\left(\mathcal{D}, E_{2}^{\prime}\right)}{\left(\mathcal{D}, E_{1} \oplus E_{2}\right) \longrightarrow\left(\mathcal{D}, E_{1} \oplus E_{2}^{\prime}\right)}(\text { SR-ChoBodY-L }) \\
\text { (SR-ChoBodY-R })
\end{gathered}
$$

Fig. 6. Additional rules for the non-standard reduction relation

$$
\begin{array}{cc}
\frac{\mathcal{D} \unlhd \mathcal{D}}{}(\mathrm{D}-\mathrm{ID}) & \frac{\mathcal{D}=\mathcal{D}_{1} \cup \mathcal{D}_{2}}{\mathcal{D} \unlhd \mathcal{D}_{1}}(\text { D-SPLT }) \\
\frac{\mathcal{D}=\left(\text { let } \tilde{x}=\tilde{\star} \operatorname{in~} \mathcal{D}^{\prime}\right)}{\mathcal{D} \unlhd[\tilde{v} / \tilde{x}] \mathcal{D}^{\prime}} \operatorname{len}(\tilde{x})=\operatorname{len}(\tilde{v}) \\
\hline & (\mathrm{D}-\mathrm{ND}) \\
\frac{\mathcal{D}_{1} \unlhd \mathcal{D}_{1}^{\prime}}{\mathcal{D}_{1} \cup \mathcal{D}_{2} \unlhd \mathcal{D}_{1}^{\prime} \cup \mathcal{D}_{2}}(\mathrm{D}-\mathrm{MRG}) \frac{\mathcal{D}_{1} \unlhd \mathcal{D}_{2} \quad \mathcal{D}_{2} \unlhd \mathcal{D}_{3}}{\mathcal{D}_{1} \unlhd \mathcal{D}_{3}}(\text { D-TRNS })
\end{array}
$$

Fig. 7. Preorder on function definitions

For the proof of the soundness of our transformation (given in Appendix B), we also prepare a relation $\mathcal{D} \unlhd \mathcal{D}^{\prime}$, which intuitively means that $\mathcal{D}$ can simulate $\mathcal{D}^{\prime}$ so that if $(\mathcal{D}, E)$ is terminating, so is $\left(\mathcal{D}^{\prime}, E\right)$ (cf. Lemma 1$)$.

Lemma 1. Suppose that $\mathcal{D} \unlhd \mathcal{D}^{\prime}$ and $\left(\mathcal{D}^{\prime}, E\right) \rightarrow\left(\mathcal{D}^{\prime}, E^{\prime}\right)$. Then $(\mathcal{D}, E){\rightarrow-{ }^{+}}^{+}$ $\left(\mathcal{D}, E^{\prime}\right)$.

Proof. By induction on the derivation of $\mathcal{D} \unlhd \mathcal{D}^{\prime}$.

## B Proof of the Soundness

Here we prove the soundness of the translation (Theorem 1) saying that if the sequential program $(\mathcal{D}, E)$ obtained by translating $P$ is terminating, $P$ is also terminating. The proof is split into two steps. First, we show that reductions from $P$ can be simulated by non-standard reductions from $(\mathcal{D}, E)$ (Lemma 4). This implies that if $(\mathcal{D}, E)$ is terminating with respect to the non-standard reduction, then $P$ is terminating. Then we show that if $(\mathcal{D}, E)$ is terminating with respect to the standard reduction, then $(\mathcal{D}, E)$ is terminating with respect to the nonstandard reduction (Lemma 6).

We start by preparing some auxiliary lemmas that are used to show the simulation relation.

Lemma 2 (substitution). If $\Gamma ; \Delta \vdash \tilde{v}: \tilde{\iota}, \Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa}$ and $\Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash$ $P \Rightarrow(\mathcal{D}, E)$, then $\Gamma ; \Delta \vdash[\tilde{v} / \tilde{y}, \tilde{w} / \tilde{z}] P \Rightarrow([\tilde{v} / \tilde{y}] \mathcal{D},[\tilde{v} / \tilde{y}] E)$.

Proof. By induction on the derivation of $\Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P \Rightarrow(\mathcal{D}, E)$.

Lemma 3. If $P \equiv{ }_{\pi} P^{\prime}$ and $\Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$, then there exists $E^{\prime}$ such that $E \equiv_{E} E^{\prime}$ and $\Gamma ; \Delta \vdash P^{\prime} \Rightarrow\left(\mathcal{D}, E^{\prime}\right)$.

Proof. By induction on the construction of $P \equiv_{\pi} P^{\prime}$.
Now we prove the simulation relation.
Lemma 4. If $P \rightarrow P^{\prime}$ and $\Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$, then there exist $\mathcal{D}^{\prime}, E^{\prime}$ such that $\mathcal{D} \unlhd \mathcal{D}^{\prime},\left(\mathcal{D}^{\prime}, E\right) \longrightarrow \rightarrow^{+}\left(\mathcal{D}^{\prime}, E^{\prime}\right)$ and $\Gamma ; \Delta \vdash P^{\prime} \Rightarrow\left(\mathcal{D}^{\prime}, E^{\prime}\right)$.

Proof. By induction on the construction of $P \rightarrow P^{\prime}$. We only give detailed proofs for interesting cases; the other cases are sketched.

Case R-Comm: In this case $P \rightarrow P^{\prime}$ must be of the form

$$
x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\left|x!(\tilde{v} ; \tilde{w}) \cdot P_{2} \rightarrow[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1}\right| P_{2}
$$

where $\operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{v}), \operatorname{len}(\tilde{z})=\operatorname{len}(\tilde{w})$ and $\tilde{v} \Downarrow \tilde{i}$. Also $\Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$ must be the form of

$$
\Gamma ; \Delta \vdash P \Rightarrow\left(\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup \mathcal{D}_{2},\left(\text { let } \tilde{y}=\tilde{\star} \text { in } E_{1}\right) \oplus\left(f_{\rho}(\tilde{v}) \oplus E_{2}\right)\right)
$$

where

$$
\begin{align*}
& \Gamma ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma ; \Delta \vdash \tilde{v}: \tilde{\iota} \quad \Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa} \\
& \Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P_{1} \Rightarrow\left(\mathcal{D}_{1}, E_{1}\right)  \tag{1}\\
& \Gamma ; \Delta \vdash P_{2} \Rightarrow\left(\mathcal{D}_{2}, E_{2}\right) . \tag{2}
\end{align*}
$$

By applying Lemma 2 to (1) with $\Gamma ; \Delta \vdash \tilde{i}: \tilde{\iota}, \Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa}$, we obtain $\Gamma ; \Delta \vdash[\tilde{i} / \tilde{y}, \tilde{w}) / \tilde{z}] P_{1} \Rightarrow\left([\tilde{i} / \tilde{y}] \mathcal{D}_{1},[\tilde{i} / \tilde{y}] E_{1}\right)$. From this and (2), we have

$$
\Gamma ; \Delta \vdash[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1} \mid P_{2} \Rightarrow\left([\tilde{i} / \tilde{y}] \mathcal{D}_{1} \cup \mathcal{D}_{2},[\tilde{i} / \tilde{y}] E_{1} \oplus E_{2}\right)
$$

by applying the rule SX-PAR. Observe that we also have

$$
\mathcal{D}=\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup \mathcal{D}_{2} \unlhd[\tilde{i} / \tilde{y}] \mathcal{D}_{1} \cup \mathcal{D}_{2}
$$

Therefore, for $\left(\mathcal{D}^{\prime}, E^{\prime}\right)$ we can take $\left([\tilde{i} / \tilde{y}] \mathcal{D}_{1} \cup \mathcal{D}_{2},[\tilde{i} / \tilde{y}] E_{1} \oplus E_{2}\right)$ with the following matching reduction sequence:

$$
\begin{array}{rlr}
E & =\left(\text { let } \tilde{y}=\tilde{\star} \text { in } E_{1}\right) \oplus f_{\rho}(\tilde{v}) \oplus E_{2} \\
& --\rightarrow_{\mathcal{D}^{\prime}}[\tilde{i} / \tilde{y}] E_{1} \oplus f_{\rho}(\tilde{v}) \oplus E_{2}  \tag{SR-LETND}\\
& --\mathcal{D}^{\prime},[\tilde{i} / \tilde{y}] E_{1} \oplus() \oplus E_{2} \quad\left(\text { by }(\text { SR-APP }) \text { and } \lambda \tilde{y} .() \in \mathcal{D}^{\prime}\left(f_{\rho}\right)\right) \\
& \equiv_{\mathrm{E}}[\tilde{i} / \tilde{y}] E_{1} \oplus E_{2} .
\end{array}
$$

Case R-RComm: In this case $P \rightarrow P^{\prime}$ is of the form

$$
* x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\left|x!(\tilde{v} ; \tilde{w}) \cdot P_{2} \rightarrow * x ?(\tilde{y} ; \tilde{z}) \cdot P_{1}\right|[(\tilde{i}, \tilde{w}) /(\tilde{y}, \tilde{z})] P_{1} \mid P_{2}
$$

where $\operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{v}), \operatorname{len}(\tilde{z})=\operatorname{len}(\tilde{w})$ and $\tilde{v} \Downarrow \tilde{i}$. Moreover, the judgment $\Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$ must be of the form

$$
\Gamma ; \Delta \vdash P \Rightarrow\left(\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup \mathcal{D}_{2},() \oplus f_{\rho}(\tilde{v}) \oplus E_{2}\right)
$$

where

$$
\begin{align*}
& \Gamma ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{\iota} ; \tilde{\kappa}) \quad \Gamma ; \Delta \vdash \tilde{v}: \tilde{\iota} \quad \Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa} \\
& \Gamma ; \Delta \vdash * x ?(\tilde{y} ; \tilde{z}) . P_{1} \Rightarrow\left(\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right),()\right)  \tag{3}\\
& \Gamma, \tilde{y}: \tilde{\iota} ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P_{1} \Rightarrow \mathcal{D}_{1} ; E_{1}  \tag{4}\\
& \Gamma ; \Delta \vdash P_{2} \Rightarrow\left(\mathcal{D}_{2}, E_{2}\right) . \tag{5}
\end{align*}
$$

Since $\Gamma ; \Delta \vdash \tilde{i}: \tilde{\iota}$ and $\Gamma ; \Delta \vdash \tilde{w}: \tilde{\kappa}$, we can apply the substitution lemma (Lemma 2) to (4) and obtain

$$
\Gamma ; \Delta \vdash[\tilde{i} / \tilde{y}, \tilde{w} / \tilde{z}] P_{1} \Rightarrow\left([\tilde{i} / \tilde{y}] \mathcal{D}_{1},[\tilde{i} / \tilde{y}] E_{1}\right)
$$

From this, (3) and (5), we have

$$
\Gamma ; \Delta \vdash P^{\prime} \Rightarrow \begin{aligned}
& \left(\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup[\tilde{i} / \tilde{y}] \mathcal{D}_{1} \cup \mathcal{D}_{2}\right. \\
& \left.() \oplus[\tilde{i} / \tilde{y}] E_{1} \oplus E_{2}\right)
\end{aligned}
$$

So we can take $\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\right.$ let $\tilde{y}=\tilde{\star}$ in $\left.\mathcal{D}_{1}\right) \cup[\tilde{i} / \tilde{y}] \mathcal{D}_{1} \cup \mathcal{D}_{2}$ as $\mathcal{D}^{\prime}$ and ()$\oplus[\tilde{i} / \tilde{y}] E_{1} \oplus E_{2}$ as $E^{\prime}$. Now it remains to show that $\mathcal{D} \unlhd \mathcal{D}^{\prime}$ and that there is a reduction sequence from $\left(\mathcal{D}^{\prime}, E\right)$ to $\left(\mathcal{D}^{\prime}, E^{\prime}\right)$. The relation $\mathcal{D} \unlhd \mathcal{D}^{\prime}$ holds because

$$
\begin{align*}
\mathcal{D} & =\left(\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup \mathcal{D}_{2}\right. \\
& =\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup \mathcal{D}_{2} \\
& \unlhd\left\{f_{\rho}(\tilde{y})=E_{1}\right\} \cup\left(\text { let } \tilde{y}=\tilde{\star} \text { in } \mathcal{D}_{1}\right) \cup[\tilde{i} / \tilde{y}] \mathcal{D}_{1} \cup \mathcal{D}_{2}  \tag{D-ND}\\
& =\mathcal{D}^{\prime}
\end{align*}
$$

Finally, by SR-App, we obtain

$$
E=() \oplus f_{\rho}(\tilde{v}) \oplus E_{2} \rightarrow \mathcal{D}^{\prime}() \oplus[\tilde{i} / \tilde{y}] E_{1} \oplus E_{2}=E^{\prime}
$$

as desired.
Case R-If-T: In this case $P \rightarrow P^{\prime}$ and $\Gamma ; \Delta \vdash P \Rightarrow(\mathcal{D}, E)$ must be of the form

$$
\begin{aligned}
& \text { if } v \text { then } P_{1} \text { else } P_{2} \rightarrow P_{1} \\
& \Gamma ; \Delta \vdash \text { if } v \text { then } P_{1} \text { else } P_{2} \Rightarrow\left(\mathcal{D}_{1} \cup \mathcal{D}_{2} \text {, if } v \text { then } E_{1} \text { else } E_{2}\right)
\end{aligned}
$$

where

$$
\begin{array}{r}
v \Downarrow i \neq 0 \quad \Gamma ; \Delta \vdash v: \iota \\
\Gamma ; \Delta \vdash P_{1} \Rightarrow\left(\mathcal{D}_{1}, E_{1}\right) \\
\Gamma ; \Delta \vdash P_{2} \Rightarrow\left(\mathcal{D}_{2}, E_{2}\right) .
\end{array}
$$

We can take $\left(\mathcal{D}_{1}, E_{1}\right)$ for ( $\mathcal{D}^{\prime}, E^{\prime}$ ) because $\mathcal{D}_{1} \cup \mathcal{D}_{2} \unlhd \mathcal{D}_{1}$, and $E \rightarrow \mathcal{D}_{1} E_{1}$, which is trivial from SR-If-T.
Case R-IF-F: Similar to the previous case.
Case R-Cong: In this case $P \rightarrow P^{\prime}$ must be of the form

$$
P \equiv_{\pi} P_{1} \rightarrow P_{1}^{\prime} \equiv_{\pi} P^{\prime} .
$$

By Lemma 3, we have

$$
\Gamma, \Delta \vdash P_{1} \Rightarrow\left(\mathcal{D}, E_{1}\right) \text { and } E \equiv_{\mathrm{E}} E_{1}
$$

for some $E_{1}$. Thus, by the induction hypothesis, we have

$$
\begin{align*}
& \Gamma, \Delta \vdash P_{1}^{\prime} \Rightarrow\left(\mathcal{D}^{\prime}, E_{1}^{\prime}\right)  \tag{6}\\
& \left(\mathcal{D}^{\prime}, E_{1}\right){\rightarrow-\rightarrow^{+}}^{\left(\mathcal{D}^{\prime}, E_{1}^{\prime}\right)} \tag{7}
\end{align*}
$$

where $\mathcal{D} \unlhd \mathcal{D}^{\prime}$. By applying Lemma 3 to (6), we obtain

$$
\Gamma, \Delta \vdash P^{\prime} \Rightarrow\left(\mathcal{D}^{\prime}, E^{\prime}\right) \text { and } E_{1}^{\prime} \equiv_{\mathrm{E}} E^{\prime}
$$

for some $E^{\prime}$. It remains to show that ( $\left.\mathcal{D}^{\prime}, E\right){\rightarrow-{ }^{+}}^{+}\left(\mathcal{D}^{\prime}, E^{\prime}\right)$, but this is easily shown by repeatedly applying the rule SR-Cong along the reduction sequence (7).
Case R-Par, R-Nu and R-LetND: Similar to the previous case, i.e. follows from the definition of the translation and the induction hypothesis together with Lemma 1.

Lemma 5. Suppose that $\emptyset ; \emptyset \vdash P \Rightarrow(\mathcal{D}, E)$. If $(\mathcal{D}, E)$ is terminating with respect to $\rightarrow$, then $P$ is terminating.
Proof. We show the contraposition. Assume that $P$ is not terminating, i.e. assume that there exists an infinite reduction sequence $P=P_{0} \rightarrow P_{1} \rightarrow \cdots$. Let $\mathcal{D}_{0}=\mathcal{D}$ and $E_{0}=E$. By applying Lemma 4, for each natural number $k \geq 1$, we obtain $\mathcal{D}_{k}, E_{k}$ such that $\emptyset ; \emptyset \vdash P_{k} \Rightarrow\left(\mathcal{D}_{k}, E_{k}\right),\left(\mathcal{D}_{k}, E_{k-1}\right) \rightarrow+\left(\mathcal{D}_{k}, E_{k}\right)$ and $\mathcal{D} \unlhd \mathcal{D}_{k}$. Hence, by Lemma 1 there exists an infinite reduction sequence $(\mathcal{D}, E)=\left(\mathcal{D}, E_{0}\right) \rightarrow^{+}\left(\mathcal{D}, E_{1}\right) \rightarrow^{+} \ldots$.

We now show the relation between standard and non-standard reductions.
Lemma 6. Assume that $\emptyset ; \emptyset \vdash P \Rightarrow(\mathcal{D}, E)$. If $(\mathcal{D}, E)$ is terminating with respect to the standard reduction $\rightsquigarrow$, then $(\mathcal{D}, E)$ is also terminating with respect to the non-standard reduction relation $\rightarrow$.

$$
\begin{align*}
& \frac{\operatorname{len}(\tilde{x})=\operatorname{len}(\tilde{i})}{(\mathcal{D}, \operatorname{let} \tilde{x}=\tilde{\star} \text { in } E) \rightarrow_{\epsilon}(\mathcal{D},[\tilde{i} / \tilde{x}] E) \quad \text { (NSR-LETND) }}  \tag{NSR-LEtND}\\
& \frac{(\lambda \tilde{y} \cdot E) \in \mathcal{D}(f) \quad \operatorname{len}(\tilde{y})=\operatorname{len}(\tilde{v}) \quad \tilde{v} \Downarrow \tilde{i}}{(\mathcal{D}, f(\tilde{v})) \rightarrow-\rightarrow_{\epsilon}(\mathcal{D},[\tilde{i} / \tilde{y}] E)} \\
& v \Downarrow i \quad i \neq 0 \\
& \overline{\left(\mathcal{D}, \text { if } v \text { then } E_{1} \text { else } E_{2}\right) \rightarrow_{\epsilon}\left(\mathcal{D}, E_{1}\right)}  \tag{NSR-If-T}\\
& \frac{v \Downarrow 0}{\left(\mathcal{D}, \text { if } v \text { then } E_{1} \text { else } E_{2}\right) \rightarrow_{\epsilon}\left(\mathcal{D}, E_{2}\right)} \\
& \frac{\left(\mathcal{D}, E_{1}\right) \rightarrow_{\gamma}\left(\mathcal{D}, E_{1}^{\prime}\right)}{\left(\mathcal{D}, E_{1} \oplus E_{2}\right) \rightarrow \rightarrow_{1 \cdot \gamma}\left(\mathcal{D}, E_{1}^{\prime} \oplus E_{2}\right)} \\
& \frac{\left(\mathcal{D}, E_{2}\right) \rightarrow \rightarrow_{\gamma}\left(\mathcal{D}, E_{2}^{\prime}\right)}{\left(\mathcal{D}, E_{1} \oplus E_{2}\right) \rightarrow \rightarrow_{2 \cdot \gamma}\left(\mathcal{D}, E_{1} \oplus E_{2}^{\prime}\right)} \\
& \frac{v \Downarrow i \quad i \neq 0}{(\mathcal{D}, \operatorname{Assume}(v) ; E) \rightarrow_{\epsilon}(\mathcal{D}, E)} \\
& \frac{v \Downarrow 0}{(\mathcal{D}, \operatorname{Assume}(v) ; E) \rightarrow \rightarrow_{\epsilon}(\mathcal{D},())} \\
& \text { (NSR-App) } \\
& \text { (NSR-If-F) } \\
& \text { (NSR-ChoBody-L) } \\
& \text { (NSR-ChoBody-R) } \\
& \text { (NSR-Ass-T) } \\
& \text { (NSR-Ass-F) }
\end{align*}
$$

Fig. 8. A variation of the non-standard reduction relation

To prove the lemma above, we introduce a slight variation of the non-standard
 change during the reduction.) It is defined by the rules in Figure 8.

The only differences of $(\mathcal{D}, E) \rightarrow \gamma\left(\mathcal{D}^{\prime}, E^{\prime}\right)$ from $(\mathcal{D}, E) \rightarrow\left(\mathcal{D}^{\prime}, E^{\prime}\right)$ are that the reduction is annotated with the position $\gamma$ that indicates where the reduction occurs, and that the rule SR-Cong for shuffling expressions is forbidden. Since the rule SR-Cong does not affect the reducibility, we can easily observe the following property. (We omit the proof since it is trivial.)

Lemma 7. If $(\mathcal{D}, E)$ has an infinite reduction sequence with respect to $\rightarrow$, $(\mathcal{D}, E)$ has an infinite reduction sequence also with respect to $\rightarrow_{\gamma}$.

It remains to show that if $(\mathcal{D}, E)$ has an infinite reduction sequence

$$
(\mathcal{D}, E) \rightarrow \gamma_{1}\left(\mathcal{D}, E_{1}\right) \rightarrow \gamma_{2}\left(\mathcal{D}, E_{2}\right) \rightarrow \gamma_{3}\left(\mathcal{D}, E_{3}\right) \rightarrow \gamma_{4} \cdots,
$$

then $(\mathcal{D}, E)$ has an infinite reduction sequence also with respect to $\rightsquigarrow$.
We write $\gamma \preceq \gamma^{\prime}$ if $\gamma$ is a prefix of $\gamma^{\prime}$. We have the following property.
Lemma 8. If

$$
(\mathcal{D}, E) \rightarrow \gamma_{\gamma_{1}}\left(\mathcal{D}, E_{1}\right) \rightarrow \gamma_{\gamma_{2}}\left(\mathcal{D}, E_{2}\right) \rightarrow \gamma_{3}\left(\mathcal{D}, E_{3}\right) \rightarrow \gamma_{4} \cdots,
$$

then there exists an infinite sequence $i_{1}<i_{2}<i_{3}<\cdots$ such that $\gamma_{i_{j}} \preceq \gamma_{i_{k}}$ for any $j<k$.

Proof. The required property obviously holds if the set $\left\{\gamma_{i} \mid i \geq 1\right\}$ is finite. So, assume that $\left\{\gamma_{i} \mid i \geq 1\right\}$ is infinite. Let $T$ be the least binary tree that contains, for every $\gamma_{i}$, the node whose path from the root is $\gamma_{i}$. By the assumption that $\left\{\gamma_{i} \mid i \geq 1\right\}$ is infinite, $T$ is an infinite tree. Thus, by König's lemma, $T$ must have an infinite path, which implies that there exists an infinite sequence

$$
\gamma_{i_{1}} \preceq \gamma_{i_{2}} \preceq \gamma_{i_{3}} \preceq \cdots,
$$

as required.
For an expression $E$ and a position $\gamma \in\{1,2\}^{*}$, we write $E \downarrow_{\gamma}$ for the subexpression at $\gamma$. It is inductively defined by:

$$
\begin{aligned}
& E \downarrow_{\epsilon}=E \\
& E \downarrow_{i \cdot \gamma}= \begin{cases}E_{i} \downarrow_{\gamma} & \text { if } E \text { is of the form } E_{1} \oplus E_{2} \\
\text { undefined } & \text { otherwise }\end{cases}
\end{aligned}
$$

The following lemma states the correspondence between $\rightarrow_{\gamma}$ and $\rightsquigarrow$.
Lemma 9. 1. If $(\mathcal{D}, E) \rightarrow \rightarrow_{\gamma}\left(\mathcal{D}, E^{\prime}\right)$, then $\left(\mathcal{D}, E \downarrow_{\gamma}\right) \rightsquigarrow\left(\mathcal{D}, E^{\prime} \downarrow_{\gamma}\right)$.
2. Suppose $E \downarrow_{\gamma^{\prime}}$ is defined and $\gamma^{\prime} \npreceq \gamma$. If $(\mathcal{D}, E){\rightarrow-\rightarrow_{\gamma}}\left(\mathcal{D}, E^{\prime}\right)$, then $E \downarrow_{\gamma^{\prime}}=$ $E^{\prime} \downarrow \gamma^{\prime}$.
3. If $(\mathcal{D}, E) \rightarrow_{\gamma}\left(\mathcal{D}, E^{\prime}\right)$, and $\gamma^{\prime} \preceq \gamma$, then $\left(\mathcal{D}, E \downarrow_{\gamma^{\prime}}\right) \rightsquigarrow^{*}\left(\mathcal{D}, E \downarrow_{\gamma}\right)$.

Proof. The properties follow by a straightforward induction on the derivation of $(\mathcal{D}, E) \rightarrow_{\gamma}\left(\mathcal{D}, E^{\prime}\right)$.

We are now ready to prove Lemma 6.
Proof (of Lemma 6). We show the contraposition. Suppose ( $\mathcal{D}, E$ ) has an infinite reduction sequence with respect to $\rightarrow$. By Lemma 7, there exists an infinite reduction sequence

$$
(\mathcal{D}, E) \rightarrow \gamma_{\gamma_{1}}\left(\mathcal{D}, E_{1}\right) \rightarrow \gamma_{\gamma_{2}}\left(\mathcal{D}, E_{2}\right) \rightarrow \gamma_{3}\left(\mathcal{D}, E_{3}\right) \rightarrow \gamma_{\gamma_{4}} \cdots .
$$

By Lemma 8, there exists an infinite sequence:

$$
\gamma_{i_{1}} \preceq \gamma_{i_{2}} \preceq \gamma_{i_{3}} \preceq \cdots
$$

such that $i_{1}<i_{2}<i_{3}<\cdots$. Let us choose a maximal one among such sequences, i.e., a sequence

$$
\gamma_{i_{1}} \preceq \gamma_{i_{2}} \preceq \gamma_{i_{3}} \preceq \cdots
$$

such that, for any $i_{j}, \gamma_{k} \preceq \gamma_{i_{j}}$ implies $k=i_{j^{\prime}}$ for some $j^{\prime} \leq j$. Consider the fragment of the infinite reduction sequence:

$$
\left(\mathcal{D}, E_{i_{\ell-1}}\right) \rightarrow \gamma_{i_{\ell-1}+1}\left(\mathcal{D}, E_{i_{\ell-1}+1}\right) \rightarrow \gamma_{i_{\ell-1}+2} \cdots \rightarrow_{\gamma_{i_{\ell}-1}}\left(\mathcal{D}, E_{i_{\ell}-1}\right) \rightarrow \gamma_{i_{\ell}}\left(\mathcal{D}, E_{i_{\ell}}\right)
$$

for each $\ell>0$. (Here, we define $\gamma_{0}=\epsilon, i_{0}=0$ and $E_{0}=E$.) By Lemma 9 (1) and $\left(\mathcal{D}, E_{i_{\ell}-1}\right) \rightarrow \gamma_{i_{\ell}}\left(\mathcal{D}, E_{i_{\ell}}\right)$, we have

$$
\left(\mathcal{D}, E_{i_{\ell}-1} \downarrow \gamma_{i_{\ell}}\right) \rightsquigarrow\left(\mathcal{D}, E_{i_{\ell} \downarrow \gamma_{i_{\ell}}}\right) .
$$

By Lemma 9 (2) (note that since none of $\gamma_{i_{\ell-1}+1}, \ldots, \gamma_{i_{\ell}-1}$ is a prefix of $\gamma_{i_{\ell}}$ by the assumption on maximality, $E_{i_{\ell-1}} \downarrow \gamma_{i_{\ell}}$ is defined), we have

$$
E_{i_{\ell-1}} \downarrow_{{i_{\ell}}_{\ell}}=E_{i_{\ell-1}+1} \downarrow \gamma_{i_{\ell}}=\cdots=E_{i_{\ell}-1} \downarrow_{\gamma_{i_{\ell}}}
$$

Thus, together with Lemma 9 (3), we obtain:

$$
\left(\mathcal{D}, E_{i_{\ell-1}} \downarrow \gamma_{i_{\ell-1}}\right) \rightsquigarrow{ }^{*}\left(\mathcal{D}, E_{i_{\ell-1}} \downarrow \gamma_{i_{\ell}}\right) \rightsquigarrow\left(\mathcal{D}, E_{i_{\ell}} \downarrow \gamma_{i_{\ell}}\right) .
$$

Therefore, we have an infinite reduction sequence

$$
(\mathcal{D}, E)=\left(\mathcal{D}, E_{i_{0}} \downarrow \gamma_{i_{0}}\right) \rightsquigarrow{ }^{+}\left(\mathcal{D}, E_{i_{1}} \downarrow \gamma_{i_{1}}\right) \rightsquigarrow+\left(\mathcal{D}, E_{i_{2}} \downarrow \gamma_{i_{2}}\right) \rightsquigarrow+\left(\mathcal{D}, E_{i_{3}} \downarrow \gamma_{i_{3}}\right) \rightsquigarrow{ }^{+} \cdots,
$$

as required.
Finally, the soundness (Theorem 1) follows from Lemmas 5 and 6.

## C Complete Definition of the Refinement Type System

This section shows the complete definition of the refinement type system we discussed in Section 4.

First, we define the well-formedness conditions for types and type environments. We write $\mathbf{F V}(\phi)(\mathbf{F V}(\Phi)$, resp.) for the set of variables occurring in $\phi$ ( $\Phi$, resp.), and $\operatorname{dom}(\Gamma)$ for the domain of $\Gamma$, i.e., $\{x \mid x: \iota \in \Gamma\}$. The relations $\Gamma \vdash \kappa \mathbf{o k}$ and $\Gamma ; \Phi ; \Delta \vdash \mathbf{o k}$ are defined by:

$$
\begin{gathered}
\mathbf{F V}(\phi) \subseteq \operatorname{dom}(\Gamma) \cup\{\tilde{x}\} \\
\frac{\Gamma, \tilde{x}: \tilde{\iota} \vdash \kappa_{i} \text { ok for each } i \in\{1, \ldots, k\}}{\Gamma \vdash \mathbf{c h}_{\rho}\left(\tilde{x} ; \phi ; \kappa_{1}, \ldots, \kappa_{k}\right) \mathbf{o k}} \\
\frac{\Gamma \vdash \kappa \mathbf{o k} \text { for every } x: \kappa \in \Delta}{} \frac{\mathbf{F V}(\Phi) \subseteq \operatorname{dom}(\Gamma)}{\Gamma ; \Phi ; \Delta \vdash \mathbf{o k}}
\end{gathered}
$$

For example, $x: \iota \vdash \boldsymbol{c h}_{\rho}(y ; y<x ; \epsilon)$ : ok holds but $\emptyset \vdash \boldsymbol{c h}_{\rho}(y ; y<x ; \epsilon)$ : ok does not.

For every type judgment of the form $\Gamma ; \Phi ; \Delta \vdash P$, we implicitly require that $\Gamma ; \Phi ; \Delta \vdash$ ok holds. Similarly, for $\Gamma ; \Phi ; \Delta \vdash v: \kappa$, we require that $\Gamma ; \Phi ; \Delta \vdash$ ok and $\Gamma \vdash \kappa$ ok hold.

The complete list of typing rules is given in Figure 9.

## D Refinement Type System with Subtyping

As mentioned in Section 5, the implementation is based on the following extension of the refinement type system in Section 4.1.

$$
\begin{aligned}
& \overline{\Gamma ; \Phi ; \Delta \vdash \mathbf{0}}(\mathrm{RT}-\mathrm{NiL}) \quad \frac{\Gamma ; \Phi ; \Delta \vdash P_{1} \quad \Gamma ; \Phi ; \Delta \vdash P_{2}}{\Gamma ; \Phi ; \Delta \vdash P_{1} \mid P_{2}} \text { (RT-PAR) } \\
& \frac{\Gamma ; \Phi ; \Delta \vdash x: \boldsymbol{c h}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Phi, \phi ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P}{\Gamma ; \Phi ; \Delta \vdash x ?(\tilde{y} ; \tilde{z}) . P}(\mathrm{RT}-\mathrm{In}) \\
& \Gamma ; \Phi ; \Delta \vdash x: \operatorname{ch}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma ; \Phi ; \Delta \vdash \tilde{v}: \tilde{\iota} \quad \Phi \vDash[\tilde{v} / \tilde{y}] \phi \\
& \frac{\Gamma ; \Phi ; \Delta \vdash \tilde{w}:[\tilde{v} / \tilde{y}] \tilde{\kappa} \quad \Gamma ; \Phi ; \Delta \vdash P}{\Gamma ; \Phi ; \Delta \vdash x!(\tilde{v} ; \tilde{w}) . P} \text { (RT-OUT) } \\
& \frac{\Gamma ; \Phi ; \Delta, x: \kappa \vdash P}{\Gamma ; \Phi ; \Delta \vdash(\nu x: \kappa) P}(\text { RT-Nu }) \\
& \frac{\Gamma ; \Phi ; \Delta \vdash x: \boldsymbol{\operatorname { c h }}_{\rho}(\tilde{y} ; \phi ; \tilde{\kappa}) \quad \Gamma, \tilde{y}: \tilde{\iota} ; \Phi, \phi ; \Delta, \tilde{z}: \tilde{\kappa} \vdash P}{\Gamma ; \Phi ; \Delta \vdash * x ?(\tilde{y} ; \tilde{z}) . P} \text { (RT-RIn) } \\
& \frac{\Gamma ; \Phi ; \Delta \vdash v: \iota \quad \Gamma ; \Phi, v \neq 0 ; \Delta \vdash P_{1} \quad \Gamma ; \Phi, v=0 ; \Delta \vdash P_{2}}{\Gamma ; \Phi ; \Delta \vdash \text { if } v \text { then } P_{1} \text { else } P_{2}} \text { (RT-IF) } \\
& \frac{\Gamma, \tilde{x}: \tilde{\iota} ; \Phi ; \Delta \vdash P}{\Gamma ; \Phi ; \Delta \vdash \text { let } \tilde{x}=\tilde{\star} \text { in } P} \text { (RT-LETND) } \\
& \frac{x: \iota \in \Gamma}{\Gamma ; \Phi ; \Delta \vdash x: \iota}(\mathrm{RT}-\mathrm{VAR}-\mathrm{INT}) \quad \frac{x: \kappa \in \Delta}{\Gamma ; \Phi ; \Delta \vdash x: \kappa}(\mathrm{RT}-\mathrm{VAR}-\mathrm{CH}) \\
& \overline{\Gamma ; \Phi ; \Delta \vdash i: \iota}(\mathrm{RT}-\mathrm{InT}) \quad \frac{\Gamma ; \Phi ; \Delta \vdash \tilde{v}: \tilde{\iota}}{\Gamma ; \Phi ; \Delta \vdash o p(\tilde{v}): \iota}(\mathrm{RT}-\mathrm{Op})
\end{aligned}
$$

Fig. 9. Typing rules of the refinement type system for the $\pi$-calculus

The set of refinement $i / o$ channel types, ranged over by $\kappa$, is given by:

$$
\kappa::=\operatorname{ch}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right)
$$

Here, $\boldsymbol{c h}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right)$ is the type of channels used for receiving tuples $(\tilde{x} ; \tilde{y})$ such that $\tilde{x}$ satisfies $\phi_{I}$ and $\tilde{y}$ have types $\tilde{\kappa}_{I}$, and for sending tuples ( $\left.\tilde{x} ; \tilde{y}\right)$ such that $\tilde{x}$ satisfies $\phi_{O}$ and $\tilde{y}$ have types $\tilde{\kappa}_{O}$. The distinction between the types of input (i.e. received) values and those of output (i.e. sent) values has been inspired by the type system of Yoshida and Hennessy [30]. It leads to a more precise type system than Pierce and Sangiorgi's subtyping, and is convenient for automatic refinement type inference [23] (because we need not infer input/output modes and perform case analysis on the modes).

The subtyping relation on channel types is defined by:

$$
\begin{array}{cl}
\Phi, \phi_{I} \vDash \phi_{I}^{\prime} & \Gamma, \tilde{x}: \tilde{u} ; \Phi, \phi_{I} \vdash \tilde{\kappa}_{I}<\tilde{\kappa}_{I}^{\prime} \\
\Phi, \phi_{O}^{\prime} \vDash \phi_{O} & \Gamma, \tilde{x}: \tilde{u} ; \Phi, \phi_{O}^{\prime} \vdash \tilde{\kappa}_{O}^{\prime}<: \tilde{\kappa}_{O}  \tag{RT-Sub-Ch}\\
\Gamma ; \Phi \vdash \operatorname{ch}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right)<: \boldsymbol{c h}_{\rho}\left(\tilde{x} ; \phi_{I}^{\prime} ; \tilde{\kappa}_{I}^{\prime} ; \phi_{O}^{\prime} ; \tilde{\kappa}_{O}^{\prime}\right)
\end{array}
$$

Note that the channel type $\boldsymbol{\operatorname { c h }}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right)$ is covariant on $\phi_{I}$ and $\tilde{\kappa}_{I}$, and contravariant on $\phi_{O}$ and $\tilde{\kappa}_{O}$.

We make the following two modifications to the typing rules.

1. We add the following subsumption rule.

$$
\begin{equation*}
\frac{\Gamma ; \Phi ; \Delta \vdash v: \kappa \quad \Gamma ; \Phi \vdash \kappa<: \kappa^{\prime}}{\Gamma ; \Phi ; \Delta \vdash v: \kappa^{\prime}} \tag{RT-Sub}
\end{equation*}
$$

2. We refine the well-formedness condition on types and type environments by:

$$
\begin{gathered}
\mathbf{F V}(\phi) \subseteq \operatorname{dom}(\Gamma) \cup\{\tilde{x}\} \\
\Gamma, \tilde{x}: \tilde{\iota} \vdash \tilde{\kappa}_{I} \mathbf{o k} \\
\Gamma, \tilde{x}: \tilde{\iota} \vdash \tilde{\kappa}_{O} \mathbf{o k} \\
\frac{\mathbf{F V}}{\Gamma \vdash \mathbf{c h}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right) \mathbf{o k}} \\
\frac{\mathbf{F V}(\Phi) \subseteq \operatorname{dom}(\Gamma)}{\Gamma ; \Phi ; \emptyset \vdash \mathbf{o k}} \\
\frac{\Gamma \vdash \operatorname{ch}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right) \mathbf{o k}}{\Gamma \models \phi_{I} \quad \Gamma, \tilde{x}: \tilde{\iota} ; \Phi, \phi_{O} \vdash \tilde{\kappa}_{O}<: \tilde{\kappa}_{I}} \\
\hline \Gamma ; \Phi ; \Delta, y: \mathbf{c h}_{\rho}\left(\tilde{x} ; \phi_{I} ; \tilde{\kappa}_{I} ; \phi_{O} ; \tilde{\kappa}_{O}\right) \vdash \mathbf{o k}
\end{gathered}
$$

The requirement for the subtyping relation in the last rule ensures the consistency between the types of values expected by a receiver process and those actually output by a sender process; for example, the channel type $\boldsymbol{c h}_{\rho}(x ; x>0 ; \epsilon ; x<0 ; \epsilon)$ is judged to be ill-formed, because the type indicates that a receiver process expects a positive value $x$ but a sender will output a negative value on the channel.

The following example demonstrates the usefulness of subtyping for refinement channel types.

Example 5. Let us consider the following process:

$$
\begin{aligned}
& * \operatorname{pred} ?(x ; r) \cdot r!(x-1) \\
& \mid * f ?(x) . \text { if } x<0 \text { then } 0 \text { else }(\nu s)(\text { pred! }(x ; s) \mid s ?(y) \cdot f!(y)) \\
& \mid f!(100) \\
& \mid * c ?(x ; r) . \text { let } y=\star \text { in } r!(y) \\
& \mid d!(\text { pred }) \mid d!(c)
\end{aligned}
$$

The process consisting of the first three lines is a variation of the process in Example 3, which is obviously terminating. Without the fourth and fifth lines, we would be able to assign the type $\boldsymbol{c h}_{\rho_{1}}\left(x ; \boldsymbol{\operatorname { t r u e }} ; \boldsymbol{\operatorname { c h }}_{\rho_{2}}(y ; y<x ; \epsilon)\right)$ in the refinement type system in Section 4, and reduce the process to a terminating program.

The processes on the fifth line, however, force us to assign the same type to pred and $c$ in the refinement type system in Section 4, and thus we can assign only $\boldsymbol{c h}_{\rho_{1}}\left(x ; \boldsymbol{\operatorname { t r u e }} ; \boldsymbol{c h}_{\rho_{2}}(y ; \boldsymbol{\operatorname { t r u e }} ; \epsilon)\right)$ to pred, failing to transform the process to a non-terminating program.

With subtyping, we can assign the following types to pred, $c$, and $d$ :

```
pred : }\mp@subsup{\boldsymbol{ch}}{\mp@subsup{\rho}{1}{}}{(x;}\mathrm{ (true; }\mp@subsup{\boldsymbol{ch}}{\mp@subsup{\rho}{2}{}}{(}(y;\boldsymbol{\operatorname{true};\epsilon;y<x;\epsilon);\boldsymbol{true};\mp@subsup{\boldsymbol{ch}}{\mp@subsup{\rho}{2}{}}{}(y;\mathrm{ true ; }\epsilon;y<x;\epsilon))
```



```
d:\mp@subsup{\boldsymbol{ch}}{\mp@subsup{\rho}{0}{}}{(\epsilon; true; }\kappa;\mathrm{ true; }\kappa)
where
```



Note that the types of pred and $c$ are subtypes of $\kappa$. Here, the type of pred indicates that the value $y$ sent along the second argument $r$ should be smaller than the first argument $x$. Thus, the process on the second line is translated to the following function definition:

$$
\begin{aligned}
f_{\rho_{f}}(x)= & \text { if } x<0 \text { then }() \text { else } \\
& \left(f_{\text {pred }}(x) \oplus\left(\text { let } y=\star \text { in Assume }(y<x) ; f_{\rho_{f}}(y)\right)\right)
\end{aligned}
$$


[^0]:    ${ }^{3}$ The actual translation given later is a little more complex.

[^1]:    ${ }^{4}$ Thus, the simple type system with "regions" introduced in the previous section is used here as a simple may-alias analysis. If $x$ and $y$ may be bound to the same channel during reductions, the type system assigns the same region to $x$ and $y$, hence $x$ and $y$ are mapped to the same function name $f_{\rho_{x}}$ by our transformation.

[^2]:    ${ }^{5}$ The program written here has been simplified for the sake of readability. For instance, we removed some redundant (), trivial function definitions, and unused nondeterministic integers. The other examples that will appear in this paper are also simplified in the same way.

[^3]:    ${ }^{6}$ The rule for replicated inputs is also modified in a similar manner.

[^4]:    ${ }^{7}$ Unfortunately, there are no standard benchmark set for the termination analysis for the $\pi$-calculus.

