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Michel Barès • Éloi Bossé

# Relational Calculus for Actionable Knowledge



Springer

Michel Barès  
Data Science Department  
Expertises Parafuse Inc.  
Jouy en Josas, France

Éloi Bossé  
Image and Information  
Processing Department  
IMT Atlantique  
Quebec City, QC, Canada

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# Preface

One of the major challenges of a newly created scientific domain called “data science” is to turn data into actionable knowledge to exploit the increasing data volumes and deal with their inherent complexity (Big data and IoT). The advances in networking capabilities have created the conditions of complexity by enabling richer, real-time interactions between and among individuals, objects, systems, and organizations. Networking involves relations of all kinds and presents challenges of complexity especially when the objective is to provide technological supports to human decision-making.

Actionable knowledge has been qualitatively and intensively studied in management, business, and social sciences but for computer sciences and engineering, recently, there has been a connection with data mining and its evolution “Knowledge Discovery and Data Mining (KDD).” The ambition of our book is to present advanced knowledge concepts and its formalization to support the analytics and information fusion (AIF) processes that aim at delivering actionable knowledge. The book offers four major contributions: (1) the concept of “relation” and its exploitation (relational calculus) for the AIF processes, (2) the formalization of certain dimensions of knowledge to achieve a semantic growth along the AIF processes, (3) the modeling of the interrelations within the couple (knowledge, action) to gain sense, and finally (4) the exploitation of relational calculus to support the AIF core technological processes that allow to transform data into actionable knowledge.

This book addresses two main poles: computations with relations (relational calculus) and creation of actionable knowledge. In the first three chapters, we explore basic properties of knowledge, knowledge representations, and knowledge processes from scientific and practical perspectives emphasizing existing directions and areas in knowledge studies. We also examine the fundamental role of information and define the relationship that exists between data, information, and knowledge. We discuss the need for formalization. Any automatic process geared to support human decision-making must be indeed endowed with reasoning ability, depending on the circumstances and the context of its employment. A suitable formalism to represent knowledge and information remains the required essential

for any subsequent artificial reasoning that is achieved throughout a knowledge processing chain, the AIF processes.

The subsequent three chapters (4–6) address the understanding of the couple (knowledge, action) and how to support the processing chain in the creation of actionable knowledge using relational calculus. Chapter 4 presents preliminaries of crisp and fuzzy relational calculus to support the discussion in the subsequent chapters. The question of how to deal with knowledge imperfections is addressed. Chapter 5 examines the couple (knowledge, action). Knowledge is a prerequisite to taking any reasoned action or course of action according to rational rules. The questions are what facilitates the relevant decision-making and what are the modalities that can make the action (effect) more efficient. There is a strong dependency between the notion of knowing about a given world and the decisions that can be made and consecutively the potential actions that can be undertaken. The notion of *mastering knowledge* for efficient actions is treated. Analysis and synthesis of information, a prerequisite to any decision-making and action, is supported by AIF technologies. Chapter 6 addresses the usage of relational calculus when applied to the AIF core processes that perform the multiple transformations required along the processing chain from data to actionable knowledge.

Jouy en Josas, France  
Quebec City, QC, Canada

Michel Barès  
Éloi Bossé

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# About the Authors

**Michel Barès Ph.D.** is an independent researcher in Artificial Intelligence (AI). He is a former researcher at DRET (Direction des Recherches Et Technologies) under the French Ministry of Defence. There, he conducted different innovative research projects relevant to military motivations concerning symbolic reasoning, decision support, and distributed AI. He has been a pioneer to introduce distributed AI in military command decision-making aids. In parallel to his military research work, he acted as professor for several universities and specialized French schools. Since 2017, he is a scientific advisor for Expertises Parafuse inc., a small research firm located in Québec City, Canada. He holds a Ph.D. degree from the university Paris 6, France.

Michel Barès, Ph.D., received the degree Expert en Traitement de l'Information (ETI) from Institut de Programmation (computer sciences) (71), a DEA in maths-physics (75), and a Ph.D. in mathematics (78) from university Paris 6. In addition, he received the informatics engineering degree from Conservatoire National des Arts et Métiers Paris (76) and an '*Habilitation à Diriger des Recherches*' (HdR) from the University of Nancy (97). In 1970, he joined '*l'Institut en Informatique et en Automatique (INRIA)*' where he worked on operational research problems related to large organizations. In 1975, he joined '*La Documentation Française*' to work on database management. In 1984, he joined DRET (Direction des Recherches et Technologies under the French Ministry of Defence) to hold a head position for the division of computer sciences and numerical analysis. There, he conducted different innovative research projects related to symbolic reasoning, intelligent interfaces, and distributed AI. He was one of the pioneers to introduce distributed AI in military command decision-making aids. As professor, Dr. Barès has taught at several universities and specialized French schools: Ecole Nationale Supérieure des Techniques Avancées (ENSTA), Ecole Supérieure de l'Aéronautique (Sup Aéro), Université de Versailles, Université de Bretagne, Université de Rouen, Ecole Militaire et Centre Interarmées de Défense (CID), and George Mason University (visiting professor, Washington, DC). Over several years, Dr. Barès has been responsible for specialized sessions and acted as chair of the international Avignon

conference on AI and expert systems. He represented France in several NATO Research Technology Organization (RTO) panels and research groups on information sciences: NATO RSG 10 workshop “speech processing,” TG 006 “modelling of organization and decision architecture,” ET 014 C4ISRS interoperability,” the NATO Data Fusion Demonstrator, and others. He has published 8 academic books on computer sciences and AI and more than 150 scientific publications in conference proceedings, technical reports, and journal papers. The application domains of Dr. Barès’ research activities range from civilian to military ones: data sciences, complex systems, analytics and information fusion (AIF), and finally, knowledge systems.

**Éloi Bossé Ph.D.** is a researcher on decision support and analytics and information fusion (AIF). He possesses a vast research experience in applying them to defense-and security-related problems and more recently to civilian domains. He is currently president of Expertise Parafuse inc., a consultant firm on AIF and decision support. He holds an academic position as an associate researcher at IMT-Atlantique, Brest, France, and a Ph.D. degree from Université Laval, Québec City, Canada.

Éloi Bossé, Ph.D., received B.A.Sc. (79), M.Sc. (81), and Ph.D. (90) degrees in Electrical Engineering from Université Laval, QC. In 1981, he joined the Communications Research Centre, Ottawa, Canada, where he worked on signal processing and high-resolution spectral analysis. In 1988, he was transferred to the Defence Research Establishment Ottawa to work on radar target tracking in multipath. In 1992, he moved to Defence Research and Development Canada Valcartier (DRDC Valcartier) to lead a group of four to five defense scientists on information fusion and resource management. He has published over 200 papers in journals, book chapters, conference proceedings, and technical reports. Dr. Bossé has held adjunct professor positions at several universities from 1993 to 2013 (Université Laval, University of Calgary, and McMaster University). He headed the C2 Decision Support Systems Section at DRDC Valcartier from 1998 till 2011. Dr. Bossé was the Executive Chair of the 10th International Conference on Information Fusion (FUSION^07), held in July 2007 in Québec City. He represented Canada (as DRDC member) in numerous international research fora under various cooperation research programs (NATO, TTCP, and bi- and tri-laterals) in his area of expertise. He is coauthor and coeditor of five to six books on analytics and information fusion. He left DRDC in September 2011. Since then, he has conducted some research activities under NATO Peace and Security Program, as researcher in Mathematics and Industrial Engineering Department at Polytechnic of Montreal, as associate researcher at IMT-Atlantique, and as researcher at McMaster University, Canada. Since 2015, concurrently with the activities just mentioned, he acts as president of Expertise Parafuse inc., a consultant research firm on analytics and information fusion (AIF) technologies, a great component of data sciences.

# List of Abbreviations

AD	Archetypal Dynamics
AI	Artificial Intelligence
AIF	Analytics and Information Fusion
Big Data 5Vs	(Velocity, Volume, Veracity, Value, Variety)
C2	Command Control
C4ISR	Command Control, Communications, Computer, Intelligence, Surveillance and Reconnaissance
CI	Contextual Information
CoA	Course of Action
CPS	Cyber-physical Systems
CPSS	Cyber-physical and Social Systems
CSE	Cognitive System Engineering
DF	Data Fusion
DIK	Data-Information-Knowledge
DM	Decision-Making
DQ	Data Quality
DSS	Decision Support Systems
DS	Dempster–Shafer
DST	Dempster–Shafer’s Theory
EBDI	Entity-Based Data Integration
ER	Entity Resolution
ES	Epistemic Structure
ETURWG	Evaluation of Technologies for Uncertainty Representation Working Group
GIT	Generalized Information Theory
GTI	General Theory of Information
HLIF	High-Level Information Fusion
H2M	Human-to-Machine
H2S	Human-to-System
IBM	International Business Machine

ICN	Information Centric Networking
ICT	Information and Communication Technology
ID	IDentification
IF	Information Fusion
IFS	Intuitionistic Fuzzy Set
IG	Interoperable Groups
Intel	Military Intelligence cycle
IoT	Internet of Things
IoE	Internet of Everything
IS	Information Systems
ISIF	International Society of Information Fusion
JDL	Joint Directors of Laboratories
JDL DIFG	Joint Directors of Laboratories' Data and Information Fusion Group
KDD	Knowledge Discovery in Databases
KID	Knowledge, Information and Data
KIME	Knowledge-Information-Matter-Energy
KS	Knowledge System
MAPE	Monitor-Analyze-Plan-Execute
MAS	Multi-agent Systems
MCDA	Multi-criteria Decision Analysis
MCDM	Multi-criteria Decision-Making
MS	Management Science
M2M	Machine-to-Machine
NATO	North Atlantic Treaty Organization
NATO SAS RG	NATO Systems Analysis and Studies Research Group
ORBAT	ORder of BATtle
OODA	Observe-Orient-Decide-Act
QoI	Quality of Information
SA	Situation Analysis
SAW	Situation Awareness
SM	Sense-Making
STO	Socio-technical Organizations
TER	Total Entity Resolution
TQM	Total Quality Management
TU	Total Uncertainty
UMM	Uncertainty Management Methods
UN	United Nations
URREF	Uncertainty Representation and Reasoning Evaluation Framework
WoT	Web of Things

## List of Symbols

$N = \{1, 2, 3, \dots\}$	The set of natural numbers
$\mathbb{R}$	The set of real numbers
$ A $	The cardinality of a set $A$
$A^C$	The complement of $A$
$\in$	Membership sign; belongs to
$\subseteq$	Subset; inclusion sign
$\subset$	Proper subset; strict inclusion
$\emptyset$	Empty set
$\cup$	Union
$\cap$	Intersection
$X$	Cartesian product
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to
$\sup$	Supremum
$\inf$	Infimum
$\max$	Maximum
$\min$	Minimum
$::$ or $\stackrel{\text{def}}{=}$	Defined as; given by
$\therefore$	Therefore
$\sim$ or $\neg$	Negation
$\Rightarrow$	Implication
$\rightarrow$	Correspond to
$\forall$	Universal quantifier; for all
$\exists$	Existencial quantifier; there exists
$asc_{\uparrow}$	Ascendant of
$desc_{\downarrow}$	Descendent of
$\vdash$	Conclusion; turnstile symbol; assertion sign
$\Leftrightarrow$	Equivalence
$\text{Dom}(R)$	Domain of relation $R$
$\text{Rng}(R)$ or $\text{Im}(R)$	Range or image of relation $R$
$coR$	The <i>complement</i> relation $coR$ of $R$
$R^{-1}$	The reverse or inverse of relation $R$
$\neq$	Not equal
$P$ -relation	A relation with property $P$
$aRb$	$a$ is related to $b$
$\text{poset } P$	<i>a partial ordering or a partial order of <math>P</math></i>
$\text{lub}$	or $\sup$ or $\sqcup$ <i>Least upper bound</i>
$glb$ or $inf$ or $\sqcap$	<i>Greatest lower bound</i>
$P \oplus Q$	The ordinal sum of two posets

$\mathbb{L} = (P, \sqcup, \sqcap, 0, 1)$	A lattice as a <i>poset</i> $P$
$R \circ S$	Composition of relations $R$ and $S$
$R \triangleleft S$	Subcomposition of relations $R$ and $S$
$R \triangleright S$	Supercomposition of relations $R$ and $S$
$R \diamond S$	Ultracomposition of relations $R$ and $S$
$x \notin A$	$x$ is not element of $A$
$\mu$	membership function
$\nu$	non-membership function
$\mu_{A(x)}$	degree of membership of element $x$ in $A$
$\nu_{A(x)}$	degree of non-membership of element $x$ in $A$
${}^\alpha A$	The $\alpha$ -cut of $A$
${}^{\alpha+} A$	The strong $\alpha$ -cut of $A$
${}^{0+} A$	The <i>support</i> of $A$
${}^1 A$	The <i>core</i> of $A$
$\text{hgt}(A)$	The <i>height</i> of $A$
$\text{plth}(A)$	The <i>plinth</i> of $A$