

# **Studies in Fuzziness and Soft Computing**

Volume 418

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Janusz Kacprzyk, Systems Research Institute, Polish Academy of Sciences,  
Warsaw, Poland

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Oscar Castillo · Juan R. Castro · Patricia Melin

# Interval Type-3 Fuzzy Systems: Theory and Design

Oscar Castillo  
Division of Graduate Studies  
Tijuana Institute of Technology  
Tijuana, Baja California, Mexico

Juan R. Castro  
School of Engineering  
UABC University  
Tijuana, Baja California, Mexico

Patricia Melin  
Division of Graduate Studies  
Tijuana Institute of Technology  
Tijuana, Baja California, Mexico

ISSN 1434-9922

ISSN 1860-0808 (electronic)

Studies in Fuzziness and Soft Computing

ISBN 978-3-030-96514-3

ISBN 978-3-030-96515-0 (eBook)

<https://doi.org/10.1007/978-3-030-96515-0>

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# Preface

The use of type-2 fuzzy systems has become widespread in the leading economy sectors, especially in industrial and application areas, such as services, health, defense, and so on. Recent studies and research have focused on type-2 fuzzy systems that enable handling uncertainty in many applications, such as intelligent control, robotics, pattern recognition, time series prediction, and medical diagnosis.

However, more recently, the use of interval type-3 fuzzy systems has been receiving increasing attention, and some successful applications have been developed in the last year. These issues were taken into consideration when defining the scope of this book, as we did realize that there was a need to offer the main theoretical concepts of type-3 fuzzy logic theory, as well as methods to design, develop, and implement the type-3 fuzzy systems.

A review of basic concepts and their use in the design and implementation of interval type-3 fuzzy systems, which are relatively new models of uncertainty and imprecision, are considered in this book. The main focus of this work is based on the basic reasons of the need for interval type-3 fuzzy systems in different areas of application. Recently, type-2 fuzzy systems have emerged as powerful approach for solving complex problems, in a wide range of application areas. In this book, we briefly review the basic concepts of type-2 fuzzy systems and then describe the proposed definitions for interval type-3 fuzzy sets and relations, also interval type-3 inference and systems. In addition, we describe methods for designing interval type-3 fuzzy systems and illustrate this with some examples and simulations. We also provide a comparison of the type-3 fuzzy approach with respect to type-2 fuzzy systems.

This book is intended to be an important reference for scientists and engineers interested in applying interval type-3 fuzzy logic techniques for solving problems in diverse areas of application. This book can also be used as a reference for graduate courses like the following: soft computing, fuzzy logic, hybrid intelligent systems, and others. We consider that this book can also be used to obtain interesting original ideas for new lines of research, or to continue the lines of research proposed by the authors of this work.

The structure of the book is as follows: in Chap. 1, we offer an introduction to the area of interval type-3 fuzzy systems; in Chap. 2, we put forward a review of type-2 fuzzy logic concepts; in Chap. 3, we present the definitions and methods of Interval type-3 fuzzy sets, their membership functions, and operations. Also, in Chap. 4, we describe the concepts and methods regarding interval type-3 fuzzy logic systems. Finally, in Chap. 5, the conclusions about the book and possible future works are presented.

We end this preface of the book by giving thanks to all the people who have helped or encouraged us to write this book that opens a new area of research in fuzzy logic, namely type-3 fuzzy logic. First of all, we would like to thank our colleague and friend Prof. Janusz Kacprzyk for always supporting and motivating our work. We would also like to thank our colleagues working in soft computing, which are too many to mention each by their name. Of course, we need to thank our supporting agencies, CONACYT and TecNM, in our country for their help during this project. We have to thank our institutions, Tijuana Institute of Technology and UABC University, for always supporting our projects. Finally, we thank our respective families for their continuous support during the time during the process of writing this book.

Tijuana, Mexico  
October 2021

Oscar Castillo  
Juan R. Castro  
Patricia Melin

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