

Semantic-based Data Augmentation for Math Word Problems

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Abstract. It’s hard for neural MWP solvers to deal with tiny local variances. In MWP task, some local changes conserve the original semantic while the others may totally change the underlying logic. Currently, existing datasets for MWP task contain limited samples which are key for neural models to learn to disambiguate different kinds of local variances in questions and solve the questions correctly. In this paper, we propose a set of novel data augmentation approaches to supplement existing datasets with such data that are augmented with different kinds of local variances, and help to improve the generalization ability of current neural models. New samples are generated by knowledge guided entity replacement, and logic guided problem reorganization. The augmentation approaches are ensured to keep the consistency between the new data and their labels. Experimental results have shown the necessity and the effectiveness of our methods.

Keywords: Math word problem · Data augmentation

1 Introduction

Automatically solving Math Word Problem (MWP) has attracted more and more research attention in recent years. The MWP solvers are fed in with a natural language description of a mathematical question, and output a solution equation as the answer. In most cases, these questions are short narratives comprised of several known quantities and a query about an unknown quantity, whose value is the answer we desire. Table 1 shows a typical example of MWP, where x in the equation refers to the unknown quantity, and is calculated from the known quantities and specific constants such as π , 1, 2.

Numerous efforts have been devoted to solving this challenging task. Early studies relying on hand-crafted features [14, 19, 20] and predefined patterns [21] have limitations in generalization. Deep learning methods have become popular to solve the MWP task in recent years [23, 25, 29, 33] due to their better capability

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Table 1: An example of math word problem and new samples generated by semantic-based data augmentation approaches.

Original
Question 1: There are <i>390</i> kilograms of <i>pears</i> in the <i>store</i> , which is <i>40%</i> less than the weight of <i>apples</i> . <i>x</i> kilograms of <i>apples</i> are there in the <i>store</i> .
Equation 1: $x = 390 \div (1 - 40\%)$
Answer 1: 650
Knowledge guided entity replacement
Question 2: There are <i>390</i> kilograms of <i>bananas</i> in the <i>kitchen</i> , which is <i>40%</i> less than the weight of <i>watermelon</i> . <i>x</i> kilograms of <i>watermelon</i> are there in the <i>kitchen</i> .
Equation 2: $x = 390 \div (1 - 40\%)$
Answer 2: 650
Logic guided problem reorganization
Question 3: There are <i>390</i> kilograms of pears in the store, which is <i>x</i> less than the weight of apples. <i>650</i> kilograms of apples are there in the store.
Equation 3: $x = 1 - 390 \div 650$
Answer 3: 0.4

of generalization. [25] first modeled the MWP task as an equation generation task, and various works have followed this framework since then. Recent works in MWP mostly focus on designing complex generation models to capture more features from limited data. For example, [29] proposed a tree-structured decoder to imitate human behaviour, [33] utilized GCN, and [13] adopted the attention mechanism to better capture the relationship between quantities.

However, current MWP solvers still have weaknesses in terms of robustness and generalization. A superior MWP solver should understand a problem precisely in two ways. First, it is able to generate the same equation for a transformed question with only unimportant entity replacement. For example, the equation generated for *Q1* in Table 1 should not be changed when the *pears* in the question is replaced with *bananas*. Second, excellent models should be capable of generating a different equation when the logic of the question changes even if **the text of the question only changes slightly**. For example, in Table 1, the only difference between *Q1* and *Q3* is the position of token *x*. But their underlying logic is totally different and thus the corresponding equations are different. These two kinds of tiny local variances in questions lead to totally different results, one of which conserves the underlying logic, while the other one changes it completely. Humans are able to disambiguate these local variances easily while it’s hard for most neural models to deal with discrete local variances. Previous works on MWP hardly consider this challenge.

The limitation of existing datasets for MWP is a main reason for the above-mentioned weaknesses. Since labeling MWP data is time-consuming, existing MWP datasets are all too small compared to datasets for other natural lan-

guage processing tasks. Besides, few challenging samples with similar questions but different equations can be found in MWP datasets, which makes it hard for neural models to learn to deal with tiny local variances. The most popular and largest single-equation MWP dataset Math23k contains only 23,161 problems, which is rather limited compared to datasets in other field like sQuAD [18] with 150,000 questions. The weaknesses of existing datasets, especially the limited coverage of challenging samples, motivated us to augment the dataset with questions of minor variances leading to heterogeneous equations.

In this paper, we propose a set of semantic-based data augmentation approaches suitable for MWP task, namely knowledge guided entity replacement and logic guided problem reorganization. And two kinds of local variances are provided accordingly. Neural MWP solvers can benefit from our augmentation strategies in terms of generalization and the ability of dealing with tiny local variances. Unlike other popular augmentation approaches [26, 28, 31], which may cause inconsistency of the questions and equations in MWP task, our augmentation methods are carefully designed for MWP task to ensure consistency.

Knowledge augmentation (knowledge guided entity replacement). As shown in Table 1, *pears* in the original question is replaced with another fruit *bananas*. And it is obvious that the replacement does not change the original logic, thus the new question conserves the original label. Our knowledge guided entity replacement method randomly replaces several entities in questions with other entities that belong to the same concept as the original ones. And we guide the replacement with knowledge base that contains much taxonomy knowledge.

Different from synonym replacement [4, 26] which replaces random words with their synonyms, replacing entities could largely avoid semantic shifting since most entities in MWP are not crucial for the logic inference.

Logical augmentation (logic guided problem reorganization). As the example shown in Table 1, quantities *390* and *40%* are known while quantity *650* is unknown in *Q1*. In the generated *Q3*, we let *40%* be the unknown one given *390* and *650*. The equation is changed accordingly to keep consistency. x in Eq 3 is substituted for *650* and *40%* is replaced with x . Afterwards, we transform the new equation to its equivalent equation of the form $x = 1 - 390 \div 650$. To enrich the problem types of the training data, our logical augmentation iteratively set the known quantities in the original question and equation to the unknown. And the new equation is further transformed to its equivalent equation with mathematical properties for normalization.

It is worth mentioning that previous works only learn the equation of the unknown, while our augmentation method helps the neural models to make full use of the limited data by learning all the possible equations of the quantities in the question.

Our contributions are summarized as follows:

- To the best of our knowledge, this is the first systematical study of data augmentation for MWP task. And is easy to be applied to any neural models and extend to other math-related tasks.

- Our methods can generate coherent questions with consistent labels, which largely diversify both textual descriptions and equation templates. It also brings in massive challenging samples that existing datasets lack.
- Experimental results show the necessity and effectiveness of our methods. The performances of the additional evaluation also indicate that our methods largely enhances the generalization ability of neural models.

2 Related Work

2.1 Math Word Problem

Early works mostly utilized statistical methods or predefined rules. [19] utilized hand-crafted features to predict the lowest common ancestor operator for each quantity pair. [20] proposed a unit dependency graph based on [19]. [14] predefined a group of logic forms and converted the math question into them.

[25] first proposed to utilize a seq2seq model with recurrent neural network to generate equation template sequence. [22] proposed equation normalization to unify duplicated representations of equivalent expressions. [3] utilized a seq2seq model with the help of stack to align the semantic with the operator. [23] proposed a two-stage algorithm to predict a template tree. [13] applied group attention mechanism to extract more features.

[16, 29, 33] replaced the recurrent neural network based decoder with a tree-structured decoder and achieved satisfactory results. [32] leveraged the framework of knowledge distillation.

Above-mentioned works all focused on the model architectures while the weakness of lacking in labeled data has been ignored. [15] is another augmentation work for MWP and is very similar to our logic guided augmentation method. However, since [15] re-order the sub-sentences of the original problem, it is likely to generate incoherent questions while our method avoid this weakness.

2.2 Data Augmentation for Natural Language Processing (NLP)

We categorize data augmentation approaches for NLP into two types. The first type changes only the text while the second one changes both the text and labels.

Some research adds random noise to input data [30] or hidden states [12] making the models less sensitive to small perturbations. [26] systematically examined some basic augmentation methods including random synonyms replacement, word insertion, etc. [28] utilized tf-idf to help determine which words to replace. [24] adopted k-nearest neighbors to find synonyms in word embedding space. The Noise brought in by these methods could be tolerated in some tasks but not in MWP task due to the strict requirement of preciseness.

[9, 27] fine tuned a pre-trained language model with text and label to generate new sentences given specific labels. [4] replaced words in the source and target sentences with rare words. Other approaches mainly applied generative models like VAE [7], seq2seq model [6], GPT [1], etc., to generate new text given a

specific label. All of the methods above generating new text given a specific label are inappropriate to MWP task, because the equation is a sequence and is not enumerable.

3 Methodology

3.1 Problem Statement

The MWP dataset contains training data D_{train} and testing data D_{test} , both of which are comprised of numerous questions Q and equations Eq . Neural models take Q as the input and generate the Eq sequence. In previous works, the neural model \mathcal{M} is trained with the training data D_{train} and tested with the testing data D_{test} . Due to the limitations of D_{train} introduced in Sec. 1, we generate new samples D_{aug} from D_{train} by our augmentation approaches. And the neural model \mathcal{M} is trained with both original and augmented data $D_{train} \cup D_{aug}$.

The question Q consists of a sequence of tokens $\mathcal{W} = \{w_i\}_{i=1}^{|\mathcal{W}|}$ including known quantities $\mathcal{N} = \{n_i\}_{i=1}^{|\mathcal{N}|}$ and entities $\mathcal{E} = \{e_i\}_{i=1}^{|\mathcal{E}|}$. Since quantities are fairly sparse, we replace quantities in Q with symbol n_i according to the occurrence order of the quantities during preprocessing phase. Quantities in Eq are replaced accordingly and we denote the answer of Eq as \hat{n} .

The target of MWP is to generate the Eq sequence which is composed of $\mathcal{N} \cup \mathcal{O} \cup \mathcal{C}$. Among them, \mathcal{O} is the set of the operators (such as \times) and \mathcal{C} is the constant set.

3.2 Knowledge Augmentation

The category information of entities is introduced to generate new questions. Given a sample (Q, Eq) , we randomly choose θ entities mentioned in Q to be replaced with other entities belonging to the same concept as the original ones. If an entity e_i belonging to concept c is selected, the alternative entities are $e_j \in \mathcal{E}_c (i \neq j)$. Similar to [26], we set $\theta = \max(1, \alpha l)$, among which l refers to the length of Q and α is a hyper-parameter used to manage the replacement ratio. More entities are replaced for longer questions considering they tolerate noise better. We notice that an entity may appear more than once in a question, and all of them should be replaced if the entity is selected. For each question Q , the new questions generated by this means are denoted as Q_K . As an example in Table 2, entities *Ming Zhang* and *apples* are replaced with a random entity of *person* and *fruit* accordingly. Since entities are less informative for MWP inference, the generated questions conserve the logic of the original question. And the new equation matched with Q_K is still Eq .

Recognize entities. There are two kinds of entities in MWP, namely **real-world entities** and **named entities**, and we take different strategies to recognize them. For detecting named entities like fake person names *Alice*, *Bob*, etc. which are common in MWP, there are numerous models and tools work very well

Table 2: An example of the question generated with knowledge augmentation.

Q	<i>Ming Zhang</i> ([PER]) bought n_0 <i>apples</i> ([FRU]).
Q_K	<i>Hong Li</i> ([PER]) bought n_0 <i>blueberries</i> ([FRU]).

Table 3: An example of the question generated with logical augmentation.

Q	There are n_1 kilograms of pears in the store, which is n_2 less than the weight of apples. How many kilograms of apples are there in the store?
Q'	There are n_1 kilograms of pears in the store, which is n_2 less than the weight of apples. x kilograms of apples are there in the store.
Q_L	There are n_1 kilograms of pears in the store, which is x less than the weight of apples. \hat{n} kilograms of apples are there in the store.

on this task. And we follow the method of [2] to recognize named entities. Another kind of entities is real-world entities like *apple*, *car*, etc. These entities can be easily recognized by referring to KGs. In this paper, we use WordNet [17] to guide our knowledge augmentation. Entities are linked to the WordNet syn-sets, and the direct hypernyms are viewed as their concepts. Besides, we restrict that only *physical entities* in questions could be replaced to avoid possible semantic shifting. *Abstract entities* like *time* should not be replaced in our case.

3.3 Logical Augmentation

Given a sample (Q, Eq) , several (Q_L, Eq_L) pairs are generated by setting a known quantity in the original question to the unknown one in the new question. And the new equation is normalized with mathematical properties. Since the textual description of Q_L is similar to that of Q , neural models tend to generate similar equation sequences for them even though their ground truth solutions are totally different. With logical augmentation, neural models are forced to learn the different equations from the slight variations of input text, which to some extent enhances the inference ability of neural models.

Question generation In MWP, the letter x is usually assigned to represent the unknown. The value of it is the answer to the question Q (denoted as \hat{n}). The unknown quantities in questions are usually indicated by question words such as *how many*. Considering both question words and letter x refer to the same unknown, we replace the question words in Q with the letter x . And then the original question could be viewed as an assertive sentence about the logical relationship between the known and unknown quantities $\mathcal{N} \cup \hat{n}$. Previous training without augmentation only learns how to get \hat{n} given \mathcal{N} , for Eq is in $x = f(\mathcal{N} \cup C)$ form, in which x refers to \hat{n} as \hat{n} is the unknown of Q . However, the logical relationship between other quantity pairs remains unlearned. To make full use of each data sample, we make the neural models additionally learn how to get $n_i \in \mathcal{N}$ given the other quantities $n_j \in \mathcal{N} \cup \hat{n} (i \neq j)$. As the letter x indicates

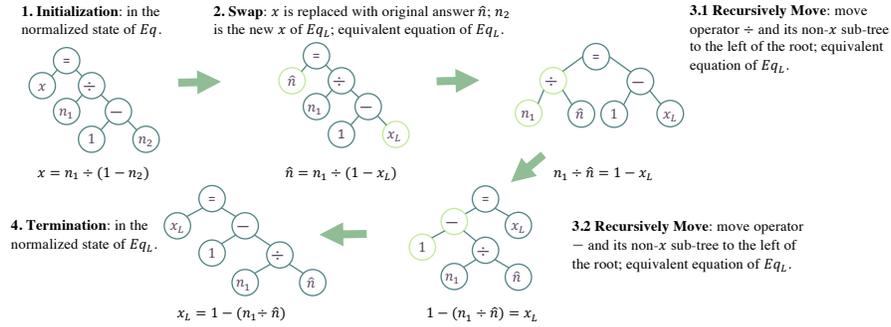


Fig. 1: The process of generating a new equation Eq_L . As illustrated in Table 3, the new unknown quantity of Q_L and Eq_L is n_2 with n_1 and \hat{n} known. The equation tree in the initialization step is built from Eq in Table 3. Equation trees from step 2 to step 4 are equivalent equations of Eq_L in different forms. And the tree in the termination step is the normalized form of Eq_L .

the quantity we desire, quantities in Q are iteratively set to x . The generated questions are denoted as Q_L .

Table 3 shows an example of questions generated with logical augmentation. Notably, when training with augmented data, original questions Q are replaced with Q' to ensure that all questions are expressed in an uniform form.

Equation generation with equation tree conversion Since the known and unknown have been changed in the generated question Q_L , the corresponding equation Eq_L should be changed accordingly to keep consistency. To normalize the form of equations, the generated equations are transformed to their equivalent equations of a specific form based on mathematical properties with the help of equation trees. In the final normalized state, the term x is isolated on the left side of the equation.

Fig. 1 shows an example of how a consistent and normalized equation Eq_L is generated from the original equation Eq .

Equation tree Equation tree is built from an equation whose root node is always the equal sign $=$. The left and right sub-trees of the root are expression trees of the left and right sides of an equation, thus the leaf nodes are operands and the inner nodes (except the root) are binary operators.

In the normalized state of an equation tree, the left sub-tree of the root only contains a leaf node x . The same equation tree in different states refers to equivalent equations of different forms. To normalize the equation, we transform the equation tree to its normalized state with two main actions based on the addition-subtraction property, the division property, and the multiplication property. 1) Move an operator node o (an inner node) and its non- x sub-tree

(the sub-tree of o that does not contain x) to another side. 2) Switch o to its inverse operator, for example, switch $+$ to $-$ or \times to \div .

The process of generating Eq_L As shown in Fig 1, there are four steps to generate Eq_L . 1) Initialization: a normalized equation tree is built from the original Eq . 2) Swap: x is substituted for \hat{n} , and the quantity n_i which is the new unknown is replaced with x_L (to distinguish with x). Since the new equation tree of Eq_L is not in normalized state, we take the available actions to transform the new equation tree to its normalized state. 3) Recursively Move: All of the operator nodes and their non- x sub-trees are recursively moved to the left of the root node in a top-down manner, leaving only x_L in the right. Different actions are chosen in different situations, all based on the natural mathematical properties. 4) Termination: Simply swapping the left and right sub-trees of the root node will make the equation tree normalized. And Eq_L could be restored from the equation tree.

4 Experiment

In this section, we conduct experiments to measure the scale of challenging samples in existing datasets. Besides, we evaluate our semantic-based augmentation strategies (denoted as s.based aug. for simplification) with three typical neural MWP solvers to show the improvements of the generalization ability brought in by s.based aug.. Extensive ablation studies are also conducted to verify the effectiveness of each augmentation strategy.

4.1 Dataset Analysis

Existing MWP datasets such as AI2 [5], SingleEQ [10], and AllArith [20] only have hundreds of samples. While relatively large-scale MWP datasets such as MAWPS [11] contains very few challenging samples as it is constructed with low lexical and template overlap. So we conduct our experiments on the largest and most popular MWP dataset Math23k [25], a Chinese MWP dataset with 23,161 pairs of (Q, Eq) .

To make the neural models able to deal with discrete tiny local variances, it's necessary for the dataset to contain a great ratio of challenging samples that have similar questions but different equations. In this section, we will analyze Math23k from the amount and the quality of challenging samples in the training and the testing set.

Notably, the analysis in this section are all based on equation templates, which means only the structures of the equations are considered rather than the actual values. For example, equations $x = 3 + 2 + 1$ and $x = 5 + 4 + 2$ are viewed as the same in template.

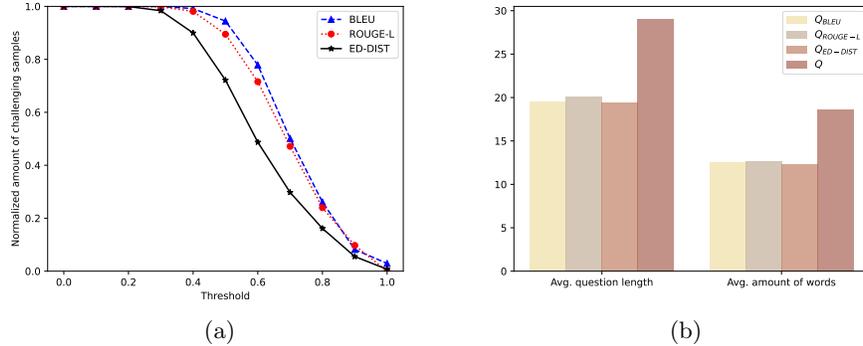


Fig. 2: (a) the amount of challenging samples filtered with different similarity score thresholds in the training set. (b) the quality of existing challenging samples.

The amount of challenging samples in the training set For a question $Q_i \in D_{train}$, if $\exists Q_j \in D_{train} (i \neq j)$ with different equation but similar text, Q_i and Q_j are considered as challenging samples. The similarity of two questions are measured with three similarity scores, namely BLEU, ROUGE-L, and normalized reverse edit distance defined below (referred to as ED-DIST).

$$ED - DIST(Q_i, Q_j) = 1 - \frac{edit - dist(Q_i, Q_j)}{\max(l_{Q_i}, l_{Q_j})}$$

l refers to the length of the question and $edit - dist(\cdot)$ is the Levenshtein distance of the given pair of questions. The more similar Q_i and Q_j are, the closer $ED - DIST(Q_i, Q_j)$ is to 1. Afterwards, we count the amount of challenging samples with the similarity thresholds set to different values as shown in Fig. 2 (a). The amount of samples in Fig. 2 (a) is normalized with respect to the size of the training set $|D_{train}|$. It's obvious that the amount of challenging samples in Math23k is rather limited as no more than 15% of the questions meet the condition when the threshold score is 0.9.

The quality of the challenging samples High-quality MWP questions are supposed to be longer sentences with rich background descriptions. Thus we analyze the average length and words diversity of challenging samples in D_{train} . The threshold of similarity scores is set to 0.9 to filter the challenging samples. In Fig. 2 (b), Q refers to all questions in D_{train} , while Q_{metric} refers to the challenging samples filtered by certain metrics. e.g. Q_{BLEU} are challenging questions that have another similar question with BLEU score higher than 0.9. As shown in Fig. 2 (b), the challenging samples filtered by all metrics are much shorter than the average question length, and so is the diversity of words. Table 4 shows some examples of existing challenging samples in D_{train} . The experimental re-

Table 4: Examples of challenging samples.

	Q_i	Q_j
challenging samples in D_{train}	The divisor is 8 and the quotient is 2, how about the dividend?	The dividend is 24 and the divisor is 3, how about the quotient?
	3 times a number is 300, this number is equal to ?	A number is 7 times 21, this number is equal to ?
	Number A is equal to 150, and number B is 20% more than A. B = ?	Number A is 10.78, and number B is 3 more than B. B = ?
challenging samples in D_{test}^*	In a parking lot, totally 48 cars and motorcycles are parked. Each car has 4 wheels, and each motorcycle has 3 wheels. If there are 20 motorcycles in the parking lot, how many wheels are there in total?	In a parking lot, totally 48 cars and motorcycles are parked. Each car has 4 wheels, and 172 wheels are there in total. If there are 20 motorcycles in the parking lot, how many wheels does a motorcycle have?

sults indicate that most of the challenging samples in D_{train} are too short and lack background descriptions.

Since both the amount and quality of existing challenging samples are not quite satisfactory, we propose knowledge and logical augmentation to generate questions with tiny local variances, which brings in massive high-quality challenging samples.

Testing set analysis A good testing set requires low lexical and template overlap between the testing and training set [10]. Besides, as the ability to disambiguate tiny local variances in questions largely reflects the generalization ability of neural models, the testing set is supposed to have more challenging samples to better evaluate the ability of neural models. As shown in Table 5, we count the number of equation templates that only appeared in the D_{test} (New eq. template in Table 5) to measure the template overlap between the testing and training set. Meanwhile, the textual similarity between questions in the testing set and the training set is calculated with ED-DIST as introduced above. Specifically, for each Q_{te} in the testing set, we calculate a similarity score with $s = \max\{ED - DIST(Q_{te}, Q_{tr})\}, \forall Q_{tr} \in D_{train}$. The challenging samples in the testing set are counted as Sec. 4.1 with the threshold set to 0.9. Results in Table 5 are normalized with respect to the size of the testing set.

As shown in Table 5, the original testing set shares a high template (low new eq. templates) and lexical (high q. similarity) overlap with the training set, and contains limited challenging samples (low num. challenging samples). It indicates that the testing set is to some extent similar to the training set, making it hard to evaluate the actual inference ability of neural models perfectly. Not to mention the ability of neural models dealing with challenging samples, the limited scope of challenging samples indicates D_{test} is not able to evaluate neural models from this aspect.

Table 5: Comparison of the two testing sets.

	New eq. template	Mean q. similarity	Num. challenging samples		
			BLEU	ROUGE-L	ED-DIST
D_{test}	0.0401	0.694	0.0211	0.0361	0.0201
D_{test}^*	0.155	0.593	0.574	0.282	0.0421

Considering the weaknesses mentioned above, we manually labeled an additional testing set D_{test}^* with 380 samples in total which contains a great deal of **high-quality** challenging samples (Table 4 shows a pair of example challenging sample in D_{test}^*). Besides, the additional testing set D_{test}^* holds lower lexical and template overlap with D_{train} , and also contains more challenging samples as shown in Table 5.

4.2 Experimental Setup

Dataset We conduct our experiments on Math23k [25]. Besides training with the whole training set D_{train} , to evaluate the performance of our augmentation approaches on different data sizes, we randomly picked three training sets of sizes 500, 2k, and 20k (the whole training set) from the D_{train} as what have done in [26]. The neural models are evaluated on both original testing set D_{test} and the manually labeled D_{test}^* .

Neural model \mathcal{M} We evaluate s.-based aug. strategies with three most typical MWP neural models.

- **Vanilla seq2seq** (marked as seq2seq). In this paper, we adopt a seq2seq model as [25] whose encoder is a 2-layer BiGRU and the decoder is a 2-layer LSTM. Besides, to help the model learn the slight variation of our augmented data, we utilize the attention mechanism before the feed-forward network.

$$\alpha_i = \frac{e^{V \tanh(W_1 h_{n_i} + W_2 \hat{h}_t + b)}}{\sum_{j=1}^{|\mathcal{N}|} e^{V \tanh(W_1 h_{n_j} + W_2 \hat{h}_t + b)}}$$

$$h_{\mathcal{N}} = \sum_{i=1}^{|\mathcal{N}|} \alpha_i h_{n_i}$$

$$P(y_t | y_0, \dots, y_{t-1}) = f(\hat{h}_t \oplus h_{\mathcal{N}})$$

V, W_1, W_2, b are all parameters, and h_{n_i} is the encoder hidden vector of the quantity n_i , while \hat{h}_t refers to the decoder hidden vector of the time step t .

- **GTS** [29] is a tree-based neural model which has outperformed previous works significantly.
- **Graph2Tree** [33] is the state-of-the-art neural model for MWP.

Baseline models for comparison We compare the performances of the three neural models trained with s.based aug. strategies with an extensive set of related work. **Math-EN** [22] proposed equation normalization based on a vanilla seq2seq model to effectively reduce the target space. **TRNN** [23] applied a seq2seq model to predict a tree-structured template in a bottom-up manner. **GROUP-ATT** [13] proposed a group attention mechanism to extract intra-relation features. **AST-Dec** [16] proposed to generate abstract syntax tree of the equation in a top-down manner.

Baseline augmentation approach We also compare the specifically designed s.based aug. with one of the most popular text augmentation strategy back translation [31], which is used to generate paraphrase of original sentence. The original sentence is first translated into a pivot language, and back to its original language. Other augmentation approaches such as synonymy replacement [26], generation-based methods [7], etc. are omitted here due to the intolerable noise for mathematical scenario.

Implementation details The baseline models are trained with D_{train} while the augmentation models are trained with $D_{train} \cup D_{aug}$. All models are evaluated on the same testing set D_{test} . To verify the ability of dealing with challenging samples, the three neural models trained with s.based aug. are additionally evaluated on D_{test}^* . Our evaluation metric is answer accuracy, which is calculated by comparing the answer of the predicted equation and the ground truth one rather than comparing the equation sequence. The parameters of the vanilla seq2seq model are set as [25]: The hidden units of both the encoder and decoder are 512. The word embedding dimension is set to 50 and the dropout for GRU and LSTM are set to 0.5. The number of epochs and mini-batch size are 80 and 32 respectively. We adopt an early stop policy after the accuracy of the validation set not increasing for 10 epochs. Adam optimizer [8] is used with learning rate set to 0.001, $\beta_1 = 0.9$ and $\beta_2 = 0.999$, and the learning rate is halved every 10 epochs. Settings of GTS and Graph2Tree are the same as that in [29] and [33].

4.3 Results

Comparison results Table 6 shows the answer accuracy of neural models trained with s.based aug. strategies compared with back translation and various baseline models. It’s obvious that specifically designed s.based.aug. performs much better than back translation on MWP task. Since MWP has a strict requirement of precision, the noises brought in by back translation are sometimes unacceptable. We notice that many of the generated questions by the means of back translation are inconsistent with their equations. Trained with s.based aug., the accuracy of the seq2seq model increases by 5.1%, which is able to beat many other complex neural models with 71.2% answer accuracy. Performance gains of simple neural networks like seq2seq are much more than that of complex neural models like GTS and Graph2Tree. Our explanation is that complex models have

Table 6: **Answer accuracy** (%) of neural models evaluated on the two testing sets. We evaluate s.based aug. on three most typical neural models. The improved performances w/ augmentation are shown in bold.

Model	D_{test}			D_{test}^*			
	w/o aug.	w/ s.based aug.	w/ back trans.	All samples		Challenging samples	
				w/o aug.	w/ s.based aug.	w/o aug.	w/ s.based aug.
Math-EN	66.7	-	-	-	-	-	-
TRNN	66.9	-	-	-	-	-	-
AST-Dec	69.0	-	-	-	-	-	-
GROUP-ATT	69.5	-	-	-	-	-	-
seq2seq	66.1	71.2 $\uparrow 5.1$	66.4 $\uparrow 0.3$	30.0	53.2 $\uparrow 23.2$	11.4	27.3 $\uparrow 15.9$
GTS	75.6	76.1 $\uparrow 0.5$	75.5 $\downarrow 0.1$	33.2	53.2 $\uparrow 20.0$	11.4	36.4 $\uparrow 25.0$
Graph2Tree	77.4	77.0 $\downarrow 0.4$	76.9 $\downarrow 0.5$	30.3	52.4 $\uparrow 22.1$	11.4	40.9 $\uparrow 29.5$

better inference ability, making them able to learn more useful features from limited data, while simpler models have to learn these features from more diverse data. However, as analysed in Sec. 4.1, considering the original testing set D_{test} holds a high overlap with the training set, we reasonably suspect that the performance decrease is caused by the complex models likely to be overfitting and learns some dataset-specific features before augmentation. And the massive challenging samples brought in by our augmentation strategies may confuse the neural models, since none of them have specially designed to handle the tiny local variances. We’ll further verify our hypothesis in Sec. 4.4.

As described in Sec. 4.1, the manually labeled testing set D_{test}^* holds lower overlap with the training set and contains 380 samples with a significant ratio of challenging samples. The three neural models trained w/ and w/o s.based aug. are tested on D_{test}^* to further evaluate the generalization ability of them. As shown in Table 6, we calculate the answer accuracy for all the questions and the challenging samples respectively. The challenging samples are filtered as 4.1 with the threshold set to 0.9. For the accuracy of the challenging samples, a challenging sample Q_i is viewed to be correctly answered only if all its similar questions Q_j has been correctly answered.

As shown in Table 6, s.based aug. strategies significantly boost the performances of all the three neural models with more than 20% increments. Besides, the ability of neural models dealing with discrete tiny local variances has also been largely improved. The results suggest that the s.based aug. strategies successfully benefit the generalization ability of the neural models. However, since existing MWP neural models hardly considered the discrete local variances which lead to respectively low accuracy, there is still a large space for future work to improve the ability of neural models dealing with such challenges.

Ablation study As illustrated in Tabel 7, both knowledge and logic guided augmentation methods contribute to the performance gains. And logical aug-

Table 7: The ablation study on datasets with different sizes. The best performance on each dataset is shown in bold.

Model	Training Set Size		
	500	2k	20k
seq2seq	9.52	31.6	66.1
seq2seq +knowledge	12.3 \uparrow 2.78	33.5 \uparrow 1.90	67.0 \uparrow 0.90
seq2seq +logic	13.0 \uparrow 3.48	34.1 \uparrow 2.50	67.3 \uparrow 1.20
seq2seq +s.based aug.	16.7 \uparrow 7.18	36.0 \uparrow 4.40	71.2 \uparrow 5.10

Table 8: Examples of error cases that are predicted wrongly after augmentation.

$Q_{tr} \in D_{train}$	$Q_{te} \in D_{test}$	pred. Eq of Q_{te}	tgt. Eq of Q_{te}
A dictionary is priced at n_1 yuan, and after n_2 of the sale, the price is still n_3 higher than the purchase price. The purchase price of this dictionary is ?	A dictionary is priced at n_1 yuan, and after n_2 of the sale, it will earn n_3 . The purchase price of this dictionary is ?	$x = n_1 \times (1 + n_3) \div n_2$	$x = n_1 \times n_2 \div (1 + n_3)$
A project can be completed in n_1 days if n_2 people come to do it. If n_3 people do it, how many days can it be done?	n_1 workers will complete a project within n_2 days. If it takes n_3 days to complete, how many workers are needed?	$x = 1 \div (n_1 \times n_2) \div n_3$	$x = (n_1 \times n_2) \div n_3$

mentation performs better than knowledge augmentation on all three datasets. This gap is more obvious on smaller datasets. We guess that on smaller datasets, the lack of problem types is more severe, thus enriching the equation templates is more needed. Besides, the results have shown that smaller datasets benefit more from the augmentation strategies than larger ones.

4.4 Case study

In this section, we'll analyze the questions that are predicted correctly before the neural models trained with augmented data but are predicted wrongly afterwards. For each error case, we search for a most similar question in D_{train} to see whether these problems have occurred during training phase. And it turns out that most of the error cases have a nearly the same question in D_{train} as shown in Table 8. The $Q_{te} \in D_{test}$ are error predicted questions in the testing set. $Q_{tr} \in D_{train}$ are their similar questions found in the training set. In the first row of Table 8, the target Eq of the Q_{tr} is $x = n_1 \times n_2 \div (1 + n_3)$. Accord-

ing to the logical augmentation described in Sec. 3.3, one of the Eq_L could be $(\hat{n} \times (1 + n_3)) \div n_2$, which is equivalent to the mistakenly predicted equation in template. This result further support our hypothesis in Sec. 4.3 and explain the performance decreases of complex models on D_{test} .

5 Conclusion & Discussion

In this paper, we argue that discrete tiny local variances are a big challenge for neural models which previous works have ignored. And we propose a set of novel semantic-based data augmentation methods to supplement existing datasets with challenging samples. Both augmentation methods we proposed are able to generate coherent questions with consistent labels and largely diversify both textual descriptions and equation templates. Extensive experimental results have shown the necessity and effectiveness of the approaches we proposed. Besides, the idea we proposed could also be transferred to other math-related tasks like MWP generation.

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