

Are Edge Weights in Summary Graphs Useful? - A Comparative Study

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Abstract

Which one is better between two representative graph summarization models with and without edge weights? From web graphs to online social networks, large graphs are everywhere. Graph summarization, which is an effective graph compression technique, aims to find a compact *summary graph* that accurately represents a given large graph. Two versions of the problem, where one allows edge weights in summary graphs and the other does not, have been studied in parallel without direct comparison between their underlying representation models. In this work, we conduct a systematic comparison by extending three search algorithms to both models and evaluating their outputs on eight datasets in five aspects: (a) reconstruction error, (b) error in node importance, (c) error in node proximity, (d) the size of reconstructed graphs, and (e) compression ratios. Surprisingly, using unweighted summary graphs leads to outputs significantly better in all the aspects than using weighted ones, and this finding is supported theoretically. Notably, we show that a state-of-the-art algorithm can be improved substantially (specifically, $8.2\times$, $7.8\times$, and $5.9\times$ in terms of (a), (b), and (c), respectively, when (e) is fixed) based on the observation.

1 Introduction and Related Works

Relationships between objects, such as friendships in online social networks, co-appearance of tags, and hyperlinks between web pages, are universal. They are naturally represented as graphs, whose sizes have grown at a tremendous rate due to advances in web technology. For example, the number of web pages (i.e., nodes in web graphs) increased by about $500\times$ in the past two decades.

Graph compression is a useful technique for efficient utilization of such large graphs. Many techniques have been developed for various purposes, including storage [5, 7, 9, 15, 17, 19, 23, 26, 28], query processing [6, 13, 18, 25], influence analysis [21, 22], pattern mining [16, 27], anomaly/outlier detection [3, 8], privacy preservation [29, 30], visualization [10, 11], and representation learning [12, 32]. We refer to surveys [4, 20] for details of them. Their common goal is to find a compact representation that exactly or approximately describes a given graph.

Among them, we focus on *graph summarization* [2, 13, 14, 15, 17, 18, 23, 25, 28], whose objective is to find a concise *summary graph* G' that accurately describes a given large graph G , or equivalently, concise G' from which we can restore a graph close to G .¹ Each node in G' is interpreted as a group of nodes in G , and each edge in G' is interpreted as the presence of edges between all pairs of nodes in two groups. Since the output G' is in the form of a graph, other graph compression methods can be applied to G' for further compression [28]. That is, graph summarization can be used as a preprocessing step of other compression methods. Moreover, a wide range of graph algorithms can be approximately executed on G' without full reconstruction (see Appendix A and [25]).

There are two representative graph summarization models: a summary graph with edge weights [2, 17, 18, 25] and one without edge weights [13, 14, 15, 23, 28]. While the latter is typically used with edge corrections

¹While we use the term “graph summarization” to refer to this specific way of compression, the term has also been used more generally, as surveyed in [20].

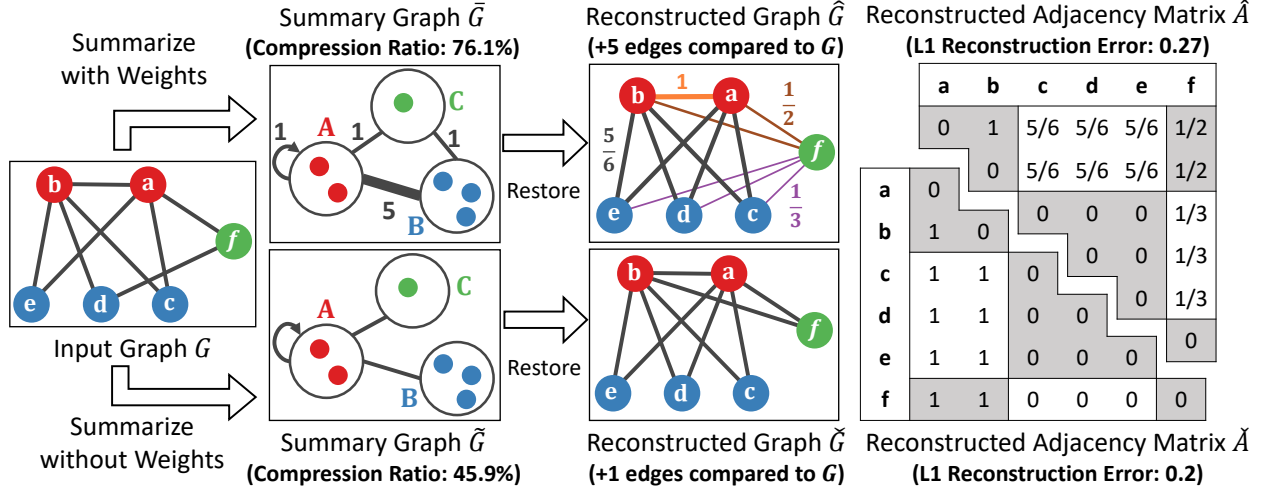


Figure 1: Examples of weighted and unweighted graph summarization and reconstruction. The summary graph \tilde{G} without edge weights is more concise with smaller reconstruction error and reconstruction size than \tilde{G} with edge weights.

for lossless compression, this work focuses on G' . While a number of search algorithms aiming at finding a high-quality summary graph under a given constraint have been developed for each model, there was no systematic comparison between the two models.

Which one is better between the two graph summarization models? Are edge weights in summary graphs useful? For a systematic comparison between the two models, we extend three search algorithms [15, 17, 18] to both models and evaluate their outputs in eight real-world graphs in five aspects: (a) reconstruction error, (b) error in node importance [24], (c) error in node proximity [31], (d) the number of edges in reconstructed graphs, and (e) compression ratios.

Counterintuitively, we find out that using unweighted summary graphs gives a significantly better trade-off among (a)-(e) than using weighted ones, regardless of search algorithms and datasets (See Fig. 1 for an example). Notably, adapting a state-of-the-art algorithm for the weighted model [17] to the unweighted model leads to $8.2\times$, $7.8\times$, and $5.9\times$ improvements in terms of (a), (b), and (c) (when (e) is fixed) and $2.2\times$ improvements in terms of (d) (when (a) is similar).

Our contributions are three-fold:

- **Systematic Comparison:** We conduct a systematic comparison between two extensively-studied graph summarization models using three search algorithms, eight datasets, and five evaluation metrics.
- **Unexpected Observation:** Our comparison leads to a surprising observation that using unweighted models is significantly better than using weighted ones in all considered aspects. We support this finding theoretically (see Theorem 1).
- **Improvement of the State of the Art:** By exploiting the observation, we can improve a state-of-the-art algorithm [17] substantially in all considered aspects (see Figs. 2-4).

Reproducibility: The source code and the datasets are available at [1].

Roadmap: In Sect. 2, we introduce graph summarization models. In Sect. 3, we define problems and present algorithms. In Sect. 4, we provide empirical results. In Sect. 5, we present theoretical results. In Sect. 6, we offer conclusions.

2 Graph Summarization Models

We introduce weighted and unweighted graph summarization models, which are compared throughout this work. See Table 1 for frequently-used symbols.

Table 1: Symbols and definitions.

Symbol	Definition	Symbol	Definition
V	set of subnodes	$G = (V, E)$	input graph
E	set of subedges	$\overline{G} = (S, P, \omega)$	weighted summary graph
S	set of supernodes	$\tilde{G} = (S, P)$	unweighted summary graph
P	set of superedges	$\hat{G} = (V, \hat{E}, \hat{\omega})$	graph reconstructed from \overline{G}
ω	superedge weight function	$\check{G} = (V, \check{E})$	graph reconstructed from \tilde{G}
\hat{E}, \check{E}	sets of reconstructed edges	A, \hat{A}, \check{A}	adjacency matrix of G, \hat{G} , and \check{G}
$\hat{\omega}$	subedge weight function	E_{AB}	# of subedges between $A, B \in S$
S_i	supernode containing $i \in V$	Π_{AB}	# of subnode pairs between $A, B \in S$

Input Graph. Consider an undirected graph $G = (V, E)$ with a set of *subnodes* $V = \{1, \dots, |V|\}$ and a set of *subedges* $E \subseteq \binom{V}{2}$. We use $A \in \mathbb{R}^{|V| \times |V|}$ to denote its *adjacency matrix*. Each entry $A_{ij} = 1$ if $\{i, j\} \in E$ and $A_{ij} = 0$ otherwise.

2.1 Weighted Graph Summarization Model

Definition. A *weighted summary graph* $\overline{G} = (S, P, \omega)$ of $G = (V, E)$ consists of a set of *supernodes* S , a set of *superedges* P , and a *superedge weight function* ω . The set S is a partition of V . That is, supernodes are disjoint sets of subnodes whose union is V . Each superedge $\{A, B\} \in P$ joins two supernodes $A \in S$ and $B \in S$. The function ω takes each superedge $\{A, B\} \in P$ and returns its *weight* ω_{AB} , which is equal to $E_{AB} := |\{\{i, j\} \in E : i \in A, j \in B\}|$, i.e., the number of subedges in G that join subnodes between $A \in S$ and $B \in S$.

Reconstruction. The *reconstructed graph* $\hat{G} = (V, \hat{E}, \hat{\omega})$ obtained from $\overline{G} = (S, P, \omega)$ consists of the set of subnodes V , the set of reconstructed subedges $\hat{E} \subseteq \binom{V}{2}$, and a subedge weight function $\hat{\omega}$. If we let $S_i \in S$ be the supernode containing each subnode $i \in V$ and let Π_{AB} be the number of possible pairs of subnodes between supernodes A and B . That is, $\Pi_{AB} := \binom{|A|}{2}$ if $A = B$ and $\Pi_{AB} := |A| \cdot |B|$ otherwise. The adjacency matrix $\hat{A} \in \mathbb{R}^{|V| \times |V|}$ of \hat{G} is defined as

$$\hat{A}_{ij} = \hat{\omega}_{ij} := \begin{cases} \frac{\omega_{S_i S_j}}{\Pi_{S_i S_j}}, & \text{if } i \neq j \text{ and } \{S_i, S_j\} \in P, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

2.2 Unweighted Graph Summarization Model

Definition. An *unweighted summary graph* $\tilde{G} = (S, P)$ of G consists of a set of supernodes S and a set of superedges P . Note that, unlike \overline{G} , \tilde{G} does not have the superedge weight function ω .

Reconstruction. The adjacency matrix $\check{A} \in \mathbb{R}^{|V| \times |V|}$ of the graph $\check{G} = (V, \check{E})$ reconstructed from \tilde{G} is defined as

$$\check{A}_{ij} := \begin{cases} 1, & \text{if } i \neq j \text{ and } \{S_i, S_j\} \in P \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

While \check{G} is typically used with edge corrections for lossless compression [15, 23, 28], this work focuses on \hat{G} .

3 Problem Formulation and Algorithms

Based on the graph summarization models, we formulate graph summarization as optimization problems. Then, we present six search algorithms for the problems.

3.1 Optimization Problem Formulation

Given a graph G , we aim to minimize the difference between a reconstructed adjacency matrix A' (i.e., \check{A} or \hat{A}) and the adjacency matrix A of G . Specifically, we aim to minimize the L_p reconstruction error, i.e.,

$$RE_p(A, A') := \|A - A'\|_p, \quad (3)$$

Table 2: The outlines of the considered search algorithms are given in Algorithms 1 and 2, and the details of each algorithm are provided in the table below.

Algorithm	Outline	G'	T	$size()$	$groups()$	$loss()$	$sparsify?()$
k-Grass (W)	Alg. 1	\bar{G}	Infinite	$ S $	$\{S\}$	Eq. (3)	False
k-Grass (U)	Alg. 1	\tilde{G}	Infinite	$ S $	$\{S\}$	Eq. (3)	False
SSumM (W)	Alg. 1	\bar{G}	Finite	Eq. (4)	Clusters [6]	Eq. (6)	True
SSumM (U)	Alg. 1	\tilde{G}	Finite	Eq. (5)	Clusters [6]	Eq. (7)	True
MoSSo-Lossy (W)	Alg. 2	\bar{G}	N/A	N/A	Clusters [6]	Eq. (6)	N/A
MoSSo-Lossy (U)	Alg. 2	\tilde{G}	N/A	N/A	Clusters [6]	Eq. (7)	N/A

Algorithm 1: Batch computation of a summary graph

Input: (1) input graph G , (2) budget k , and (3) # iters: T
Output: summary graph G'
1 initialize G' ; $t \leftarrow 1$
2 **while** $size(G') > k$ and $t < T$ **do**
3 $C \leftarrow groups(); t \leftarrow t + 1$
4 **for each** $C_i \in C$ **do**
5 merge one or more pairs within C_i to minimize $loss()$
6 **if** $sparsify?()$ **then** sparsify G' until $size(G') \leq k$
7 **return** G'

Algorithm 2: Incremental update of a summary graph

Input: (1) summary graph G' and (2) change in $\{src, dst\}$
Output: updated G'
1 $C \leftarrow groups();$
2 **for each** $u \in \{src, dst\}$ **do**
3 $\hat{N}_u \leftarrow$ sample neighbors of u
4 **for each** $w \in \hat{N}_u$ **do**
5 $P \leftarrow C' \in C$ where $w \in C'$
6 $v \leftarrow$ draw one in $\hat{N}_u \cap P$
7 **if** $loss()$ drops **then** move w to S_v
8 **return** G'

while constraining the size of the output summary graph G' (i.e., \bar{G} or \tilde{G}) to be at most a given constant. The size can be (a) the number of supernodes in G' [2, 18, 25], (b) the number of superedges in G' , or (c) the size of G' in bits [17].

Size in Bits of Summary Graphs. The size of a weighted summary graph $\bar{G} = (S, P, \omega)$ in bits is defined as

$$size_{bits}(\bar{G}) := 2|P| \log_2 |S| + |P| \log_2 \omega_{max} + |V| \log_2 |S|, \quad (4)$$

where ω_{max} is the largest superedge weight in \bar{G} , and in our experiments in Sect. 4, $\omega_{max} \ll |S|$. The three terms on the right side in Eq. (4) correspond to $|P|$ superedges in bits, $|P|$ superedge weights in bits, and the supernode membership of $|V|$ subnodes in bits, respectively. Similarly, the size of an unweighted summary graph \tilde{G} in bits is defined as

$$size_{bits}(\tilde{G}) := 2|P| \log_2 |S| + |V| \log_2 |S|. \quad (5)$$

3.2 Weighted Graph Summarization Algorithms

We introduce three searching algorithms for finding a weighted summary graph $\bar{G} = (S, P, \omega)$ of the input graph G . See Algorithms 1 and 2 for their outlines and Table 2 for details.

k-Grass. K-GRASS [18] first initializes the set S of supernodes so that each subnode forms a singleton supernode. Then, it repeats greedily merging a supernode pair until $|S|$ reaches the target number (i.e., the given constraint). Specifically, in each step, among all supernode pairs, K-GRASS merges a pair whose merger

Table 3: Summary of the eight real-world graphs used in the paper. They are obtained from emails (EE), collaborations (DB), co-purchases (A6), computer networks (SK), online social networks (LJ), and hyperlinks (WS, DP, and WL).

Name	# Nodes	# Edges	Name	# Nodes	# Edges
Email-Enron (EE)	36,692	183,831	DBLP (DB)	317,080	1,049,866
Amazon-0601 (A6)	403,394	2,443,408	WebSmall (WS)	325,557	2,738,969
Skitter (SK)	1,696,415	11,095,298	LiveJournal (LJ)	3,997,962	34,681,189
DBPedia (DP)	18,268,991	126,890,209	WebLarge (WL)	18,483,186	261,787,258

increases Eq. (3) least. During the entire process, K-GRASS creates a superedge between each supernode pair A and B (i.e., $\{A, B\} \in P$) if and only if $E_{AB} > 0$.

SSUMM. SSUMM [17] initializes S as in K-GRASS. Then, SSUMM divides S into disjoint groups of supernodes with similar connectivity to find pairs to be merged efficiently. After that, in each group, SSUMM repeats merging a pair of supernodes whose merger decreases Eq. (6) most.

$$size_{bits}(\bar{G}) + \sum_{\{A,B\} \in P} \Pi_{AB} \cdot \mathcal{H}\left(\frac{E_{AB}}{\Pi_{AB}}\right) + \sum_{\{A,B\} \notin P} 2E_{AB} \log_2 |V|, \quad (6)$$

where $\mathcal{H}(\cdot)$ is the entropy function. Eq. (6) considers both the size of a summary graph and the reconstruction error. Specifically, the second term is the number of bits for exactly restoring the subedges between supernodes that are joined by superedges, and the third term is that for the other subedges (see [17] for details). During the process, the superedge between each supernode pair exists only when it decreases Eq. (6). If $size_{bits}(\bar{G})$ (i.e., Eq. (4)) cannot satisfy the given constraint (i.e., the target size) within the given number of iterations, SSUMM sparsifies \bar{G} greedily based on Eq. (3) to satisfy the constraint.

MoSSo-Lossy. MoSSo-LOSSY is a lossy variant of MoSSo [15], which is a lossless graph compression algorithm. While processing subedges incrementally, it updates $\bar{G} = (S, P, \omega)$. Specifically, for each subedge $\{u, v\}$, it samples a fixed number of neighbors of u and v . Then, for each such neighbor w , MoSSo-LOSSY moves w from S_w to the supernode which another sampled subnode with similar connectivity belongs to if this change decreases Eq. (6). As in SSUMM, for each pair of supernodes, a superedge joins them only when it decreases Eq. (6).

3.3 Unweighted Graph Summarization Algorithms

We extend the above algorithms for obtaining an unweighted summary graph $\tilde{G} = (S, P)$ of the input graph G . The differences are highlighted in Table 2 with the outlines in Algorithms 1 and 2.

k-Grass (Unweighted). This variant is different from K-GRASS in that Eq. (2) is used, instead of Eq. (1), when Eq. (3) is computed. Moreover, for each supernode pair, the superedge between them exists only when it decreases Eq. (3).

SSumM (Unweighted). Instead of Eq. (6) used in SSUMM, this variant uses Eq. (7), whose second term is the number of bits for exactly restoring the subedges between supernodes that are joined by unweighted superedges.

$$size_{bits}(\tilde{G}) + \sum_{\{A,B\} \in P} 2(\Pi_{AB} - E_{AB}) \log_2 |V| + \sum_{\{A,B\} \notin P} 2E_{AB} \log_2 |V|. \quad (7)$$

MoSSo-Lossy (Unweighted). This variant uses Eq. (7), instead of Eq. (6), which is used in MoSSo-LOSSY.

4 Experiments

We review our experiments for comparing weighted and unweighted graph summarization in five aspects. We describe the settings and then present the results.

4.1 Experimental Settings

Machines: We performed our experiments on a desktop with a 3.80GHZ Intel i7-10700K CPU and 64GB memory.

Datasets: We used the eight datasets summarized in Table 3.

Search Algorithms: We used the six algorithms described in Sect. 3. We implemented them commonly in OpenJDK 12 and set their target size to $\{0.1, 0.2, \dots, 0.9\}$ of the size in the input graph. We fixed T to 20 in both versions of SSUMM. We excluded [15, 23, 28] from the comparison since they assume extra components (e.g., edge corrections) in addition to a summary graph.

Evaluation Metric: Given the input graph $G = (V, E)$ and a summary graph G' (i.e., \hat{G} or \tilde{G}), the compression ratio is defined in bits as $\frac{size_{bits}(G')}{2|E|\log_2|V|}$.

4.2 Results

Reconstruction Error: The L_1 and L_2 reconstruction error (i.e., $p = \{1, 2\}$ in Eq. (3)) is compared in Fig. 2. Unweighted summary graphs described the input graph more accurately (specifically, up to $8.2\times$ when comparing SSUMM and its variant) than weighted ones, when compression ratios were the same.

Error in Node Importance: We used PageRank [24] (with the damping factor 0.85) to measure the importance of subnodes. In Fig. 3(a)-(h), we report the sum of absolute difference between PageRank scores obtained from input and summary graphs (see Appendix A for how to compute PageRank scores on a summary graph). When the compression ratios were the same, unweighted summary graphs maintained the node importance more accurately (specifically, up to $7.8\times$ when comparing SSUMM and its variant) than weighted ones.

Error in Node Proximity: We used Random Walk with Restart (RWR) [31] (with the damping factor 0.95) to measure the proximity between subnodes. For each query node, we compute the RWR scores between the query node and the others on input and summary graphs, and we compute the sum of absolute difference (see Appendix A for how to compute the RWR scores on a summary graph). In Fig. 3(i)-(p), we report the difference averaged over 100 randomly-sampled query nodes. Unweighted summary graphs preserved the proximity between nodes more accurately (specifically, up to $5.9\times$ when comparing SSUMM and its variant) than weighted ones, when the compression ratios were the same.

Size of Reconstructed Graphs: As shown in Fig. 4, when L_1 reconstruction errors were similar, graphs reconstructed from unweighted summary graphs had significantly fewer (specifically, up to $2.2\times$ fewer when comparing SSUMM and its variant) subedges than those reconstructed from weighted ones. When reconstruction errors are similar, fewer reconstructed edges, which lead to faster query processing (see [25] and Appendix A for examples), are preferred.

5 Discussion: Why Can Edge Weights be Harmful?

As answers to this question, we provide an example in Figure 1, and we prove in Theorem 1 that at least when the L_1 reconstruction error is the objective, the superedge weight function ω is not useful and even harmful. The theorem, however, is not generalized to other objectives.

Theorem 1 *Consider a graph G and its weighted summary graph $\bar{G} = (S, P, \omega)$. Assume ω is not fixed but variable. When $RE_1(A, A')$ is minimized, for each superedge $\{A, B\} \in P$, the weight $\frac{\omega_{AB}}{\Pi_{AB}}$ of subedges reconstructed from it is either 1 or 0, just as in Eq. (2), where an unweighted summary graph is used.*

Proof 1 *The L_1 reconstruction error can be written as follows:*

$$RE_1(A, A') = \sum_{\{A, B\} \in P} \sum_{\{i, j\} \in \Pi_{AB}} |A_{ij} - A'_{ij}| + \sum_{\{A, B\} \notin P} \sum_{\{i, j\} \in \Pi_{AB}} |A_{ij}|,$$

Since the second term on the right side does not depend on ω , we focus on the first term where

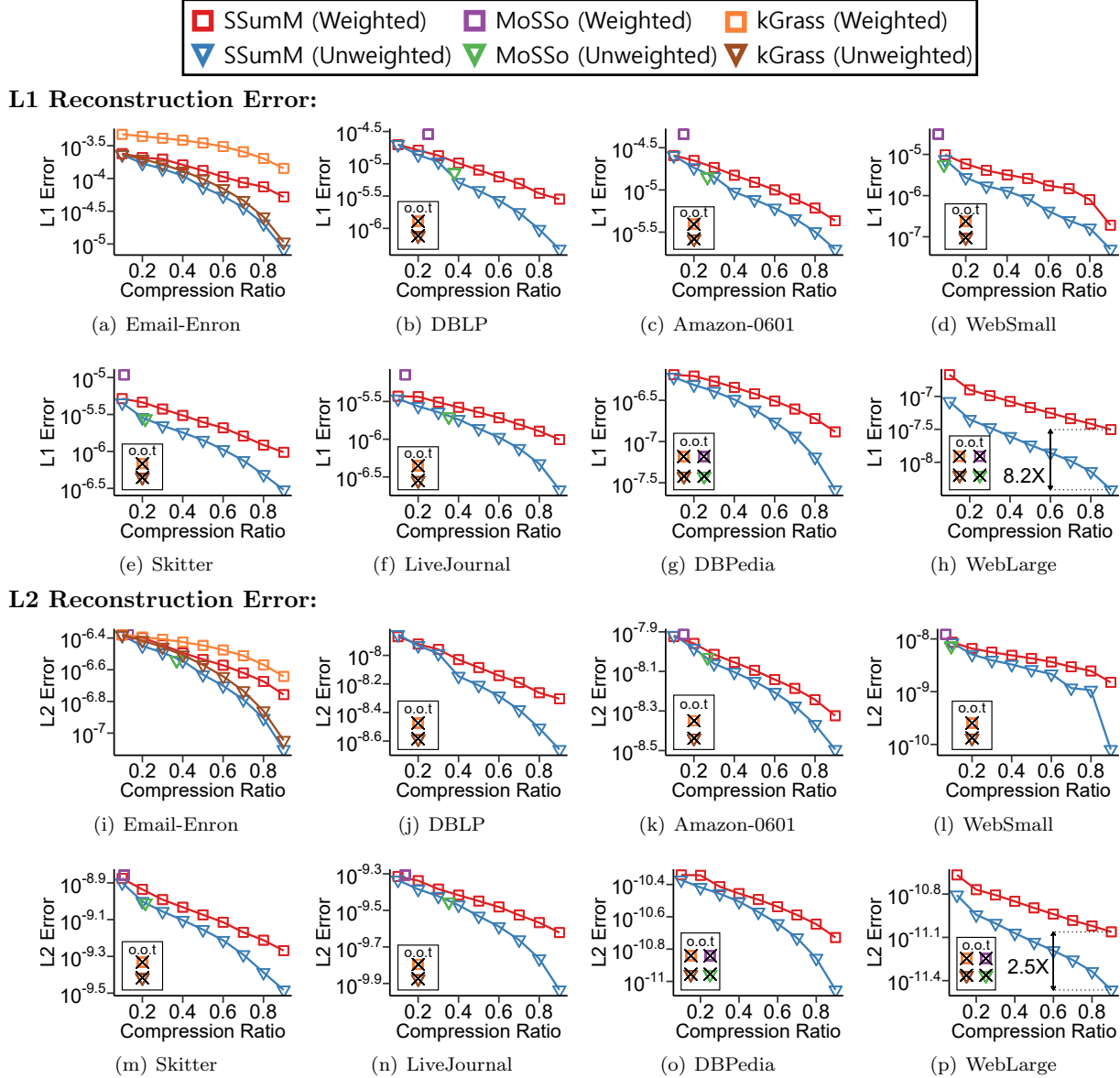


Figure 2: The reconstruction error is significantly lower in unweighted graph summarization than in weighted summarization. **o.o.t.**: out of time (≥ 48 hours).

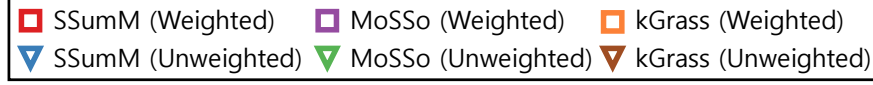
$$\sum_{\{i,j\} \in \Pi_{AB}} |A_{ij} - A'_{ij}| = E_{AB} \left| 1 - \frac{\omega_{AB}}{\Pi_{AB}} \right| + (\Pi_{AB} - E_{AB}) \left| 0 - \frac{\omega_{AB}}{\Pi_{AB}} \right|. \quad (8)$$

Note that Eq. (8) is strictly larger when $\omega_{AB} > \Pi_{AB}$ than when $\omega_{AB} = \Pi_{AB}$. Moreover, Eq. (8) is strictly larger when $\omega_{AB} < 0$ than when $\omega_{AB} = 0$. Thus, for the purpose of minimization, we can focus on when $\omega_{AB} \in [0, \Pi_{AB}]$, and thus Eq. (8) can be rewritten as

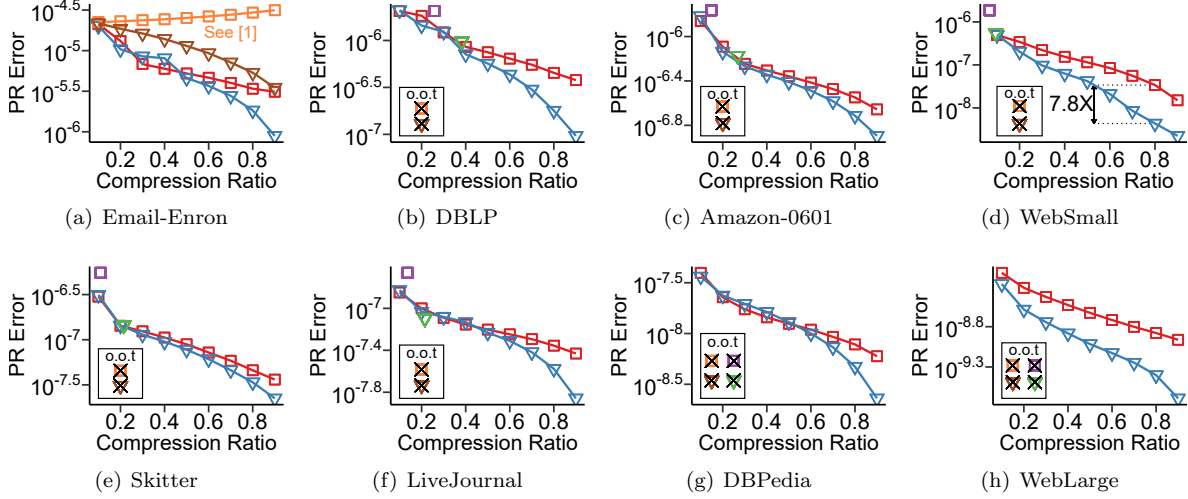
$$\sum_{\{i,j\} \in \Pi_{AB}} |A_{ij} - A'_{ij}| = E_{AB} + \frac{\omega_{AB}}{\Pi_{AB}} (\Pi_{AB} - 2E_{AB}). \quad (9)$$

We consider two cases depending on the sign of $\Pi_{AB} - 2E_{AB}$.

- **Case 1.** $\Pi_{AB} < 2E_{AB}$: Since the derivative w.r.t. ω_{AB} is negative between 0 and Π_{AB} , Eq. (9) is minimized when $\omega_{AB} = \Pi_{AB}$, i.e., when A'_{ij} is 1.



Error in Node Importance (Spec., PageRank Scores [24]):



Error in Node Proximity (Spec., Random Walk with Restart Scores [31]):

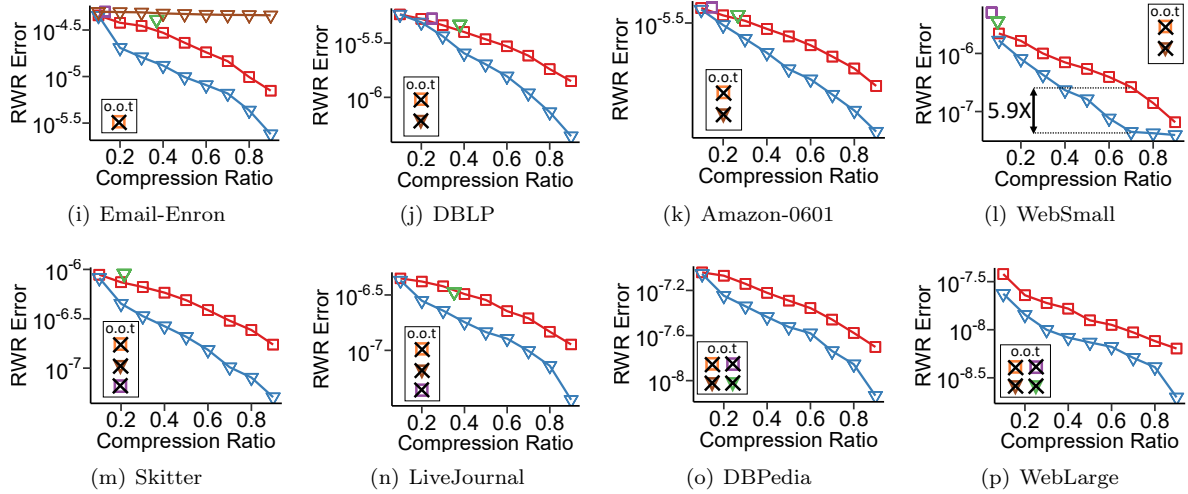


Figure 3: Importance of nodes and proximity between nodes are preserved more accurately in unweighted graph summarization than in weighted summarization. **o.o.t.**: summarization or RWR computation ran out of time (≥ 48 hours).

- **Case 2.** $\Pi_{AB} \geq 2E_{AB}$: Since the derivative w.r.t. ω_{AB} is non-negative between 0 and Π_{AB} , Eq. (9) is minimized when $\omega_{AB} = 0$, i.e., when A'_{ij} is 0,

Therefore, when Eq. (8) is minimized, for each superedge $\{A, B\} \in P$, the weight of subedges reconstructed from it (i.e., $\frac{\omega_{AB}}{\Pi_{AB}}$) is either 1 or 0, as in the unweighted model. ■

6 Conclusion and Future Directions

In this work, we conducted a systematic comparison between two extensively-studied graph summarization models with and without superedge weights. To this end, we extended three search algorithms to both models (Algorithms 1-2 and Table 2) and compared their outputs from eight real-world graphs in five aspects (Figs. 2-4). Our empirical comparison revealed a surprising finding that removing superedge weights leads to significant improvements in all five aspects, as in the example in Fig. 1. Then, we developed a

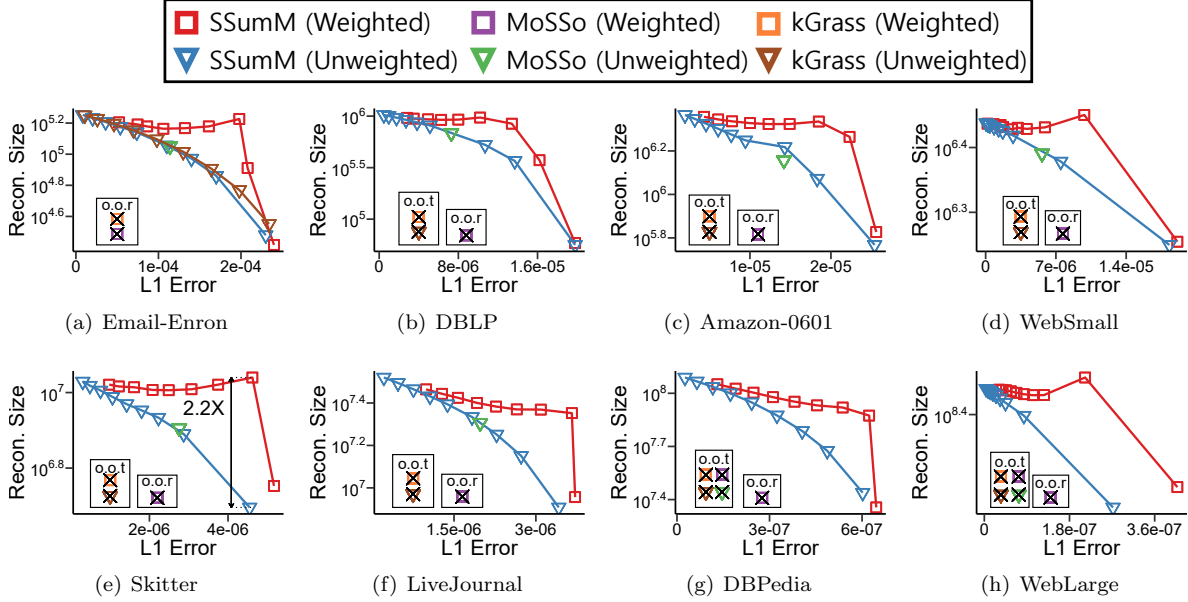


Figure 4: When reconstruction errors are similar, more concise graphs are reconstructed from unweighted summary graphs than from weighted summary graphs. **o.o.t.**: out of time (≥ 48 hours). **o.o.r.**: out of range with too many subedges.

theoretical analysis to shed light on this counterintuitive observation (Theorem 1). Noteworthy, we showed in Figs. 2-4 that SSumM [17], a state-of-the-art graph-summarization algorithm, can be improved substantially (specifically, up to $8.2\times$, $7.8\times$, and $5.9\times$ in terms of reconstruction error, error in node importance, and error in node proximity, respectively, when the compression ratio was fixed; and $2.2\times$ in terms of the size of reconstructed graphs, when the reconstruction error was similar) based on the observation. As future work, we would like to explore (a) better superedge weighting schemes and (b) combinations of weighted and unweighted superedges.

Reproducibility: The source code and the datasets are available at [1].

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A Appendix: Graph Algorithms on Summary Graphs

Given a summary graph G' (i.e., \bar{G} or \tilde{G}) and a query node $u \in V$, an approximate set of neighbors of u can be retrieved from G' without reconstructing the entire graph, as described in Algorithm 3. In other words, neighborhood queries can be answered approximately from G' . A wide range of graph algorithms (e.g., DFS, BFS, PageRank, and Dijkstra’s) access the input graph only through neighborhood queries, and thus they can be executed approximately on summary graphs without restoring the entire graph. See Algorithm 4 for examples.

Algorithm 3: $\text{getNeighbors}(G', u)$

Input: (1) summary graph G' (\bar{G} or \tilde{G}) and (2) query subnode u
Output: approximate neighborhood \hat{N}_u of u with subedge weights

```

1  $\hat{N}_u \leftarrow \emptyset$ 
2 for each  $A$  where  $\{A, S_u\} \in P$  do
3   for each  $v \in A$  do
4     if  $v \neq u$  then
5       if  $G' = \bar{G}$  then
6          $\text{add } v \text{ to } \hat{N}_u \text{ with weight } \frac{\omega_{AS_u}}{\Pi_{AS_u}}$ 
7       if  $G' = \tilde{G}$  then
8          $\text{add } v \text{ to } \hat{N}_u \text{ with weight } 1$ 
9 return  $\hat{N}_u$ 
```

Algorithm 4: PageRank [24] and Random Walk with Restart (RWR) [31] on G'

Input: (1) summary graph G' , (2) damping factor d , and (3) (only for RWR) query subnode u
Output: score vector $r^{new} \in \mathbb{R}^{|V|}$

```

1  $V \leftarrow \bigcup_{A \in S} A$ 
2  $r^{old} \leftarrow \mathbf{0}$ ;  $r^{new} \leftarrow \frac{1}{|V|} \cdot \mathbf{1}$ 
3  $q \leftarrow \frac{1}{|V|} \cdot \mathbf{1}$ 
4 (only for RWR)  $q \leftarrow \mathbf{0}$ ;  $q_u \leftarrow 1$ 
5 while  $r^{new} \neq r^{old}$  do
6    $r^{old} \leftarrow r^{new}$ ;  $r^{new} \leftarrow \mathbf{0}$ 
7   for each  $v \in V$  do
8      $\hat{N}_v \leftarrow \text{getNeighbors}(G', v)$ 
9      $w_{sum} \leftarrow \text{sum of weights in } \hat{N}_v$ 
10    for each neighbor  $l$  with weight  $w$  in  $\hat{N}_v$  do
11       $r_l^{new} \leftarrow r_l^{new} + \frac{w}{w_{sum}} r_v^{old}$ 
12     $r^{new} \leftarrow d \cdot r^{new} + (1 - d) \cdot \sum_{v \in V} r_v^{new} \cdot \mathbf{q}$ 
13 return  $r^{new}$ 
```

$\triangleright \mathbf{0}$ is the zero vector of size $|V|$
 $\triangleright \mathbf{1}$ is the one vector of size $|V|$