On the undecidability of the Panopticon detection problem

V. Liagkou^{1,4}, P.E. Nastou⁵, P. Spirakis², and Y.C. Stamatiou^{1,3}

¹ Computer Technology Institute and Press - "Diophantus", University of Patras Campus, 26504, Greece

² Department of Computer Science, University of Liverpool, UK and Computer Engineering and Informatics Department, University of Patras, 26504, Greece

³ Department of Business Administration, University of Patras, 26504, Greece

⁴ University of Ioannina, Department of Informatics and Telecommunications, 47100 Koatakioi Arta, Greece

⁵ Department of Mathematics, University of the Aegean, Applied Mathematics and Mathematical Modeling Laboratory, Samos, Greece

e-mails: liagkou@cti.gr, pnastou@aegean.gr, P.Spirakis@liverpool.ac.uk, stamatiu@ceid.upatras.gr

Abstract. The Panopticon (which means "watcher of everything") is a well-known structure of continuous surveillance and discipline proposed by Bentham in 1785. This device was, later, used by Foucault and other philosophers as a paradigm and metaphor for the study of constitutional power and knowledge as well as a model of individuals' deprivation of freedom. Nowadays, technological achievements have given rise to new, non-physical (unlike prisons), means of constant surveillance that transcend physical boundaries. This, combined with the confession of some governmental institutions that they actually collaborate with these Internet giants to collect or deduce information about people, creates a worrisome situation of several co-existing Panopticons that can act separately or in close collaboration. Thus, they can only be detected and identified through the expense of (perhaps considerable) effort. In this paper we provide a theoretical framework for studying the detectability status of Panopticons that fall under two theoretical, but not unrealistic, definitions. We show, using Oracle Turing Machines, that detecting modern day, ICT-based, Panopticons is an undecidable problem. Furthermore, we show that for each sufficiently expressive formal system, we can effectively construct a Turing Machine for which it is impossible to prove, within the formal system, either that it is a Panopticon or it is not a Panopticon.

Keywords: Formal Methods \cdot Security \cdot Privacy \cdot Undecidability \cdot Panopticon \cdot Turing Machine \cdot Oracle Computations

1 Introduction

In 1785, the English philosopher and social theorist Jeremy Bentham (see [1]) envisaged an, admittedly, unprecedented (for that era) institutional punishment establishment, the *Panopticon*. The architecture of this establishment consisted of a circular building dominated by an "observation tower" in the center of which a single guard was continuously watching the inmates, imprisoned in cells arranged around the circular building. Standing on the observation tower, the prison's inspector was able to observe the interior of the cells at any time. Moreover, the prisoners, themselves, could never be able to see the inspector, who remained for ever "invisible" to them.

In the '70s, Foucault studied, deeply, Bentham's concepts, pointing to the Panopticon as a generic model that denotes a way of defining and discussing *power* relations in terms of their impact on people's everyday life. He, also, described the Panopticon as a mechanism of power enforcement that is reduced to its ideal form: "a figure of political technology that may and must be detached from any specific use" ([6]). According to Foucault, disciplinary power was increasingly permeating, in his era, the social body in schools, factories, hospitals, asylums and military barracks constituting "new prison regimes" in the emergent capitalist society. His own conceptualization and usage of Panopticon has allowed for several more or less metaphorical, yet extensive, usages of the *all-seeing* abilities that the "panopticized" state of affairs offers in diverse core field areas. This was, in retrospect, an indirect reference to the modern day, information technology based, surveillance and people monitoring operations conducted, openly or covertly, by several agencies and organizations worldwide.

Nowadays, our "digital selves" and personal digital assets and information transcend physical bounds and, literally, are diffused over the vast, uncontrollable, Internet territory. This dispersion of our digital assets and personal information provides unlimited opportunities for massive data collection and surveillance of our daily actions by state agencies, intelligence institutions and Internet service providers. Several voices exist that equate this situation to an information era *Panopticon* or state of massive surveillance concept. In particular, contrary to the "brick-and-mortar" Panopticon of Bentham, the surveillance actors often remain invisible and covert since modern surveillance methods and devices are hard to detect, unlike the classical Panopticon physical structure of whose existence and objectives all its subjects are aware.

In this paper we investigate the complexity of detecting Panopticons using the *Turing Machine* formalism of an effective computational procedure. We provide two different, but not unrealistic, theoretical models of a Panopticon and show that there is no algorithm, i.e. Turing machine, that can detect, systematically, all Panopticons under these two definitions. In other words, detecting Panopticons, at least the ones that fall under these two plausible definitions, is an undecidable problem, in principle.

More specifically, the first formal model we examine studies Panopticons whose Panopticon behaviour is manifested through the *execution* of states (actions) that belong in a specific set of states that characterizes Panopticon behaviour. In some sense, since the focal point of this model is the *execution* of states of a particular type, the model captures the *visible behaviour* of the Panopticon, according to the action ir perform, and, thus we call this model *behavioural*.

The second formal model focuses on the *impact* or *consequences* of the actions of the Panopticon and not the actions themselves. In particular, this model captures an essential characteristic of Panopticons, that of acquiring, rather, *effortlessly* information through *surveillance* and *eavesdropping*. We model this characteristic using *Oracle* Turing Machines with the oracle having the role of information acquired "for free" based on surveillance and/or eavesdropping actions. This model is in some sense based on the information that a Panopticon deduces using "free" information and, thus, we call it *deductive*. Essentially, this model focuses on the *semantics* of a Turing Machine, i.e. outcomes of operation, while the first model focuses on the *syntax*, i.e. definition, of a Turing Machine. Last, we show that for any formal system, we can demonstrate a Turing Machine whose Panopoticon status, under the second formal model, cannot be proved within the formal system. That is, no proof can be produced by the formal system that this Turing Machine is a *Panopticon* and no proof that it is not a *Panopticon*. In other words, given any formal system, one can provide a procedure that generates a Turing Machine which is impossible to have its Panopticon or non-Panopticon status within the formal system.

2 Definitions and notation

In this section we briefly state the relevant definitions and notation that will be used in the subsequent sections. We, first, define a simple extension of a Turing Machine, following the notation in [8].

Definition 1 (Turing machines). A Turing machine is an octuple, defined as $M=(Q, Q_{pan}, \Sigma, \Gamma, \delta, q_0, B, F)$ where Q is a finite set of normal operation states, Γ is a finite set called the tape alphabet, where Γ contains a special symbol B that represents a blank, Σ is a subset of $\Gamma - \{B\}$ called the input alphabet, δ is a partial function from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R\}$ called the transition function, $q_0 \in Q$ is a distinguished state called the start state, $F \subset Q$ is a set of final states, and $Q_{pan} \subset Q$, $Q_{pan} \cap F = \emptyset$, is a distinguished set of states linked to Panopticon behaviour. We assume that transitions from states in Q_{pan} do not change the Turing machine's tape contents, i.e. they are purely interactions with the external environment of the Turing machine and can affect only the environment.

Notation-wise, given M we denote by $\langle M \rangle$ its *code*, i.e. an encoding of its description elements as stated in Definition 1 using any fixed alphabet, usually the alphabet $\{0, 1\}$ (binary system). The details can be found in, e.g., [5,8] but they are inessential for our arguments.

One of the main outcomes of Turing's pioneering work [13] was that there exist problems that Turing machines cannot solve. The first, such, problem was the, so called, *Halting problem* (see, also, [4] for an excellent historic account):

The Halting Problem

Input: A string $x = \langle M, w \rangle$ which is actually the encoding (description) of a Turing machine $\langle M \rangle$ and its input w.

Output: If the input Turing M machine halts on w, output True. Otherwise, output False.

The language corresponding to the Halting problem is $L_u = \{ \langle M, w \rangle | w \in L(M) \}$. In other words, the language L_u contains all possible *Turing machine-input* pair encodings $\langle M, w \rangle$ such that w is accepted by M. This is why L_u is also called *universal language* since the problem of deciding whether a given Turing machine M accepts a given input w is equivalent to deciding whether $\langle M, w \rangle \in L_u$. The language L_u was the first language proved to be non-recursive or undecidable by Turing.

In order to discuss Panopticons, we need an important variant of Turing machines, called *oracle* Turing Machines. Such a machine has a special tape on which it can write

queries to which they obtain the answer instantaneously in one step, no matter what query it is.

This type of Turing Machines was, first, discussed, briefly, by Turing himself in [14] under the name *o-machine*. Post developed further this concept in a series of papers [10,11,12] and his collaboration with Kleene in [9] resulted to the definition that is used today in computability theory.

Below, we give a formal definition of an Oracle Turing Machine:

Definition 2 (Oracle Turing Machine). Let A be a language, $A \subseteq \Sigma^*$. A Turing machine with oracle A is a single-tape Turing machine with three special states $q_?, q_y$ and q_n . The special state $q_?$ is used to ask whether a string is in the set A. When the Turing machine enters state $q_?$ it requests an answer to the question: "Is the string of non-blank symbols to the right of the tape head in A?" The answer is provided by having the state of the Turing machine change on the next move to one of the states q_y or q_n . The computation proceeds normally until the next time $q_?$ is reached, at which point the Turing machine requests another answer from the oracle.

With respect to notation, we denote by M^A the Turing machine M with oracle A. Also, a set (language) L is recursive with respect to A if $L = L(M^A)$ for some Turing machine M^A that *always* halts while two oracle sets (languages) are called *equivalent* if each of them is recursive in the other (see [8]).

3 Our contributions

Our approach is different for each of the two Panopticon models we propose since they are of a different nature, i.e syntactic (for the behavioural model) vs. semantic (for the deductive model).

For the behavioural model, we provide a simple adaptation of Cohen's piooneering formal model of a *virus* and prove a Panopticon detection impossibility result much like Cohen's result for virus detection.

For the deductive model, we follow a completely different approach using Oracle Turing Machines and a technique that can be applied to prove undecidability results for this type of machines. More specifically, in Chapter 8 of [8] a technique from [7] is presented that establishes an hierarchy of undecidable problems for Oracle Turing Machines. In particular, The technique targets the oracle set $S_1 = \{ \langle M \rangle | L(M) = \emptyset \}$, with $\langle M \rangle$ denoting the encoding of Turing machine M, as we discussed before. Then, the sets $S_{i+1} = \{ \langle M \rangle | L^{S_i}(M) = \emptyset \}$ can be, recursively, defined and the following can be proved (see [7,8]):

Theorem 1. The membership problem for TM's without oracles is equivalent to S_1 (i.e. L_u is equivalent to S_1).

Theorem 2. The problem of deciding whether $L(M) = \Sigma^*$ is equivalent to S_2 .

Our first contribution is to propose a plausible Panopticon model which incorporates the *information deduction* element of its behaviour (see Definition 4). We accomplish this as follows: information deduction takes place whenever the Turing machine under scrutiny for Panopticon behaviour produces a *completely new* information set given a set of fixed, finitely many, already *known* information sets. This set models the information that the Panopticon *already knows* through surveillance and observation, without (usually) expending considerable effort since it, merely, intercepts or eavesdrops information.

More formally, let $N_i = \{L_1^i, L_2^i, \ldots, L_k^i\}$ be a set of recursively enumerable languages, for some fixed integer $k \geq 1$, such that $\emptyset \notin N_i$ for all *i*. Also, let $M_1^i, M_2^i, \ldots, M_k^i$ Turing machines that, correspondingly, accept these languages. These Turing machines and their corresponding languages model the fixed, finitely many, information sets already known to the Panopticon. We, also, say that a set is *disjoint* from a collection of sets if it is disjoint from all the sets in the collection. set is *disjoint* from a collection of sets if it is disjoint from all the sets in the collection.

We will, now, define the oracle set $S_1 = \{ \langle M \rangle | L(M) \text{ is disjoint from } N_1 \}$, with $\langle M \rangle$ denoting the encoding of Turing machine M, and, recursively, in analogy with [7,8], the sets $S_{i+1} = \{ \langle M \rangle | L(M^{S_i}) \text{ is disjoint from } N_{i+1} \}$. The sets S_1 and $S_2 = \{ \langle M \rangle | L(M^{S_1}) \text{ is disjoint from } N_2 \}$, in particular, are central to our approach.

Based on this framework, in Section 4.2 we prove two theorems analogous to Theorems 1 and 2 on the undecidability of the problem of detecting a deductive Panopticon. The first one, Theorem 4, is focused on the weaker form of the deductive Panopticons, related to the set S_1 , while the more powerful one, based on oracle computation for "free" information gathering, related to the set S_2 , is handled by Theorem 5. In particular, in Theorem 4 we prove that L_u is equivalent to S_2 and in Theorem 5 we prove that the problem of whether $L(M) = \Sigma^*$ is equivalent to S_2 .

Finally, in Section 5 we show that for any sufficiently expressive formal system \mathcal{F} , such as Set Theory, we can effectively construct a Turing Machine which is impossible to classify it as a Panopticon or non-Panopticon within \mathcal{F} . In other words no formal system is powerful enough so that given any Turing Machine, it can provide either a proof that it is a Panopticon or a proof that it is not a Panopticon.

Before continuing, we should remark that the essential element of the proposed definition of deductive Panopticons is that the oracle consultations model the "effortless", through surveillance, interception or eavesdropping, information gathering by Internet surveillance agencies and organizations. In this context, the sets S_{i+1} define an infinite *hierarchy* of deductive Panopticons in which a Panopticon whose accepted language belongs in S_{i+1} operates by consulting a (weaker) lower-level Panopticon whose language belongs in S_i , with the weakest Panopticons being the ones whose accepted languages belong in S_1 . These Panopticons do not have oracle consultations or effortless information gathering capabilities.

4 The Panopticon detection problem

Formal proofs about the impossibility of detecting, in a *systematic* (i.e. algorithmic) and *general* way, malicious entities, such as the Panopticons in our case, already exist for a

long time for a very important category of such entities, the *computer viruses* or *malware* in general.

In Cohen's pioneering work (see [2,3]) a natural, formal, definition of a virus is provided based on Turing machines. Specifically, Cohen defined a virus to be a program, or Turing machine, that simply copies itself to other programs, or more formally, injects its transition function into other Turing machines' transition functions (see 1) replicating, thus, itself indefinitely. Then, he proves that L_u reduces to the problem of deciding whether a given Turing Machine behaves in this way proving that detecting viruses is an undecidable problem.

Following Cohen's paradigm, we will propose two, rather restricted (so as to be amenable to a theoretical analysis) but reasonable and precise definitions of a Panopticon. We, first, define the *behavioural Panopticons*:

Definition 3. (Behavioral Panopticons) A Panopticon is a Turing machine that when executed will demonstrate a specific, recognizable, behaviour particular to Panopticons manifested by the execution (not simply the existence in the Turing machine's description) of a sequence of actions, e.g. it will publish secret information about an entity, it will download information illegally etc., actions reflected by reaching, during its operation, states in the set P (see Definition 1).

This is much like Cohen's definition of a virus since it characterizes Panopticons according to their *displayed* or *manifested* behaviour. We stress the word *execution* in order to preclude situations where a false alarm is raised for "Panopticons" which merely list actions that are characteristic of Panopticon behaviour without *ever* actually invoking them during their operation. Instead, they operate normally without any actions taking place that manifest Panopticon behaviour.

Beyond displayed behaviour, however, Panopticons can be reasonably assumed to also possess *deductive* powers, not directly visible or measurable. In other words, one type of such Panopticons may operate by gathering or computing totally new information, distinct from the information already known to it. Thus, another type of such Panopticons can be based on given *easily* acquired, or even *stolen*, freely provided (in some sense) information. In other words, based on information the Panopticon acquires for free, in a sense, it deduces further information, perhaps expending some computational effort this time. We model the characteristic Panopticon action, i.e. *observation* or *surveillance*, using oracle Turing machines, where the freely acquired information is modeled by the oracle set of the machine. Based on this information, the Turing machine deduces, through its normal computation steps, *further* information about its targets. Below, we describe both these two types of Panopticons.

Definition 4. (Deductive Panopticons) A Panopticon is a computer program that by itself or based on observed or stolen (and, thus, acquired without expending computational effort to deduce or produce it) information, deduces (perhaps with computational effort) further information about entities.

In the definition above, the Panopticon operating by itself, i.e. without oracles, is weaker than the one with oracles since the latter is allowed to obtain free advice or information, in the form of an oracle. Naturally, many other definitions would be reasonable or realistic. Our main motivation behind the ones stated above was a balance of theoretical simplicity and plausibility in order to spark interest on the study on formal properties of Panopticons as well as the difficulty of detecting them algorithmically.

Based on the two formal Panopticon definitions we gave above, we can define the corresponding Panopticon detection problems. The aim of a Panopticon detection algorithm or Turing machine, is to take as input the encoding of another Turing machine and decide whether it is Panopticon or not based on the formal definition.

The Panopticon Detection Problem 1

Input: A description of a Turing machine (program).

Output: If the input Turing machine behaves like a Panopticon according to Definition 3 output True. Otherwise, output False.

More formally, if by L_b we denote the language consisting of Turing machine encodings $\langle M \rangle$ which are Panopticons according to Definition 3, then we want to decide L_b , i.e. to design a Turing machine that, given $\langle M \rangle$, decides whether $\langle M \rangle$ belongs in L_b or not.

The Panopticon Detection Problem 2

Input: A description of a Turing machine (program).

Output: If the input Turing machine behaves like a Panopticon according to Definition 4 output True. Otherwise, output False.

More formally, if by L_d we denote the language consisting of Turing machine encodings $\langle M \rangle$ which are Panopticons according to Definition 4, then we want to decide L_d , i.e. to design a Turing machine that, given $\langle M \rangle$, decides whether $\langle M \rangle$ belongs in L_d or not.

4.1 Behavioral Panopticons

Let Q_{pan} be the set of actions which, when *executed*, manifest Panopticon behaviour (see Definition 3). We will show below that L_u is recursive in L_b . This implies that if we had a decision procedure for L_b then this procedure could also be used for deciding L_u which is undecidable. Thus, no decision procedure exists for L_b too.

Theorem 3. (Impossibility of detecting behavioural Panopticons) The language L_b is undecidable.

Proof. Our proof is similar to Cohen's proof about the impossibility of detecting viruses. Let $\langle M, w \rangle$ be an instance of the Halting problem. We will show how we can decide whether $\langle M, w \rangle$ belongs in L_u or not using a hypothetical decision procedure (Turing machine) for the language L_b . In other words, we will show that L_u is recursive in L_b .

Given $\langle M, w \rangle$ we design a Turing machine M^{u-b} that modifies the transition function (see Definition 1) of M so as when a final state is reached (i.e. a state in the set F of M) a transition takes place that essentially starts the execution of the actions in Q_{pan} . In a sense, M is now a new Turing machine M' containing the actions of M followed by actions (any of them) described by the states in P. Now, M' is given as input the input of M, i.e. w, and operates as described above.

Let us assume that there exists a Turing machine M_b that decides L_b . Then we can give to it as input M'. Suppose that M_b answers that $M' \in L_b$. Since a state in Q_{pan} was finally activated, as M_b decided, this implies that M halted on w since M' initially simulated M on w. Then we are certain that M halts on w.

Assume, now, that M_b decides that M' is not a Panopticon. Then a state in Q_{pan} was never invoked, which implies that no halting state is reached by M on w since a state Q_{pan} is invoked, in M', only from halting states of M, which is simulated by M'. Thus, M does not halt on w.

It appears that M' is a Panopticon if and only if M halts on w and, thus, we have shown that L_u is recursive in L_b . There is a catch, however, that invalidates this reasoning: if M itself can exhibit the Panopticon behaviour, i.e. it can reach a state in Q_{pan} before reaching a final state. Then Panopticon behaviour can be manifested without ever M reaching a final state that would lead M' to invoke a Panopticon state in Q_{pan} , by its construction. A solution to this issue is to remove the states in Q_{pan} from the transition function of M, giving this new version to M^{u-b} to produce M'. This action would validate the equivalence M' is a Panopticon if and only if M halts on w, completing the proof.

More formally, we create a new set of dummy ("harmless" or "no-operation") "Panopticon" states Q'_{pan} which contains a new state for each of the states in Q_{pan} . Then we replace the states from Q_{pan} that appear in the transition function of M with the corresponding states in Q'_{pan} . Actually, this transformation removes from a potential Panopticon the actions that *if* executed would manifest a Panopticon. We stress, again, the fact the mere existence of Panopticon actions is not considered Panopticon behaviour.

With this last transformation, M' is a Panopticon if and only if M halts on w and, thus, L_u is recursive in L_b .

4.2 Deductive Panopticons

We, first, prove the undecidability of S_1 , i.e. the impossibility of deciding for a given Turing machine (its encoding, to be precise) whether it accepts a language disjoint from a given, fixed, finite set of languages. In other words, it is impossible to detect Turing machines that decide, perhaps with effort, new information sets given some known ones.

Theorem 4. The Halting Problem for Turing machines without oracles, i.e. L_u , is equivalent to S_1 .

Proof. We first prove that given an oracle for the S_1 we can solve the Halting problem (or, equivalently, recognize the language L_u). We construct a Turing machine M^{S_1} such that given $\langle M, w \rangle$ constructs a Turing machine M' which operates as follows. It ignores its input and simulates, internally, M on w. If M accepts w, M' accepts its input. Then, $L(M') = \emptyset$ if M does not accept w while $L(M') = \Sigma^*$ if M accepts w. Then, M^{S_1} asks the oracle whether $M' \in S_1$. If the answer is yes, i.e. let $L(M') = \emptyset$, then M does not accept w. If the answer is no, then $L(M') = \Sigma^*$ and, thus, M accepts w. We, thus, can recognize L_u .

For the other direction, we show that we can recognize S_1 given an oracle for the Halting problem (more precisely, L_u). We will construct a Turing machine M'' such that, given M, it constructs another Turing machine M' that operates as follows. M' ignores its own input and uses a generator of triples (i, j, l), $1 \leq l \leq k + 1$, for simulating the *l*th Turing machine, M_l , with $M_{k+1} = M$, on the *i*th string for *l* steps. Each time one of the Turing machines M_1, M_2, \ldots, M_k accepts a particular input, this fact is recorded on M'''s tape. Each time M_{k+1} accepts an input, M'' checks whether the same input was accepted earlier by one of the M_1, M_2, \ldots, M_k . If no, the process continues. If yes, M' stops the simulation and M' accepts its own input. Thus, $L(M) \in S_1$ if $L(M') = \emptyset$ while $L(M) \notin S_1$ if $L(M') = \Sigma^*$, i.e. M' accepts all its inputs, ε in particular. Then, $M''L_u$ may query its oracle set L_u for $\langle M', \varepsilon \rangle$. If the answer is yes then M'' rejects M, otherwise it accepts it.

Theorem 5. The problem of deciding whether $L(M) = \Sigma^*$ is equivalent to S_2 .

Proof. We first show that deciding whether $L(M) = \Sigma^*$ is recursive in S_2 . We construct a Turing machine $M^{'''S_2}$ that takes as input a Turing machine M and constructs from it a Turing machine \hat{M}^{S_1} , that is a Turing machine with oracle set S_1 , that operates in the following way. It enumerates strings x over the alphabet Σ , and for each such string it uses oracle S_1 in order to decide whether M accepts x. This can be accomplished in the way described in the first part of the proof of Theorem 4.

Then \hat{M}^{S_1} accepts its own input if and only if a string x is found *not* accepted by M, or

$$L(\hat{M}^{S_1}) = \begin{cases} \emptyset, \text{ if } L(M) = \Sigma^*\\ \Sigma^* \text{ otherwise.} \end{cases}$$

Now M'''^{S_2} asks its oracle S_2 whether $L(\hat{M}^{S_1}) \in S_2$, i.e. whether $L(\hat{M}^{S_1})$ is disjoint from all sets in N'. If the answer is yes, then $L(\hat{M}^{S_1}) = \emptyset$. Consequently, $L(M) = \Sigma^*$. If the answer is no, on the other hand, then $L(\hat{M}^{S_1}) = \Sigma^*$ and, thus, $L(M) \neq \Sigma^*$. Thus, deciding whether $L(M) = \Sigma^*$ is recursive in S_2 .

We not turn to showing that S_2 is recursive in the problem of whether $L(M) = \Sigma^*$. In other words, if by L_* we denote the codes of the Turing machines which accept all their inputs, then there exists a Turing machine $M'''L_*$, i.e. a Turing machine with oracle set L_* , which accepts S_2 .

Given a Turing machine M^{S_1} , we define the notion of a valid computation of M^{S_1} using oracle S_1 in a way similar to notion defined in [7,8]. A valid computation is a sequence of computation steps such that the next one follows from the current one after a computational (not oracle query) step, according to the internal operation details (i.e. program) of the Turing machine. If a query step is taken, however, i.e. the Turing machine M^{S_1} enters state $q_?$, and the next state is q_n this means that M^{S_1} submitted a query to the oracle S_1 with respect to whether some given Turing machine, say T, belongs to the set S_1 , receiving the answer no. In other words, the oracle replied that $L(T)\notin S_1$ or, equivalently, L(T) is not disjoint from all sets in N_1 . As evidence for the correctness of this reply from the oracle, we insert a valid computation of the ordinary (i.e. with no oracle) Turing machine T that shows that a *particular* string is accepted by, both, T and one of the Turing machines accepting a language in N_1 . If, however, after q_2 the state q_y follows, no computation is inserted. Intuitively, such a computation would be infinite.

We, now, describe the operation of M'''^{L_*} with M^{S_1} as input. Given M^{S_1} , M'''^{L_*} constructs a Turing machine M' to accept valid computations of M^{S_1} leading to acceptance, whenever the accepted string, also, belongs to at least one of the sets $L_1^2, L_2^2, \ldots, L_k^2$. The easy case is when the given computation is malformed, when one step does not follow from the previous one according to the internals of the Turing machine, or when the added computation inserted in the $q_{?}$ - q_n case is not valid. In all these cases M' rejects its input.

However, there is some difficulty in the $q_2 - q_y$ case since, as we stated above, there is no obvious *finite* computation evidence for the correctness of the reply. Now the Turing machine M' must decide on its own whether the reply is correct. The reply q_y means that the language accepted by the queried Turing machine T belongs to S_1 or, in other words, it is disjoint from all the sets in S_1 . As in the proof of Theorem 4, M'generates all triples $(i, j, l), 1 \leq l \leq k + 1$, for simulating the *l*th Turing machine, $M_1^1, M_2^2, \ldots, M_k^1, M_{k+1} = T$, on the *i*th string for *l* steps. Each time one of the Turing machines M_1, M_2, \ldots, M_k accepts a particular input, this fact is recorded on M''s tape. Each time T accepts an input, M'checks whether the same input was accepted earlier by one of the Turing machines $M_1^1, M_2^1, \ldots, M_k^1$. Also, each time one of these machines accepts an input, M'checks whether the same input was accepted earlier by one of these two cases apply, the process continues. If one of these two cases, however, holds M' stops the simulation and rejects the computation since it was invalid. It was invalid because a common element was found between the language accepted by T and the language of one the Turing machines accepting languages in N_1 .

If, however, the computation is valid and it ends at an accepting state for a particular string x which was given as input to M^{S_1} , then M' starts generating pairs (j, l), $1 \le l \le k$, simulating the *l*th Turing machine, M_l^2 , on x for j steps. If x is accepted by any of these Turing machines, then M' accepts its own input.

Based on the above, M' accepts all input strings, that is $L(M') = \Sigma^*$, if $L(M^{S_1}) \notin S_2$, i.e. when M^{S_1} has valid computations of strings that, also, belong to at least one of the sets $L_1^2, L_2^2, \ldots, L_k^2$. Otherwise, it accepts the empty set, i.e. $L(M') = \emptyset$. Thus, $L(M') = \Sigma^*$ if and only if $L(M^{S_1}) \notin S_2$.

Finally, $M'''L_*$ asks its oracle whether $L(M') = \Sigma^*$ or not deciding, in this way, S_2 and, thus, detecting deductive Panopticons.

5 Weaknesses of formal systems in characterizing Panopticons

Based on the Recursion Theorem, the following, central to our approach in this Section theorem is proved in [8]:

Theorem 6. Given a formal system \mathcal{F} , we can construct a Turing Machine for which no proof exists in \mathcal{F} that it either halts or does not halt on a particular input. This Turing Machine, denoted by M_G , is the following:

$$g(i,j) = \begin{cases} 1, & \text{if there is a proof in } F \text{ that} f_i(j) \text{ is not defined} \\ & (i.e. \text{ does not halt}) \text{ or, in other words if there is} \\ & a \text{ proof that the ith Turing Machine does not} \\ & \text{halt, given input } j \end{cases}$$
(1)
undefined, otherwise

We now prove the following, based on Theorem 6 and the, effectively constructible, Turing Machine M_G given in (1):

Theorem 7 (Imposibility of proving Panopticon status within formal systems). Let \mathcal{F} be a consistent formal system. Then we can construct a Turing Machine for which there is no proof in \mathcal{F} that it behaves as a Panopticon and no proof that it does not behave as a Panopticon, based on Definition 4 and the set intersection property described in Sections 3 and 4.2.

Proof. For some fixed k, we define a set $N = \{L_1, L_2, \ldots, L_k\}$ of recursively enumerable languages and a set of corresponding Turing Machines M_1, M_2, \ldots, M_k which accept them. We also assume there exists a recursively enumerable language L, accepted by a Turing Machine M, which is disjoint from N.

These elements can be effectively constructed. For instance, for k = 2, we can have as $L_1 = \{\text{The set of multiples of } 2\}, L_2 = \{\text{The set of multiples of } 3\}$, and $L = \{\text{The set of primes}\}$. For each of these languages (in particular L) we may construct a corresponding Turing Machine that accepts it. Given these elements, we proceed as follows.

We construct a Turing Machine $M_{?}$ which, given w as input, simulates M on w and M_{G} , as defined in (1), on some *fixed* input w_{G} , independently of what w is, by alternating between them in a way similar to the alternation technique applied in Theorem 4. Then $M_{?}$ accepts if either of them accepts at some step of the simulation process.

We, now, observe that $L(M_?) = L(M)$, if M_G does not halt on w_G and $L(M_?) = \Sigma^*$ if M_G halts on w_G . Then, according to the definition of a deductive Panopticon (see Definition 4 and Section 3), $M_?$ is a Panopticon if and only if M_G does not halt on the particular input w_G . But, now, if a proof existed in the formal system \mathcal{F} that $M_?$ either is a Panopticon or it is not a Panopticon then the same proof could be used to prove that M_G either halts or does not halt, correspondingly, contradicting, thus, Theorem 6. \Box

6 Discussion and directions for future research

Theorems 3, 4, and 5 in Section 4 show that, even for Panopticons with the simple behaviours described in Definition 3 and Definition 4, it is impossible, in principle,

to detect them. Potential Panopticons, naturally, can have any imaginable, complex, behaviour but then the problem of detecting them may become harder compared to our definitions.

Comparing, now, Theorems 3, 4, and 5, Theorem 3 examines the detection of Panopticons based on the execution of *specific* visible or detectable actions, i.e. on a *behavioural level*, such as connecting to a server and sending eavesdropped information or sending an email to the unlawful recipient. Theorems 4 and 5 examine Panopticon detection not based on their visible behaviour but from what languages they may accept, without having any visible clue of behaviour or actions, only their *descriptions* as Turing machines (i.e. programs or systems). These theorems, that is, examine the detection of Panopticons at a *metabehavioural level*.

With respect to the difference between Theorems 4 and 5, we first observe that L_u is recursively enumerable but not recursive while the $\{ \langle M \rangle | L(M) = \Sigma^* \}$ language is not recursively enumerable (see, e.g., [8]). Although they are, both, not recursive (i.e. not decidable), their "undecidabilities" are of different levels, with the $\{ < M > | L(M) = \Sigma^* \}$ language considered "more difficult" than L_u in restricted types of Turing machines (Panopticons). For example, the L_u language is decidable for Context-free Grammars (i.e. for Turing machines modeling Context-free Grammars) while the $\{ \langle M \rangle | L(M) = \Sigma^* \}$ language is still undecidable. Also, for regular expressions, the problem of deciding L_u is solvable efficiently (i.e. by polynomial time algorithms) while the $\{ \langle M \rangle | L(M) = \Sigma^* \}$ language has been shown, almost certainly, to require exponential time (in the length of the given regular expression) to solve (see, e.g., [8]). Therefore, a similar decidability complexity status is expected from S_1 (deductive Panopticons without external advice) and S_2 (deductive Panopticons with external advice in the form of an oracle) since they are equivalent to the languages L_u and $\{ \langle M \rangle | L(M) = \Sigma^* \}$ respectively. That is, when we consider more restricted definitions of Panopticons that render the detection problem decidable, then deciding which Panopticons belong in S_1 is expected to be easier than deciding which Panopticons belong in S_2 .

Finally, Theorem 7 shows that for any formal system \mathcal{F} , we can, effectively, exhibit a particular Turing Machine for which there is no proof in \mathcal{F} , that it is either a Panopticon or it is not a Panopticon, emphasizing the difficulty of recognizing Panopticons by formal means.

As a next step, it is possible to investigate the status of the Panopticon detection problem under other definitions, either targeting the behaviour (i.e. specific actions) of the Panopticon or its information deducing capabilities (e.g. accepting languages with specific closure properties or properties describable in some formal system such as second order logic). Our team plans to pursue further Panopticon definitions in order to investigate their detection status, especially for the decidable (and, thus, more practical) cases of suitably constrained Panipticons.

In conclusion, we feel that the formal study of the power and limitations of massive surveillance establishments and mechanisms of today's as well as of the future Information Society can be, significantly, benefitted from fundamental concepts and deep results of computability and computational complexity theory. We hope that our work will be one step towards this direction.

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