Lecture Notes in Computer Science 13359

Advanced Research in Computing and Software Science

Subline of Lecture Notes in Computer Science

More information about this series at https://link.springer.com/bookseries/558

Ulrich Berger · Johanna N. Y. Franklin ·
Florin Manea · Arno Pauly (Eds.)

Revolutions and Revelations in Computability

18th Conference on Computability in Europe, CiE 2022
Swansea, UK, July 11–15, 2022
Proceedings

Springer

Editors
Ulrich Berger
Department of Computer Science
Swansea University
Swansea, UK

Johanna N. Y. Franklin
Department of Mathematics
Hofstra University
Hempstead, NY, USA

Florin Manea
Georg-August University Göttingen
Göttingen, Germany

Arno Pauly
Department of Computer Science
Swansea University
Swansea, UK

ISSN 0302-9743 ISSN 1611-3349 (electronic)
Lecture Notes in Computer Science
ISBN 978-3-031-08739-4 ISBN 978-3-031-08740-0 (eBook)
https://doi.org/10.1007/978-3-031-08740-0

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

Computability in Europe (CiE) is an annual conference organized under the auspices of the Association Computability in Europe (ACiE), a European association of researchers from a broad variety of backgrounds who are connected to one another through their work in computability. The conference series has built up a strong tradition of developing an interdisciplinary scientific program that brings together researchers in all aspects of computability, foundations of mathematics, and computer science as well as the interplay of these theoretical areas with practical issues in computer science and with other disciplines such as biology, mathematics, philosophy, and physics. Its purpose is not only to allow researchers to report on their own ongoing work but also to broaden their own perspectives by engaging with the work of others from different backgrounds.

The motto of CiE 2022 was "Revolutions and Revelations in Computability". This alludes to the revolutionary developments we have seen in computability theory, starting with Turing's and Gödel's discoveries of the uncomputable and the unprovable and continuing to the present day with the advent of new computational paradigms such as quantum computing and bio-computing, which have dramatically changed our view of computability and revealed new insights into the multifarious nature of computation. The motto also hints at the historic role of the host city, Swansea, in the Industrial Revolution, as the world center of copper smelting in the 18th and 19th centuries.

CiE 2022 was the 18th conference in the series, and this was the second time it has been held in Swansea. Previous meetings have taken place in Amsterdam (2005), Swansea (2006), Siena (2007), Athens (2008), Heidelberg (2009), Ponta Delgada (2010), Sofia (2011), Cambridge (2012), Milan (2013), Budapest (2014), Bucharest (2015), Paris (2016), Turku (2017), Kiel (2018), Durham (2019), and, virtually, in Salerno (2020) and Ghent (2021). After two online CiE conferences, we were very happy to be able to hold CiE 2022 as a largely in-person meeting with some online elements.

The conference series has become a major event and is the largest international conference that brings together researchers focusing on computability-related issues. The CiE conference series is coordinated by the ACiE Conference Series Steering Committee consisting of Alessandra Carbone (Paris), Liesbeth De Mol (Lille), Gianluca Della Vedova (Executive Officer, Milan), Nataša Jonoska (Tampa), Benedikt Löwe (Amsterdam), Florin Manea (Chair, Göttingen), Klaus Meer (Cottbus), Russell Miller (New York), Mariya Soskova (Madison), and ex-officio members Elvira Mayordomo (President of the Association, Zaragoza) and Marcella Anselmo (Treasurer, Salerno).

Conference Structure and Program

The conference program was centered around tutorials, invited lectures, and a set of special sessions ranging over a variety of topics as well as contributed papers and informal presentations. The Program Committee of CiE 2022 consisting of 32 members, selected the invited and tutorial speakers and the special session organizers and coordinated the reviewing process and the selection of submitted contributions. The Program Committee selected 19 of the 34 non-invited submitted papers for publication in this volume. Each paper received at least three reviews by the Program Committee and their subreviewers. In addition to the contributed papers, the volume contains seven invited papers and 23 abstracts.

Invited Tutorials

- Dora Giammarresi (Università di Roma, Italy), *Two-Dimensional Languages and Models*
- Noam Greenberg (Victoria University of Wellington, New Zealand), *Recent Interactions Between Computability and Set Theory*

Invited Lectures

- Erika Ábrahám (RWTH Aachen University, Germany), *SMT Solving: Historical Review and New Developments*
- Thierry Coquand (University of Gothenburg, Sweden), *Sheaf Cohomology in Univalent Foundation*
- Liesbeth De Mol (Université de Lille, France), *Towards a Diversified Understanding of Computability, or Why We Should Care More about Our Histories*
- Damir Dzhafarov (University of Connecticut, USA), *Reverse Mathematics 2021*
- Harvey M. Friedman (The Ohio State University, USA), *String Replacement Systems*
- Svetlana Selivanova (KAIST, South Korea), *Computational Complexity of Classical Solutions of Partial Differential Equations*

Special Sessions

At the intersection of computability and other areas of mathematics. Organizers: Denis Hirschfeldt (University of Chicago) and Karen Lange (Wellesley College)

- Meng-Che Ho (California State University, Northridge), *A Computable Functor from Torsion-Free Abelian Groups to Fields*
- Bjørn Kjos-Hanssen (University of Hawai`i at Mānoa), *An Incompressibility Theorem for Automatic Complexity*
- Elvira Mayordomo (Universidad de Zaragoza), *Algorithmic Dimensions, the Point-to-Set Principles, and the Complexity of Oracles*
- Alexandra Shlapentokh (East Carolina University), *A Connection Between Inverse Galois Problem of a Field and Its First-Order Theory*

Computability theory of blockchain technology. Organizers: Arnold Beckmann (Swansea University) and Anton Setzer (Swansea University)

- Eli Ben-Sasson (StarkWare), *Ultra Scaling Blockchains with ZK-STARKs*
- Maurice Herlihy (Brown University), *Blockchains and Related Technologies: Which Ideas Are Likely to Endure?*
- Philip Wadler (University of Edinburgh), *Smarter Contracts: Applications of Haskell and Agda at IOG*

Computing Language: Love Letters, Large Models and NLP. Organizers: Liesbeth De Mol (Université de Lille) and Giuseppe Primiero (University of Milan) for the Council of the HaPoC Commission

- Troy Astarte (Swansea University), *'My avid fellow feeling' and 'Fleas': Playing with Words on the Computer*
- Juan-Luis Gastaldi (ETH Zürich), *Mathematics as Natural Language: Principles, Consequences and Challenges of the Application of NLP Models to the Treatment of Mathematical Knowledge*
- Maël Pégny (Universität Tübingen), *Are Large Language Models Models (of Language)?*
- Jacopo Tagliabue (Coveo Labs), *Are We There Yet? Meaning in the Age of Large Language Models*

Computing with bio-molecules. Organizers: Jérôme Durand-Lose (Université d'Orléans) and Claudio Zandron (University of Milano-Bicocca)

- Giuditta Franco (University of Verona), *DNA Library Evidence Strings*
- Maria Dolores Jiménez-López (University of Tarragona), *Processing Natural Language with Biomolecules: Where Linguistics, Biology and Computation Meet*
- Nicolas Schabanel (CNRS - LIP, École Normale Supérieure de Lyon), *Turedo a New Computational Model for Molecular Nanobots?*
- Petr Sosík (Silesian University in Opava), *Computability and Complexity in Morphogenetic Systems*

Constructive and reverse mathematics. Organizers: Samuele Maschio (Università di Padova) and Takako Nemoto (Hiroshima Institute of Technology)

- Makoto Fujiwara (Tokyo University of Science), *An Extension of the Equivalence Between Brouwer's Fan Theorem and Weak König's Lemma with a Uniqueness Hypothesis*
- Takayuki Kihara (Nagoya University), *Computability Theory and Reverse Mathematics via Lawvere-Tierney Topologies*
- Robert Lubarsky (Florida Atlantic University), *On the Necessity of Some Topological Spaces*
- Huishan Wu (BLCU Beijing), *Reverse Mathematics and Semisimple Rings*

Reachability problems. Organizers: Paul Bell (Liverpool John Moores University) and Igor Potapov (University of Liverpool)

- Kitty Meeks (University of Glasgow), *Reducing Reachability in Temporal Graphs: Towards a More Realistic Model of Real-World Spreading Processes*
- Olivier Bournez (École Polytechnique), *Programming with Ordinary Differential Equations: Some First Steps Towards a Programming Language*
- Véronique Bruyère (Université de Mons), *A Game-Theoretic Approach for the Automated Synthesis of Complex Systems*
- James Worrell (University of Oxford), *The Skolem Landscape*

Women in Computability Workshop

ACiE and this conference have had a strong tradition of encouraging women to participate in computability-related research since CiE 2007. In 2016, a Special Interest Group for Women in Computability was established, and in 2021, Mariya Soskova set up an online mentorship program associated with this group to connect junior researchers in computability theory with women mentors not just at the conference but throughout the year. These initiatives are anchored in the annual Women in Computability workshop, held this year with the following speakers:

- Troy Astarte (Swansea University, Wales)
- Dora Giammarresi (Università di Roma, Italy)
- Svetlana Selivanova (KAIST, South Korea)

Organization and Acknowledgements

The CiE 2022 conference was organized by the Theory Group of the Department of Computer Science at Swansea University.

We are happy to acknowledge and thank the following for their financial support: Association Computability in Europe, the Institute of Coding in Wales, the London Mathematical Society, and technocamps.

We are also happy to announce that CiE 2022 was held in cooperation with the Association for Women in Mathematics for the first time in the history of this conference series and supports its Welcoming Environment Statement.

The high quality of the conference was achieved through the careful work of the Program Committee, the Special Session organizers, and all of the referees, and we are very grateful for their help in creating an exciting program for CiE 2022.

May 2022

Ulrich Berger
Johanna N. Y. Franklin
Florin Manea
Arno Pauly

Organization

Program Committee

Marcella Anselmo	University of Salerno, Italy
Verónica Becher	University of Buenos Aires, Argentina
Ulrich Berger (Co-chair)	Swansea University, UK
Katie Brodhead	Florida State University, USA
Laura Crosilla	University of Oslo, Norway
Joel Day	Loughborough University, UK
Gianluca Della Vedova	University of Milan, Italy
Leah Epstein	University of Haifa, Israel
Maribel Fernández	King's College London, UK
Ekaterina Fokina	TU Wien, Austria
Johanna Franklin (Co-chair)	Hofstra University, USA
Lorenzo Galeotti	Amsterdam University College, The Netherlands
Sandra Kiefer	RWTH Aachen University, Germany
Susana Ladra	University of Coruna, Spain
Maria Emilia Maietti	University of Padua, Italy
Florin Manea	University of Göttingen, Germany
Klaus Meer	Brandenburg University of Technology Cottbus-Senftenberg, Germany
Dale Miller	LIX, École Polytechnique, France
Russell Miller	Queens College, CUNY, USA
Arno Pauly (Co-chair)	Swansea University, UK
Nadia Pisanti	University of Pisa, Italy
Solon Pissis	CWI Amsterdam, The Netherlands
Giuseppe Primiero	University of Milan, Italy
Ramyaa Ramyaa	New Mexico Tech, USA
Monika Seisenberger	Swansea University, UK
Anton Setzer	Swansea University, UK
Alexandra Shlapentokh	East Carolina University, USA
Reed Solomon	University of Connecticut, USA
Mariya Soskova	University of Wisconsin-Madison, USA
Peter Van Emde Boas	University of Amsterdam, The Netherlands
Andreas Weiermann	Ghent University, Belgium
Andreas Wichert	University of Lisbon, Portugal

Organizing Committee

Troy Astarte	Swansea University, UK
Arnold Beckmann	Swansea University, UK
Ulrich Berger	Swansea University, UK

Tonicha Crook	Swansea University, UK
Faron Moller	Swansea University, UK
Bertie Mueller	Swansea University, UK
Arno Pauly	Swansea University, UK
Olga Petrovska	Swansea University, UK
Markus Roggenbach	Swansea University, UK
Monika Seisenberger	Swansea University, UK
Anton Setzer	Swansea University, UK
John Tucker	Swansea University, UK

Additional Reviewers

Eric Allender
Uri Andrews
Stefano Berardi
Olivier Bournez
Vasco Brattka
Gabriele Buriola
Wesley Calvert
Douglas Cenzer
Francesco Ciraulo
Pieter Collins
Chris Conidis
Thierry Coquand
Walter Dean
Matthew de Brecht
Nuiok Dicaire
Jacques Duparc
Arnaud Durand
Guido Fiorino
Emanuele Frittaion
Samuel Frontull
Ziyuan Gao
Guido Gherardi
Daniel Graça
Ferruccio Guidi
Valentina Harizanov
Stefan Hoffmann
Mathieu Hoyrup
Hajime Ishihara
Takayuki Kihara
Bjørn Kjos-Hanssen
Manfred Kufleitner
Oliver Kullmann
Karen Lange
Steffen Lempp
Robert Lubarsky
Elvira Mayordomo
Tim McNicholl
Alexander Melnikov
Kenshi Miyabe
Larry Moss
Carl Mummert
Matthias Naaf
Takako Nemoto
Reino Niskanen
Hugo Nobrega
Dag Normann
Victor Ocasio González
Mizuhito Ogawa
Sebastian Ordyniak
Benedikt Pago
Fedor Pakhomov
Robert Passmann
Guillermo Perez
Iosif Petrakis
Christopher Porter
Pavel Pudlák
Michael Rathjen
Jan Rooduijn
Dino Rossegger
Luca San Mauro

Manfred Schmidt-Schauss
Matthias Schröder
Frank Stephan
Wolfgang Thomas
Daniel Turetsky
Ionut Tutu
Christian Urban
Yde Venema
Oleg Verbitsky
Albert Visser
Guohua Wu
Francesca Zaffora Blando
Martin Ziegler
Marius Zimand

Invited Tutorials

Two-Dimensional Languages and Models

Dora Giammarresi

Dipartimento di Matematica, Università Roma “Tor Vergata”, via della Ricerca Scientifica, 00133 Rome, Italy
giammarr@mat.uniroma2.it
https://www.mat.uniroma2.it/∼giammarr/

A picture, defined as a rectangular array of symbols chosen from a given alphabet, is the two-dimensional counterpart of a string. Researchers were inspired by the attempt to reproduce Chomsky’s hierarchy for picture languages. In the past and more so in recent years, the classical methods used to define string languages have been essayed for picture languages, thus obtaining various formal models and picture language families.

The tutorial presents the state of the art of formal definitions for picture languages. The formal models considered are: 2D regular expressions, tiling systems, automata and grammars of different types. Each picture language family will be presented by means of typical examples that illustrate its expressiveness. Moreover each 2D formal model will be compared with the corresponding string model to point out similarities and differences. The two-dimensional perspective will show up with its intrinsic richness whose we will analyze drawbacks and benefits.

Reference

1. Reghizzi, S.C., Giammarresi, D., Lonati, V.: Two-dimensional models. In: J.-É., Pin (ed.) Handbook of Automata Theory, vol. 1, pp. 303–333, EMS Publishing House, Berlin (2021)

MIUR Excellence Department Project awarded to the Department of Mathematics, University of Rome Tor Vergata, CUP E83C18000100006 and OBPOBS Project funded by Tor Vergata University, CUP E83C22001660005.

Recent Interactions Between Computability and Set Theory

Noam Greenberg

School of Mathematics and Statistics, Victoria University of Wellington, Wellington, New Zealand

Since very early days, there has been a certain overlap between computability theory and set theory: one can view both fields as inhabiting two parts of a spectrum that starts with regular languages and polynomial-time computation, continues with partial computable functions and Turing reducibility, and then the hyperarithmetic realm, effective descriptive set theory, fine structure of the constructible hierarchy, and inner models for large cardinals. Thus the same diagonal argument was used by Cantor for the unctounability of the reals, by Gödel for the incompleteness theorem, and by Turing for the undecidability of the halting problem.

I plan to survey three areas which have seen recent activity.

1. **Higher randomness**. Both Martin-Löf and Sacks suggested strenghtening the notions of effective randomness to obtain nice closure properties. They considered randomness with respect to effectively Borel (hyperarithmetic) sets, and effectively co-analytic (Π^1_1) sets. This subject was later picked up and developed by Hjorth and Nies, and then Chong and Yu. In parallel, Hamkins, Welch and others have considered infinite-time Turing machines and related notions of higher randomness. At the extreme end we find randomness over Gödel's L studied originally by Solovay. I will discuss relativising randomness in the higher setting, and what this tells us about the different equivalent definitions of ML-randomness.
2. **Uncountable structures**. Computable algebra and computable model theory investigate the interplay between information and structure: what information can be stored in a structure or in its isomorphism type. By the nature of computability, this study is restricted to countable or separable structures. *Admissible computability* is a generalisation of computability to domains beyond the natural numbers, and can be used to study the effective properties of uncountable, well-ordered structures. I will in particular examine the case of free and almost-free abelian groups, related to Shelah's work on the subject.
3. **Effective Borel sets**. Shoenfield's limit lemma says that membership in a Δ^0_2 set can be understood as an approximation process, involving finitely many mind-changes. To understand membership in more complicated Borel sets, the extra ingredient needed is the Turing jump. Montalbán's "true stages" machinery allows us to dynamically approximate membership in Borel sets. It was one of the ingredients in Day and Marks's recent resolution of the decomposability problem. I will discuss other applications to descriptive set theory.

Invited Lectures

SMT Solving: Historical Review and New Developments

Erika Ábrahám

RWTH Aachen
abraham@informatik.rwth-aachen.de

Satisfiability modulo theories (SMT solving) is a relatively recent research thread in computer science, with the aim to provide algorithms and tools for checking the satisfiability of (usually quantifier-free) first-order logic formulas over different theories. Starting with relatively easy theories like equalities and uninterpreted functions, state-of-the-art SMT solvers nowadays provide support for numerous theories, including (quantifier-free) real arithmetic. For real arithmetic, some exciting recent developments combine traditional SMT solving ideas with a kind of counterexample-guided abstraction refinement using methods from computer algebra.

In this talk we give a historical review of SMT solving with a focus on arithmetic theories, describe our own solver SMT-RAT and discuss some of these fascinating new research directions.

Sheaf Cohomology in Univalent Foundation

Thierry Coquand

Computer Science Department, University of Gothenburg
coquand@chalmers.se
http://www.cse.chalmers.se/~coquand

Abstract. In the introduction of his book on Higher Topos Theory, Jacob Lurie motivates this theory by the fact that it allows an elegant and general treatment of sheaf cohomology. It was realised early on that these ideas could be expressed in the setting of univalent foundations/homotopy type theory. I will try to explain in my presentation recent insights which show that this can be done in a maybe suprisingly direct way. Furthermore, all this can be formulated in a constructive meta theory, avoiding the non effective notion of injective resolutions.

Keywords: Univalent Foundation · Homotopy Type Theory · Constructive Mathematics

Towards a Diversified Understanding of Computability or Why We Should Care More About Our Histories

Liesbeth De Mol

CNRS, UMR 8163 Savoirs, Textes, Langage, Université de Lille
liesbeth.de-mol@univ-lille.fr

Abstract. In this talk I will argue that we should care more for and be more careful with the history of computability making a plea for a more diverse and informed understanding. The starting point will be the much celebrated Turing machine model. Why is it that within the computability community, this model is often considered as thé model? In the first part of this talk I review some of those reasons, showing how and why they are in need of a revision based, mostly, on historical arguments. On that basis I argue that, while surely, the Turing machine model is a basic one, part of its supposed superiority over other models is based on socio-historical forces. In part II then, I consider a number of historical, philosophical and technical arguments to support and elaborate the idea of a more diversified understanding of the history of computability. Central to those arguments will be the differentiation between, on the one hand, the logical equivalence between the different models with respect to the computable functions, and, on the other hand, some basic intensional differences between those very same models. To keep the argument clear, the main focus will be on the different models provided by Emil Leon Post but I will also include references to the work by Alonzo Church, Stephen C. Kleene and Haskell B. Curry.

Supported by the PROGRAMme project, ANR-17-CE38-0003-01.

Reverse Mathematics 2021

Damir Dzhafarov

Department of Mathematics, University of Connecticut
damir.dzhafarov@uconn.edu

Reverse mathematics is a foundational program in logic aimed at measuring the complexity of mathematical proofs and constructions according to the strength of the axioms needed to carry them out. Founded by Friedman in the 1970s, and principally developed by him and Simpson throughout the 1980s, it has become an incredibly active and far reaching area. Part of its appeal comes from its close, nearly inseparable connection to computability theory. The initial focus of the subject was a classificatory one, of categorizing different parts of mathematics into one of a handful of benchmark subsystems of second order arithmetic. Over time, interest has shifted to examples that defy this classification, giving rise to a zoo of mathematical principles with a rich and intricate web of strengths and interconnections. More recently, the subject has expanded to include notions and techniques from computable analysis, giving an even finer gauge with which to calibrate mathematical complexity, and offering new insights along the way. The talk will survey a bit of the history of the subject, some of the recent and ongoing developments, and offer a view of where it may be headed next.

Abstracts of Invited Talks

A Computable Functor from Torsion-Free Abelian Groups to Fields

Meng-Che "Turbo" Ho[1], Julia Knight[2], and Russell Miller[3]

[1] California State University, Northridge, Northridge, CA 91330, USA
[2] University of Notre Dame, Notre Dame, IN 46556, USA
[3] City University of New York, New York, NY 10017, USA

In descriptive set theory, complexities of classes of countable structures are studied. A classical example is the isomorphism problem $\cong_r$ on the class of torsion-free abelian groups of rank r. Baer [1] gave a simple invariant for $\cong_1$, i.e., when two torsion-free abelian groups of rank 1 are isomorphic. However, Hjorth [3] showed that $\cong_1 <_B \cong_2$ and Thomas [6] generalized this to show that $\cong_n <_B \cong_{n+1}$. Recently, Paolini and Shelah [5] showed that the class of torsion-free abelian group with domain ω is Borel complete.

We compare the class of torsion-free abelian groups and the class of fields using the notion of computable functors defined by Miller, Poonen, Schoutens, and Shlapentokh [4], and the notion of effective interpretability. Harrison-Trainor, Melnikov, Miller, and Montalban [2] showed that the presence of a uniform effective interpretation between two classes implies the presence of a computable functor between them and vice versa. Writing $\mathfrak{TFAb}_\mathrm{r}$ to be the class of torsion-free abelian groups of rank r and $\mathfrak{TD}_\mathrm{r}$ to be the class of fields of transcendence degree r with characteristic 0, we show the following:

Theorem 1 (Ho, Knight, and Miller)

1. *There is a Turing-computable reduction from* $\mathfrak{TFAb}_\mathrm{r}$ *to* $\mathfrak{TD}_\mathrm{r}$ *that is uniform in r. That is, there is a Turing functional (uniform in r)* $\Phi_r : \mathfrak{TFAb}_\mathrm{r} \to \mathfrak{TD}_\mathrm{r}$ *such that for every* $G, H \in \mathfrak{TFAb}_\mathrm{r}$, $G \cong H$ *if and only if* $\Phi_r(G) \cong \Phi_r(H)$.
2. *There is a uniform effective interpretation of* $\Phi_r(G)$ *in G. Thus,* Φ_r *can be completed to a computable functor in the sense of [4].*

References

1. Baer, R.: Abelian groups without elements of finite order. Duke Math. J. **3**(1), 68–122 (1937)
2. Harrison-Trainor, M., Melnikov, A., Miller, R., Montalbán, A.: Computable functors and effective interpretability. J. Symb. Log. **82**(1), 77–97 (2017)
3. Hjorth, G.: Around nonclassifiability for countable torsion free abelian groups. In: Eklof, P.C., Göbel, R. (eds.) Abelian Groups and Modules, pp. 269–292. Trends in Mathematics, Birkhäuser (1999)
4. Miller, R., Poonen, B., Schoutens, H., Shlapentokh, A.: A computable functor from graphs to fields. J. Symb. Log. **83**(1), 326–348 (2018)
5. Paolini, G., Shelah, S.: Torsion-free abelian groups are borel complete (2021). https://arxiv.org/abs/2102.12371
6. Thomas, S.: The classification problem for torsion-free abelian groups of finite rank. J. Amer. Math. Soc. **16**(1), 233–258 (2003)

An Incompressibility Theorem for Automatic Complexity

Bjørn Kjos-Hanssen

University of Hawai'i at Mānoa
bjoernkh@hawaii.edu
https://math.hawaii.edu/wordpress/bjoern/

Abstract. Shallit and Wang showed that the automatic complexity $A(x)$ satisfies $A(x) \geq n/13$ for almost all $x \in \{0,1\}^n$.

They also stated that Holger Petersen had informed them that the constant 13 can be reduced to 7.

Here we show that it can be reduced to $2+\varepsilon$ for any $\varepsilon > 0$.

The result also applies to nondeterministic automatic complexity $A_N(x)$. In that setting the result is tight inasmuch as $A_N(x) \leq n/2+1$ for all x.

Algorithmic Dimensions, The Point-To-Set Principles, and the Complexity of Oracles

Elvira Mayordomo

Departamento de Informática e Ingeniería de Sistemas, Instituto de Investigación en Ingeniería de Aragón, Universidad de Zaragoza, Spain

Effective and resource-bounded dimensions were defined by Lutz in [5] and [4] and have proven to be useful and meaningful for quantitative analysis in the contexts of algorithmic randomness, computational complexity and fractal geometry (see the surveys [1, 2, 6, 12] and all the references in them).

The point-to-set principle of J. Lutz and N. Lutz [8] fully characterizes Hausdorff and packing dimensions in terms of effective dimensions in the Euclidean space, enabling effective dimensions to be used to answer open questions about fractal geometry, with already an interesting list of geometric measure theory results (see [3, 11] and more recent results in [7, 13–15]). This characterization has been recently extended to separable spaces [10] and to resource-bounded dimensions [9].

In this talk I will review the point-to-set principles focusing on the importance of the oracle that achieves the characterization of classical dimension in terms of an algorithmic dimension. For instance Stull [15] has been able to improve the Marstrand projection theorem by analyzing the optimality of the oracles in the point-to-set principles. I will discuss some open problems on the complexity of the oracles involved in the point-to-set principles for both the effective and resource-bounded cases.

Bibliography

1. Downey, R.G., Hirschfeldt, D.R.: Algorithmic Randomness and Complexity. Springer, New York (2010). https://doi.org/10.1007/978-0-387-68441-3
2. Hitchcock, J.M., Lutz, J.H., Mayordomo, E.: The fractal geometry of complexity classes. SIGACT News Complex. Theor. Col. **36**, 24–38 (2005)
3. Lutz, J.H., Lutz, N.: Who asked us? how the theory of computing answers questions about analysis. In: Du, D.Z., Wang, J. (eds.) Complexity and Approximation. LNCS, vol. 12000. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-41672-0_4
4. Lutz, J.H.: Dimension in complexity classes. SIAM J. Comput. **32**(5), 1236–1259 (2003)

Research supported in part by Spanish Ministry of Science and Innovation grant PID2019-104358RB-I00 and by the Science dept. of Aragon Government: Group Reference T64_20R (COSMOS research group).

5. Lutz, J.H.: The dimensions of individual strings and sequences. Inf. Comput. **187** (1), 49–79 (2003)
6. Lutz, J.H.: Effective fractal dimensions. Math. Logic Q. **51**(1), 62–72 (2005)
7. Lutz, J.H.: The point-to-set principle, the continuum hypothesis, and the dimensions of hamel bases. Computability (2022). To appear
8. Lutz, J.H., Lutz, N.: Algorithmic information, plane Kakeya sets, and conditional dimension. ACM Trans. Comput. Theor. **10** (2018). Article 7
9. Lutz, J.H., Lutz, N., Mayordomo, E.: Dimension and the structure of complexity classes. Technical report (2021). https://arxiv.org/2109.05956
10. Lutz, J.H., Lutz, N., Mayordomo, E.: Extending the reach of the point-toset principle. In: Berenbrink, P., Monmege, B. (eds.) 39th International Symposium on Theoretical Aspects of Computer Science (STACS 2022), vol. 219 of Leibniz International Proceedings in Informatics (LIPIcs), pp. 48:1–48:14. Schloss Dagstuhl Leibniz-Zentrum für Informatik (2022)
11. Lutz, J.H., Mayordomo, E.: Algorithmic fractal dimensions in geometric measure theory. In: Brattka, V., Hertling, P. (eds.) Handbook of Computability and Complexity in Analysis. Theory and Applications of Computability. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-59234-9_8
12. Mayordomo, E.: Effective fractal dimension in algorithmic information theory. In: Cooper, S.B., Löwe, B., Sorbi, A. (eds.) New Computational Paradigms. Springer, New York (2008). https://doi.org/10.1007/978-0-387-68546-5_12
13. Slaman, T.: Personal communication (2021)
14. Stull, D.: The dimension spectrum conjecture for planar lines. Technical report (2021). https://arxiv.org/arXiv:2102.00134
15. Stull, D.: Optimal oracles for point-to-set principles. In: Berenbrink, P., Monmege, B., (eds.), 39th International Symposium on Theoretical Aspects of Computer Science (STACS 2022) of Leibniz International Proceedings in Informatics (LIPIcs), vol. 219, pp. 57:1–57:17. Schloss Dagstuhl Leibniz-Zentrum für Informatik (2022)

A Connection Between Inverse Galois Problem of a Field and Its First-Order Theory

Alexandra Shlapentokh

Department of Mathematics, East Carolina University, Greenville, NC 27858
shlapentokha@ecu.edu
myweb. ecu. edu/shlapentokha

Given a countable field F and a finite group G the Inverse Galois Problem for F and G is the problem of determining whether the field F has a finite Galois extension L such that $Gal(L/F) \cong G$. We show that the Turing degree of this problem is less or equal to the Turing degree of the first-order theory of the field.

Ultra Scaling Blockchains with ZK-STARKs

Eli Ben-Sasson

StarkWare Industries Ltd.
eli@starkware.co
https://www.starkware.co/

Abstract. Scalable and Transparent ARguments of Knowledge (STARKs) are practically efficient cryptographic proofs that use minimal cryptographic assumptions and are capable of improving the scalability and privacy of blockchains. By now, there have been weeks during which STARK-based systems settled 33% more transactions than Ethereum, while using only 1% of Ethereum's computational resources.

This talk explains why STARKs and blockchains blend nicely together like wine and cheese, and will describe the "theory-to-practice" journey of STARK technology from the early days of PCP theory to blockchain rollups, layer 2/3 systems, and beyond.

Blockchains and Related Technologies: Which Ideas Are Likely to Endure?

Maurice Herlihy

Brown University
herlihy@cs.brown.edu
https://cs.brown.edu/ mph/

Abstract. Blockchains and distributed ledgers have become the focus of much recent attention. Like many innovations, this field emerged from outside mainstream computer science, although almost all the component ideas were already well-known. As a new area driven mostly by technological and financial innovations, it can be difficult to distinguish accomplishment from aspiration, and especially difficult to tell which ideas are of transient versus lasting interest.

This talk surveys the theory and practice of blockchain-based distributed systems from the point of view of classical distributed computing, along with opinions about promising future research directions for our community.

Smarter Contracts: Applications of Haskell and Agda at IOG

Philip Wadler[1,2]

[1] School of Informatics, University of Edinburgh
wadler@inf.ed.ac.uk
https://homepages.inf.ed.ac.uk/wadler/
[2] IOG
https://iohk.io/en/team/philip-wadler

Abstract. Cardano is a proof-of-stake blockchain platform developed by IOG. Its smart contract language, Plutus, is based on Haskell, and supports both on-chain and off-chain components with a single sourcce language; and much of the software of Cardano is implemented in Haskell. On-chain components of smart contracts are compiled to Plutus Core, which is a variant of System F and has been formally specified in Agda. Property-based testing is used to compare the production implementation of Plutus Core with an evaluator derived from the proof of soundness of System F written in Agda. Astonishingly, IOG is one of the few firms to insist its products be based on peer-reviewed research.

Keywords: Smart contracts · blockchain · Haskell · Agda

References

1. Manuel M.T., et al.: The Extended UTXO Model. Financial Cryptography Workshops, pp. 525–539 (2020)
2. Manuel M.T., et al.: Native Custom Tokens in the Extended UTXO Model. ISoLA (3), 89–111 (2020)
3. Chapman, J., Kireev, R., Nester, C., Wadler, P.: System F in agda, for fun and profit. MPC, 255–297 (2019)
4. Peyton Jones, M., Gkoumas, V., Kireev, R., MacKenzie, K., Nester, C., Wadler, P.: Unraveling recursion: compiling an IR with recursion to system F. MPC, 414–443 (2019)

‘My Avid Fellow Feeling’ and ‘Fleas’

Playing with Words on the Computer

Troy Kaighin Astarte

Swansea University

An early non-numerical application of computers was processing human language, such as in the field of machine translation; natural language processing remains a significant field today. But almost as soon as they were employed for serious language applications, computers were applied to playful or artistic endeavours as well. This talk explores two historical examples: one fairly well-studied, the other a new archival discovery. One is Strachey’s 1952 program for randomly generating love letters, and the other a poetry programming competition held at Newcastle University in 1968.

In the early 1950s Christopher Strachey, a schoolmaster at Harrow, visited Manchester University to write some programs for its Ferranti Mark 1. This included a program randomly generate (rather mawkish) love letters. The letters were based on a template with blanks of particular types (adjectives, nouns, adverbs), and a pool of words which were inserted into the appropriate spaces at random. This program, which Strachey may have written with help from both Turing and his literary sister Barbara, represents a very early example of digital combinatory literature.

The University of Newcastle upon Tyne’s Computing Laboratory underwent a period of change in the late 1960s: expanding teaching, experiments in new networked computing, and growing breadth of research. Such research topics included automatic typesetting, medical literature information retrieval, and bibliographic data processing—manipulating language. In this context the lab director, Prof. Ewan Page, announced a competition for the production of limericks or poems, written *as programs*, such that their output was also poetic. The competition saw entries by a number of PhD students and the ingenuity on display was high. Programs humorously explored existing algorithms, reflected on life as a PhD student, or referenced classic works of literature.

Various analyses of Strachey’s work discuss its position as an early form of digital art, as a parody of attitudes towards love written by a gay man, and as a technical exploration. In my talk, I extend these analyses and consider how they apply to the Newcastle poetry competition. I argue that both examples show the crucial role of play in the practice of programming, and discuss how considering humans and machines together provides a better perspective on the perennial questions of the form “Will a computer ever write a symphony as good as Beethoven?”

Mathematics as Natural Language: Principles, Consequences and Challenges of the Application of NLP Models to the Treatment of Mathematical Knowledge

Juan Luis Gastaldi

ETH Zürich, Zürich ZH 8092, Switzerland
juan.luis.gastaldi@gess.ethz.ch

Recent years have seen a remarkable development of deep neural network (DNN) techniques for data analysis, along with their increasing application in scientific research across different disciplines. The field of mathematics has not been exempted from this general trend. Indeed, various works have suggested the relevance of possible applications of DNNs to the treatment of mathematical knowledge at multiple levels [1, 2, 4–9, 12, 14–16]. Significantly, the vast majority of those results resort to neural models specifically developed for the processing of natural language (NLP), from word embeddings [10, 11, 13] to seq2seq [17] and Transformers [3, 18].

This circumstance is remarkable for several reasons. Starting with the fact that, while the computational treatment of natural language traditionally implied an effort toward the latter's mathematization, it is now the mathematical knowledge that needs to be conceived as a kind of natural language, thus suggesting novel and non-trivial articulations between both. Furthermore, these contemporary neural approaches entail a renewed interest in textual aspects of mathematics and their representational capabilities. More precisely, since mathematical texts (statements, expressions, symbols) are all DNNs can rely on to perform tasks involving mathematical knowledge, the success of these methods would imply a new fundamental role of mathematical texts, going far beyond the usual understanding of mathematical writing as a simple notation for a pre-existing mathematical content, or a more or less arbitrary syntax for an independently determined semantics. Finally, even more than any other field of application, these attempts in mathematics raise critical epistemological questions since the formal (i.e., non-empirical) nature generally attributed to mathematical knowledge contrasts with the radically empirical position assumed by connectionist approaches guiding the application of DNNs and characterizing the practice of natural language.

After reviewing the most relevant literature in the field, this paper assesses the philosophical stakes of recent attempts to apply NLP models to mathematical knowledge. It concludes by indicating the conceptual and technical challenges and orientations to be drawn from such applications for a linguistically-driven philosophy of mathematics.

This project has received funding from the *European Union's Horizon 2020 research and innovation programme* under grant agreement No 839730.

References

1. Alemi, A.A., Chollet, F., Een, N., Irving, G., Szegedy, C., Urban, J.: Deepmath- deep sequence models for premise selection. In: Proceedings of the 30th International Conference on Neural Information Processing Systems, NIPS 2016, pp. 2243–2251. Curran Associates Inc., Red Hook, New York (2016)
2. Bansal, K., Loos, S.M., Rabe, M.N., Szegedy, C., Wilcox, S.: Holist: an environment for machine learning of higher-order theorem proving (extended version) (2019). http://arxiv.org/abs/1904.03241
3. Brown, T.B., et al.: Language models are few-shot learners (2020)
4. Charton, F.: Linear algebra with transformers (2021). https://arxiv.org/abs/2112.01898
5. Charton, F., Hayat, A., Lample, G.: Learning advanced mathematical computations from examples. In: International Conference on Learning Representations (2021). https://openreview.net/forum?id=-gfhS00XfKj
6. Davies, A., et al.: Advancing mathematics by guiding human intuition with AI. Nature **600** (7887), 70–74 (2021). https://doi.org/10.1038/s41586-021-04086-x, https://doi.org/10.1038/s41586-021-04086-x
7. Kaliszyk, C., Chollet, F., Szegedy, C.: Holstep: A machine learning dataset for higher-order logic theorem proving (2017). http://arxiv.org/abs/1703.00426
8. Lample, G., Charton, F.: Deep learning for symbolic mathematics (2019)
9. Loos, S.M., Irving, G., Szegedy, C., Kaliszyk, C.: Deep network guided proof search (2017). http://arxiv.org/abs/1701.06972
10. Mikolov, T., Chen, K., Corrado, G., Dean, J.: Efficient estimation of word representations in vector space (2013). http://arxiv.org/abs/1301.3781
11. Mikolov, T., Yih, W.t., Zweig, G.: Linguistic regularities in continuous space word representations. In: Proceedings of the 2013 Conference of the North American Chapter of the ACL: Human Language Technologies. pp. 746–751. ACL (2013)
12. Peng, S., Yuan, K., Gao, L., Tang, Z.: Mathbert: a pre-trained model for mathematical formula understanding (2021). https://arxiv.org/abs/2105.00377
13. Pennington, J., Socher, R., Manning, C.D.: Glove: Global vectors for word representation. In: EMNLP. vol. 14, pp. 1532–1543 (2014)
14. Polu, S., Han, J.M., Zheng, K., Baksys, M., Babuschkin, I., Sutskever, I.: Formal mathematics statement curriculum learning (2022). https://arxiv.org/abs/2202.01344
15. Polu, S., Sutskever, I.: Generative language modeling for automated theorem proving (2020). https://arxiv.org/abs/2009.03393
16. Saxton, D., Grefenstette, E., Hill, F., Kohli, P.: Analysing mathematical reasoning abilities of neural models (2019)
17. Sutskever, I., Vinyals, O., Le, Q.V.: Sequence to sequence learning with neural networks. In: Proceedings of the 27th International Conference on Neural Information Processing Systems, NIPS 2014, vol. 2, pp. 3104–3112. MIT Press, Cambridge (2014)
18. Vaswani, A., et al.: Attention is all you need (2017)

Are Large Language Models Models (of Language)?

Maël Pégny

Universität Tübingen
maelpegny@gmail.com

Abstract. Large Language Models (LLMs) are extremely large deep neural networks trained on humongous amount of data. Primarily trained for part-of-speech prediction, i.e. roughly the task of predicting what comes next in a text, they have been able to display not only state-of-the art performances for this task, but also have shown a great versatility to be fine-tuned for many other NLP tasks. Recently, they have even shown a surprising ability to be used as a basis to build efficient models for non-NLP tasks on multi-modal data. LLMs are thus slowly emerging as some of the most crucial models in all of AI. In this presentation, I will try to articulate the epistemological consequences of this evolution. First, I will first examine the respective consequences of two different paths towards task-agnosticity in NLP and beyond (in-context learning and transfer learning), and then see whether they can be seen as a true road towards, if not the great General AI, at least a more general AI. Finally, I will try to articulate how this question of task-agnosticity relates to the nature of the knowledge produced by large opaque models.

Are We There Yet? Meaning in the Age of Large Language Models

Jacopo Tagliabue

Coveo Labs, New York NY, USA
jtagliabue@coveo.com

1 Long Abstract

There is no time like the present. With the advent of the "golden age of Natural Language Processing" [1] (NLP), a contagious enthusiasm on the capabilities of large language models (LLMs) started spreading from research institutions into the general public [2]. While critics, mostly from academia, have repeatedly argued that LLMs show limited "understanding" [3, 4], the pace of development of increasingly larger models doesn't seem to slow down [5]: are we there yet?

In *this* talk, we briefly review the two dominant traditions on *meaning* of the last century:

- the "symbolic" tradition, where *meaning* is mostly about deductive composition of atomic components which are *given*; for example, see the model-theoretic semantics in [8];
- the "neural" tradition, where *meaning* is mostly about statistical association of atomic components which are *learned*; for example, see the distributional semantics in [9].

While a satisfactory unification of the two approaches is ultimately desirable [7], we argue that the duality of meaning – which sometimes behaves like a function, sometimes like a vector – is here to stay, at least for the time being. Contrary to the symbolic camp, we stress the importance of a theory of lexical acquisition and analogical reasoning; contrary to the neural camp, we stress the importance of *true* zero-shot generalization and a more rounded (and less naive) view of what counts as "grasping the meaning" of something [6].

In particular, the two camps seem to fundamentally disagree not (only) on what counts as a good explanation, but what is there to explain in the first place. In this perspective, we discuss the famous architecture behind recent LLMs – i.e. the transformer [10] –, which has been successfully adapted to many sequential problems that have nothing to do with the original NLP problem [11]: whether language is simply yet another sequence prediction problem is a contentious issue with deep ramifications into linguistics and cognitive sciences.

Finally, we conclude showing recent progress made in *grounded* language models – including our own research –, and sketch a roadmap for investigating meaning in more ecological settings.

References

1. A golden age for natural language. https://www.forbes.com/sites/louisaxu/2021/12/01/a-golden-age-for-natural-language/. Accessed 21 Jan 2022
2. A robot wrote this entire article. Are you scared yet, human. https://www.theguardian.com/commentisfree/2020/sep/08/robot-wrote-this-article-gpt-3. Accessed 31 Jan 2022
3. Bender, E.M., Koller, A.: Climbing towards NLU: on meaning, form, and understanding in the age of data. In: Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics, Online (2020)
4. GPT-3, Bloviator: OpenAI's language generator has no idea what it's talking about. https://www.technologyreview.com/2020/08/22/1007539/gpt3-openai-language-generator-artificial-intelligence-ai-opinion/. Accessed 23 Jan 2022
5. Using deepspeed and megatron to train megatron-turing nlg 530b, the world's largest and most powerful generative language model. https://www.microsoft.com/en-us/research/blog/using-deepspeed-and-megatron-to-train-megatron-turing-nlg-530b-the-worlds-largest-and-most-powerful-generative-language-model/. Accessed 29 Jan 2022
6. Lake, B.M., Murphy, G.L.: Word meaning in minds and machines. Psychological review (2021)
7. Baroni, M., Bernardi R., Zamparelli R.: Frege in space: a program of compositional distributional semantics. linguistic issues in language technology (2014)
8. Montague, R.: The proper treatment of quantification in ordinary english. In: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics, Dordrecht (1973)
9. Mikolov, T., Kai C., Gregory S.C., Dean J.: Efficient Estimation of Word Representations in Vector Space. ICLR (2013)
10. Vaswani, A., et al.: Attention is All you Need. ArXiv (2017)
11. Janner, M., Li, Q., Levine, S.: Reinforcement learning as one big sequence modeling problem. ArXiv (2021)

DNA Library Evidence Strings

Giuditta Franco

Department of Computer Science, University of Verona, Italy
giuditta.franco@univr.it

Generation of a combinatorial library of n-long binary strings, especially by starting from few sequences and by applying yet efficient basic DNA operations, has a biological, technological, and algorithmic relevance. Namely, some DNA based research, including tools and technologies from life science, aim at improving the cost, speed and efficiency of technologies for writing and combining DNA or other information-storing bio-polymers. DNA-based digital data storage provides a valid alternative to current technologies: it is promising in terms of information density (orders of magnitude higher than traditional memories) and of stability (millennia versus years). From a computational perspective, the design of efficient molecular algorithms is a challenge for the development of new biotechnologies generating DNA libraries.

In the context of design and development of string algorithms for computational biotechnology, this talk revolves around a simple DNA library generation algorithm, which starts from four specific DNA strings and efficiently produces a library of 2^n different strings in linear time. It consists in an iterative application of specific null context splicing rules, which recombine a couple of strings (in which one given substring occurs) by producing a new couple of chimerical strings, and may be implemented in laboratory by an XPCR procedure (a variant of the well known PCR). Of course such an algorithm is correct (and complete) iff it produces the whole library of DNA strings. Correctness is proved from a theoretical viewpoint while the experimental feasibility needs to be demonstrated independently. In an experimental context, the algorithm outcome is proved by the existence, in the final pool, of two specific patterns called library *evidence strings*, which are specific cyclic strings with a motif four characters long. If they (both) are present in the pool after the execution of the algorithm, we are guaranteed that each single instruction (an XPCR based string recombination) had produced the expected result, then the whole library has been generated (with no experimental drawbacks).

The algorithm with the experimental work validating all the procedures are presented in the talk, as well as the concept of evidence strings, with their combinatorial properties. This is a nice showcase where a string combinatorial property allows us to assess the experimental success of a DNA algorithm.

Keywords: Computational biotechnology · DNA library · Molecular computing · Periodic strings · XPCR

Turedo a New Computational Model for Molecular Nanobots?

Daria Pchelina[1], Nicolas Schabanel[2], Shinnosuke Seki[3], and Guillaume Theyssier[4]

[1] LIPN, Université Paris 13
daria.pchelina@normalesup.org
[2] CNRS - École Normale Supérieure de Lyon
nicolas.schabanel@ens-lyon.fr
http://perso.ens-lyon.fr/nicolas.schabanel/
[3] The University of Electro-Communications
shinsek@gmail.com
[4] Institut de Mathématiques de Marseille (CNRS - Université Aix-Marseille)
guillaume.theyssier@cnrs.fr

Abstract. Different models have been proposed to understand natural phenomena at the molecular scale from a computational point of view. Oritatami systems are a model of molecular co-transcriptional folding: the transcript (the "molecule") folds as it is synthesized according to a local energy optimisation process, in a similar way to how actual biomolecules such as RNA fold into complex shapes and functions. We introduce a new model, called turedo, which is a self-avoiding Turing machine on the plane that evolves by marking visited positions and that can only move to unmarked positions. Any oritatami can be seen as a particular turedo. We show that any turedo with lookup radius 1 can conversely be simulated by an oritatami, using a universal bead type set. Our notion of simulation is strong enough to preserve the geometrical and dynamical features of these models up to a constant spatio-temporal rescaling (as in intrinsic simulation). As a consequence, turedo can be used as a readable oritatami "higher-level" programming language to build readily oritatami "smart robots", using our explicit simulation result as a compiler. Furthermore, as our gadgets are simple enough, this might open the way to a readable oritatami programming, and these ingredients could be regarded as a promising direction to implement computation in co-transcribed RNA nanostructures in wetlab.

As an application of our simulation result, we prove three new complexity results on the (infinite) limit configurations of oritatami systems (and radius-1 turedos), assembled from a finite seed configuration. First, we show that such limit configurations can embed any recursively enumerable set, and are thus exactly as complex as aTAM limit configurations. Second, we characterize the possible densities of occupied positions in such limit configurations: they are exactly the Π_2-computable numbers between 0 and 1. We also show that all such limit densities can be produced by one single oritatami system, just by changing the finite seed configuration. Third, we exhibit a universal turedo (and consequently a universal oritatami system) that is able to build any finite shape

up to some upscaling from an asymptotically minimum size seed, and show conversely that uncomputably large upscaling is needed in general in this regards.

None of these results is implied by previous constructions of oritatami embedding tag systems or 1D cellular automata, which produce only computable limit configurations with constrained density.

Note that, reframing our results, we prove that doodling without lifting the pen nor intersecting lines and using only a 1-local view to decide for the drawing directions produce drawings as complex and as dense as can be.

Keywords: Molecular self-assembly · Co-transcriptional folding · Intrinsic simulation · Arithmetical hierarchy of real numbers · 2D Turing machines · Computability

Computability and Complexity in Morphogenetic Systems

Petr Sosík

Institute of Computer Science, Faculty of Philosophy and Science,
Silesian University in Opava, Czech Republic
petr.sosik@fpf.slu.cz

Keywords: Morphogenetic system · Membrane computing · Self-assembly · Turing universality · **P** versus **NP** · **PSPACE**

Extended Abstract

Morphogenesis, literally meaning *generation of the form*, lies at the core of many processes of both organic and inorganic nature, described in biology or geology. Morphogenesis inspires many ideas of human creation, e.g., in architecture, design and art. Basic principles of morphogenesis, in a nutshell, are controlled growth and self-assembly. Both these topics are often understood as (semi)-algorithmic processes, for a mutual benefit of biology, chemistry and computer science.

To capture algorithmic aspects of morphogenesis, we have created a formal model of morphogenetic growth called the *morphogenetic (M) system* [5]. The model unfolds in a 3D (or generally, dD) continuous space in discrete time steps. Spatial structure of the model is determined by the underlying so-called polytopic tile system in $\mathbb{R}^d$. It is based on a generalization of Wang tiles to arbitrary d'-dimensional polytopes of specified sizes and shapes, $1 \le d' \le d-1$. Unlike Wang tiling or algorithmic tile assembly (aTAM) [1], the tiles (polytopes) are not present in an arbitrary many copies, but they are created by reactions of simpler shapeless atomic objects. These objects can mutually react and pass through a specific *protion channels* in tiles. Their "metabolism" is controlled by a set of evolutionary rules inspired by membrane systems with proteins on membranes [2]. Every object, either a tile or a floating object, has at each moment its specific position and orientation in $\mathbb{R}^d$, possibly changing as the system evolves. The combination of self-assembly and evolutionary rules provides the M system with feedback loops and hence with the ability of a surprisingly complex behaviour. We refer the reader to [5] for a formal description and detailed examples of M systems formation.

In previous publications we have used M systems as models of bacterial growth, self-healing properties and resistance to damages caused by, e.g., antibiotics [3]. Computer simulations with our freely available software package Cytos

Supported by the Silesian University in Opava under the Student Funding Scheme, project SGS/8/2022.

(https://github.com/mmaverikk/Cytos) were in a very good agreement with biological experiments. Here we focus on computational aspects of morphogenetic systems. After reviewing previous results related to their computational universality (in the Turing sense), we present two new results on minimal universal M systems.

Theorem 1. *There exists a universal M system in 2D with three tiles, 26 floating objects, one protion and 26 rules.*

We further extend the result to the case of self-healing M system which can recover from injuries to their structure.

Theorem 2. *There exists an M system in 2D with 8 tiles, 28 floating objects, 4 protions and 100 rules, that simulates a universal Turing machine M on any given input in linear time, and it is self-healing of degree 1, provided that injuries at each step only affect tiles and objects belonging to a single tape cell.*

Then we focus on computational complexity of M systems and we demonstrate how they can characterize the **P** versus **NP** borderline. M systems under standard definition can solve NP-complete problems in randomized polynomial time. We introduce *M systems with mass*, where mutual pushing of objects are at each step limited by a certain certain distance, due to their nonzero mass.

Theorem 3. *M systems with mass can solve in polynomial time exactly the class of problems* **P**.

Finally, we also discuss a possible relation of M systems to the class **PSPACE** and we conjecture that, even under the standard definition, they most likely cannot solve **PSPACE**-complete problems in a polynomial time. For a more detailed description of the results presented here we refer the reader to [4].

References

1. Krasnogor, N., Gustafson, S., Pelta, D., Verdegay, J.: Systems self-assembly: multidisciplinary snapshots. studies in multidisciplinarity, Elsevier Science (2011)
2. Paun, A., Popa, B.: P systems with proteins on membranes. Fundamenta Informaticae. **72**(4), 467–483 (2006)
3. Smolka, V., Drastík, J., Bradík, J., Garzon, M., Sosík, P.: Morphogenetic systems: models and experiments. Biosystems **198**, Art. No. 104270 (2020). https://doi.org/10.1016/j.biosystems.2020.104270
4. Sosík, P.: Morphogenetic computing: computability and complexity results. Submitted (2022)
5. Sosík, P., Smolka, V., Drastík, J., Moore, T., Garzon, M.: Morphogenetic and homeostatic self-assembled systems. In: Patitz, M., Stannett, M. (eds.) Unconventional Computation and Natural Computation. UCNC 2017. LNCS, vol. 10240. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-58187-3_11

Computability Theory and Reverse Mathematics via Lawvere-Tierney topologies

Takayuki Kihara

Department of Mathematical Informatics, Graduate School of Informatics,
Nagoya University, Japan
kihara@i.nagoya-u.ac.jp

We present a new perspective of oracle: We consider an oracle to be an "*operation on truth-values*" that may cause a transformation of one world into another: A mathematical statement φ may be false in computable mathematics, but φ can be true in computable mathematics relative to an oracle α. This means that the oracle α caused a change in the truth value of the statement φ, and also caused a change from the computable world to the α-relative computable world. One might say that this is based on the idea that there is a correspondence between "*computations using oracles*" and "*proofs using transcendental axioms*". Such an idea is used as a very standard technique in, for example, classical reverse mathematics. Our approach is similar, but with a newer perspective that deals more directly with operations on truth-values. More explicitly, it is formulated using topos-theoretic notions such as Lawvere-Tierney topology, which is a kind of generalization of Grothendieck topology to an arbitrary topos.

In this talk, we will connect the structure of the Lawvere-Tierney topologies on a certain relative realizability topos (e.g., the effective topos; the Kleene-Vesley topos) with a certain degree structure in computability theory, based on previous work by Lee and van Oosten. For this purpose, we introduce a new computability-theoretic reducibility notion, which is a common extension of the notions of Turing reducibility and generalized Weihrauch reducibility. This notion can be thought of as a fusion of the notions of generalized Weihrauch reducibility and Bauer's extended Weihrauch reducibility. We introduce a realizability predicate relative to a "extended generalized Weihrauch degree", which is identical to the realizability relative to the corresponding Lawvere-Tierney topology, and then show some separation results on constructive reverse mathematics.

References

1. Takayuki K.: Degrees of incomputability, realizability and constructive reverse mathematics. arXiv:2002.10712 (2020)

The author's research was partially supported by JSPS KAKENHI Grant Numbers 19K03602 and 21H03392.

2. Takayuki K.: Lawvere-Tierney topologies for computability theorists. arXiv:2106.03061 (2021)
3. Takayuki K.: Rethinking the notion of oracle: A link between synthetic descriptive set theory and effective topos theory. arXiv:2202.00188 (2022)

Reverse Mathematics and Semisimple Rings

Hishan Wu

School of Information Science, Beijing Language and Culture University,
15 Xueyuan Road, Haidian District, Beijing 100083
huishanwu@blcu.edu.cn

We study rings and modules from the standpoint of reverse mathematics. In this talk, we mainly focus on semisimple rings and semisimple modules. Semisimple modules are often defined as modules that can be written as direct sums of simple submodules. In 2013, Yamazaki initiated the study of semisimple modules as well as other kinds of modules like projective modules and injective modules in reverse mathematics; he showed that the statement "every submodule of a semisimple module is a direct summand" is equivalent to ACA_0 over RCA_0. Semisimple modules have various equivalent definitions in classical algebra. We first discuss equivalent characterizations of semisimple modules in reverse mathematics. By choosing a different characterization for semisimple modules, We first discuss equivalent characterizations of semisimple modules in reverse mathematics. By choosing a different characterization for semisimple modules, we define a left R-module M over a ring R to be semisimple if every submodule of it is a direct summand. We view a ring R as left semisimple if the left regular module ${}_RR$ is semisimple. Based on such definitions of semisimple modules and semisimple rings, we study characterizations of left semisimple rings in terms of projective modules and injective modules in reverse mathematics. For instance, we show that ACA_0 is equivalent to the statement "any left module over a left semisimple ring is projective" over RCA_0 and that ACA_0 proves the statement "if every cyclic left R-module is injective, then R is a left semisimple ring". For more details of the work, refer to a recent paper in Archive for Mathematical Logic https://doi.org/10.1007/s00153-021-00812-4.

The Skolem Landscape

James Worrell

Department of Computer Science, University of Oxford, UK
jbw@cs.ox.ac.uk

Abstract. We overview the Skolem and Positivity Problems for *C*-finite and *P*-finite recurrence sequences. We describe the history of these problems, their relevance to computer science, and the state of the art as regards decidability.

Keywords: Recurrence sequences · Skolem-Mahler-Lech theorem · Skolem problem · Positivity problem

1 A Landscape of Decision Problems

This talk aims to paint a landscape of decision problems for recurrence sequences. We consider sequences that satisfy linear recurrences with constant coefficients (the so-called *C*-finite sequences) and, more generally, we consider those that satisfy recurrences with polynomial coefficients (the so-called *P*-finite sequences). For example, the Fibonacci sequence is *C*-finite, while the sequence of harmonic numbers is *P*-finite. Many authors use the term holonomic in place of *P*-finite.

The two main decision problems that we investigate are the ***Skolem Problem*** (does the sequence have a zero term?) and the ***Positivity Problem*** (are all terms of the sequence positive?). From a computer science perspective, we consider these as canonical reachability problems for linear systems.

Decidability of the Skolem and Positivity Problems are open, both for *C*-finite sequences and for *P*-finite sequences. In the talk we will survey the history of the two problems, starting with the celebrated Skolem-Mahler-Lech theorem which characterises the structure of the set of zeros of a *C*-finite sequence. We will mention also subsequent variations and generalisations of this theorem to *P*-finite sequences, both in finite and zero characteristic. For further motivation, we will describe some of the many different guises in which the Skolem and Positivity Problems appear in automata theory, logic and model checking, analysis of algorithms, combinatorics, and related areas.

In the second half of the talk, we will describe partial decidability results for variations of the problems, including recent developments (of ourselves and others). We will, in particular, mention a recent proof that the Skolem Problem for simple *C*-finite sequences is decidable subject to two well known number-theoretic conjectures: the *p*-adic Schanuel Conjecture and the exponential local-global principle. In general, we will attempt to give a flavour of some of the relevant mathematics, which ranges from classical results in Diophantine geometry, such as the Subspace theorem, to more speculative number-theoretic conjectures, such as the periods conjecture of Kontsevich and Zagier.

In summary, the talk aims to give an idea of a landscape of decision problems for recurrence sequences, to explain why the problems are important, what is currently known in terms of partial decidability results, and why decidability of the central problems in this landscape remain open.

Contents