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Partitioning Dense Graphs with Hardware Accelerators

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Abstract. Graph partitioning is a fundamental combinatorial optimization problem that attracts a lot of attention from theoreticians and practitioners due to its broad applications. From multilevel graph partitioning to more general-purpose optimization solvers such as Gurobi and CPLEX, a wide range of approaches have been developed. Limitations of these approaches are important to study in order to break the computational optimization barriers of this problem. As we approach the limits of Moore's law, there is now a need to explore ways of solving such problems with special-purpose hardware such as quantum computers or quantum-inspired accelerators. In this work, we experiment with solving the graph partitioning on the Fujitsu Digital Annealer (a special-purpose hardware designed for solving combinatorial optimization problems) and compare it with the existing top solvers. We demonstrate limitations of existing solvers on many dense graphs as well as those of the Digital Annealer on sparse graphs which opens an avenue to hybridize these approaches.

Keywords: Graph Partitioning \cdot Dense Graphs \cdot Digital Annealer \cdot Quantum-Inspired

1 Introduction

There are several reasons to be optimistic about the future of quantum-inspired and quantum devices. However, despite their great potential, we also need to acknowledge that state-of-art classical methods are extremely powerful after years of relentless research and development. In classical computing, the development of algorithms, the rich mathematical framework behind them, and sophisticated data structures are relatively mature, whereas the area of quantum computing is still at its nascent stage. Many existing classical algorithms do not have provable or good enough bounds on the performance (e.g., they might not have ideal performance in the worst case), but in many applications, the worst-case scenarios are rather rarely seen. As a result, such algorithms, many of which heuristics, can achieve excellent results in terms of the solution quality or speed. Therefore, when utilizing emerging technologies such as quantum-inspired hardware accelerators and quantum computers to tackle certain problems, it is important to compare them not only with possibly slow but provably strong algorithms but also with the heuristic algorithms that exhibit reasonably good results on the instances of interest.

The graph partitioning [3] is one of the combinatorial optimization problems for which there exists a big gap between rigorous theoretical approaches that ensure best known worst-case scenarios, and heuristics that are designed to cope with application instances exhibiting a reasonable quality-speed trade-off. Instances that arise in practical applications often contain special structures on which heuristics are engineered and tuned. Because of its practical importance, a huge amount of work has been done for a big class of graphs that arise in such areas as combinatorial scientific computing, machine learning, bioinformatics, and social science, namely, *sparse graphs*. Over the years, there were several benchmarks on which the graph partitioning algorithms have been tested and compared with each other to mention just a few [2,7,41]. However, *dense graphs* can be rarely found in them. The situation with general dense linear algebra instances, many of which are used to test graph partitioners, is just a little bit better. In many cases, working with dense graphs requires very different algorithms and advanced computational resources. As a result, most existing excellent graph partitioning heuristics do not perform well in practice on dense graphs, while provable algorithms with complexity that depends on the number of edges (or non-zeros in the corresponding matrix) are extremely slow. As we also show in computational results, a graph sparsification does not necessarily practically help to achieve high-quality solutions.

Multilevel Algorithms This class of heuristics is one of the most successful for a variety of cut-based graph problems such as the minimum linear arrangement [31], and vertex separator [11]. Specifically for a whole variety of (hyper)graph partitioning versions [21, 22, 32, 34] these heuristics exhibit best quality/speed trade-off [3]. In multilevel graph partitioning frameworks, a hierarchy of coarse graph representations is constructed in such a way that each next coarser graph is smaller than the previous finer one, and a solution of the partitioning for the coarse graph can approximate that of the fine graph and be further improved using fast local refinement. Multilevel algorithms are ideally suited for sparse graphs and suffer from the same problems as the algebraic multigrid (which generalizes, to the best of our knowledge, all known multilevel coarsening for partitioning) on dense matrices. In addition, a real scalability of the existing refinement for partitioning is achieved only

2 X. Liu et al.

for sparse local problems. Typically, if the density is increasing throughout the hierarchy construction, various ad-hoc tricks are used to accelerate optimization sacrificing the solution quality. When such things happen at the coarse levels, an error is quickly accumulated. Here we compare our results with KaHIP [33] which produced the best results among several multilevel solvers [3].

Hardware Accelerators for Combinatorial Problems Hardware accelerators such as GPU have been pivotal in the recent advancements of fields such as machine learning. Due to the computing challenges arising as a result of the physical scaling limits of Moore's law, scientists have started to develop special-purpose hardware for solving combinatorial optimization problems. Examples of such hardware include adiabatic quantum computers [19], complementary metal-oxide-semiconductor (CMOS) annealers [1] and coherent Ising machines [17]. The gatebased universal quantum computers can also be used to solve such optimization problems [26]. These novel technologies are all unified by an ability to solve the Ising model or, equivalently, the quadratic unconstrained binary optimization (QUBO) problem. The general QUBO is NP-hard and many problems can be formulated as QUBO [28]. Previous work on using QUBO based models include areas such as clustering and community detection [4, 5, 20, 29] [35, 36, 40], chemistry [15, 16, 39], finance [30], and machine learning [6, 14, 23, 25]. It is also often used as a subroutine to model large neighborhood local search [27]. The Fujitsu Digital Annealer (DA) [8], used in this work, utilizes application-specific integrated circuit hardware for solving fully connected QUBO problems. Internally the hardware runs a modified version of the Metropolis-Hastings algorithm for simulated annealing. The hardware utilizes massive parallelization and a novel sampling technique. The novel sampling technique speeds up the traditional Markov Chain Monte Carlo by almost always moving to a new state instead of being stuck in a local minimum. Here, we use the third generation DA, which is a hybrid software-hardware configuration that supports up to 100,000 binary variables. DA also supports users to specify inequality constraints and special equality constraints such as 1-hot and 2-way 1-hot constraints.

Our contribution The goal of this paper is twofold. First, we demonstrate that existing scalable graph partitioning dedicated solvers are struggling with the dense graphs not only in comparison to the special-purpose hardware accelerators but even sometimes if compared to generic global optimization solvers that are not converged. At the same time, we demonstrate a clear superiority of classical dedicated graph partitioning solvers on sparse instances. Second, this work is a step towards investigating what kind of problems we can solve using combinatorial hardware accelerators. Can we find problems that are hard for existing methods, but can be solved more efficiently with novel hardware and specialized algorithms? As an example, we explore the performance of Fujitsu Digital Annealer (DA) on graph partitioning and compare it with general-purpose solver Gurobi, and also graph partitioning solver KaHIP.

We do not attempt to achieve an advantage for every single instance, especially since at the current stage, the devices we have right now are still facing many issues on scalability, noise, and so on. However, we advocate that hybridization of classical algorithms and specialized hardware (e.g., future quantum and existing quantum-inspired hardware) is a good candidate to break the barriers of the existing quality/speed trade-off.

2 Graph Partitioning Formulations

Let G = (V, E) be an undirected, unweighted graph, where V denotes the set of n vertices, and E denotes the set of m edges. The goal of perfect balanced k-way graph partitioning (GP), is to partition V into k parts, V_1, V_2, \dots, V_k , such that the k parts are disjoint and have equal size, while minimizing the total number of *cut* edges. A *cut edge* is an edge that has two end vertices assigned to different parts. Sometimes, the quality of the partition can be improved if we allow some imbalance between different parts. In this case, we allow some imbalance factor $\epsilon > 0$, and each part can have at most $(1 + \epsilon) [n/k]$ vertices.

Binary Quadratic Programming Formulation of GP We first review the integer quadratic programming formulation for k-way GP [12, 40]. When k = 2, we introduce binary variables $x_i \in \{0, 1\}$ for each vertex $i \in V$, where $x_i = 1$ if vertex *i* is assigned to one part, and 0 otherwise. We denote by **x** the column vector $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$. The quadratic programming is then given by

$$\min_{\mathbf{x}} \mathbf{x}^T L \mathbf{x} \quad \text{such that } x_i \in \{0, 1\}, \ \forall i \in V,$$
(1)

where L is the Laplacian matrix of graph G. The Laplacian matrix L is defined as L = D - A, where D is a diagonal matrix, with the degree of each node on the diagonal entries, and A is the adjacency matrix of graph G, with $A_{ij} = 1, \forall (i, j) \in E$ and 0 otherwise. For perfect balance GP, we have the following equality constraint:

$$\mathbf{x}^T \mathbb{1} = \left\lceil \frac{n}{2} \right\rceil,\tag{2}$$

where 1 is the column vector with ones. For the imbalanced case, we have the following inequality constraint:

$$\mathbf{x}^T \mathbf{1} \le (1+\epsilon) \left\lceil \frac{n}{2} \right\rceil. \tag{3}$$

When k > 2, we introduce binary variables $x_{i,j} \in \{0,1\}$ for each vertex $i \in V$ and part j, where $x_{i,j} = 1$ if vertex i is assigned to part j, and 0 otherwise. Let \mathbf{x}_j denote the column vector $\mathbf{x}_j = (x_{1,j}, x_{2,j}, \cdots, x_{n,j})^T$ for $1 \leq j \leq k$. The quadratic programming formulation is then given by

$$\min_{\mathbf{x}} \quad \frac{1}{2} \sum_{j=1}^{k} \mathbf{x}_{j}^{T} L \mathbf{x}_{j}$$
s.t.
$$\sum_{j=1}^{k} x_{i,j} = 1, \quad \forall i \in V,$$

$$x_{i,j} \in \{0,1\}, \quad \forall i \in V, \quad 1 \le j \le k$$

Again, for perfect balance GP, we have another set of equality constraints:

$$\mathbf{x}_j^T \mathbb{1} = \left\lceil \frac{n}{k} \right\rceil, \quad 1 \le j \le k.$$

For the imbalance case, we have the following inequality constraints:

$$(1-\epsilon)\left\lceil \frac{n}{k}\right\rceil \le \mathbf{x}_j^T \mathbb{1} \le (1+\epsilon)\left\lceil \frac{n}{k}\right\rceil, \quad 1\le j\le k.$$

QUBO Formulation To convert the problem into QUBO model, we will need to remove the constraints and add them as penalty terms to the objective function [28]. For example, in the quadratic programming (1) with the equality constraint (2), we obtain the QUBO model as follows:

$$\min_{\mathbf{x}} \mathbf{x}^T L \mathbf{x} + P\left(\mathbf{x}^T \mathbb{1} - \left\lceil \frac{n}{2} \right\rceil\right)^2$$

s.t. $x_i \in \{0, 1\}, \quad \forall i \in V,$

where P > 0 is a postive parameter to penalize the violation of constraint (2). For inequality constraints, we will introduce additional slack variables to first convert the inequality to equality constraints, and then add them as penalty terms to the objective function.

3 Computational Experiments

The goal of the experiments was to identify the class of instances that is more suitable to be solved using the QUBO framework and the current hardware. We compare the performance of DA with exact solver Gurobi [9], and the state-of-the-art multilevel graph partitioning solver KaHIP [33]. We set the time limit for DA and Gurobi to be 15 minutes. For KaHIP, we use KaFFPaE, a combination of distributed evolutionary algorithm and multilevel algorithm for GP. KaFFPaE computes partitions of very high quality when the imbalance factor $\epsilon > 0$, but does not perform very well for the perfectly balanced case when $\epsilon = 0$. Therefore we also enable KaBaPE, which is recommended by the developers. We run KaFFPaE with 24 processes in parallel, and set the time limit to be 30 minutes.

To evaluate the quality of the solution, we compare the approximation ratio, which is computed using the GP cut found by each solver divided by the best-known value. For some graphs, we have the best-known provided from the benchmark [41], otherwise we use the best results found by the three solvers as the best known. Since this is a minimization problem, the minimum possible value of the approximation ratio is 1, the smaller the better. For each graph and each solver used, we also provide the objective function value, i.e., the number of cut edges.

Graph Partitioning on Sparse Graphs We first test the three solvers on instances from the Walshaw graph partitioning archive [41]. The information of the graphs is given in Table 1, where |V| is the number of nodes of the graph, and $d_{avg} = |E|/|V|$ describes the density of the graph. We present the summary of the results with box plots in Fig. 1 (a), (d), (g) and (j). We also provide the objective function value of each graph obtained by the three solvers in Table 1. We observe that in Figure 1 (g) and (j), where we compare DA and Gurobi, DA can find the best-known partition for most instances, and perform better compared to Gurobi. However, for several sparse graphs, i.e., $d_{avg} < 3$, for example, uk, add32 and 4elt, DA can not find the best-known solutions. For these sparse graphs, multilevel graph partitioning solvers such as KaHIP can usually perform

an effective coarsening and uncoarsening procedure based on local structures of the graph and therefore find good solutions quickly. As shown in Fig. 1 (a), (d) and Table 1, KaHIP performs better than DA. Based on the numerical results, we conclude that for the sparse graphs, generic and hardware QUBO solvers do not lead to many practical advantages. However, graphs with more complex structures, that bring practical challenges to the current solvers might benefit from using the QUBO and hardware accelerators.



Fig. 1: Comparison of DA with KaHIP (dedicated GP solver), and Gurobi (general-purpose solver) for sparse and dense graphs respectively. The y-axis represents the approximation ratio (solution to best-solution ratio), the minimum possible value of the approximation ratio is 1, the smaller the better. The x-axis represents the imbalance factor as percentage

Graph Partitioning on Dense Graphs To validate our conjecture, in the next set of experiments, we examine dense graphs from the SuiteSparse Matrix Collection [7] (see details in Table 2). The experimental results are presented in Fig. 1 (b), (e), (h) and (k), and the objective function values of each graph obtained by the three solvers are provided in Table 2. We observe that for these dense graphs, in general, DA is able to find solutions that are usually at least as good as those produced by KaHIP and Gurobi. In particular, we find that for one

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k = 2		d	00	% imba	lance		19	% imba	lance	
graph		u_{avg}	Best known	DA	KaHIP	Gurobi	Best known	DA	KaHIP	Gurobi
add20	2395	3.12	596	596	613	596	585	586	591	585
data	2851	5.29	189	189	189	212	188	188	188	193
3elt	4720	2.91	90	90	90	91	89	89	89	90
uk	4824	1.42	19	19	19	19	19	43	19	19
add32	4960	1.91	11	11	11	11	10	41	10	10
bcsstk33	8738	33.37	10171	10171	10171	11674	10097	10097	10097	53045
whitaker3	9800	2.96	127	127	127	640	126	126	126	669
crack	10240	2.97	184	184	184	331	183	195	183	336
wing_nodal	10937	6.90	1707	1707	1707	13739	1695	1695	1695	10274
fe_4elt2	11143	2.95	130	266	130	746	130	330	130	629
vibrobox	12328	13.40	10343	10343	10343	31029	10310	10310	10310	31598
bcsstk29	13992	21.64	2843	2843	2843	26144	2818	2826	2818	25966
4elt	15606	2.94	139	256	139	592	138	278	138	564
fe_sphere	16386	3.00	386	386	386	1082	386	386	386	1128
cti	16840	2.86	334	334	334	2443	318	318	318	2471
memplus	17758	3.05	5499	7286	5550	14030	5452	7293	5476	11249

Table 1: Objective value (number of cut edges) of graphs from Walshaw graph partition archive, k = 2, 3.

k = 2	V	d	39	% imbal	lance		5% imbalance					
graph		$ u_{avg} $	Best known	DA	KaHIP	Gurobi	Best known	DA	KaHIP	Gurobi		
add20	2395	3.12	560	560	568	560	536	536	540	536		
data	2851	5.29	185	185	185	212	181	188	181	190		
3elt	4720	2.91	87	87	87	87	87	87	87	87		
uk	4824	1.42	18	36	18	18	18	38	18	18		
add32	4960	1.91	10	70	10	10	10	49	10	10		
bcsstk33	8738	33.37	10064	10064	10064	43173	9914	9914	10010	11105		
whitaker3	9800	2.96	126	126	126	195	126	126	126	176		
crack	10240	2.97	182	195	182	571	182	201	182	254		
wing_nodal	10937	6.90	1678	1678	1678	11392	1668	1668	1668	9379		
fe_4elt2	11143	2.95	130	261	130	132	130	287	137	130		
vibrobox	12328	13.40	10310	10310	10310	34490	10310	10310	10310	31173		
bcsstk29	13992	21.64	2818	3273	2818	25601	2818	2823	2818	25084		
4elt	15606	2.94	137	293	137	482	137	360	137	507		
fe_sphere	16386	3.00	384	384	384	1120	384	384	384	762		
cti	16840	2.86	318	343	318	1424	318	324	318	$14\overline{55}$		
memplus	17758	3.05	5352	7137	5362	11676	5253	7001	5274	10923		

k = 3	0%	0% imbalance			1% imbalance			6 imbala	ance	5% imbalance			
graph	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	
add20	936	949	1148	941	937	1093	924	923	1067	950	894	1029	
data	261	261	5129	259	259	4539	265	255	6528	253	250	6348	
3elt	168	162	5023	185	162	4292	177	162	4855	162	159	4449	
uk	175	32	50	210	31	154	233	31	144	202	29	33	
add32	133	28	28	135	25	25	184	22	22	172	19	19	
bcsstk33	16247	16247	189894	16155	16059	27722	15960	15819	178712	15679	15552	47890	
whitaker3	253	253	12842	412	253	12160	422	253	12198	258	252	19294	
crack	492	288	20263	570	287	20304	459	281	20304	573	281	20304	
wing_nodal	2864	2850	50296	3187	2844	50370	2978	2828	50370	3228	2813	50370	
fe_4elt2	515	248	21800	618	248	21865	636	248	21865	643	248	21865	

h = 2			007	imbolon		1% imbalance				
$\kappa = 2$	V	<i>d</i>	07		æ	1/		e		
graph		u_{avg}	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi		
exdata_1	6001	188.59	2000	28646	2000	1980	33004	1980		
TSC_OPF_1047	8140	123.39	1188	1187	1763	1381	1170	1586		
nd3k	9000	181.71	149880	149880	817727	149829	149829	818433		
nemeth26	9506	79.02	3298	3298	4842	3284	3284	3297		
mycielskian14	12287	150.38	553735	553735	924567	545355	545355	923991		
human_gene2	14340	629.50	544938	546204	4514331	542850	543834	4514154		
opt1	15449	61.98	24725	24725	479054	24030	24030	448245		
gupta3	16783	277.26	1143782	1143782	2325983	1137072	1137072	2323035		
ramage02	16830	84.66	80940	80940	712906	80912	80912	707102		
pkustk07	16860	71.23	66852	66852	600140	66834	66834	588200		
k = 2		d	3%	6 imbaland	e	5%	6 imbalanc	e		
graph		u_{avg}	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi		
exdata_1	6001	188.59	1940	19776	1940	1900	19736	1900		
TSC_OPF_1047	8140	123.39	1339	1146	1647	1185	1140	1591		
nd3k	9000	181.71	148935	148935	803125	148403	148403	727512		
nemeth26	9506	79.02	3284	3284	3290	3284	3284	-1		
mycielskian14	12287	150.38	527882	527883	923991	509891	509891	923991		

Table 2: Objective value (number of cut edges) of graphs from SuiteSparse Matrix Collection, k = 2, 3.

k = 3	0% imbalance			1% imbalance			39	% imbala	nce	5% imbalance			
graph	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	
exdata_1	2668	17550	2840	2654	14576	2664	2628	13066	2628	2600	16004	2678	
TSC_OPF_1047	78608	78369	668861	72610	72066	669320	72055	71688	669320	71243	70727	669320	
nd3k	230329	230329	1088853	230099	230042	1053084	229898	229652	1035165	229417	229309	1077240	
nemeth26	6748	6748	141759	6736	6736	122437	6736	6736	41057	6736	6736	20107	

14340 629.50

15449 61.98

16783 277.26

16830 84.66

16860 71.23

human_gene2

opt1

gupta3

ramage02

pkustk07

instance, exdata_1, KaHIP fails significantly. We therefore use a graph generator MUSKETEER [10] to generate similar instances³. The details of the parameters used to generate the graphs can be found in the appendix. In short, MUSKETEER applies perturbation to the original graph with a multilevel approach, the local editing preserves many network properties including different centralities measures, modularity, and clustering. The information about generated instances is given in Table 3. The experiment results are presented in Fig. 1 (c), (f), (i) and (l), and the objective function value of each graph obtained by the three solvers are provided in Table 3. We find that in most instances, DA outperforms KaHIP and Gurobi, demonstrating that in this class of problems, specialized hardware such as DA is having an advantage.

Currently, to tackle GP on dense graphs, the main practical solution is to first sparsify the graphs [13, 18, 24, 37] (hoping that the sparsified graph still preserves the structure of the original dense graph), solve GP on the sparsified graph, and finally project the obtained solution back to the original graph. We have applied the Forest Fire sparsification [13] available in Networkit [38]. This sparsification is based on random walks. The vertices are burned starting from a random vertex, and fire may spread to the neighbors of a burning vertex. The intuition is that the edges that are visited more often during the random walk are more important in the graph. In our experiments, we eliminate 30% of the edges. Then we solve GP using KaHIP (KaffpaE version) and project the obtained solution back to the original dense graph. We repeat the entire procedure 10 times for each graph, and compare the best results obtained with DA and KaHIP. As shown in Fig. 2, for dense graphs with complex structures, KaHIP does not outperform DA, and graph sparsification does not help to achieve this goal. In this case, we advocate the use of the QUBO framework and specialized hardware.

4 Conclusion and Discussion

As novel technologies for solving computational combinatorial optimization problems emerge, it is important to identify areas in which these technologies outperform both existing state-of-the-art general-purpose and also

³ The exdata graph files are available here: https://github.com/JoeyXLiu/dense-graph-exdata

Table 3: Objective value (number of cut edges) of graphs generated	from exdata_1,	k = 2,	3
					_

k = 2		d	0%	imbala	ince	1%	imbala	nce	3%	imbala	nce	5%	imbala	nce
graph		u_{avg}	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi
exdata_2	6016	184.22	31769	32716	554144	31319	32373	554060	30421	31536	554060	29554	30462	554060
$exdata_3$	6225	187.60	32002	32841	583986	31573	32527	584154	30736	31790	583975	29913	30746	583975
exdata_4	6026	172.27	25446	26142	519131	25128	26015	518990	24504	25338	518990	23904	24708	518990
exdata_5	6052	183.74	28422	29466	556394	28051	29104	556311	27325	28333	556311	26593	27648	556311
$exdata_6$	6056	183.22	658	668	555451	552	564	5374	310	320	555398	169	169	555398
exdata_7	6028	186.86	30612	31520	563061	30192	31118	562987	29355	30285	562987	28545	29511	562987
exdata_8	6074	185.45	30682	31071	563758	30242	30996	563668	29425	30112	563657	28641	29353	563657
$exdata_9$	6046	184.24	627	655	27805	492	511	557185	256	268	1268	117	117	7910
$exdata_{10}$	6139	192.34	29434	30298	588719	29059	29897	588986	28296	29162	588729	27561	28591	588729
$exdata_{11}$	6091	130.47	1052	1364	1054	1033	1282	1736	990	1215	990	954	1197	950
$exdata_12$	6390	169.91	1290	1585	2087	1270	1509	541868	1250	1395	541810	1206	1330	541810
$exdata_13$	6026	108.19	813	1160	820	843	1118	841	799	983	781	733	921	713
$exdata_14$	5827	99.43	610	766	608	616	732	598	584	668	582	582	646	560
$exdata_{15}$	6380	153.01	1295	1496	1295	1277	1468	1275	1235	1368	1232	1189	1340	1189
$exdata_16$	6686	176.23	838	5816	838	826	5806	816	796	1710	828	752	1081	728
$exdata_17$	5813	118.20	1044	1220	1043	1038	1238	1024	996	1094	985	956	1117	946
$exdata_{18}$	5769	136.95	1058	1288	1090	1050	1317	1048	1002	1164	1000	979	1135	962
$exdata_19$	6062	108.49	950	1041	329090	932	1003	2651	890	951	3973	853	925	851
exdata_20	5990	159.65	1070	1474	1093	1051	1382	1051	1015	1620	1025	977	1688	978

k = 3		d	0%	imbala	nce	1%	imbala	nce	3%	imbala	nce	5% imbalance			
graph		u_{avg}	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	DA	KaHIP	Gurobi	
exdata_2	6016	184.22	47529	48008	738783	47215	47519	738784	46579	47179	738784	46005	46460	738784	
exdata_3	6225	187.60	46893	47433	778088	46649	47234	778091	45911	46556	778091	45289	45844	778091	
exdata_4	6026	172.27	37468	38530	692385	37258	38365	692383	36692	37649	692383	36186	37186	692383	
$exdata_5$	6052	183.74	42031	42749	741759	41806	42451	741761	41202	41916	741761	40552	41232	741761	
exdata_6	6056	183.22	14306	15219	740057	14193	14814	740056	13794	13974	740056	13109	13217	740056	
exdata_7	6028	186.86	45890	46420	749893	45573	46123	749886	45140	45530	749886	44471	44947	749886	
exdata_8	6074	185.45	44025	44489	751243	43738	44243	751254	43133	43662	751254	42560	42920	751254	
exdata_9	6046	184.24	14627	15582	742520	14752	15176	742522	14032	14305	742522	13278	13635	742522	
$exdata_{10}$	6139	192.34	43057	43983	787039	42858	43600	787032	42182	43010	787032	41596	42404	787032	
$exdata_{11}$	6091	130.47	1866	2214	529928	1838	2046	529927	1795	2078	529927	1745	1985	529927	
$exdata_12$	6390	169.91	2723	3232	724215	2742	2971	724224	2534	2789	724224	2485	2801	724224	
$exdata_13$	6026	108.19	1499	1606	1993	1505	1605	3272	1579	1561	2421	1474	1504	2470	
$exdata_14$	5827	99.43	1191	1369	2666	1194	1314	1872	1163	1286	2784	1133	1277	1287	
$exdata_{15}$	6380	153.01	2571	3113	18380	2520	3045	5906	2416	2873	3727	2317	2816	4651	
$exdata_16$	6686	176.23	1585	3323	1882	1814	1825	2110	1644	3237	1909	1606	1683	2268	
$exdata_17$	5813	118.20	1970	1993	3925	1981	1958	4096	1982	1943	3619	1915	1923	3342	
$exdata_18$	$57\overline{69}$	136.95	1908	2198	2878	1976	2103	2887	1842	2059	3622	1833	1920	2772	
exdata_19	6062	108.49	2125	2402	3485	2095	2371	9231	2011	2347	3008	1908	2176	2502	
exdata_20	5990	159.65	1844	2097	2598	1822	2075	2559	1810	1934	2888	1691	1885	2876	



Fig. 2: Comparison of DA, KaHIP and KaHIP with sparsification. The y-axis represents the approximation ratio, the x-axis represents the imbalance factor as percentage

8 X. Liu et al.

problem dedicated solvers. In this work, we have focused on demonstrating practical advantage of software and hardware approaches for the Graph Partitioning problem. We found that dense graphs exhibit limitations of the existing algorithms. By experimenting with the Fujitsu Digital Annealer (DA), a quantum-inspired device, we show graphs on which the DA significantly outperforms current state-of-the-art solvers that are run for identical or longer time. In particular, we run experiments on instances from three datasets, namely the Walshaw graph partitioning dataset, which represents well-known sparse graphs, graphs from the Sparse-Suite Matrix collection, and lastly synthetically generated graphs. We observe that on sparse graphs (from the Walshaw benchmark) partitioned into two parts with 0% imbalance, the DA returns results identical to the state-of-art graph partitioning solver KaHIP. However, as we increase the imbalance factor and number of parts, we notice that KaHIP outperforms the DA for this dataset. In the Sparse-Suite dataset, we however observe that DA and KaHIP return similar results with a few cases where the DA significantly outperforms KaHIP. Lastly, in our last dataset of synthetically generated graphs, we observe that the DA outperforms KaHIP in almost all cases. With regards to the general-purpose solver, we observe that KaHIP and the DA outperform Gurobi in almost all cases. Our results demonstrate instances where both the DA and KaHIP perform well individually which suggests an opportunity to hybridize state-of-the-art algorithms and emerging technologies to achieve the best quality/time trade-off.

Appendix

In this appendix we give the parameters we used with $MUSKETEER^4$ [10] to generate the exdata instances in Table 4:

exdata	node growth rate	edge edit rate	node edit rate
2-4	[0.01, 0.001]	[0.05, 0.04, 0.03]	[0.07, 0.06, 0.05]
5-7	[0.009, 0.001]	[0.05, 0.04, 0.03]	[0.07, 0.06, 0.05]
8-10	[0.01, 0.001]	[0.05, 0.04, 0.03]	[0.07, 0.06, 0.05]
11-15	[0, 0, 0, 0, 0, 0, 0, 0.01, 0.001]	[0, 0, 0, 0, 0, 0.05, 0.04, 0.03]	[0, 0, 0, 0, 0, 0.07, 0.06, 0.05]
16-20	[0, 0, 0, 0, 0, 0, 0, 0, 0.02, 0.002]	[0, 0, 0, 0, 0, 0, 0, 0, 0.06, 0.05, 0.04, 0.03]	[0, 0, 0, 0, 0, 0, 0, 0, 0.08, 0.07, 0.06, 0.05]

 Table 4: Parameters used with MUSKETEER

References

- 1. Aramon, M., Rosenberg, G., Valiante, E., Miyazawa, T., Tamura, H., Katzgrabeer, H.: Physics-inspired optimization for quadratic unconstrained problems using a digital annealer. Frontiers in Physics 7, 48 (2019)
- 2. Bader, D.A., Meyerhenke, H., Sanders, P., Wagner, D.: 10th dimacs implementation challenge-graph partitioning and graph clustering (2011), https://www.cc.gatech.edu/dimacs10/
- Buluç, A., Meyerhenke, H., Safro, I., Sanders, P., Schulz, C.: Recent advances in graph partitioning. In: Algorithm Engineering: Selected Results and Surveys. LNCS 9220, Springer-Verlag, pp. 117–158. Springer (2016)
- Cohen, E., Mandal, A., Ushijima-Mwesigwa, H., Roy, A.: Ising-based consensus clustering on specialized hardware. In: International Symposium on Intelligent Data Analysis. pp. 106–118. Springer (2020)
- Cohen, E., Ushijima-Mwesigwa, H., Mandal, A., Roy, A.: Unified clustering and outlier detection on specialized hardware. In: ICASSP 2021-2021 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). pp. 3770–3774. IEEE (2021)
- 6. Crawford, D., Levit, A., Ghadermarzy, N., Oberoi, J.S., Ronagh, P.: Reinforcement learning using quantum boltzmann machines. arXiv preprint arXiv:1612.05695 (2016)
- 7. Davis, T.A., Hu, Y.: The university of florida sparse matrix collection. ACM Transactions on Mathematical Software (TOMS) 38(1), 1–25 (2011), https://sparse.tamu.edu/
- 8. Fujitsu: Fujitsu Digital Annealer (2022), https://www.fujitsu.com/global/services/business-services/ digital-annealer/
- 9. Gurobi Optimization, I.: Gurobi optimizer reference manual (2018), https://www.gurobi.com/
- Gutfraind, A., Safro, I., Meyers, L.A.: Multiscale network generation. In: 2015 18th international conference on information fusion. pp. 158–165. IEEE (2015)
- 11. Hager, W.W., Hungerford, J.T., Safro, I.: A multilevel bilinear programming algorithm for the vertex separator problem. Computational Optimization and Applications **69**(1), 189–223 (2018)
- Hager, W.W., Krylyuk, Y.: Graph partitioning and continuous quadratic programming. SIAM Journal on Discrete Mathematics 12(4), 500–523 (1999)

⁴ https://github.com/sashagutfraind/musketeer

- Hamann, M., Lindner, G., Meyerhenke, H., Staudt, C.L., Wagner, D.: Structure-preserving sparsification methods for social networks. Social Network Analysis and Mining 6(1), 22 (2016)
- 14. Henderson, M., Novak, J., Cook, T.: Leveraging adiabatic quantum computation for election forecasting. arXiv preprint arXiv:1802.00069 (2018)
- 15. Hernandez, M., Aramon, M.: Enhancing quantum annealing performance for the molecular similarity problem. Quantum Information Processing 16(5), 133 (2017)
- 16. Hernandez, M., Zaribafiyan, A., Aramon, M., Naghibi, M.: A novel graph-based approach for determining molecular similarity. arXiv preprint arXiv:1601.06693 (2016)
- Inagaki, T., Haribara, Y., Igarashi, K., Sonobe, T., Tamate, S., Honjo, T., Marandi, A., McMahon, P.L., Umeki, T., Enbutsu, K., et al.: A coherent ising machine for 2000-node optimization problems. Science 354(6312), 603–606 (2016)
- John, E., Safro, I.: Single-and multi-level network sparsification by algebraic distance. Journal of Complex Networks 5(3), 352–388 (2016)
- Johnson, M.W., Amin, M.H., Gildert, S., Lanting, T., Hamze, F., Dickson, N., Harris, R., Berkley, A.J., Johansson, J., Bunyk, P., et al.: Quantum annealing with manufactured spins. Nature 473(7346), 194 (2011)
- 20. Kalehbasti, P.R., Ushijima-Mwesigwa, H., Mandal, A., Ghosh, I.: Ising-based louvain method: clustering large graphs with specialized hardware. In: International Symposium on Intelligent Data Analysis. pp. 350–361. Springer (2021)
- 21. Karypis, G., Kumar, V.: A fast and high quality multilevel scheme for partitioning irregular graphs. SIAM Journal on Scientific Computing **20**(1) (1999)
- 22. Karypis, G., Kumar, V.: Multilevel algorithms for multi-constraint graph partitioning. In: SC'98: Proceedings of the 1998 ACM/IEEE Conference on Supercomputing. pp. 28–28. IEEE (1998)
- Khoshaman, A., Vinci, W., Denis, B., Andriyash, E., Amin, M.H.: Quantum variational autoencoder. Quantum Science and Technology 4(1), 014001 (2018)
- 24. Leskovec, J., Faloutsos, C.: Sampling from large graphs. In: Proceedings of the 12th ACM SIGKDD international conference on Knowledge discovery and data mining. pp. 631–636 (2006)
- 25. Levit, A., Crawford, D., Ghadermarzy, N., Oberoi, J.S., Zahedinejad, E., Ronagh, P.: Free energy-based reinforcement learning using a quantum processor. arXiv preprint arXiv:1706.00074 (2017)
- Liu, X., Angone, A., Shaydulin, R., Safro, I., Alexeev, Y., Cincio, L.: Layer vqe: A variational approach for combinatorial optimization on noisy quantum computers. IEEE Transactions on Quantum Engineering pp. 1–1 (2022)
- 27. Liu, X., Ushijima-Mwesigwa, H., Mandal, A., Upadhyay, S., Safro, I., Roy, A.: Leveraging special-purpose hardware for local search heuristics. Computational Optimization and Applications, to appear (2022)
- 28. Lucas, A.: Ising formulations of many np problems. Frontiers in Physics 2, 5 (2014)
- Negre, C.F., Ushijima-Mwesigwa, H., Mniszewski, S.M.: Detecting multiple communities using quantum annealing on the d-wave system. Plos one 15(2), e0227538 (2020)
- Rosenberg, G., Haghnegahdar, P., Goddard, P., Carr, P., Wu, K., De Prado, M.L.: Solving the optimal trading trajectory problem using a quantum annealer. IEEE Journal of Selected Topics in Signal Processing 10(6), 1053– 1060 (2016)
- 31. Safro, I., Ron, D., Brandt, A.: Graph minimum linear arrangement by multilevel weighted edge contractions. Journal of Algorithms **60**(1), 24–41 (2006)
- 32. Safro, I., Sanders, P., Schulz, C.: Advanced coarsening schemes for graph partitioning. ACM Journal of Experimental Algorithmics (JEA) **19**, 2–2 (2015)
- Sanders, P., Schulz, C.: Think locally, act globally: Highly balanced graph partitioning. In: Experimental Algorithms, 12th International Symposium, SEA 2013, Rome, Italy, June 5-7, 2013. Proceedings. vol. 7933, pp. 164–175. Springer (2013)
- Shaydulin, R., Chen, J., Safro, I.: Relaxation-based coarsening for multilevel hypergraph partitioning. SIAM Multiscale Modeling and Simulation 17, 482–506 (2019)
- Shaydulin, R., Ushijima-Mwesigwa, H., Safro, I., Mniszewski, S., Alexeev, Y.: Community detection across emerging quantum architectures. arXiv preprint arXiv:1810.07765 (2018)
- Shaydulin, R., Ushijima-Mwesigwa, H., Safro, I., Mniszewski, S., Alexeev, Y.: Network community detection on small quantum computers. Advanced Quantum Technologies 2(9), 1900029 (2019)
- 37. Spielman, D.A., Srivastava, N.: Graph sparsification by effective resistances. SIAM Journal on Computing 40(6), 1913–1926 (2011)
- Staudt, C.L., Sazonovs, A., Meyerhenke, H.: Networkit: A tool suite for large-scale complex network analysis. Network Science 4(4), 508–530 (2016)
- Terry, J.P., Akrobotu, P.D., Negre, C.F., Mniszewski, S.M.: Quantum isomer search. arXiv preprint arXiv:1908.00542 (2019)
- 40. Ushijima-Mwesigwa, H., Negre, C.F., Mniszewski, S.M.: Graph partitioning using quantum annealing on the D-Wave system. In: Proceedings of the Second International Workshop on Post Moores Era Supercomputing. pp. 22–29 (2017)
- 41. Walshaw, C.: The graph partitioning archive. https://chriswalshaw.co.uk/partition/ (2009)