

Formal Translation from Reversing Petri Nets to Coloured Petri Nets

Kamila Barylska¹, Anna Gogolińska¹, Łukasz Mikulski¹,
Anna Philippou², Marcin Piatkowski¹, and Kyriaki Psara²

¹ Faculty of Mathematics and Computer Science
Nicolaus Copernicus University, 87-100 Toruń, Poland

{khama,anna.gogolinska,lukasz.mikulski,marcin.piatkowski}@mat.umk.pl

² Department of Computer Science, University of Cyprus
{anna.philippou,psara.kyriaki}@ucy.ac.cy

Abstract. Reversible computation is an emerging computing paradigm that allows any sequence of operations to be executed in reverse order at any point during computation. Its appeal lies in its potential for low-power computation and its relevance to a wide array of applications such as chemical reactions, quantum computation, robotics, and distributed systems. Reversing Petri nets are a recently-proposed extension of Petri nets that implements the three main forms of reversibility, namely, backtracking, causal reversing, and out-of-causal-order reversing. Their distinguishing feature is the use of named tokens that can be combined together to form bonds. Named tokens along with a history function, constitute the means of *remembering* past behaviour, thus, enabling reversal. In recent work, we have proposed a structural translation from a subclass of RPNs to the model of Coloured Petri Nets (CPNs), an extension of traditional Petri nets where tokens carry data values. In this paper, we extend the translation to handle RPNs with token multiplicity under the individual-token interpretation, a model which allows multiple tokens of the same type to exist in a system. To support the three types of reversibility, tokens are associated with their causal history and, while tokens of the same type are equally eligible to fire a transition when going forward, when going backwards they are able to reverse only the transitions they have previously fired. The new translation, in addition to lifting the restriction on token uniqueness, presents a refined approach for transforming RPNs to CPNs through a unifying approach that allows instantiating each of the three types of reversibility. The paper also reports on a tool that implements this translation, paving the way for automated translations and analysis of reversible systems using CPN Tools.

1 Introduction

Reversible computation is a form of computing that allows operations to be seamlessly reversed at any point during the computational process.

This unique capability has been attracting growing attention due to its potential applications in low-power computing, robotics, and distributed systems as well as applications which naturally embed reversible behavior such as biological systems and quantum mechanics.

Aiming to understand the foundations of reversibility, in recent years work has been carried out towards the development of reversible models and frameworks. This study has brought forward three main forms of reversibility in the context of concurrent and distributed systems, namely, *backtracking*, allowing actions to be reversed in the exact order in which they were executed, *causal-order* reversibility, a form of reversing where an action can be undone provided that all of its effects (if any) have been undone beforehand, and *out-of-causal-order* reversibility, a form of reversing featured most notably in biochemical systems. These concepts have been studied within a variety of formalisms [8,9,24,15,28,20,16,6].

One line of research among these approaches has been the investigation of reversible behavior in Petri nets. Initial studies considered the reversal of selected transitions of conventional Petri nets [4,5] and explored decidability problems regarding reachability and coverability in the resulting Petri net. Given that this approach to reversibility violates causality, subsequent work in [18,10] investigated whether it is possible to add a complete set of effect-reverses for a given transition without changing the set of reachable markings, showing that this problem is in general undecidable. In another line of work [16] proposes a causal semantics for P/T nets by identifying the causalities and conflicts of a P/T net through unfolding it into an equivalent occurrence net and subsequently introducing appropriate reverse transitions to create a coloured Petri net (CPN) that captures a causal-consistent reversible semantics. On a similar note, [17] introduces the notion of reversible occurrence nets and associates a reversible occurrence net to a causal reversible prime event structure, and vice versa.

In this work, we focus on Reversing Petri Nets [19,20] (RPNs), a reversible model inspired by Petri nets that allows the modelling of reversibility as realised by backtracking, causal-order, and out-of-causal-order reversing. A key challenge when reversing computations in Petri nets is handling *backward conflicts*. These conflicts arise when tokens occur in a certain place due to different causes making unclear which transitions ought to be reversed. To handle this ambiguity, RPNs introduce the notion of a *history* of transitions, which records causal information of executions. Furthermore, inspired by biochemical systems as well as other resource-aware applications, the model employs named tokens that can

be connected together to form bonds, and are preserved during execution. The usefulness of the framework and its extensions [21,22] was illustrated in a number of examples including the modelling of long-running transactions with compensations [25], a signal-passing mechanism used by the ERK pathway, and an application to Massive MIMO [23], and they have been translated to Answer Set Programming (ASP), a declarative programming framework with competitive solvers [11]. One of the extensions of RPNs concerns the introduction of multiple tokens [21] according to the individual-token interpretation. The aim of this extension has been to allow multiple tokens of the same type to exist within a net while distinguishing them based on their causal path. This implies that tokens of the same type are equally eligible to fire a transition when going forward, however, when going backwards they are able to reverse only the transitions they have previously fired.

A challenge that arises is to explore the relationship between RPNs and classical Petri nets. Of particular interest is how the global control imposed via the history construct in RPNs can be captured by the strictly local semantics of Petri nets. To this effect, in our previous work [1] we have provided a translation from a subclass of acyclic RPNs into coloured Petri nets (CPNs) [12], an extension of traditional Petri nets where tokens can carry data values. The study establishes that RPNs can be encoded into CPNs, demonstrating that the principles of reversible computation can be directly encoded in the traditional model. The translation involves a structural transformation from RPNs to CPNs, introducing both forward and backward instances for each transition. The translation succeeds in maintaining a local approach by carefully storing histories and causal dependencies of executed transition sequences in additional places. Furthermore, the introduction of cycles to the RPN model and the consequences to the CPN translation were considered in [3]. In the current work, we go a step further and consider the RPN extension with multiple tokens. Specifically, we propose a translation of a subclass of RPNs with multiple tokens to CPNs. Similarly to previous work, the resulting translation succeeds in capturing the global control merely at the local level of additional places, while lifting restrictions on token uniqueness. Furthermore, the current translation extends beyond prior work by presenting a refined approach that unifies the transformation process for the three types of reversibility, making it more flexible and comprehensive. The correctness of the translation is formally proved establishing the correspondence between the states of an RPN and its CPN translation as well as how a transition in a state of an RPN can be matched by a transition of

any corresponding state of the CPN, leading to equivalent states, and vice versa. An additional contribution of the present paper lies in the practical implementation of the transformation, realized through a tool. This tool facilitates the analysis of reversible systems using CPN Tools [26], providing a bridge between theoretical models and practical applicability. We note that a previous version of this work has appeared in [2]. The present paper constitutes an extension of that work with a refinement of the machinery and the complete proofs of the results.

Paper organization Following some preliminary definitions, Sections 3 and 4 provide an overview of reversing Petri nets and a description of coloured Petri nets. Sections 5 and 6 define the necessary machinery for transforming RPNs to CPNs and establish formally the correctness of the translation. Section 7 presents the tool that has been developed implementing the transformation and providing a connection between RPNs to CPN Tools. The paper concludes by a summary of the contributions and a discussion of future work.

2 Preliminaries

The set of non-negative integers is denoted by \mathbb{N} . Given a set X , the cardinality (number of elements) of X is denoted by $\#X$, the powerset (set of all subsets) by 2^X – the cardinality of the powerset is $2^{\#X}$. Multisets over X are members of \mathbb{N}^X , i.e., functions from X into \mathbb{N} .

In what follows every function $f : X \rightarrow Y$ might be extended in a natural way to the domain 2^X .

Definition 1. *The monoid \mathbb{N}^X , for a set X , is the set of multisets over X with componentwise addition $+$.*

If $y, z \in \mathbb{N}^X$ then $(y + z)(x) = y(x) + z(x)$ for every $x \in X$. For $Y, Z \subseteq \mathbb{N}^X$ we define $Y + Z = \bigcup\{y + z \mid y \in Y, z \in Z\}$. The partial order \leq is understood componentwise, while $<$ means \leq and \neq .

3 Reversing Petri nets

In this section we recall the model of reversing Petri nets (RPNs) defined in [20] and specifically their extension with multiple tokens of [21], together with descriptions of the three reversibility semantics which these nets support in addition to the standard forward execution, namely *backtracking*, *causal reversing* and *out-of-causal-order reversing*. Due to the lack of space, we will only focus here on the most important concepts

and facts. We refer the reader to [20,21,25], where RPNs are presented in detail.

A reversing Petri net is built on the basis of a set of bases or simply tokens that correspond to the basic entities that occur in a system. These tokens are persistent, cannot be consumed, and can be combined together as the effect of transitions via so-called *bonds* into coalitions (also called *molecules*) that record the computational history of each token. This approach is similar to reaction systems from biochemistry but can be applied to a wide range of systems that feature reversible behaviour. Based on this intuition, reversing Petri nets are defined as follows:

Definition 2. A *reversing Petri net* (RPN) is a tuple (P, T, F, A, B) where:

1. P is a finite set of *places*,
2. T is a finite set of *transitions*,
3. $F : (P \times T \cup T \times P) \rightarrow \mathbb{N}^{A \cup B \cup \bar{A} \cup \bar{B}}$ is a set of directed *arcs*,
4. A is a finite set of *base* or *token types* ranged over by a, b, \dots . Instances of tokens of type a are denoted by a_i where i is unique and the set of all instances of token types in A is denoted by \mathcal{A} . Furthermore, we write $\bar{A} = \{\bar{a} \mid a \in A\}$ for the set containing a “negative” form for each token type³,
5. $B \subseteq A \times A$ is a set of undirected *bond types*. We assume that the relation B is symmetric. The two elements $(a, b), (b, a) \in B$ would be represented by one element only and denoted by $\langle a, b \rangle$. We also use the notation $a - b$ for a bond type $\langle a, b \rangle \in B$ ⁴. Similarly, instances of bonds are elements of $\mathcal{A} \times \mathcal{A}$ and denoted by $\langle a_i, b_j \rangle$ or $a_i - b_j$ for $a_i, b_j \in \mathcal{A}$. The set of all instances of bond types in B is denoted by \mathcal{B} . $\bar{B} = \{\bar{\beta} \mid \beta \in B\}$ contains the corresponding “negative” form for each bond type.

The first two clauses of the definition state the sets of *places* and *transitions*, which are understood in a standard way (see [27]). To preserve individuality whilst adding token multiplicity we introduce \mathcal{A} and \mathcal{B} which are the sets of token and bond instances, where for each token type a of a set of types A , instances of that type are indicated as a_i , i being a unique index. Intuitively, the incoming and outgoing arcs of a transition indicate the requirements of a transition firing and the effects of its execution,

³ Elements of A signify the presence of the base type, when elements of \bar{A} the absence of it.

⁴ $\langle a, b \rangle$ may be also understood as two elements multiset over A .

respectively. The negative types of tokens \bar{A} or \bar{B} indicate the requirement of token absence and they make sense only on incoming arcs of a transition, hence $F(t, t\bullet) \cap \bar{A} = \emptyset$ and $F(t, t\bullet) \cap \bar{B} = \emptyset$, where for transition $t \in T$ we define sets $\bullet t = \{p \in P \mid F(p, t) \neq \emptyset\}$, $t\bullet = \{p \in P \mid F(t, p) \neq \emptyset\}$ (sets of input and output places of t), and $\text{pre}(t) = \bigcup_{p \in P} F(p, t)$, $\text{post}(t) = \bigcup_{p \in P} F(t, p)$ (unions of labels of the incoming/outgoing arcs of t), as well as $\text{effect}(t) = \text{post}(t) \setminus \text{pre}(t)$.

As with standard Petri nets the association of tokens to places is called a *marking* such that $M : P \rightarrow 2^{\mathcal{A} \cup \mathcal{B}}$. In addition, we employ the notion of a *history*, which assigns a memory to each transition $H : T \rightarrow 2^{\mathbb{N} \times 2^{P \times \mathcal{A}}}$. Intuitively, a history of $H(t) = \emptyset$ for some $t \in T$ captures that the transition has not taken place, and a history of $(k, S) \in H(t)$, captures that the transition was executed as the k^{th} transition occurrence and it has not been reversed and S contains the information of transferred token instances and places from where they were taken. Note that $|H(t)| > 1$ may arise due to different token instances firing the same transition. A pair of a marking and a history, $\langle M, H \rangle$, describes a *state* of an RPN with $\langle M_0, H_0 \rangle$ the initial state, where $H_0(t) = \emptyset$ for all $t \in T$.

Finally, for $a_i \in \mathcal{A}$ and $C \subseteq 2^{\mathcal{A} \cup \mathcal{B}}$ (usually C is a marking of a given place) we define $\text{con}(a_i, C)$ to be the set containing the tokens connected to a_i as well as the bonds creating these connections according to set C . In order to do that let us first define, for a given $a_i \in \mathcal{A}$ and $C \subseteq 2^{\mathcal{A} \cup \mathcal{B}}$, the set $\text{paths}(a_i, C)$ as follows: $\text{paths}(a_i, C) = \{(c_1, c_2, \dots, c_k) \mid c_i \in \mathcal{A} \cap C, i \in 1, \dots, k; \exists_{((c_1, c_2), (c_2, c_3), \dots, (c_{k-1}, c_k))} c_1 = a_i; (c_{j-1}, c_j) \in \mathcal{B} \cap C, j \in 2, \dots, k\}$.

We are ready to define: $\text{con}(a_i, C) = (\{a_i\} \cap C) \cup \{\langle b_k, c_l \rangle, c_l \mid \exists_{(a_i, \dots, b_k, c_l)} (a_i, \dots, b_k, c_l) \in \text{paths}(a_i, C)\}$.

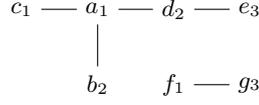
More intuitively, following graph theory, we can treat a molecule as a graph, with sets of vertices \mathcal{A} and edges \mathcal{B} . Then, $\text{paths}(a_i)$ is analogous to the set of all paths from vertex a_i and $\text{con}(a_i, C)$ to the connected component of a_i , both within the graph C .

Let $X \subseteq \mathcal{A} \cup \mathcal{B}$ then by $X|_x$ we denote the subset of X containing all its elements of type x , where x can be either a base or a bond type. Let us also define a function *type* as $\ell : \mathcal{A} \rightarrow A$, which for a given base instance (i.e. element of \mathcal{A}) returns its type (i.e. corresponding element of A).

Example 1. Figure 1 depicts a marking $M(p)$ for some place p . We compute the sets $\text{paths}(a_1, M(p))$ and $\text{con}(a_1, M(p))$ as follows:

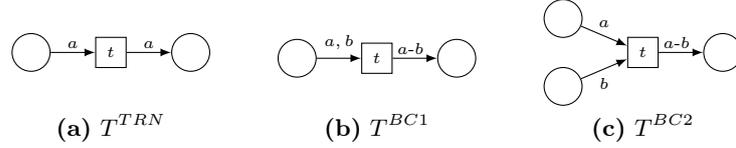
$$\begin{aligned} \text{paths}(a_1, M(p)) &= \{(a_1, c_1), (a_1, d_2), (a_1, d_2, e_3), (a_1, b_2)\}, \\ \text{con}(a_1, M(p)) &= \{a_1, b_2, c_1, d_2, e_3, \langle a_1, b_2 \rangle, \langle a_1, c_1 \rangle, \langle a_1, d_2 \rangle, \langle d_2, e_3 \rangle\}. \end{aligned}$$

Let us define the following three sets of transitions (see Fig 2):

**Fig. 1.** Exemplary molecules.

- $T^{TRN} = \{t \in T \mid \#(\bullet t) = \#(t\bullet) = 1; \exists_{a \in A}(F(\bullet t, t) = \{a\}; F(t, t\bullet) = \{a\})\}$ – the set of transitions that transfer molecules.
- $T^{BC1} = \{t \in T \mid \#(\bullet t) = \#(t\bullet) = 1; \exists_{a,b \in A}(F(\bullet t, t) = \{a, b\}; F(t, t\bullet) = \{\langle a, b \rangle\})\}$ – the set of transitions that create a bond in a molecule.
- $T^{BC2} = \{t \in T \mid \exists_{p_1, p_2 \in P} \bullet t = \{p_1, p_2\}; \#(t\bullet) = 1; \exists_{a,b \in A}(F(p_1, t) = \{a\}; F(p_2, t) = \{b\}; F(t, t\bullet) = \{\langle a, b \rangle\})\}$ – the set of transitions with two input places that create a bond in a molecule.

Note that a transition $t \in T^{BC1}$ may create a bond even between bases which are already parts of the same molecule. For a transition $t \in T^{TRN}$, we say that t is of *type TRN*. Analogously, $t \in T^{BC1}$ is of *type BC1* and $t \in T^{BC2}$ is of *type BC2*.

**Fig. 2.** The decomposition of the transitions set into the set of transitions only transferring molecules (a), creating a bond with a single input place (b), or two input places (c).

Definition 3. A reversing Petri net (P, T, F, A, B) is called *low-level*, if it satisfies the following conditions for all $t \in T$:

1. $A \cap \text{pre}(t) = A \cap \text{post}(t)$,
2. if $a-b \in \text{pre}(t)$ then $a-b \in \text{post}(t)$,
3. T can be decomposed into three sets: $T = T^{BC1} \cup T^{BC2} \cup T^{TRN}$, where $T^{TRN}, T^{BC1}, T^{BC2}$ are defined above,
4. if $a, b \in F(p, t)$ and $\beta = a-b \in F(t, q)$ then $\bar{\beta} \in F(p, t)$.

We say that a transition is forward enabled when the following conditions are satisfied:

Definition 4. Consider an RPN (P, T, F, A, B) , a transition $t \in T$, and a state $\langle M, H \rangle$. We say that t is *forward enabled* in $\langle M, H \rangle$ if there exist $S : P \rightarrow 2^A$, such that following hold:

1. $\forall p \in \bullet t, a \in A F(p, t)(a) = \#(S(p) \cap M(p))|_a$,
2. $\forall p \in \bullet t \bar{a} \in F(p, t) \implies \left(\bigcup_{\alpha \in S(p) \cap M(p)} \text{con}(\alpha, M(p)) \right) \Big|_a = \emptyset$,
3. $\forall p \in \bullet t \overline{a_1 - a_2} \in F(p, t) \implies \left(\bigcup_{\alpha \in S(p) \cap M(p)} \text{con}(\alpha, M(p)) \right) \Big|_{a_1 - a_2} = \emptyset$,
4. $\forall a \in A F(t, t\bullet)(a) = F(\bullet t, t)(a)$,
5. $F(t, t\bullet)(a_1 - a_2) = 1 \implies \exists_{\alpha_1, \alpha_2 \in S(\bullet t), \alpha_1 \neq \alpha_2, \ell(\alpha_1) = a_1; \ell(\alpha_2) = a_2; (\alpha_1 - \alpha_2) \notin M(\bullet t)}$.

In other words, a transition t is forward enabled in a state $\langle M, H \rangle$ if (1) all token instances selected in the set $S(p)$ exist in the respective marking $M(p)$ and correspond to the requirements of the incoming arc $F(p, t)$ (i.e. tokens required by the action for execution), (2),(3) none of the tokens or bonds whose absence is required exist in the connected components that have been selected in S , (4) tokens are preserved according to the labels of the arcs, and (5) indicates that a transition can create a bond only if its input contains token instances of suitable types and those instances are not bonded yet.

We may now define the firing rule of forward execution in RPNs:

Definition 5. Given a reversing Petri net (P, T, F, A, B) , a state $\langle M, H \rangle$, and a transition t enabled in $\langle M, H \rangle$ with $S : P \rightarrow 2^A$ selected in the definition of forward enabledness, we write $\langle M, H \rangle \xrightarrow{t} \langle M', H' \rangle$:

$$M'(p) = \begin{cases} M(p) \setminus \bigcup_{\alpha_i \in S(p)} \text{con}(\alpha_i, M(p)) & \text{if } p \in \bullet t \\ M(p) \cup \bigcup_{\alpha_i \in S(\bullet t)} \text{con}(\alpha_i, M(\bullet t)) \cup \\ \quad \bigcup_{\ell(\alpha_1) - \ell(\alpha_2) \in F(t, p)} \{ \langle \alpha_1, \alpha_2 \rangle \mid \\ \quad \alpha_1 \neq \alpha_2 \in S(\bullet t); (\alpha_1 - \alpha_2) \notin M(\bullet t) \} & \text{if } p \in t\bullet \\ M(p), & \text{otherwise} \end{cases}$$

and

$$H'(t') = \begin{cases} H(t') \cup \{ (\max\{k' \mid (k', S') \in H(t'), t' \in T\} + 1, S) \} & \text{if } t' = t \\ H(t'), & \text{otherwise} \end{cases}$$

After the forward execution of the transition t , all tokens and bonds selected in S and occurring in its incoming arcs are transferred from the input places to the output place of t . Any newly created bonds will also be

added to the output places. Moreover, the history function H is changed by assigning the next available integer number to the transition along with the selected set of token instances.

Since we have added token multiplicity we need to dynamically define the *bonding effect* $\mathbf{b.effect}(t, S)$ of transition t concerning the particular set S of token instances, that are involved in the occurrence of the transition.

$$\mathbf{b.effect}(t, S) = \bigcup_{a_1 - a_2 \in F(t, t \bullet)} \{ \langle \alpha_1, \alpha_2 \rangle \mid \alpha_1 \neq \alpha_2 \in S(\bullet t), \ell(\alpha_1) = a_1, \ell(\alpha_2) = a_2 \}$$

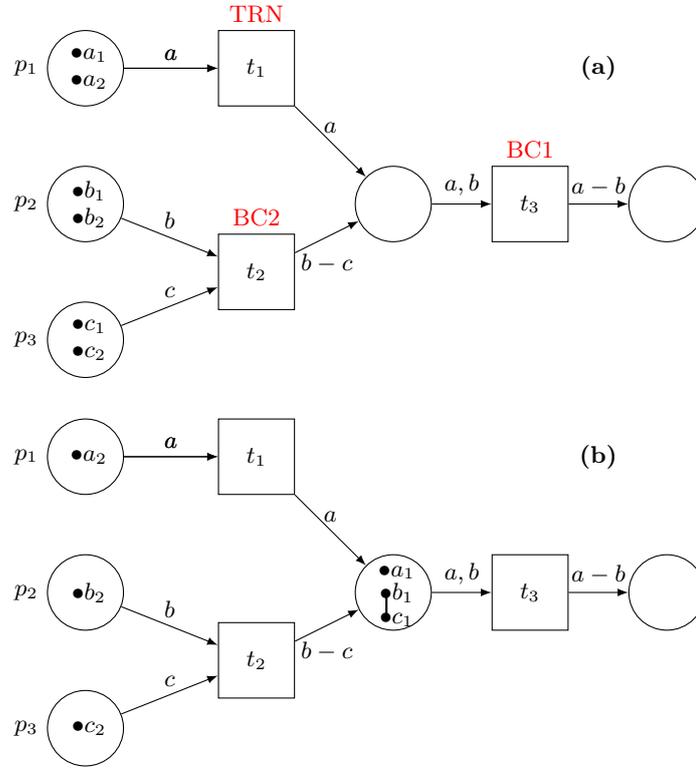


Fig. 3. (a) Reversing Petri Net N in its initial marking. (b) Reversing Petri net N from part (a) after the execution of sequence $t_1 t_2$, or, equivalently, after the execution of sequence $t_2 t_1$.

The example depicted in Figure 3a shows a reversible Petri net in its initial marking.

Note that all kinds of transitions occurs in the net: transition t_1 is a transporting one, hence $t_1 \in T^{TRN}$ (as indicated in red in the figure), while t_2 and t_3 create bonds; moreover $t_2 \in T^{BC2}$ and $t_3 \in T^{BC1}$. In addition, note that, for simplicity, we only indicate the positive token and bond types that are necessary for the transition to fire.

Initially, the history of all transitions equals \emptyset . We can see that transitions t_1 and t_2 are enabled and might be executed in arbitrary order. After the executions of sequence t_1t_2 or t_2t_1 we obtain the marking presented in Figure 3b. But the history function varies depending on the order of the executed transitions. Namely, after t_1t_2 we have $H(t_1) = \{(1, \{(p_1, a_1)\})\}$, $H(t_2) = \{(2, \{(p_2, b_1), (p_3, c_1)\})\}$, $H(t) = \emptyset$ for $t \in \{t_0, t_3\}$, while after t_2t_1 : $H(t_1) = \{(2, \{(p_1, a_1)\})\}$, $H(t_2) = \{(1, \{(p_2, b_1), (p_3, c_1)\})\}$, $H(t) = \emptyset$ for $t \in \{t_0, t_3\}$.

3.1 Backtracking

We now present the semantics for three forms of reversibility as proposed in [20], starting with the simplest form of reversibility namely, backtracking.

Definition 6. Consider an RPN (P, T, F, A, B) a state $\langle M', H' \rangle$ and a transition t where $t \in T$. We say that t is *bt-enabled* in $\langle M', H' \rangle$ if $(k, S) \in H(t)$, with $k = \max\{k' \mid (k', S') \in H(t'), t' \in T\}$.

We say that a transition is backward enabled only if its the last transition executed during a forward computation, i.e. it has the highest H value. The effect of backtracking a transition in a reversing Petri net is defined as follows:

Definition 7. Given an RPN (P, T, F, A, B) , a state $\langle M', H' \rangle$, and a transition t with $(k, S) \in H'(t)$ *bt-enabled* in $\langle M', H' \rangle$, we write $\langle M', H' \rangle \xrightarrow{t_b} \langle M, H \rangle$:

$$M(p) = \begin{cases} M'(p) \cup \bigcup_{q \in t \bullet, \alpha_i \in S(p)} \text{con}(\alpha_i, M'(q) \setminus \text{b.effect}(t, S)), & \text{if } p \in \bullet t \\ M'(p) \setminus \bigcup_{\alpha_i \in S(\bullet t)} \text{con}(\alpha_i, M'(p)), & \text{if } p \in t \bullet \\ M'(p), & \text{otherwise} \end{cases}$$

and

$$H(t') = \begin{cases} H'(t') \setminus \{(k, S)\}, & \text{if } t' = t \\ H'(t'), & \text{otherwise} \end{cases}$$

Thus, when a transition t is reversed in a backtracking fashion all tokens and bonds in the output place of the transition, as well as their connected components, are transferred to the incoming places of the transition and any newly-created bonds are broken.

3.2 Causal Reversing

The ability to reverse in a causal manner is defined as follows.

Definition 8. Consider an RPN (P, T, F, A, B) , a state $\langle M', H' \rangle$, and a transition $t \in T$. Then t is *co-enabled* in $\langle M', H' \rangle$ if $(k, S) \in H'(t)$ and, for all $\alpha_i \in S$, if $\text{con}(\alpha_i, M(q)) \cap S' \neq \emptyset$ for some q and some t' where $(k', S') \in H'(t')$ then $k' \leq k$.

Note that the causal reversing of a transition $t \in T$ is allowed if all transitions executed causally after t have been reversed. The effect of causally reversing a transition in an reversing Petri net is as follows:

Definition 9. Given an reversing Petri net (P, T, F, A, B) , a state $\langle M', H' \rangle$, and a transition t with $(k, S) \in H'(t)$ *co-enabled* in $\langle M', H' \rangle$, we write $\langle M', H' \rangle \xrightarrow{t}_c \langle M, H \rangle$ where M is defined as in Definition 7 and H as:

$$H(t) = \{(k', S') \mid t' \in T, (k', S') \in H'(t'), k' < k\} \\ \cup \{(k' - 1, S') \mid t' \in T, (k', S') \in H'(t'), k' > k\}$$

Reversing a transition in a causally-respecting order is implemented in exactly the same way as in backtracking (Definition 7), i.e., the tokens are moved from the out-places to the in-places of the transition, all bonds created by the transition are broken, and the reversal adjusts the history function. The history function is updated by removing history (k, S) of the reversed transition and shifting down all identifiers that are greater than k by one.

3.3 Out-of-causal-order Reversibility

We are now ready to define out-of-causal-order reversing enabledness.

We begin by noting that in out-of-causal-order reversibility any executed transition can be reversed at any time.

Definition 10. Consider an RPN (P, T, F, A, B) , a state $\langle M', H' \rangle$, and a transition $t \in T$. We say that t is *o-enabled* in $\langle M', H' \rangle$, if $H'(t) \neq \emptyset$.

According to the above definition, in this setting, every executed transition can be reversed. We can use the history function to identify which token instances have been used for the firing of a particular transition. The following notion helps to define the last executed transition manipulating a given set of tokens, where we write \perp to express that the value is undefined.

Definition 11. Given a reversing Petri net (P, T, F, A, B) , a history H , and a set of bases and bonds instances $C \subseteq \mathcal{A} \cup \mathcal{B}$ we write:

$$\text{last}(C, H) = \begin{cases} t, & \text{if } \exists t, (k, S) \in H(t), S \cap C \neq \emptyset, \text{ and} \\ & \forall t', (k', S') \in H(t'), S' \cap C \neq \emptyset, k' \leq k \\ \perp, & \text{otherwise} \end{cases}$$

The effect of reversing a transition in out-of-causal order is as follows:

Definition 12. Given a reversing Petri net (P, T, F, A, B) with initial marking M_0 , a state $\langle M', H' \rangle$ and a transition t with (k, S) that is o-enabled in $\langle M', H' \rangle$, we write $\langle M', H' \rangle \xrightarrow{t}_o \langle M, H \rangle$ where H is adjusted as in Definition 9 and M as follows for all $p \in P$:

$$\begin{aligned} M(p) = & M'(p) \setminus \text{b.effect}(t, S) \setminus \{C_{\alpha,p} \mid (\exists t' \in T) p \in t' \bullet, \alpha \in M'(p), t' \neq \text{last}(C_{\alpha,p}, H)\} \\ & \cup \{C_{\alpha,q} \mid (\exists t' \in T) p \in t' \bullet, \alpha \in M'(q), \text{last}(C_{\alpha,q}, H) = t'\} \\ & \cup \{C_{\alpha,q} \mid \alpha \in M'(q), \text{last}(C_{\alpha,q}, H) = \perp, C_{\alpha,q} \subseteq M_0(p)\} \end{aligned}$$

where we use the abbreviation $C_{\alpha,z} = \text{con}(\alpha, M'(z) \setminus \text{b.effect}(t, S))$ for $\alpha \in S, z \in P$.

In the above formula, $C_{\alpha,z}$ indicates the part of a molecule containing instance α obtained from place z in marking M' by removing the bond created by transition t . To compute marking $M(p)$ we have to check whether a place p is an exit from the last executed transition manipulating an instance α in the current history H or, in case the last such place is undefined, whether p was the place in which the component of α existed in the initial marking. We illustrate the definition with the following example.

Example 2. Let us look at Figure 4a. We can execute the sequence $t_1 t_2 t_3 t_4$ at the initial marking M_0 , obtaining marking M' as follows: place p_9 contains a molecule depicted in the circle (Figure 4b), other places are empty.

Assume now that t_3 is the transition to be reversed. (Note that this reversing transition is described as t in the formula of Definition 12.) Let us focus on place p_9 first. We want to determine the marking $M(p_9)$ obtained after reversing t_3 , hence for p from the definition we take p_9 . $M'(p)$ contains the whole molecule from Figure 4b. According to the formula, we have to remove the bond created by the forward execution of transition t_3 , i.e. $\text{b.effect}(t, S)$, namely: the bond $a_1 - c_1$. This results in partitioning the whole molecule into two parts: $e_1 - a_1 - b_1$ and $c_1 - d_1$.

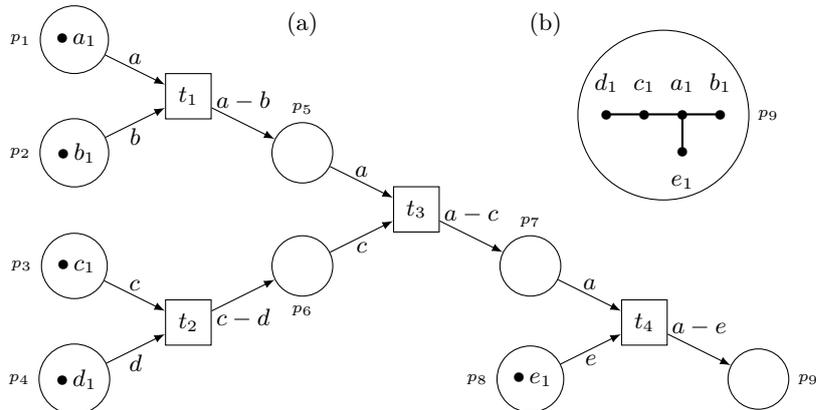


Fig. 4. Example of a reversing Petri net working according to the out-of-causal-order semantics. (a) Initial marking M_0 . (b) The content of place p_9 in marking M' obtained from M_0 after the execution of the sequence $t_1 t_2 t_3 t_4$, other places are empty.

For each base instance α we have to analyse its connected component $C_{\alpha,p}$ from the formula. Base instance a_1 (and also e_1) has been used by t_4 , hence the instance together with its whole connected component $C_{a_1,p} = \{e_1, a_1, b_1, \langle e_1, a_1 \rangle, \langle a_1, b_1 \rangle\}$ stays a part of $M(p)$. For $C_{c_1,p}$ we have to analyse the next part of the formula from Definition 12. Base instance c_1 was present in p_9 at M' . However, t_4 has not been the last transition (according to history H) using c_1 (and d_1 from the same connected component). That is why molecule $c_1 - d_1$ should be removed from p_9 at M . Other parts of the formula do not apply in this case. Assume next, that the reversed transition t is still t_3 , but we focus on place p_6 (i.e. p from the formula equals p_6 now), and we want to determine $M(p_6)$. At M' (Figure 4b) place p_6 is empty and we cannot subtract anything from its marking. Base instances a_1, c_1 ⁵ and their connected components at M' are located in place p_9 , hence for q we take p_9 . (As said before, $C_{a_1,q}$ stays in place p_9 at M .) For $C_{c_1,q} = \{c_1, d_1, \langle c_1, d_1 \rangle\}$ we have to determine transition t' , for which place p is the output place - it is transition t_2 . This transition has been the last one (according to history H) using c_1 , hence $C_{c_1,q}$ should be added to $M(p)$ (which is in this case $M(p_6)$). The last part of the formula from Definition 12 does not apply here - p_6 is not an initial

⁵ We mention only those two because after reversing t_3 we have two molecules in p_9 , the first one can be described as $con(a_1, M(p_9))$, the second $con(c_1, M(p_9))$. Any other base instance from those two connected components can be taken instead.

place for any base instance. The marking obtained from M' (depicted in Figure 4a) by reversing transition t_3 is presented in Figure 5a.

In the last case, we want to reverse transition t_1 (not t_3) starting from the marking M' from Figure 4b and determine the obtained marking M – for t from the formula from Definition 12 we take t_1 . Transition t_1 has created bond $a_1 - b_1$. Let us now take for p (from the definition) the place p_2 , which is an initial place for b_1 and it is not an output place for any transition $t' \in T$. All base instances from the analysed RPN in M' were located in p_9 , hence for all of them q (from the Definition 12 formula) equals p_9 . According to the formula, we have to remove the bond created by the forward execution of transition t_1 , i.e. $\mathbf{b.effect}(t, S)$, namely: the bond $a_1 - b_1$. The connected component for a_1 obtained from p_9 in M' after removing the bond created by t_1 is as follows: $C_{a_1, q} = \{a_1, e_1, c_1, d_1, \langle d_1, c_1 \rangle, \langle c_1, a_1 \rangle, \langle a_1, e_1 \rangle\}$. The last transition that used those instances exists. However, for $C_{b_1, p_9} = \{b_1\}$ we cannot determine the last transition that used b_1 in accordance with history H - the last one was t_1 but now is being reversed. b_1 in the initial marking M_0 was located in p_2 , hence it goes back to that place - it is described by the last part of the formula. The marking obtained from M' (depicted in Figure 4b) by reversing transition t_1 is presented in Figure 5b.

4 Coloured Petri Nets

Recall that RPNs constitute a model in which transitions can be reversed according to three semantics: backtracking, causal, and out-of-causal-order reversing. A main characteristic of RPNs is the concept of a *history*, which assigns a memory to each transition. While this construct enables transition reversal, it imposes the need of a global control during computation. Our goal is to recast the model of RPNs into one without any form of global control while establishing the expressiveness relation between RPNs and the model of bounded coloured Petri nets. In this section we recall the notion of coloured Petri nets.

Definition 13 ([12]). A (non-hierarchical) *coloured Petri net* is a nine-tuple $CPN = (P, T, D, \Sigma, V, C, G, E, I)$, where:

- P and T are finite, disjoint sets of *places* and *transitions*;
- $D \subseteq P \times T \cup T \times P$ is a set of *directed arcs*;
- Σ is a finite set of non-empty *colour sets*;
- V is a finite set of *typed variables* such that $Type[v] \in \Sigma$ for all $v \in V$, where $Type$ is a function returning a colour of variable;

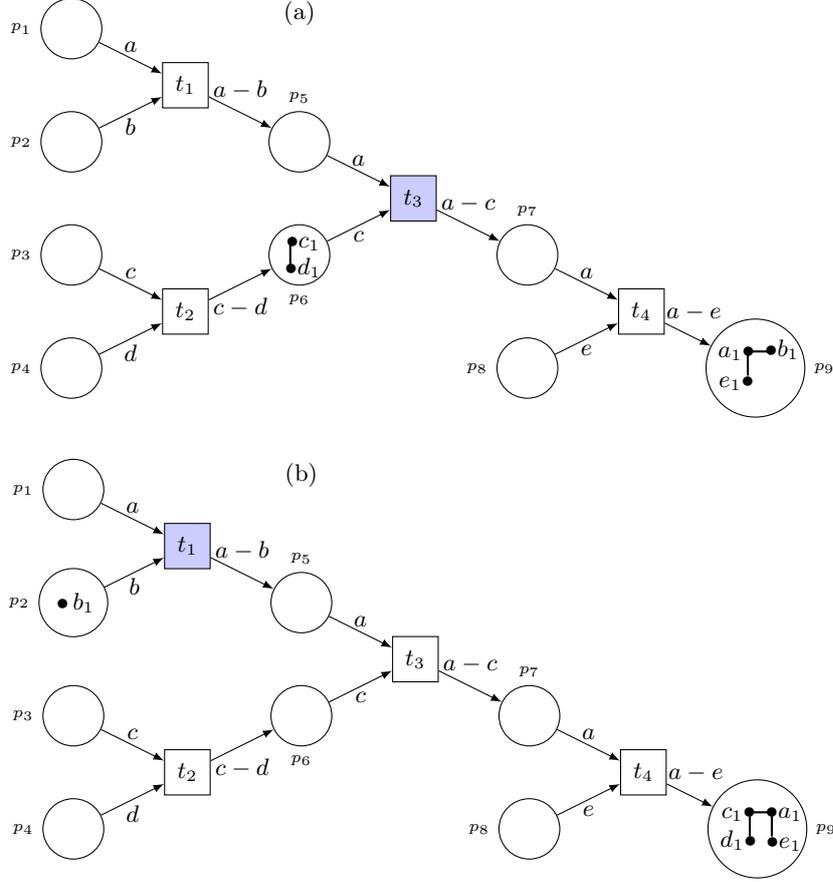


Fig. 5. (a) Marking obtained after reversing transition t_3 at marking M' described in Figure 4b. (b) Marking obtained after reversing transition t_1 at marking M' described in Figure 4b.

- $C : P \rightarrow \Sigma$ is a *colour set function* that assigns colour sets to places;
- $G : T \rightarrow \text{EXPR}_V$ is a *guard function* that assigns a guard to each transition t such that $\text{Type}[G(t)] = \text{Bool}$;
- $E : D \rightarrow \text{EXPR}_V$ is an *arc expression function* that assigns an arc expression to each arc $d \in D$ such that $\text{Type}[E(d)] = \mathbb{N}^{C(p)}$, where p is the place connected with the arc d ;
- $I : P \rightarrow \text{EXPR}_0$ is an *initialisation function* that assigns an initialisation expression to take each place p such that $\text{Type}[I(p)] = \mathbb{N}^{C(p)}$.

Note that, according to the utilised CPN-Tools [7], EXPR_V is the set of *net inscriptions* (over a set of variables V , possibly empty, i.e., using only

constant values) provided by CPN ML. Moreover, by $Type[e]$ we denote the type of values obtained by the evaluation of expression e . The set of *free variables* in an expression e is denoted by $Var[e]$. The setting of a particular value to free variable v is called a *binding* $b(v)$. We require that $b(v) \in Type[v]$ and denote with the use of $\langle \rangle$ filled by the list of valuations and written next to the element to whom it relates. The set of bindings of t is denoted by $B(t)$. The *binding element* is a transition t together with a valuation $b(t)$ of all the free variables related to t . We denote it by (t, b) , for $t \in T$ and $b \in B(t)$.

A *marking* M in coloured Petri nets is a function which assigns a set of tokens $M : P \rightarrow \mathbb{N}^\Sigma$ consistently with C . An initial marking is denoted by M_0 and defined for each $p \in P$ as follows: $M_0(p) = I(p)\langle \rangle$.

A binding element (t, b) is *enabled* at a marking M if $G(t)\langle b \rangle$ is true and at each place $p \in P$ there are enough tokens in M to fulfil the evaluation of the arc expression function $E(p, t)\langle b \rangle$. The resulting marking is obtained by removing the tokens given by $E(p, t)\langle b \rangle$ from $M(p)$ and adding those given by $E(t, p)\langle b \rangle$ for each $p \in P$.

We define the *enabledness* of transitions in CPN as follows: a transition $t \in T$ is *enabled* in M and its execution leads to marking M' (denoted $M[t]M'$) if there exists a binding $b \in B(t)$, such that the binding element (t, b) is enabled at M .

5 Transformation

Let us recall that for a given low-level RPN $N_R = (P_R, T_R, F_R, A_R, B_R)$ with \mathcal{A} and \mathcal{B} for bases and bonds instances, we have the following decomposition: $T_R = T_R^{BC1} \cup T_R^{BC2} \cup T_R^{TRN}$. We assume that we number transitions and the enumeration is consistent with \prec (i.e. if $t_i \prec t_j$ then $i < j$).

For such an RPN we define the following:

- a relation \rightarrow on $P_R \cup T_R$ as follows, $x \rightarrow y$ if $F(x, y)$ is not empty and we call it *direct order*;
- a relation \prec on $P_R \cup T_R$ as a transitive (but irreflexive) closure of \rightarrow ;
- a bounded set of integers $\mathbb{N}_b = \{0, \dots, nb\}$, where nb is an integer number which is larger than the number of transitions and the number of instances of bases and bonds;
- $ConCom(X)$ - a connected component of the set $X \subseteq \mathcal{A} \cup \mathcal{B}$; having a set X consisting of bases and bonds instances and treating X as an undirected graph, the function $ConCom(X)$ returns a set of connected components of X ;

- mol is a pair (V, E) called a molecule, where $V \subseteq \mathcal{A}$ and $E \subseteq \mathcal{B}$, such that (V, E) is a connected graph, by mol_α we understand a molecule (V, E) such that $\alpha \in V$;
- if a multiset $X \in \mathbb{N}^Y$ contains only one element (is a singleton) we denote it by this only element $y \in Y$, such that $X(y) = 1$.

Remark 1. Note that the relation \rightarrow could be defined only between places and transitions. On the other hand, such a restriction does not hold for \prec .

Furthermore, for any element $x \in P_R \cup T_R$ or $t \in T_R$, we consider the following five sets:

- a set of *neighborhood* of an element $x \in P_R \cup T_R$ as $nei(x)$;
- a set of *dependency counters* of a transition $t \in T_R$ as $dpc(t)$ – a set of transitions used to decide whether a reversing transition is enabled;
- a set *dependency histories* of a transition $t \in T_R$ as $dph(t)$ – a set of transitions which would be affected by the execution of the given transition or its reverse;
- a set of *reversing input places* of a transition $t \in T_R$ as $rin(t)$ – the set of places in which one needs to search for molecules, while reversing transition t ;
- a set *reversing output places* of a transition $t \in T_R$ as $rout(t)$ – the set of places where a molecule might be placed after reversing transition t .

During the transformation from an RPN to a CPN two types of new places are added to the CPN: *transition history places* and *connection history places*. A transition history place is created for every transition and it contains information about history of executions of that transition. Histories are important during reversing and to reverse a transition t sometimes it is necessary to check and modify content of history places of other transitions - the set of those transitions is denoted as $dph(t)$. A connection history place is created for a pair of transitions and it contains a number (counter) which describes how many times transitions from the pair were executed. Those places are not created for every pair of transitions, but for a given transition t they are added to the net only for transition t and transitions from $dpc(t)$.

The above sets differ depending on the assumed operational semantics of reversing. We use subscripts BT , C and OCC to clearly indicate that we operate according to backtracking, causal-order reversing and out-of-causal-order reversing, respectively.

In Table 1 we present how these sets are defined depending on the relative semantics.

backtracking	
<i>nei</i>	$nei_{BT}(x) = \{y \in P_R \cup T_R \mid (x \rightarrow y \vee y \rightarrow x) \vee (\exists z \in P_R \cup T_R x \rightarrow z \rightarrow y \vee y \rightarrow z \rightarrow x)\}$
<i>dpc</i>	$dpc_{BT}(t) = T_R \setminus \{t\}$
<i>dph</i>	$dph_{BT}(t) = nei_{BT}(t) \cap T_R$
<i>rin</i>	$rin_{BT}(t) = nei_{BT}(t) \cap P_R$
<i>rout</i>	$rout_{BT}(t) = nei_{BT}(t) \cap P_R$
causal-order reversing	
<i>nei</i>	$nei_C(x) = \{y \in P_R \cup T_R \mid (x \rightarrow y \vee y \rightarrow x) \vee (\exists z \in P_R \cup T_R x \rightarrow z \rightarrow y \vee y \rightarrow z \rightarrow x)\}$
<i>dpc</i>	$dpc_C(t) = nei_C(t) \cap T_R$
<i>dph</i>	$dph_C(t) = nei_C(t) \cap T_R$
<i>rin</i>	$rin_C(t) = nei_C(t) \cap P_R$
<i>rout</i>	$rout_C(t) = nei_C(t) \cap P_R$
out-of-causal-order reversing	
<i>nei</i>	$nei_{OOC}(x) = \{y \in P_R \cup T_R \mid x \prec y \vee y \prec x\}$
<i>dpc</i>	$dpc_{OOC}(t) = nei_{OOC}(t) \cap T_R$
<i>dph</i>	$dph_{OOC}(t) = nei_{OOC}(t) \cap T_R$
<i>rin</i>	$rin_{OOC}(t) = nei_{OOC}(t) \cap P_R$
<i>rout</i>	$rout_{OOC}(t) = nei_{OOC}(t) \cap P_R$

Table 1. Sets *nei*, *dpc*, *dph*, *rin*, and *rout* for the three operational semantics of reversing.

Note that in what follows, we use places that contain exactly one token and we denote the marking by the value of this token instead of a multiset containing only this value.

Now, we are ready to define the CPN $N_C(N_R) = (P_C, T_C, D_C, \Sigma_C, V_C, C_C, G_C, E_C, I_C)$ corresponding to N_R . According to the definition of CPNs, in the following transformation we use notations: P_C - set of places, T_C - set of transitions, D_C - set of arcs, Σ_C - set of colours, V_C - set of variables, C_C - function that assigns colours to places, G_C - function that assigns guards to transitions, E_C - function that assigns arc expressions to arcs, I_C - function that assigns initial expressions to places.

Before we go into the details of the transformation, let us explain the meaning of the *max* function we use.

Remark 2. The function *max* operates over the set of transitions and returns the maximal one according to the order determined by the sequence of firings - the last, executed transition is the largest. The sequence of

firings is stored by the history. In RPNs it is conducted explicitly, with the use of the history function. On the other hand, in CPNs, the history is scattered among transitions history places. However, it is possible to reproduce the whole sequence by calculations. Appropriate formulas are presented in Section 6. In function max the information about the history of executions is obtained from places h_j , where $t_j \in (dph(t_i) \cup t_i)$ and t_i is a transition to be reversed. Moreover, not the whole sequence of firings is considered during max calculations. Only those transitions, which during their execution have used any token instance from $con(\alpha, C_\alpha)$ for given α and C , are considered. Notice, that max in CPNs is very similar to $last(C, H)$ defined in Definition 11, where C is $con(\alpha, C_\alpha)$ and H is stored in places h_j . When $last(C, H)$ exists in an RPN, it is equal to the value of max in the corresponding CPN - for the same C and H . When $last(C, H)$ equals \perp , then function max would return t_0 (the initialization transition which is added to the CPN during the transformation to generate the initial marking). Furthermore, the order of transitions determined by the history, limited to a given set of token instances $con(\alpha, C_\alpha)$ is compatible to the order determined by relation \prec over the same set of transitions. This holds because the considered PNs are acyclic, hence token instances can be transported only along the arcs, so according to \prec . Because of that, considering only executions related to some set of token instances $con(\alpha, C_\alpha)$, it is not possible for t_i to be executed after t_j when $t_i \prec t_j$.

First, let us specify precisely the set $\mathbb{N}_b = \{0, \dots, nb\}$ previously introduced. Let \mathbb{K} be the number of different instances occurring in reversing Petri net N_R in the initial marking increased by 2. Then $nb = 2\mathbb{K}$ and hence $\mathbb{N}_b = \{0, \dots, 2\mathbb{K}\}$.

Remark 3. Note that the numbers of tokens in each place is \mathbb{K} strong safe, i.e. at any marking in every possible situation \mathbb{K} tokens are placed, hence we deal with multisets over Σ_C . This approach has been used here for technical reasons - we consider special kind of tokens - idle ones denoted as (\emptyset, \emptyset) . However, wherever this does not lead to misunderstandings, we use the denotation for a single token.

In the following transformation, in some places, the CPN-Tools semantics is used. The transformation is prepared with a view to putting it into practice - implementation in the program. The most frequently used CPN-Tools semantics elements are: $++$ which means concatenation, and n' which describes quantity of elements.

$$\mathbf{P}_C = P_R \cup \{h_i \mid t_i \in T_R\} \cup \{h_{ij} \mid t_i, t_j \in T_R; i < j; t_j \in dpc(t_i)\}$$

Set of places contains places from the original RPN net, transition history places for each transition, and connection history places for pairs of transitions (set dpc determines for which transitions connection history place is added). Notice that indexes of transition history and connection history places are very important. For transitions t_i and t_j transition history places are denoted as h_i and h_j (respectively), and the connection history place is denoted as t_{ij} (for $i < j$) or t_{ji} (for $i > j$).

$$\mathbf{T_C} = T_R \cup \{tr_i \mid t_i \in T_R\} \cup \{t_0\}$$

Set of transitions contains transitions from the original RPN and reversing transitions - one for each original transition. A reversing transition for t_i is denoted as tr_i . Transition t_0 is added for technical reasons - more about this transition can be found at the end of this section.

$$\begin{aligned} \mathbf{D_C} = & \text{Domain}(F_R) \cup (\text{Domain}(F_R))^{-1} \cup \\ & \{(t_i, h_i), (h_i, t_i), (tr_i, h_i), (h_i, tr_i) \mid t_i \in T_R\} \cup \\ & \{(tr_i, h_j), (h_j, tr_i) \mid t_i \in T_R; t_j \in dph(t_i)\} \cup \\ & \{(t_i, h_{jk}), (h_{jk}, t_i), (tr_i, h_{jk}), (h_{jk}, tr_i) \mid t_i \in T_R; \{i, l\} = \{j, k\}; t_l \in dpc(t_i)\} \end{aligned}$$

This set contains all arcs: arcs from RPN N_R , arcs opposite to those from N_R , arcs between every transition t_i and its history places (in both directions), arcs between every reversing transition tr_i and the history place of t_i (in both directions), arcs between every reversing transition tr_i and history places of transitions from $dph(t_i)$ (in both directions), arcs between every transition t_i and all its connection history places (in both directions), arcs between every reversing transition tr_i and all connection history places of t_i (in both directions).

$$\mathbf{\Sigma_C} = \mathbb{N}_b \cup A_R \cup B_R \cup \overline{A_R} \cup \overline{B_R} \cup \mathcal{A} \cup \mathcal{B} \cup (2^A \times 2^B) \cup 2^{(\mathbb{N}_b \times T_R \times T_R \times 2^A)}$$

We define the following colours: a bounded set of natural numbers, base types, bond types, negative base types, negative bond types, instances of bases, instances of bonds, Cartesian product of subsets of token instances and subsets of bond instances – **molecules**, subsets of 4-tuples (one 4-tuple contains the following information: the second transition in the tuple, in the context of the first one in the tuple, was n -th in the sequence of executions and has used the given base instances).

$$\begin{aligned} \mathbf{V_C} = & \{(\mathcal{X}_i, \mathcal{Y}_i) \in (2^A \times 2^B) \mid i \in \mathbb{N}_b\} \cup (\mathcal{X}, \mathcal{Y}) \in (2^A \times 2^B) \cup \\ & \{(\alpha_i \in \mathcal{A} \mid i \in \mathbb{N}_b\} \cup \{cnt_i \in \mathbb{N}_b \mid i \in \{1, \dots, |T_R|\}\} \cup \{H_i \in \\ & 2^{(\mathbb{N}_b \times T_R \times T_R \times 2^A)} \mid i \in \{1, \dots, |T_R|\}\} \end{aligned}$$

$$\begin{aligned} \mathbf{C_C} = & \{p \mapsto (2^A \times 2^B) \mid p \in P_R\} \cup \\ & \{h_i \mapsto 2^{(\mathbb{N}_b \times T_R \times T_R \times 2^A)} \mid t_i \in T_R\} \cup \end{aligned}$$

$$\{\mathbf{h}_{ij} \mapsto \mathbb{N}_b \mid t_i, t_j \in T_R; i < j; t_j \in dpc(t_i)\}$$

This set describes which colours are assigned to which places (respectively): colour *molecule* to places from the RPN, set of 4-tuples to history places, and set of bounded natural numbers to connection history places.

$$\mathbf{G}_C = G_C^{TRN} \cup G_C^{BC1} \cup G_C^{BC2} \cup G_C^{t_0} \cup G_C^{\bar{t}_0} \cup G_C^{\overline{TRN}} \cup G_C^{\overline{BC1 \cup BC2}}$$

where

$$G_C^{BC1} = \{\mathbf{t}_i \mapsto (\alpha_1 \in \mathcal{X}_1 \wedge \alpha_2 \in \mathcal{X}_2) \vee (\alpha_1, \alpha_2 \in \mathcal{X}_1 \wedge \langle \alpha_1, \alpha_2 \rangle \notin \mathcal{Y}_1 \wedge (\mathcal{X}_2, \mathcal{Y}_2) = (\emptyset, \emptyset) \mid$$

$$t_i \in T_R^{BC1}; \{\ell(\alpha_1), \ell(\alpha_2)\} \subset F_R(\bullet t_i, t_i);$$

$$\alpha_1 \neq \alpha_2; \{(\mathcal{X}_1, \mathcal{Y}_1), (\mathcal{X}_2, \mathcal{Y}_2)\} \subseteq Var[E_C(\bullet t_i, t_i)];$$

$$F_R(\bullet t_i, t_i) \cap (\overline{A \cup B}) \cap \ell(\mathcal{X}_1 \cup \mathcal{X}_2 \cup \mathcal{Y}_1 \cup \mathcal{Y}_2) = \emptyset\}$$

– Guard of BC1 transition evaluates whether a set of base instances \mathcal{X}_1 of a molecule $(\mathcal{X}_1, \mathcal{Y}_1)$ contains an instance α_1 and a set of instances \mathcal{X}_2 of a molecule $(\mathcal{X}_2, \mathcal{Y}_2)$ contains an instance α_2 , both sets are obtained for the only input place. The types of those instances form a label of an arc between the input place and the transition in the original RPN, α_1 and α_2 differs, and the molecules do not contain negative base nor bond types. It might also happen that a new bond is created within the already existing molecule - in that case both instances α_1, α_2 are unbonded and contained in molecule $(\mathcal{X}_1, \mathcal{Y}_1)$, and the second molecule $(\mathcal{X}_2, \mathcal{Y}_2)$ is empty (an idle token).

$$G_C^{BC2} = \{\mathbf{t}_i \mapsto (\alpha_1 \in \mathcal{X}_1 \wedge \alpha_2 \in \mathcal{X}_2) \mid$$

$$t_i \in T_R^{BC2}; \{\ell(\alpha_1), \ell(\alpha_2)\} \subset \bigcup_{X \in F_R(\bullet t_i, t_i)} X;$$

$$\alpha_1 \neq \alpha_2; p_1 \neq p_2 \in \bullet t_i;$$

$$(\mathcal{X}_1, \mathcal{Y}_1) \in Var[E_C(p_1, t_i)]; (\mathcal{X}_2, \mathcal{Y}_2) \in Var[E_C(p_2, t_i)];$$

$$\bigcup_{X \in F_R(\bullet t_i, t_i)} X \cap (\overline{A \cup B}) \cap \ell(\mathcal{X}_1 \cup \mathcal{X}_2 \cup \mathcal{Y}_1 \cup \mathcal{Y}_2) = \emptyset\}$$

– Guard of BC2 transition evaluates whether a set of base instances \mathcal{X}_1 of a molecule $(\mathcal{X}_1, \mathcal{Y}_1)$ obtained from the first input place contains an instance α_1 , set of base instances \mathcal{X}_2 of a molecule $(\mathcal{X}_2, \mathcal{Y}_2)$ obtained from the second input place contains an instance α_2 , types of those instances form labels of arcs between the input places and the transition in the original RPN, α_1 and α_2 differs, there are two different input places, and the molecules obtained from input places do not contain negative base nor bond types.

$$G_C^{TRN} = \{\mathbf{t}_i \mapsto (\alpha \in \mathcal{X}) \mid$$

$$t_i \in T_R^{TRN}; \ell(\alpha) \in F_R(\bullet t_i, t_i); E_C(\bullet t_i, t_i) = \{(\mathcal{X}, \mathcal{Y})\};$$

$$F_R(\bullet t_i, t_i) \cap (\overline{A \cup B}) \cap \ell(\mathcal{X} \cup \mathcal{Y}) = \emptyset\}$$

– Guard of TRN transition evaluates whenever a set of base instances \mathcal{X} of a molecule $(\mathcal{X}, \mathcal{Y})$ obtained from the input place contains an in-

stance α , type of α form a label of an arc between the input place and the transition, and the molecule does not contain negative base nor bond types.

$$\mathbf{G}_C^{t_0} = \{\mathbf{t}_0 \mapsto \text{false}\}$$

– Guard of the initial transition t_0 - always returns *false*, hence the transition cannot be executed.

$$\mathbf{G}_C^{t_0} = \{\mathbf{tr}_0 \mapsto \text{false}\}$$

– Guard of reversal transition for the initial transition t_0 - always returns *false*, hence the transition cannot be executed.

The last two guards are a little bit more complex, that is why we include functions in their descriptions. Those functions are: *isElement* and *numOfnonEmpty*⁶. The function *isElement* returns *true* if its first argument is an element of the set given as the second argument. In the opposite case the function returns *false*. The function *numOfnonEmpty* counts how many of its arguments are equal to (\emptyset, \emptyset) and returns that number.

Both following guards have the same construction. The first element of a guard is a logical conjunction of $\#dpc(t_i)$ conditions, each of them ensures that the token describing the history of t_i and obtained from the place h_i (which is represented as $E_C(h_i, tr_i)$ in the guards) contains 4-tuple related to the execution which is reversed. In the forward execution of the transition (to be reversed) the base α was transported (for transporting transition) or a bond between instances α_1 and α_2 was created (for BC1 or BC2 transition), which from now on is denoted as $\langle \alpha_1, \alpha_2 \rangle$. This part of the guards is very important because exactly here the choice: *which execution would be reversed?* (which is equivalent to the choice: *execution related to which instances would be reversed?*) is made. In CPN examples, prepared using CPN-Tools, this choice can be made by the user or randomly. The next part of the guards checks whether the set consisting of instances of bases (for TRN transition) or bonds (for BC1 and BC2 transition) obtained from all input places of the transition (to be reversed), contains instances related to its forward execution. The last part of the guards assures that from all tokens obtained from input places only one describes a molecule, the remaining ones should be idle tokens.

$$\mathbf{G}_C^{\overline{TRN}} = \{\mathbf{tr}_i \mapsto (\bigwedge_{t_j \in dpc(t_i)} \text{isElement}((k_j, t_j, t_i, \{\alpha\}), E_C(h_i, tr_i)); \text{isElement}(\alpha, \bigcup_{p_g \in rin(t_i)} \mathcal{X}_g));$$

⁶ Exemplary implementations of those functions are included in colour Petri nets generated by our application. Their formal definitions are included in descriptions of guards.

$\text{numOfnonEmpty}(\{(\mathcal{X}_g, \mathcal{Y}_g) \mid p_g \in \text{rin}(t_i)\}) = 1$
 where
 $t_i \in T_R^{TRN}$; $\ell(\alpha) \in F_R(t_i, t_i \bullet)$; $\forall_{p_g \in \text{rin}(t_i)} (\mathcal{X}_g, \mathcal{Y}_g) = b(E_C(p_g, tr_i))$
 $\text{isElement}(q, Q) = \text{true}$ if and only if $q \in Q$;
 $\text{numOfnonEmpty}(Q) = \#\{(q_1, q_2) \in Q \mid (q_1, q_2) \neq (\emptyset, \emptyset)\}$ and for
 backtracking k_j is fixed as follows: $k_j = b(E_C(h_{ij}, tr_i))$ for $i < j$ or
 $k_j = b(E_C(h_{ji}, tr_i))$ for $i > j$ }
 $\mathbf{G}_C^{BC1 \cup BC2} = \{tr_i \mapsto (\bigwedge_{t_j \in \text{dpc}(t_i)} \text{isElement}(\langle k_j, t_j, t_i, \{\alpha_1, \alpha_2\} \rangle),$
 $E_C(h_i, tr_i)); \text{isElement}(\langle \alpha_1, \alpha_2 \rangle, \bigcup_{p_g \in \text{rin}(t_i)} \mathcal{Y}_g);$
 $\text{numOfnonEmpty}(\{(\mathcal{X}_g, \mathcal{Y}_g) \mid p_g \in \text{rin}(t_i)\}) = 1$
 where
 $t_i \in (T_R^{BC1} \cup T_R^{BC2})$; $\{\ell(\alpha_1), \ell(\alpha_2)\} \in F_R(t_i, t_i \bullet)$;
 $\forall_{p_g \in \text{rin}(t_i)} (\mathcal{X}_g, \mathcal{Y}_g) = b(E_C(p_g, tr_i))$;
 $\text{isElement}(q, Q) = \text{true}$ if and only if $q \in Q$;
 $\text{numOfnonEmpty}(Q) = \#\{(q_1, q_2) \in Q \mid (q_1, q_2) \neq (\emptyset, \emptyset)\}$ and for
 backtracking k_j is fixed as follows: $k_j = b(E_C(h_{ij}, tr_i))$ for $i < j$ or
 $k_j = b(E_C(h_{ji}, tr_i))$ for $i > j$ }

$\mathbf{E}_C = \{(\mathbf{p}, \mathbf{t}) \mapsto (\mathcal{X}, \mathcal{Y}) \mid (p, t) \in \text{Domain}(F_R); t \in T_R^{TRN} \cup T_R^{BC2}\}$
 Description of input arcs from the original RPN for *TRN* and *BC2*
 transitions - the transfer of one molecule $(\mathcal{X}, \mathcal{Y})$ obtained from the
 place p .
 \cup
 $\{(\mathbf{p}, \mathbf{t}) \mapsto (1'(\mathcal{X}_1, \mathcal{Y}_1) ++ 1'(\mathcal{X}_2, \mathcal{Y}_2)) \mid (p, t) \in \text{Domain}(F_R); t \in T_R^{BC1}\}$
 Description of input arcs from the original RPN for *BC1* transition -
 the transfer of two molecules from place p (in the guard it is assumed
 that one of those molecules may be empty).
 \cup
 $\{(\mathbf{t}, \mathbf{p}) \mapsto (\emptyset, \emptyset) \mid (p, t) \in \text{Domain}(F_R)\}; t \in T_R^{TRN} \cup T_R^{BC2}\}$
 Description of arcs opposite to input arcs from the original RPN.
 An idle token is transferred for *TRN* or *BC2* transition.
 \cup
 $\{(\mathbf{t}, \mathbf{p}) \mapsto 2'(\emptyset, \emptyset) \mid (p, t) \in \text{Domain}(F_R)\}; t \in T_R^{BC1}\}$
 Description of arcs opposite to input arcs from the original RPN.
 An idle token is transferred for *BC1* transition.
 \cup
 $\{(\mathbf{t}, \mathbf{p}) \mapsto (\mathcal{X}, \mathcal{Y}) \mid E_C(\bullet \mathbf{t}, t) = \{(\mathcal{X}, \mathcal{Y})\}; t \in T_R^{TRN}; (t, p) \in \text{Domain}(F_R)\}$
 Description of output arcs for *TRN* transitions, similar to the ones
 from the RPN. They contain transfer of the molecule obtained from
 the input place.

∪

$$\{(\mathbf{t}, \mathbf{p}) \mapsto (\mathcal{X}_1 \cup \mathcal{X}_2, \mathcal{Y}_1 \cup \mathcal{Y}_2 \cup \{\langle \alpha_1, \alpha_2 \rangle\}) \mid E_C(\bullet \mathbf{t}) = \{(\mathcal{X}_1, \mathcal{Y}_1), (\mathcal{X}_2, \mathcal{Y}_2)\}; \\ \{\ell(\alpha_1), \ell(\alpha_2)\} \in F_R(t, p); t \in T_R^{BC1} \cup T_R^{BC2}; (t, p) \in \text{Domain}(F_R)\}$$

Description of output arcs for BC1 and BC2 transitions. They describe the transfer of the molecules containing the instances of bases and bonds, obtained from the input places (BC2) or place (BC1) and the new bond. Types of the instances in the new bond should be consistent with the label of the arc in the RPN.

∪

$$\{(\mathbf{p}, \mathbf{t}) \mapsto (\emptyset, \emptyset) \mid (t, p) \in \text{Domain}(F_R); t \in T_R\}$$

Description of arcs opposite to input arcs from the original RPN. An idle token is transferred.

∪

$$\{(\mathbf{h}_{jk}, \mathbf{t}_i) \mapsto cnt_l \mid t_i \in T_R; \{i, l\} = \{j, k\}; t_l \in dpc(t_i)\}$$

Description of arc from connection history place to a transition. The value obtained from that place is represented by variable cnt_l .

∪

$$\{(\mathbf{t}_i, \mathbf{h}_{jk}) \mapsto E_C(h_{jk}, t_i) + 1 \mid t_i \in T_R; \{i, l\} = \{j, k\}; t_l \in dpc(t_i)\}$$

Description of the arc from a transition to its connection history place. It describes the transfer of the value obtained from that place (by the opposite arc) increased by 1.

∪

$$\{(\mathbf{h}_i, \mathbf{t}_i) \mapsto H_l \mid H_l \in 2^{(\mathbb{N}_b \times T_R \times T_R \times 2^A)}\}$$

Description of the arc from transition history place to the transition. The value obtained from that place is represented by variable H_l and it contains the whole history of transition t_i (a set of 4-tuples).

∪

$$\{(\mathbf{t}_i, \mathbf{h}_i) \mapsto E_C(h_i, t_i) \cup \bigcup_{t_l \in dpc(t_i)} \{(E_C(h_{jk}, t_i) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\})\}\}$$

$$\text{where } t_i \notin T_R^{TRN}; \{i, l\} = \{j, k\}; \langle \ell(\alpha_1), \ell(\alpha_2) \rangle \in F_R(t_i, t_i \bullet)\}$$

Description of the arc from BC1 or BC2 transition to its transition history place. The value obtained from the transition history place is transferred back (its described by $E_C(h_i, t_i)$) and a new 4-tuple is added for every transition from $dpc(t_i)$. Each tuple consists of 4 components: the first is a number of current execution of t_i in the sequence of executions of t_i and t_l - this value is obtained from h_{il} or h_{li} , the next two components are identifiers of transitions and the last one is the description of the bond created during the considered execution.

∪

$$\{(\mathbf{t}_i, \mathbf{h}_i) \mapsto E_C(h_i, t_i) \cup \bigcup_{t_l \in dpc(t_i)} \{E_C(h_{jk}, t_i) + 1, t_l, t_i, \{\alpha\}\}\}$$

where

$$t_i \in T_R^{TRN}; \{i, l\} = \{j, k\}; \ell(\alpha) \in F_R(t_i, t_i \bullet)$$

Description of the arc from TRN transition to its transition history place. It is very similar to the previous one, except for the last component of the 4-tuples - in this case it is the description of the base instances which were transferred during the considered execution.

∪

$$\{(h_{jk}, tr_i) \mapsto cnt_l \mid t_i \in T_R; \{i, l\} = \{j, k\}; t_l \in dpc(t_i)\}$$

Description of the arc from transition history counter place of t_i to its reversing transition tr_i . The value obtained from the place is represented by cnt_l and it is a number of executions of transitions t_i and t_l .

∪

$$\{(tr_i, h_{jk}) \mapsto E_C(h_{jk}, tr_i) - 1 \mid t_i \in T_R; \{i, l\} = \{j, k\}; t_l \in dpc(t_i)\}$$

Description of the arc from reversing transition tr_i to connection history place of t_i . It describes the transfer of the value obtained from the connection history place by the transition tr_i decreased by one.

∪

$$\{(h_j, tr_i) \mapsto H_j \mid (H_j \in 2^{\mathbb{N}_b \times T_R \times T_R \times 2^A}); (t_j \in dph(t_i) \vee j = i)\}$$

Description of the arc from transition history place of t_i to its reversing transition. The value obtained from that place is represented by variable H_j and it contains the whole history of transition t_i (a set of 4-tuples).

∪

$$\{(p, tr_i) \mapsto 2'(\emptyset, \emptyset) \mid (p \in rin(t_i); \forall t_j \in T_R p \notin t_j \bullet)\}$$

Description of the arc between a place to a reversing transition. The place has to be in a set $rin(t_i)$ and it cannot be an input place to any transition. Then two idle tokens are transferred from the place.

∪

$$\{(p, tr_i) \mapsto (1'(\mathcal{X}, \mathcal{Y}) + +1'(\emptyset, \emptyset)) \mid (p \in rin(t_i); \exists t_j \in T_R p \in t_j \bullet)\}$$

Description of the arc between a place to a reversing transition. The place has to be in a set $rin(t_i)$ and it has to be an input place to some transition from the net. Then a molecule and an idle token are transferred from the place (during execution the molecule also can be an idle token).

∪

$$\{(tr_i, h_j) \mapsto updateExtHist(k_j \in Var[G_C(tr_i)], E_C(h_j, tr_i))\}$$

where

$$t_i \in T_R; t_j \in dph(t_i); (b(k_j), t_j, t_i, Y) \in b(E_C(h_i, tr_i));$$

$$updateExtHist(k_j, E_C(h_j, tr_i)) = \bigcup_{(k, t_g \neq i, t_j, X) \in b(E_C(h_j, tr_i))} (k, t_g, t_j, X)$$

$$\cup \bigcup_{(k < k_j, t_i, t_j, X) \in b(E_C(h_j, tr_i))} (k, t_i, t_j, X) \cup \bigcup_{(k > k_j, t_i, t_j, X) \in b(E_C(h_j, tr_i))} (k -$$

$1, t_i, t_j, X)\}$

Description of the arc between the reversing transition of t_i and history places of other transitions. It contains calling of $updateExtHist()$ function. The first argument of the function is number k_j which is the first component of 4-tuple from history of transition t_j which have been binded during evaluation of the tr_i guard. The second argument of $updateExtHist()$ are elements of t_j history and the function edits them: elements not related to the pair t_i and t_j are not changed, elements related to t_i and t_j with the first component k smaller than k_j are also not changes, elements related to t_i and t_j with k greater than k_j are adjusted by decreasing k by 1.

\cup

$\{(tr_i, h_i) \mapsto$

$updateIntHist(K = \{k_j \in Var[G_C(tr_i)] \mid t_j \in dpc(t_i)\}, E_C(h_i, tr_i))$

where

$t_i \in T_R; \forall k_j \in K (b(k_j), t_j, t_i, Y) \in b(E_C(h_i, tr_i));$

$updateIntHist(K, E_C(h_i, tr_i)) = \bigcup_{(k < k_j, t_j, t_i, X) \in b(E_C(h_i, tr_i))} (k, t_j, t_i, X)$

$\cup \bigcup_{(k > k_j, t_j, t_i, X) \in b(E_C(h_i, tr_i))} (k - 1, t_j, t_i, X)\}$

\cup

Description of the arc between the reversing transition of t_i and history places of transition t_i . It contains calling of $updateIntHist()$ function. The first argument of the function are numbers k_j which are the first components of 4-tuple from t_i history related to pairs t_i and $t_j \in dpc(t_i)$ which have been binded during evaluation of the tr_i guard. The second argument of $updateIntHist()$ are elements of t_i history and the function edits them: elements related to t_i and t_j with the first component k smaller than k_j are not changes, elements related to t_i and t_j with k greater than k_j are adjusted by decreasing k by 1.

$\{(tr_i, p) \mapsto (1'(\mathcal{X}_1, \mathcal{Y}_1) + +1'(\emptyset, \emptyset))$

where

$t_i \in T_R^{TRN}; p \in rout(t_i); \{p\} = t\bullet;$

having $(\mathcal{X}_g, \mathcal{Y}_g) = E_C(p_g, tr_i)$ and $\alpha \in Var[G_C(tr_i)]:$

$(\mathcal{X}_1, \mathcal{Y}_1) = (\emptyset, \emptyset)$ if

$t \neq \max((\bigcup_{t_j \in (dph(t_i) \cup t_i)} b(E_C(h_j, tr_i)) |_{con(\alpha, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g))}); \mathcal{X}_1 \cup$

$\mathcal{Y}_1 = con(\alpha, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g))$ if

$t = \max((\bigcup_{t_j \in (dph(t_i) \cup t_i)} b(E_C(h_j, tr_i)) |_{con(\alpha, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g))})\}$

Description of the arc between reversing transition of transporting transition t_i and it output place p . Transition t_i transported base instance α in the execution which is withdrew in the current execution of tr_i - value of α is evaluated by the tr_i guard. Place p is an output

place of some transition t . Molecules obtained by tr_i from its input place p_g is denoted by $(\mathcal{X}_g, \mathcal{Y}_g)$. Transition tr_i transports an idle token and $(\mathcal{X}_1, \mathcal{Y}_1)$ token which can an idle token or a molecule. The $(\mathcal{X}_1, \mathcal{Y}_1)$ token is an idle token if t is not the maximal (last) transition of transitions from $dph(t_i)$ based on histories obtained from places h_j among those transitions which used the molecule containing α . The $(\mathcal{X}_1, \mathcal{Y}_1)$ token is equal to the molecule containing α if t is the maximal one.

\cup

$$\{(tr_i, \mathbf{p}) \mapsto (1'(\mathcal{X}_1, \mathcal{Y}_1) + +1'(\mathcal{X}_2, \mathcal{Y}_2))$$

where

$$t_i \in (T_R^{BC1} \cup T_R^{BC2}); p \in rout(t_i); \{p\} = t\bullet;$$

having $(\mathcal{X}_g, \mathcal{Y}_g) = E_C(p_g, tr_i)$ and $\langle \alpha_1, \alpha_2 \rangle \in Var[G_C(tr_i)]$:

$(\mathcal{X}_1, \mathcal{Y}_1) = (\emptyset, \emptyset)$ if

$$t \neq \max((\bigcup_{t_j \in (dph(t_i) \cup t_i)} b(E_C(h_j, tr_i)) |_{con(\alpha_1, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})});$$

$\mathcal{X}_1 \cup \mathcal{Y}_1 = con(\alpha_1, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})$ if

$$t = \max((\bigcup_{t_j \in (dph(t_i) \cup t_i)} b(E_C(h_j, tr_i)) |_{con(\alpha_1, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})});$$

$\mathcal{X}_2 \cup \mathcal{Y}_2 = (\emptyset, \emptyset)$ if

$$(t \neq \max((\bigcup_{t_j \in (dph(t_i) \cup t_i)} b(E_C(h_j, tr_i)) |_{con(\alpha_2, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})});$$

$\vee (\mathcal{X}_1, \mathcal{Y}_1) = (\mathcal{X}_2, \mathcal{Y}_2));$

$\mathcal{X}_2 \cup \mathcal{Y}_2 = con(\alpha_2, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})$ if

$$(t = \max((\bigcup_{t_j \in (dph(t_i) \cup t_i)} b(E_C(h_j, tr_i)) |_{con(\alpha_2, \bigcup_{p_g \in rin(t_i)} (\mathcal{X}_g \cup \mathcal{Y}_g) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})});$$

$(\mathcal{X}_1, \mathcal{Y}_1) \neq (\mathcal{X}_2, \mathcal{Y}_2)) \}$

Description of the arc between reversing transition of BC1 or BC2 transition t_i and its output place p . Transition t_i created a bond $\langle \alpha_1, \alpha_2 \rangle$ in the execution which is withdrawn in the current execution of tr_i - value of $\langle \alpha_1, \alpha_2 \rangle$ is evaluated by the tr_i guard. Molecules obtained by tr_i from its input place p_g is denoted by $(\mathcal{X}_g, \mathcal{Y}_g)$. Place p is an output place of some transition t . Transition tr_i transports two tokens: $(\mathcal{X}_1, \mathcal{Y}_1)$ and $(\mathcal{X}_2, \mathcal{Y}_2)$ - both can be an idle tokens. The $(\mathcal{X}_1, \mathcal{Y}_1)$ token is an idle token if t is not the maximal (last) transition of transitions from $dph(t_i)$ based on histories obtained from places h_j among those transitions which used the molecule containing α_1 after breaking bond $\langle \alpha_1, \alpha_2 \rangle$. The $(\mathcal{X}_1, \mathcal{Y}_1)$ token is equal to the molecule containing α_1 after breaking $\langle \alpha_1, \alpha_2 \rangle$ if t is the maximal one. The same for $(\mathcal{X}_2, \mathcal{Y}_2)$ but then we consider molecule containing α_2 . $(\mathcal{X}_1, \mathcal{Y}_1)$ cannot be equal to $(\mathcal{X}_2, \mathcal{Y}_2)$.

$$\begin{aligned} \mathbf{I}_C = & \{\mathbf{p} \mapsto ConCom(M_0(p)) + +(\mathbb{K} - \#ConCom(M_0(p)))'(\emptyset, \emptyset) \mid p \in P_R\} \cup \\ & \{\mathbf{h}_i \mapsto \emptyset \mid t_i \in T_R\} \cup \\ & \{\mathbf{h}_{ij} \mapsto 0 \mid t_i, t_j \in T_R; t_i \prec t_j; t_j \in dpc(t_i)\} \end{aligned}$$

Initial expressions of places: places from P_R contain molecules from the initial marking of N_R and idle tokens (to fulfill \mathbb{K} strong safeness), places h_i contain empty sets and places h_{ij} contain 0. For technical reasons those initial values in places are set by initialization transition t_0 . That transition is part of $dph(t)$ for every $t \in T_R$, is executed at the initial marking and cannot be reversed. It is added to the net so that the *max* always exists.

6 State Equivalence

A state in an RPN is a pair $\langle M_R, H_R \rangle$, where M_R is a marking and H_R is a history function. According to definitions:

- $M_R : P_R \rightarrow 2^{\mathcal{A} \cup \mathcal{B}}$,
- $H : T \rightarrow 2^{\mathbb{N} \times 2^{P \times \mathcal{A}}}$.

A state in a CPN generated from RPN is a marking M_C of coloured Petri net, $M_C : P \rightarrow \mathbb{N}^{\Sigma_C}$ is a function that assigns multisets of tokens to places consistently with C_C , as follows:

- $M_C(p) \in \mathbb{N}^{(2^{\mathcal{A}} \times 2^{\mathcal{B}})}$, for $p \in P_R$,
- $M_C(h_i) \in \mathbb{N}^{2^{(\mathbb{N}_b \times T_R \times T_R \times 2^{\mathcal{A}})}}$, for $t_i \in T_R$,
- $M_C(h_{ij}) \in \mathbb{N}^{\mathbb{N}_b}$, for $t_i, t_j \in T_R; i < j; t_j \in dpc(t_i)$.

Remark 4. In both formalisms, namely RPNs and CPNs, the history of the whole performed execution has to be stored. In RPNs it is conducted explicitly, with the use of history function: $H : T \rightarrow 2^{\mathbb{N} \times 2^{P \times \mathcal{A}}}$ – for transition $t \in T_R$, history $H(t)$ contains a set of pairs of the shape (k_X, X) , where X is a set of pairs $\{(p_1, \alpha_1), (p_2, \alpha_2), \dots, (p_l, \alpha_l)\}$. On the other hand, in CPNs the history is scattered among transitions history places. Every transition history place contains elements belonging to the following Cartesian product: $\mathbb{N}_b \times T_R \times T_R \times 2^{\mathcal{A}}$, i.e. elements of the form (k_Y, t_i, t_j, Y) . Note that during the transformation we cannot just replace X with Y (nor the other way round), because they are of different types. Having a base instance belonging to the set $Y \subseteq \mathcal{A}$, we are able to find a suitable place by indicating the last transition which has been using the given base instance and utilizing its only output place (see Section 6.2 for details). On the other hand, having X , we are able to obtain the set of Y by summing up the components located at the second coordinate of each pair: $Y = \bigcup_{(p, \alpha) \in X} \{\alpha\}$ (see Section 6.1).

6.1 Transformation of states from RPN to CPN

Now, we focus on transformation of states from an RPN to a CPN. This process is quite straightforward - the formulas presented below have to be used.

Let assume that N_R is an RPN $(P_R, T_R, F_R, A_R, B_R)$ and $N_C(N_R)$ is a CPN $(P_C, T_C, D_C, \Sigma_C, V_C, C_C, G_C, E_C, I_C)$. Then to each state in N_R we assign a single state in $N_C(N_R)$ as follows:

- $M_C(p)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(p)), p \in P_R$,
 $M_C(p)((\emptyset, \emptyset)) = \mathbb{K} - \#\text{ConCom}(M_R(p)), p \in P_R$. The number of tokens in every place $p \in P_R$ is equal to \mathbb{K} . If a place at marking M_R contains tokens, then equivalent tokens have to be present at marking M_C . To obtain a fixed number of tokens \mathbb{K} , we fulfill the place with idle tokens.
- $M_C(h_{ij}) = \#H_R(t_i) + \#H_R(t_j)$ for a connection history place h_{ij} . The content of such a place describes how many times transitions t_i and t_j have been executed. Each execution of a transition is related to one element in its H_R , hence it is sufficient to sum up the elements in H_R for both transitions.
- $M_C(h_i) = \bigcup_{(k, Y) \in H_R(t_i), t_j \in \text{dpc}(t_i)} (\#\{k' \mid (k', Z) \in (H_R(t_i) \cup H_R(t_j)) \wedge (k' < k)\} + 1, t_j, t_i, X)$, where $X = \bigcup_{(p, \alpha) \in Y} \{\alpha\}$, for a transition history place h_i . One element of such a place is a 4-tuple and describes one execution of t_i in the context of t_j . Last three components of that 4-tuple are easy to obtain - there are names of transitions and set X , which is obtainable from $H_R(t_i)$ as a sum of all instances constituting the second coordinate of each pair $(p, \alpha) \in X$. To obtain the first component, we have to calculate how many times transitions t_i and t_j have been executed before the execution involving X occurred. Those executions are indicated by elements of $H_R(t_i)$ and $H_R(t_j)$ with first components smaller than the first component in the element including X .

6.2 Transformation of states from CPN to RPN for backtracking

In the following section, we deliberate about the state transformation from a CPN to an RPN. It turns out that it is not as straightforward as the other way round. The main issue we face is that a single state of coloured Petri net constructed according the rules of transformation may correspond to many states of the initial reversible Petri net. The following example illustrates this phenomenon.

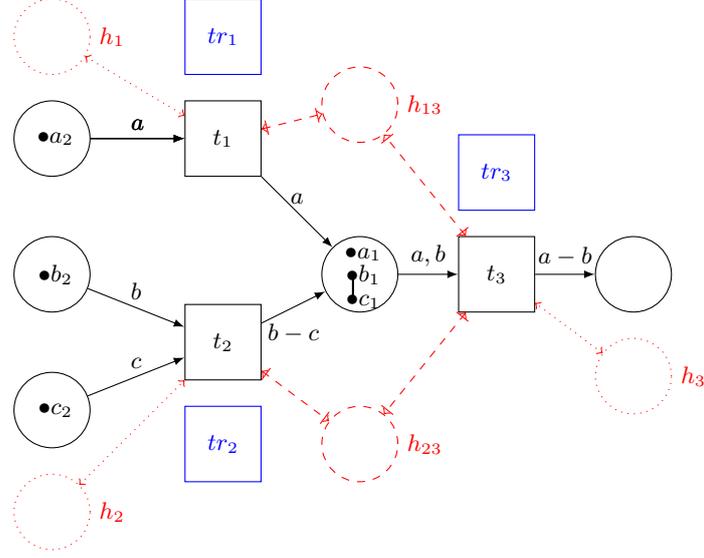


Fig. 6. Coloured Petri Net constructed on the basis of the reversible Petri net depicted in Figure 3 and according to out-of-causal-order semantics.

Example 3. Let us construct a coloured Petri net N_C on the basis of the reversing Petri net N_R depicted in Figure 3. At the very beginning, we add reverse transitions tr_1, tr_2, tr_3 to the net - they are depicted blue. For the reasons of transparency and clarity, here we only mark the reversing transitions, without specifying all their connections. We want to start with adding history places, first we add one transition history place for each transition t_1, t_2, t_3 - respectively: h_1, h_2, h_3 . The places are marked dotted red. Next we need to compute the sets of places: h_{13}, h_{23} - out-of-causal dependency counters and h_1, h_2, h_3 - out-of-causal dependency histories, for t_1, t_2, t_3 obtaining the following:

- $dpc_{OOC}(t_1) = dph_{OOC}(t_1) = \{t_3\}$
- $dpc_{OOC}(t_2) = dph_{OOC}(t_2) = \{t_3\}$
- $dpc_{OOC}(t_3) = dph_{OOC}(t_3) = \{t_1, t_2\}$.

Therefore we need to add the following connection history places: h_{13}, h_{23} - those places are depicted dashed red. Now we should add the arcs between transitions and places h_1, h_2, h_3 and h_{13}, h_{23} , according to the transformation described in Section 5. They are marked by dashed and dotted arrows. Assume that, after the execution of the sequence $t_1 t_2$, we

obtain marking M'_C , depicted in Figure 6. The content of history places is as follows:

- $M'_C(h_1) = \{(1, 3, 1, \{a_1\})\}$
- $M'_C(h_2) = \{(1, 3, 2, \{b_1, c_1\})\}$
- $M'_C(h_3) = \emptyset$
- $M'_C(h_{13}) = 1$
- $M'_C(h_{23}) = 1$.

On the other hand, exactly the same marking one can obtain after the execution of t_2t_1 . This is because in a history place of a transition we only store information concerning other transitions being in relation \prec with it. In our example the two transitions t_1 and t_2 might be considered “independent”, as they are not in the relation, namely $\neg(t_1 \prec t_2 \vee t_2 \prec t_1)$. The order of occurrences of the transitions t_1 and t_2 is not stored anywhere in the net. For comparison, let us look again at the example shown in the Figure 3, considering only the black, solid part, which is the starting RPN. The markings obtained after t_1t_2 and after t_2t_1 are equal but, according to the semantics presented in Definition 5, the history function returns different values.

Remark 5. Let us note that the above example would look significantly different when considering backtracking semantics, because in that case for a given transition t the set of dependency histories $dph(t)$ would contain all transitions different from t .

The above observations allows us formulate an extremely important **conclusion**:

One state of the coloured Petri net created from a given reversing Petri net as a result of transformation described above may correspond to many states of the original reversing Petri net. Even when, for casual and out-of-casual-order semantics, there exists no one-to-one correspondence between states in a CPN and corresponding RPN, for backtracking it is possible to obtain states in RPNs on the basis of states in CPNs. It can be done quite straightforwardly, by using the ensuing formulas. To every state in CPN $N_C(N_R)$ we assign a single state in RPN N_R as follows:

- $M_R(p) = \bigcup_{(\mathcal{X}, \mathcal{Y}) \in M_C(p)} \mathcal{X} \cup \mathcal{Y}$, for $p \in P_R$; in the RPN places contain the same base and bond instances as in the CPN.
- $H_R(t_i) = \bigcup_{(k, j, i, X) \in M_C(h_i)} [K(X) : \bigcup_{\alpha \in X} (t_\alpha \bullet, \alpha)]$ where for a particular set X :

$$K(X) = 1 + \frac{\sum_{(k_j, j, i, X) \in M_C(h_i)} (k_j - 1) - \#\{(k_g, j, i, Y) \in M_C(h_i) \mid (k_g < k_f) \wedge ((k_f, j, i, X) \in M_C(h_i))\}}{\#dpc(t_i) - 1} \cdot t_\alpha = \max((\bigcup_{t_j \in (dph(t_i) \cup \{t_i\})} M_C(h_j)) |_{con(\alpha, \bigcup_{p \in \text{rin}(t_i)} M_C(p))}).^7$$

Calculating the value of the history function in RPNs is a bit tricky. It contains two elements: a number in the sequence of executions and a set of pairs: a place from which a base instance has been taken and the instance. Those base instances are included in markings of transition history places and can be obtained from the last components of 4-tuples. Using the partial order of transitions the last transition (before the considered execution of t_i), which has been using the given base instance, can be indicated - hence its output place must be the place from which t_i obtained the instance. The most difficult to compute is $K(X)$, which denotes the index of execution of the transition t_i related to instances included in X in the sequence of all executions of transitions. In CPNs this information is splitted among 4-tuples in marking of the transition history place. The first component in each 4-tuple: k_j means that the considered execution of t_i (related to X) was k_j -th in the sequence of executions of transitions t_i and t_j . Hence, there had to occur $k_j - 1$ executions before the considered one. However, among those $k_j - 1$ executions for each 4-tuple, also previous executions of t_i are included, and they are included in each 4-tuple. That is why we have to calculate how many times t_i was executed before the considered execution and subtract the redundant information. Each previous execution of transition t_i is described by $\#dcp(t_i)$ elements of $M_C(h_i)$. Hence, to obtain the number of previous executions we have to multiply by $\frac{\#dpc(t_i) - 1}{\#dpc(t_i)}$. Number 1 is added to the obtained value to include the considered execution of t_i .

6.3 Transformation of states from CPN to RPN - all modes

The transformations described in Sections 6.1 and 6.2 determine the *correspondence* between states of an RPN and states of a corresponding CPN. If M_C is a state in the CPN obtained from M_R in the RPN as a result of transformation described in Section 6.1, or M_R in the RPN is composed according to description presented in Section 6.2 from M_C in the CPN, then we say that M_R *corresponds* to M_C and M_C *corresponds* to M_R . Have in mind that, in backtracking semantics the *correspondence* is one-to-one,

⁷ Note that in above equations i is fixed as the number of transition t_i .

while in causal and out-of-causal might be many-to-one (see Example 3). To determine corresponding states in all semantics the following theorem should be used.

The following fact follows directly from the transformation described in Section 6.2.

Lemma 1. *For each state M_C in the CPN there exists at least one reachable state $\langle M_R, H_R \rangle$ in RPN to which M_C is assigned.*

Now, we are ready to prove the main theorem of the paper.

Theorem 1. *Let N_R be an RPN and $N_C(N_R)$ the equivalent CPN. Then $\langle M_{C_0}, H_0 = \emptyset \rangle$ (an initial marking of the RPN with empty history) is equivalent to CPN marking obtained as a result of I_C/t_0 . Moreover, if $\langle M_R, H_R \rangle$ is a state reachable in N_R and M_C is a corresponding marking of N_C then M_C is reachable in N_C . In addition, if $t_i \in T_R$ is enabled in N_R at $\langle M_R, H_R \rangle$ and its execution leads to $\langle M'_R, H'_R \rangle$, then t_i is enabled in N_C at M_C and its execution leads to M'_C which corresponds to $\langle M'_R, H'_R \rangle$. If $t_i \in T_R$ can be reversed at $\langle M_R, H_R \rangle$ and its reverse leads to $\langle M'_R, H'_R \rangle$, then tr_i is enabled in N_C at M_C and its execution leads to M'_C which corresponds to $\langle M'_R, H'_R \rangle$.*

Proof. Let N_R be an RPN and $N_C(N_R)$ the equivalent CPN. In the initial marking of $N_C(N_R)$ we assign to each place $p \in P_R$ a set of tokens $ConCom(M_{C_0}(p))$ supplemented by the proper number of idle tokens (\emptyset, \emptyset) . Moreover, $M_C(h_i) = \emptyset$ since history H_0 is empty and $M_C(h_{ij}) = 0$, since all $H_R(t_i) = \emptyset$.

Let $\langle M_R, H_R \rangle$ be a reachable state, such that t_i is enabled at M_R . Let M_C be a marking corresponding to $\langle M_R, H_R \rangle$.

Part A: forward executions

Case 1: $t_i \in T_R^{TRN}$.

In this case, in the RPN we have one input arc and one output arc for t_i , with the same inscriptions equal to $a \in A$. In the CPN, transition t_i has an input arc labelled with single $(\mathcal{X}, \mathcal{Y})$. By Definition 4(1), if t_i is enabled, then in $S(\bullet t_i)$ we have only one instance α of type a . Moreover, $\alpha \in M_C(\bullet t_i)$, hence one can choose a binding b in which $\alpha \in b(\mathcal{X})$. We need to proceed with all additional negative inscriptions on the input arc. Note that, $b((\mathcal{X}, \mathcal{Y})) = con(\alpha, M_C(\bullet t_i))$, hence by Definition 4(2,3) $(\overline{A} \cup \overline{B}) \cap \ell(\mathcal{X} \cup \mathcal{Y}) = \emptyset$. This way the guard function of t_i with binding b returns true and t_i is enabled in the CPN.

After the execution of t_i in the RPN the contents of two places $\bullet t_i, t_i \bullet$ change, $M'_R(\bullet t_i) = M_R(\bullet t_i) \setminus \text{con}(\alpha, M_R(\bullet t_i))$ and $M'_R(t_i \bullet) = M_R(t_i \bullet) \cup \text{con}(\alpha, M_R(\bullet t_i))$. On the other hand, in the CPN $M'_C(\bullet t_i) = M_C(\bullet t_i) - b((\mathcal{X}, \mathcal{Y})) + (\emptyset, \emptyset)$, while $M'_C(t_i \bullet) = M_C(t_i \bullet) - (\emptyset, \emptyset) + b((\mathcal{X}, \mathcal{Y}))$. Moreover, $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in \text{dpc}(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\alpha\}\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\alpha\}\}$, while for every $t_l \in \text{dpc}(t_i)$ we have $M'_C(h_{il}) = M_C(h_{il}) + 1$ for $i < l$ and $M'_C(h_{li}) = M_C(h_{li}) + 1$ for $i > l$. The contents of the other places in both nets do not change. We are left with the determination of the value of history. In the RPN we have $H'_R(t_i) = H_R(t_i) \cup \{(k = \max\{k' \mid (k', S') \in H(t'), t' \in T\} + 1, \{(\bullet t_i, \alpha)\})\}$. Note that, M'_C is assigned to M'_R since M_C is assigned to M_R and:

- $M'_C(p) = M_C(p)$ for $p \in P_R \setminus \{\bullet t_i, t_i \bullet\}$;
- $M'_C(\bullet t_i)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(\bullet t_i)) \setminus \text{con}(\alpha, M_R(\bullet t_i)) = \text{ConCom}(M_R(\bullet t_i) \setminus \text{con}(\alpha, M_R(\bullet t_i))) = \text{ConCom}(M'_R(\bullet t_i))$,
 $M'_C(\bullet t_i)((\emptyset, \emptyset)) = M_C(\bullet t_i)((\emptyset, \emptyset)) + 1 = \#\text{ConCom}(M_R(\bullet t_i)) + 1 = \mathbb{K} - \#\text{ConCom}(M'_R(\bullet t_i))$;
- $M'_C(t_i \bullet)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(t_i \bullet)) \cup \text{con}(\alpha, M_R(\bullet t_i)) = \text{ConCom}(M_R(t_i \bullet) \cup \text{con}(\alpha, M_R(\bullet t_i))) = \text{ConCom}(M'_R(t_i \bullet))$,
 $M'_C(t_i \bullet)((\emptyset, \emptyset)) = M_C(t_i \bullet)((\emptyset, \emptyset)) - 1 = \#\text{ConCom}(M_R(t_i \bullet)) - 1 = \mathbb{K} - \#\text{ConCom}(M'_R(t_i \bullet))$;
- $M'_C(h_{jl}) = M_C(h_{jl})$ for $j \neq i, l \neq i$;
- $M'_C(h_{il}) = M_C(h_{il}) + 1 = \#H_R(t_i) + 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i < l; t_l \in \text{dpc}(t_i)$
- $M'_C(h_{li}) = M_C(h_{li}) + 1 = \#H_R(t_i) + 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i > l; t_l \in \text{dpc}(t_i)$
- $M'_C(h_j) = M_C(h_j)$ for $j \neq i$;
- $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in \text{dpc}(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\alpha\}\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\alpha\}\} = \bigcup_{(k, \{(p'_\alpha, \alpha')\}) \in H_R(t_i), t_l \in \text{dpc}(t_i)} \{(\#\{k' \mid \{(k', \{(p''_\alpha, \alpha'')\}) \in H_R(t_i) \cup H_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\alpha'\})\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\alpha\}\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\alpha\}\} = \bigcup_{(k, \{(p'_\alpha, \alpha')\}) \in H'_R(t_i), t_l \in \text{dpc}(t_i)} \{(\#\{k' \mid \{(k', \{(p''_\alpha, \alpha'')\}) \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\alpha'\})\}$, where p_α denotes a place from which α is obtained.

Case 2: $t_i \in T_R^{BC2}$.

In this case, in the RPN we have $\bullet t_i = \{p_1, p_2\}$, hence it has two input arcs with inscriptions, precisely: a_1 arc from p_1 , a_2 arc from p_2 , and one output

arc with inscription $\{\langle a_1, a_2 \rangle\}$, for $a_1, a_2 \in A$. In the CPN, transition t_i has two input arcs: one labelled with $(\mathcal{X}_1, \mathcal{Y}_1)$, and the other with $(\mathcal{X}_2, \mathcal{Y}_2)$. By Definition 4(1), in $S(\bullet t_i)$ two instances are present: α_1 of type a_1 and α_2 of type a_2 . Moreover, $\alpha_1 \in M_C(p_1)$ and $\alpha_2 \in M_C(p_2)$, hence one can choose a binding b in which $\alpha_1 \in b(\mathcal{X}_1)$ and $\alpha_2 \in b(\mathcal{X}_2)$. We need to proceed with all additional negative inscriptions on the input arc. Note that, $b((\mathcal{X}_1, \mathcal{Y}_1)) = \text{con}(\alpha_1, M_C(p_1))$ and $b((\mathcal{X}_2, \mathcal{Y}_2)) = \text{con}(\alpha_2, M_C(p_2))$ hence by Definition 4(2,3) $(\overline{A} \cup \overline{B}) \cap \ell(\mathcal{X}_1 \cup \mathcal{Y}_1) \cap \ell(\mathcal{X}_2 \cup \mathcal{Y}_2) = \emptyset$. This way the guard function of t_i with binding b returns true and t_i is enabled in the CPN.

After the execution of t_i in the RPN, the contents of three places $p_1, p_2 \in \bullet t_i$ and $t_i \bullet$ change, $M'_R(p_1) = M_R(p_1) \setminus \text{con}(\alpha_1, M_R(p_1))$, $M'_R(p_2) = M_R(p_2) \setminus \text{con}(\alpha_2, M_R(p_2))$ and $M'_R(t_i \bullet) = M_R(t_i \bullet) \cup \text{con}(\alpha_1, M_R(p_1)) \cup \text{con}(\alpha_2, M_R(p_2)) \cup \{\langle \alpha_1, \alpha_2 \rangle\}$. On the other hand, in the CPN $M'_C(p_1) = M_C(p_1) - b((\mathcal{X}_1, \mathcal{Y}_1)) + (\emptyset, \emptyset)$, $M'_C(p_2) = M_C(p_2) - b((\mathcal{X}_2, \mathcal{Y}_2)) + (\emptyset, \emptyset)$, while $M'_C(t_i \bullet) = M_C(t_i \bullet) - (\emptyset, \emptyset) + b((\mathcal{X}_1 \cup \mathcal{X}_2, \mathcal{Y}_1 \cup \mathcal{Y}_2) \cup \{\langle \alpha_1, \alpha_2 \rangle\})$. Moreover, $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in \text{dpc}(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\}$, while for every $t_l \in \text{dpc}(t_i)$ we have $M'_C(h_{il})$ for $i < l$ is equal to $M_C(h_{il}) + 1$ and $M'_C(h_{li})$ for $i > l$ is equal to $M_C(h_{li}) + 1$. The contents of the other places in both nets do not change. We are left with the determination of the value of history. In the RPN we have $H'_R(t_i) = H_R(t_i) \cup \{(k = \max\{k' \mid (k', S') \in H(t'), t' \in T\} + 1, \{(p_1, \alpha_1), (p_2, \alpha_2)\})\}$. Note that, M'_C is assigned to M'_R since M_C is assigned to M_R and:

- $M'_C(p) = M_C(p)$ for $p \in P_R \setminus \{p_1, p_2, t_i \bullet\}$;
- $M'_C(p_1)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(p_1)) \setminus \text{con}(\alpha_1, M_R(p_1)) = \text{ConCom}(M_R(p_1) \setminus \text{con}(\alpha_1, M_R(p_1))) = \text{ConCom}(M'_R(p_1))$,
 $M'_C(p_1)((\emptyset, \emptyset)) = M_C(p_1)((\emptyset, \emptyset)) + 1 = \#\text{ConCom}(M_R(p_1)) + 1 = \mathbb{K} - \#\text{ConCom}(M'_R(p_1))$;
- $M'_C(p_2)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(p_2)) \setminus \text{con}(\alpha_2, M_R(p_2)) = \text{ConCom}(M_R(p_2) \setminus \text{con}(\alpha_2, M_R(p_2))) = \text{ConCom}(M'_R(p_2))$,
 $M'_C(p_2)((\emptyset, \emptyset)) = M_C(p_2)((\emptyset, \emptyset)) + 1 = \#\text{ConCom}(M_R(p_2)) + 1 = \mathbb{K} - \#\text{ConCom}(M'_R(p_2))$;
- $M'_C(t_i \bullet)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(t_i \bullet)) \cup \text{con}(\alpha_1, M_R(p_1)) \cup \text{con}(\alpha_2, M_R(p_2)) \cup \{\langle a_1, a_2 \rangle\} = \text{ConCom}(M_R(t_i \bullet) \cup (\text{con}(\alpha_1, M_R(p_1)) \cup \text{con}(\alpha_2, M_R(p_2)) \cup \{\langle a_1, a_2 \rangle\})) = \text{ConCom}(M'_R(t_i \bullet))$,
 $M'_C(t_i \bullet)((\emptyset, \emptyset)) = M_C(t_i \bullet)((\emptyset, \emptyset)) - 1 = \#\text{ConCom}(M_R(t_i \bullet)) - 1 = \mathbb{K} - \#\text{ConCom}(M'_R(t_i \bullet))$;
- $M'_C(h_{jl}) = M_C(h_{jl})$ for $j \neq i, l \neq i$;

- $M'_C(h_{il}) = M_C(h_{il}) + 1 = \#H_R(t_i) + 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i < l; t_l \in dpc(t_i)$
- $M'_C(h_{li}) = M_C(h_{li}) + 1 = \#H_R(t_i) + 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i > l; t_l \in dpc(t_i)$
- $M'_C(h_j) = M_C(h_j)$ for $j \neq i$;
- $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in dpc(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in dpc(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} = \bigcup_{(k, \{(p'_{\alpha_1}, \alpha_1'), (p'_{\alpha_2}, \alpha_2')\}) \in H_R(t_i), t_l \in dpc(t_i)} \{(\#\{k' \mid \{(k', \{(p''_{\alpha_1}, \alpha_1''), (p''_{\alpha_2}, \alpha_2'')\})\} \in H_R(t_i) \cup H_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\langle \alpha'_1, \alpha'_2 \rangle\})\} \cup \bigcup_{t_l \in dpc(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in dpc(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} = \bigcup_{(k, \{(p'_{\alpha_1}, \alpha_1'), (p'_{\alpha_2}, \alpha_2')\}) \in H'_R(t_i), t_l \in dpc(t_i)} \{(\#\{k' \mid \{(k', \{(p'_{\alpha_1}, \alpha_1'), (p'_{\alpha_2}, \alpha_2')\})\} \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\langle \alpha'_1, \alpha'_2 \rangle\})\}$, where p_α denotes a place from which α is obtained.

Case 3: $t_i \in T_R^{BC1}$.

In this case, in the RPN t_i has one input arc with inscription $\{a_1, a_2\}$ and one output arc with inscription $\{\langle a_1, a_2 \rangle\}$ for $a_1, a_2 \in A$. In the CPN transition t_i has an input arc labelled with $1'(\mathcal{X}_1, \mathcal{Y}_1) + 1'(\mathcal{X}_2, \mathcal{Y}_2)$. By Definition 4(1), in $S(\bullet t_i)$ two instances are present: α_1 of type a_1 and α_2 of type a_2 . Moreover, $\alpha_1, \alpha_2 \in M_C(\bullet t_i)$, hence we have two possibilities of binding:

Subcase A:

α_1 and α_2 are included in the same molecule and one can choose a binding b in which $\alpha_1, \alpha_2 \in b(\mathcal{X}_1)$ and $(\mathcal{X}_2, \mathcal{Y}_2) = (\emptyset, \emptyset)$. We need to proceed with all additional negative inscriptions on the input arc. Note that, $b((\mathcal{X}_1, \mathcal{Y}_1)) = \text{con}(\alpha_1, M_C(\bullet t_i)) = \text{con}(\alpha_2, M_C(\bullet t_i))$ hence by Definition 4(2,3) $(\bar{A} \cup \bar{B}) \cap \ell(\mathcal{X}_1 \cup \mathcal{Y}_1) = \emptyset$. This way the guard function of t_i with binding b returns true and t_i is enabled in the CPN.

After the execution of t_i in the RPN the contents of two places $\bullet t_i, t_i \bullet$ change, $M'_R(\bullet t_i) = M_R(\bullet t_i) \setminus \text{con}(\alpha_1, M_R(\bullet t_i))$ and $M'_R(t_i \bullet) = M_R(t_i \bullet) \cup \bigcup_{\alpha' \in S(\bullet t_i)} \text{con}(\alpha', M_R(\bullet t_i)) \cup \{\langle \alpha_1, \alpha_2 \rangle\}$. On the other hand, in the CPN $M'_C(\bullet t_i) = M_C(\bullet t_i) - b((\mathcal{X}_1, \mathcal{Y}_1)) - (\emptyset, \emptyset) + 2'(\emptyset, \emptyset)$, while $M'_C(t_i \bullet) = M_C(t_i \bullet) - (\emptyset, \emptyset) + b((\mathcal{X}_1, \mathcal{Y}_1 \cup \{\langle \alpha_1, \alpha_2 \rangle\}))$. Moreover, $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in dpc(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in dpc(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\}$, while for every $t_l \in dpc(t_i)$ we have $M'_C(h_{il})$ for $i < l$ is equal to $M_C(h_{il}) + 1$ and $M'_C(h_{li})$ for $i > l$ is equal to $M_C(h_{li}) + 1$. The contents of the other places in both nets do not change. We are left

with the determination of the value of history: $H'_R(t_i) = H_R(t_i) \cup \{(k = \max\{k' \mid (k', S') \in H(t'), t' \in T\} + 1, \{(\bullet t_i, \alpha_1), (\bullet t_i, \alpha_2)\})\}$. Note that, M'_C is assigned to M'_R since M_C is assigned to M_R and:

- $M'_C(p) = M_C(p)$ for $p \in P_R \setminus \{\bullet t_i, t_i \bullet\}$;
- $M'_C(\bullet t_i)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(\bullet t_i)) \setminus \text{con}(\alpha_1, M_R(\bullet t_i)) = \text{ConCom}(M_R(\bullet t_i) \setminus \text{con}(\alpha_1, M_R(\bullet t_i))) = \text{ConCom}(M'_R(\bullet t_i))$,
 $M'_C(\bullet t_i)((\emptyset, \emptyset)) = M_C(\bullet t_i)((\emptyset, \emptyset)) - 1 + 2 = \#\text{ConCom}(M_R(\bullet t_i)) + 1 = \mathbb{K} - \#\text{ConCom}(M'_R(\bullet t_i))$;
- $M'_C(t_i \bullet)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(t_i \bullet)) \cup \text{con}(\alpha_1, M_R(\bullet t_i)) \cup \{\langle \alpha_1, \alpha_2 \rangle\} = \text{ConCom}(M_R(t_i \bullet) \cup \text{con}(\alpha_1, M_R(\bullet t_i))) = \text{ConCom}(M'_R(t_i \bullet))$,
 $M'_C(t_i \bullet)((\emptyset, \emptyset)) = M_C(t_i \bullet)((\emptyset, \emptyset)) - 1 = \#\text{ConCom}(M_R(t_i \bullet)) - 1 = \mathbb{K} - \#\text{ConCom}(M'_R(t_i \bullet))$;
- $M'_C(h_{jl}) = M_C(h_{jl})$ for $j \neq i, l \neq i$;
- $M'_C(h_{il}) = M_C(h_{il}) + 1 = \#H_R(t_i) + 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i < l; t_l \in \text{dpc}(t_i)$
- $M'_C(h_{li}) = M_C(h_{li}) + 1 = \#H_R(t_i) + 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i > l; t_l \in \text{dpc}(t_i)$
- $M'_C(h_j) = M_C(h_j)$ for $j \neq i$;
- $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in \text{dpc}(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} = \bigcup_{(k, \{(p'_{\alpha_1}, \alpha_1'), (p'_{\alpha_1}, \alpha_2')\}) \in H_R(t_i), t_l \in \text{dpc}(t_i)} \{(\#\{k' \mid \{(k', \{(p''_{\alpha_1}, \alpha_1''), (p''_{\alpha_1}, \alpha_2'')\}) \in H_R(t_i) \cup H_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\langle \alpha'_1, \alpha'_2 \rangle\})\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in \text{dpc}(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} = \bigcup_{(k, \{(p'_{\alpha_1}, \alpha_1'), (p'_{\alpha_1}, \alpha_2')\}) \in H'_R(t_i), t_l \in \text{dpc}(t_i)} \{(\#\{k' \mid \{(k', \{(p'_{\alpha_1}, \alpha_1'), (p'_{\alpha_1}, \alpha_2')\}) \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\langle \alpha'_1, \alpha'_2 \rangle\})\}$, where p_α denotes a place from which α is obtained.

Subcase B:

α_1 and α_2 are included in different molecules, hence one can choose a binding b in which $\alpha_1 \in b(\mathcal{X}_1)$ and $\alpha_2 \in b(\mathcal{X}_2)$. We need to proceed with all additional negative inscriptions on the input arc. Note that, $b((\mathcal{X}_1, \mathcal{Y}_1)) = \text{con}(\alpha_1, M_C(\bullet t_i))$, $b((\mathcal{X}_2, \mathcal{Y}_2)) = \text{con}(\alpha_2, M_C(\bullet t_i))$ hence by Definition 4(2,3) $(\bar{A} \cup \bar{B}) \cap \ell(\mathcal{X}_1 \cup \mathcal{Y}_1) \cap \ell(\mathcal{X}_2 \cup \mathcal{Y}_2) = \emptyset$. This way the guard function of t_i with binding b returns true and t_i is enabled in the CPN.

After the execution of t_i in the RPN the contents of two places $\bullet t_i, t_i \bullet$ change, $M'_R(\bullet t_i) = M_R(\bullet t_i) \setminus (\text{con}(\alpha_1, M_R(\bullet t_i)) \cup \text{con}(\alpha_2, M_R(\bullet t_i)))$ and $M'_R(t_i \bullet) = M_R(t_i \bullet) \cup \bigcup_{\alpha' \in S(\bullet t_i)} \text{con}(\alpha', M_R(\bullet t_i)) \cup \{\langle \alpha_1, \alpha_2 \rangle\}$. On the other

hand, in the CPN $M'_C(\bullet t_i) = M_C(\bullet t_i) - b((\mathcal{X}_1, \mathcal{Y}_1)) - b((\mathcal{X}_2, \mathcal{Y}_2)) + 2 \cdot (\emptyset, \emptyset)$, while $M'_C(t_i \bullet) = M_C(t_i \bullet) - (\emptyset, \emptyset) + b((\mathcal{X}_1 \cup \mathcal{X}_2, \mathcal{Y}_1 \cup \mathcal{Y}_2 \cup \{\langle \alpha_1, \alpha_2 \rangle\}))$. Moreover $M'_C(h_i) = M_C(h_i) \cup \bigcup_{t_l \in dpc(t_i), i < l} \{b(E_C(h_{il}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\} \cup \bigcup_{t_l \in dpc(t_i), i > l} \{b(E_C(h_{li}, t_i)) + 1, t_l, t_i, \{\langle \alpha_1, \alpha_2 \rangle\}\}$, while for every $t_l \in dpc(t_i)$ we have $M'_C(h_{il})$ for $i < l$ is equal to $M_C(h_{il}) + 1$ and $M'_C(h_{li})$ for $i > l$ is equal to $M_C(h_{li}) + 1$. The contents of the other places in both nets do not change. We are left with the determination of the value of history: $H'_R(t_i) = H_R(t_i) \cup \{(k = \max\{k' \mid (k', S') \in H(t'), t' \in T\} + 1, \{(\bullet t_i, \alpha_1), (\bullet t_i, \alpha_2)\})\}$. Note that, M'_C is assigned to M'_R since M_C is assigned to M_R and:

- $M'_C(p) = M_C(p)$ for $p \in P_R \setminus \{\bullet t_i, t_i \bullet\}$;
- $M'_C(\bullet t_i)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(\bullet t_i)) \setminus (\text{con}(\alpha_1, M_R(\bullet t_i)) \cup \text{con}(\alpha_2, M_R(\bullet t_i))) = \text{ConCom}(M_R(\bullet t_i) \setminus (\text{con}(\alpha_1, M_R(\bullet t_i)) \cup \text{con}(\alpha_2, M_R(\bullet t_i)))) = \text{ConCom}(M'_R(\bullet t_i))$,
 $M'_C(\bullet t_i)((\emptyset, \emptyset)) = M_C(\bullet t_i)((\emptyset, \emptyset)) + 2 = \#\text{ConCom}(M_R(\bullet t_i)) + 2 = \mathbb{K} - \#\text{ConCom}(M'_R(\bullet t_i))$;
- $M'_C(t_i \bullet)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(t_i \bullet)) \cup \text{con}(\alpha_1, M_R(\bullet t_i)) \cup \text{con}(\alpha_2, M_R(\bullet t_i)) \cup \{\langle \alpha_1, \alpha_2 \rangle\} = \text{ConCom}(M_R(t_i \bullet) \cup \text{con}(\alpha_1, M_R(\bullet t_i))) = \text{ConCom}(M'_R(t_i \bullet))$,
 $M'_C(t_i \bullet)((\emptyset, \emptyset)) = M_C(t_i \bullet)((\emptyset, \emptyset)) - 1 = \#\text{ConCom}(M_R(t_i \bullet)) - 1 = \mathbb{K} - \#\text{ConCom}(M'_R(t_i \bullet))$;
- Changes of markings of other places are the same as in the previous situation.

Part B: Reverse executions – equivalence of enabledness

Note that, in case of reverse executions, we have to deal with three different semantics, and the notion of transition enabledness differs depending on the chosen one. Therefore, we need to investigate three possible situations.

Case 1:

Backtracking. Notice, that in the RPN transition t is enabled to be reversed when it is the last forward executed transition in the execution of the net, that is according to Definition 3.1, if it has the greatest number in $H(t')$ for $t' \in T$. In the CPN, the last executed transition t can be established as the one with the values on the first position in elements of its history place equal to values obtained from respective connection history places related to t . Note that, by the construction of connection

history places, those values are the greatest possible. Hence, by the transformation transition t is bt -enabled.

Case 2:

Causal reversing. According to Definition 8 in the RPN transition t is co -enabled if there is no other transition t' that has been executed after t (with value k' from the history higher than k assigned to t) and has used any token instance α_i utilised by t . In other words any other transition t' dependent on t has not been fired, and the dependency is based on token instances used by transitions. In the CPN, t can be reversed, if token instances utilised by t are present in places from $rin(t)$, which is for causal semantics the set of output places of t . Since t can be reversed, any other transition t' such that $t \bullet \cap \bullet t' \neq \emptyset$ using the same token instances has not been executed. Once more, we may say that any other transition t' dependent on t has not been fired, and the dependency is based mostly on the structure of the net. However, since cycles are not allowed in RPNs, both dependency approaches are equivalent. We refer readers interested in cycles in RPNs and how they impact dependency between transition in causal reversing to [3]. Notice, that the first part of the guards of reversing transitions corresponds to the choice of the execution of t to be reversed and collect all information from the history place related to that chosen execution. This part is fulfilled after any execution of t . The same holds for out-of-causal-order semantics.

Case 3:

Out-of-causal-order reversing. In the RPN, transition t is o -enabled if it was executed, i.e. its history $H(t)$ is not empty (Definition 10). In the CPN, similarly to causal semantics case, t can be reversed if token instances transferred by t are present in places from $rin(t)$. For out-of-causal-order semantics, $rin(t)$ consists of all places to which token instances used by t could be transported. Hence, this condition is naturally fulfilled. Then the only condition necessary to satisfy is the first part of the guard, which is achieved simply by any execution of t .

Part C: Reverse executions – equivalence of markings

According to Definition 9 for causal semantics, the change in marking M'_R during reversing is the same as for backtracking semantics, which is presented in Definition 7. Moreover, the change in marking M'_R during reversing for backtracking can be described by the formula for out-of-causal-

order reversing - Definition 12. In that case let q be the only output place of t and $\alpha \in M'_R(t\bullet) = M'_R(q)$. We are faced with two possible situations: either $\bullet t = \text{last}(C_{\alpha,q}, H_R)\bullet$ or $C_{\alpha,q} \subseteq M_0(\bullet t)$. Hence, in the following paragraph, we can focus only on the formula presented in Definition 12.

Notice, that according to Remark 2, t , being the last transition according to the partial order defined by \prec , is equal to the last transition indicated by Definition 11. If $\text{last}(C, H)$ equals \perp the maximal transition equals the initial transition t_0 .

Case 1: $t_i \in T^{TRN}_R$

Assume, that during its execution, transition t_i which is reversed has transported token instance α of type a . Then, $\text{b.effect}(t_i, S) = \emptyset$ and $\{\alpha\} = S$. During reversing, only the contents of two places change, namely place q , such that $\alpha \in M_R(q)$ and place p such that either $p = \text{last}(C_{\alpha,q}, H'_R)\bullet$ or $C_{\alpha,q} \in M_{R0}(p)$ for $\text{last}(C_{\alpha,q}, H'_R) = \perp$. We use the abbreviation $C_{\alpha,q} = \text{con}(\alpha, M_R(q))$ from Definition 12.

- $M'_C(p') = M_C(p')$ for $p' \in P_R \setminus \{p, q\}$;
- $M'_C(q)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(q)) \setminus \text{con}(\alpha, M_R(q)) = \text{ConCom}(M_R(q) \setminus \text{con}(\alpha, M_R(q))) = \text{ConCom}(M'_R(q))$,
 $M'_C(q)((\emptyset, \emptyset)) = M_C(q)((\emptyset, \emptyset)) + 1 = \#\text{ConCom}(M_R(q)) + 1 = \mathbb{K} - \#\text{ConCom}(M'_R(q))$;
- $M'_C(p)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(p)) \cup \text{con}(\alpha, M_R(q)) = \text{ConCom}(M_R(p) \cup \text{con}(\alpha, M_R(q))) = \text{ConCom}(M'_R(p))$,
 $M_C(p)((\emptyset, \emptyset)) = M'_C(p)((\emptyset, \emptyset)) - 1 = \#\text{ConCom}(M'_R(p)) - 1 = \mathbb{K} - \#\text{ConCom}(M_R(p))$;
- $M'_C(h_{jl}) = M_C(h_{jl})$ for $j \neq i, l \neq i$;
- $M'_C(h_{il}) = M_C(h_{il}) - 1 = \#H_R(t_i) - 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i < l; t_l \in \text{dpc}(t_i)$
- $M'_C(h_{li}) = M_C(h_{li}) - 1 = \#H_R(t_i) - 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i > l; t_l \in \text{dpc}(t_i)$
- $M'_C(h_i) = \bigcup_{(k < k_j, t_j, t_i, \{\alpha'\}) \in M_C(h_i), (k_j, t_j, t_i, \{\alpha\}) \in M_C(h_i)} (k, t_j, t_i, \{\alpha'\}) \cup \bigcup_{(k > k_j, t_j, t_i, \{\alpha'\}) \in M_C(h_i), (k_j, t_j, t_i, \{\alpha\}) \in M_C(h_i)} (k - 1, t_j, t_i, \{\alpha'\}) \} = \bigcup_{(k, \{p'_\alpha, \alpha'\}) \in H'_R(t_i), t_l \in \text{dpc}(t_i)} \{(\#\{k' \mid \{(k', \{p''_\alpha, \alpha''\}) \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\alpha'\})\}$, where p_α denotes a place from which α is obtained. The last equality holds because H'_R is modified according to Definition 9.
- $M'_C(h_j) = M_C(h_j)$ for $t_j \notin \text{dph}(t_i)$
- For $t_j \in \text{dph}(t_i)$ and $t_j \in T^{TRN}_R$ we have:
 $M'_C(h_j) = \bigcup_{(k, t_g \neq i, t_j, \{\alpha'\}) \in M_C(h_j)} (k, t_g, t_j, \{\alpha'\}) \cup \bigcup_{(k < k_j, t_i, t_j, \{\alpha'\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha\}) \in M_C(h_i)} (k, t_i, t_j, \{\alpha'\}) \cup$

$$\bigcup_{(k > k_j, t_i, t_j, \{\alpha'\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha\}) \in M_C(h_i)} (k-1, t_i, t_j, \{\alpha'\}) =$$

$$\bigcup_{(k, \{p'_\alpha, \alpha'\}) \in H'_R(t_j), t_l \in dpc(t_j)} \{(\#\{k' \mid \{(k', \{p''_\alpha, \alpha''\}) \in H'_R(t_j) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_j, \{\alpha'\})\},$$
 where p_α denotes a place from which α is obtained. The last equality holds because H'_R is modified according to Definition 9.

– For $t_j \in dph(t_i)$ and $t_j \in T^{BC1} \cup T^{BC2}$ we have:

$$M'_C(h_j) = \bigcup_{(k, t_g \neq i, t_j, \{\alpha'_1, \alpha'_2\}) \in M_C(h_j)} (k, t_g, t_j, \{\alpha'_1, \alpha'_2\}) \cup$$

$$\bigcup_{(k < k_j, t_i, t_j, \{\alpha'_1, \alpha'_2\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha\}) \in M_C(h_i)} (k, t_i, t_j, \{\alpha'_1, \alpha'_2\}) \cup$$

$$\bigcup_{(k > k_j, t_i, t_j, \{\alpha'_1, \alpha'_2\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha\}) \in M_C(h_i)} (k-1, t_i, t_j, \{\alpha'_1, \alpha'_2\}) =$$

$$\bigcup_{(k, \{p'_{\alpha_1}, \alpha_1'\}, \{p'_{\alpha_2}, \alpha_2'\}) \in H'_R(t_i), t_l \in dpc(t_i)} \{(\#\{k' \mid \{(k', \{p'_{\alpha_1}, \alpha_1'\}, \{p'_{\alpha_2}, \alpha_2'\}) \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\alpha'_1, \alpha'_2\})\},$$
 where p_α denotes a place from which α is obtained. The last equality holds because H'_R is modified according to Definition 9.

Case 2: $t_i \in T_R^{BC2}$

Assume, that during its execution transition t_i which is reversed has created a bond $\langle \alpha_1, \alpha_2 \rangle$, where α_1 is of type a_1 and α_2 of type a_2 . Then, $\text{b.effect}(t_i, S) = \{\langle \alpha_1, \alpha_2 \rangle\}$ and $\{\alpha_1, \alpha_2\} = S$, $\text{con}(\alpha_1, M_R(q)) = \text{con}(\alpha_2, M_R(q))$ because in $M_R(q)$ α_1 and α_2 are bonded. During reversing, only the contents of three places change, namely: place q for $\langle \alpha_1, \alpha_2 \rangle \in M'_R(q)$, and place p_1 such that either $p_1 = \text{last}(C_{\alpha_1, q}, H_R) \bullet$ or $C_{\alpha_1, q} \in M_{R0}(p_1)$ if $\text{last}(C_{\alpha_1, q}, H_R) = \perp$ and place p_2 such that either $p_2 = \text{last}(C_{\alpha_2, q}, H_R) \bullet$ or $C_{\alpha_2, q} \in M_{R0}(p_2)$ if $\text{last}(C_{\alpha_2, q}, H_R) = \perp$. We use the abbreviation $C_{\alpha, q} = \text{con}(\alpha, M'_R(q) \setminus \text{b.effect}(t_i, S))$ defined in Definition 12. Here, markings of two places p_1 and p_2 change, in contrast to the case $t_i \in T_R^{TRN}$ (only one place). Since reversing of $t_i \in T_R^{BC2}$, always results in splitting of molecule containing the bond $\langle \alpha_1, \alpha_2 \rangle$ (in M'_R located in place q) into two parts: the first containing α_1 , which goes back to p_1 in M_R and the second containing α_2 , goes back to p_2 in M_R .

- $M'_C(p') = M_C(p')$ for $p' \in P_R \setminus \{p_1, p_2, q\}$;
- $M'_C(q)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(q)) \setminus \text{con}(\alpha_1, M_R(q)) = \text{ConCom}(M_R(q) \setminus \text{con}(\alpha, M_R(q))) = \text{ConCom}(M'_R(q))$,
 $M'_C(q)((\emptyset, \emptyset)) = M_C(q)((\emptyset, \emptyset)) + 1 = \#\text{ConCom}(M_R(q)) + 1 = \mathbb{K} - \#\text{ConCom}(M'_R(q))$;
- $M'_C(p_1)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(p_1)) \cup \text{con}(\alpha_1, M_R(q) \setminus \{\langle \alpha_1, \alpha_2 \rangle\}) = \text{ConCom}(M_R(p_1) \cup \text{con}(\alpha_1, M_R(q) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})) = \text{ConCom}(M'_R(p_1))$,
 $M'_C(p_1)((\emptyset, \emptyset)) = M_C(p_1)((\emptyset, \emptyset)) - 1 = \#\text{ConCom}(M_R(p_1)) - 1 = \mathbb{K} - \#\text{ConCom}(M'_R(p_1))$;
- $M'_C(p_2)(\mathcal{X}) = 1$ for $\mathcal{X} \in \text{ConCom}(M_R(p_2)) \cup \text{con}(\alpha_2, M_R(q) \setminus \{\langle \alpha_1, \alpha_2 \rangle\}) = \text{ConCom}(M_R(p_2) \cup \text{con}(\alpha_2, M_R(q) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})) = \text{ConCom}(M'_R(p_2))$,

- $$M'_C(p_2)((\emptyset, \emptyset)) = M_C(p_2)((\emptyset, \emptyset)) - 1 = \#ConCom(M_R(p_2)) - 1 = \mathbb{K} - \#ConCom(M'_R(p_2));$$
- $M'_C(h_{jl}) = M_C(h_{jl})$ for $j \neq i, l \neq i$;
 - $M'_C(h_{il}) = M_C(h_{il}) - 1 = \#H_R(t_i) - 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i < l; t_l \in dpc(t_i)$
 - $M'_C(h_{li}) = M_C(h_{li}) - 1 = \#H_R(t_i) - 1 + \#H_R(t_l) = \#H'_R(t_i) + \#H'_R(t_l)$, for $i > l; t_l \in dpc(t_i)$
 - $M'_C(h_i) = \bigcup_{(k < k_j, t_j, t_i, \{\alpha'_1, \alpha'_2\}) \in M_C(h_i), (k_j, t_j, t_i, \{\alpha_1, \alpha_2\}) \in M_C(h_i)} (k, t_j, t_i, \{\alpha'_1, \alpha'_2\}) \cup \bigcup_{(k > k_j, t_j, t_i, \{\alpha'_1, \alpha'_2\}) \in M_C(h_i), (k_j, t_j, t_i, \{\alpha_1, \alpha_2\}) \in M_C(h_i)} (k-1, t_j, t_i, \{\alpha'_1, \alpha'_2\}) = \bigcup_{(k, \{(p'_{\alpha_1}, \alpha'_1), (p'_{\alpha_1}, \alpha'_2)\}) \in H'_R(t_i), t_l \in dpc(t_i)} \{(\#\{k' \mid \{(k', \{(p'_{\alpha_1}, \alpha'_1), (p'_{\alpha_1}, \alpha'_2)\}) \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\alpha'_1, \alpha'_2\})\}$ where p_α denotes a place from which α is obtained. The last equality holds because H'_R is modified according to Definition 9.
 - $M'_C(h_j) = M_C(h_j)$ for $t_j \notin dph(t_i)$
 - For $t_j \in dph(t_i)$ and $t_j \in T^{TRN}$ we have:
$$M'_C(h_j) = \bigcup_{(k, t_g \neq i, t_j, \{\alpha'\}) \in M_C(h_j)} (k, t_g, t_j, \{\alpha'\}) \cup \bigcup_{(k < k_j, t_i, t_j, \{\alpha'\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha_1, \alpha_2\}) \in M_C(h_i)} (k, t_i, t_j, \{\alpha'\}) \cup \bigcup_{(k > k_j, t_i, t_j, \{\alpha'\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha_1, \alpha_2\}) \in M_C(h_i)} (k-1, t_i, t_j, \{\alpha'\}) = \bigcup_{(k, \{(p'_\alpha, \alpha')\}) \in H'_R(t_j), t_l \in dpc(t_j)} \{(\#\{k' \mid \{(k', \{(p''_\alpha, \alpha'')\}) \in H'_R(t_j) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_j, \{\alpha'\})\}$$
, where p_α denotes a place from which α is obtained. The last equality holds because H'_R is modified according to Definition 9.
 - For $t_j \in dph(t_i)$ and $t_j \in T^{BC1} \cup T^{BC2}$ we have:
$$M'_C(h_j) = \bigcup_{(k, t_g \neq i, t_j, \{\alpha'_1, \alpha'_2\}) \in M_C(h_j)} (k, t_g, t_j, \{\alpha'_1, \alpha'_2\}) \cup \bigcup_{(k < k_j, t_i, t_j, \{\alpha'_1, \alpha'_2\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha_1, \alpha_2\}) \in M_C(h_i)} (k, t_i, t_j, \{\alpha'_1, \alpha'_2\}) \cup \bigcup_{(k > k_j, t_i, t_j, \{\alpha'_1, \alpha'_2\}) \in M_C(h_j), (k_j, t_j, t_i, \{\alpha_1, \alpha_2\}) \in M_C(h_i)} (k-1, t_i, t_j, \{\alpha'_1, \alpha'_2\}) = \bigcup_{(k, \{(p'_{\alpha_1}, \alpha'_1), (p'_{\alpha_1}, \alpha'_2)\}) \in H'_R(t_i), t_l \in dpc(t_i)} \{(\#\{k' \mid \{(k', \{(p'_{\alpha_1}, \alpha'_1), (p'_{\alpha_1}, \alpha'_2)\}) \in H'_R(t_i) \cup H'_R(t_l)\} \wedge k' < k\} + 1, t_l, t_i, \{\alpha'_1, \alpha'_2\})\}$$
, where p_α denotes a place from which α is obtained. The last equality holds because H'_R is modified according to Definition 9.

Case 3: $t_i \in T^{BC1}$

Assume, that during its execution transition t_i which is reversed, has created a bond $\langle \alpha_1, \alpha_2 \rangle$, where α_1 is of type a_1 and α_2 of type a_2 . Here, two situations are possible. In the first, after reversing of t_i which results in breaking the bond $\langle \alpha_1, \alpha_2 \rangle$, we have $\text{con}(\alpha_1, M_R(q) \setminus \{\langle \alpha_1, \alpha_2 \rangle\}) = \text{con}(\alpha_2, M_R(q) \setminus \{\langle \alpha_1, \alpha_2 \rangle\})$ - token instances α_1 and α_2 after breaking $\langle \alpha_1, \alpha_2 \rangle$ are still connected. Now, only the contents of two places change, namely q for $\langle \alpha_1, \alpha_2 \rangle \in M_R(q)$ and p such that either $p = \text{last}(C_{\alpha_1, q}, H'_R) \bullet$ or $C_{\alpha_1, q} \in M_{R0}(p)$ if $\text{last}(C_{\alpha, q}, H'_R) = \perp$, where $C_{\alpha_1, q} = \text{con}(\alpha_1, M_R(q) \setminus$

$\{\langle\alpha_1, \alpha_2\rangle\}$). This situation is analogous to $t_i \in T_R^{TRN}$. In the second possibility, after reversing of t_i which results in breaking bond $\langle\alpha_1, \alpha_2\rangle$, we have $\text{con}(\alpha_1, M_R(q) \setminus \{\langle\alpha_1, \alpha_2\rangle\}) \neq \text{con}(\alpha_2, M_R(q) \setminus \{\langle\alpha_1, \alpha_2\rangle\})$ - molecule containing α_1 and α_2 is split into two molecules, the first containing α_1 and the second α_2 . This situation is analogous to $t_i \in T_R^{BC2}$.

It remains for us to prove the simple fact.

Lemma 2. *The following operations are correct:*

$$\begin{aligned} \text{ConCom}(M_R(\bullet t_i)) \setminus \text{con}(\alpha, M_R(\bullet t_i)) &= \text{ConCom}(M_R(\bullet t_i) \setminus \text{con}(\alpha, M_R(\bullet t_i))) \\ \text{ConCom}(M_R(\bullet t_i)) \cup \text{con}(\alpha, M_R(\bullet t_i)) &= \text{ConCom}(M_R(\bullet t_i) \cup \text{con}(\alpha, M_R(\bullet t_i))) \end{aligned}$$

Proof. Function $\text{ConCom}()$ returns a set of connected components of its argument. The molecule $\text{con}(\alpha, M_R(\bullet t_i))$ is a connected component by definition. Subtraction of $\text{con}(\alpha, M_R(\bullet t_i))$ is an operation on one connected component from $M_R(\bullet t_i)$ - it removes one connected component from $M_R(\bullet t_i)$ or a part of it. Hence, the subtracting before or after applying $\text{ConCom}()$ function will not change the result of $\text{ConCom}()$. Analogously for the addition of $\text{con}(\alpha, M_R(\bullet t_i))$.

7 Software

The transformation of RPNs to CPNs described in this paper has been implemented in a java application **RPNEditor**, which may be downloaded at the webpage <https://www.mat.umk.pl/~folco/rpneditor>.

The application provides a graphical interface for displaying and edition of low-level RPNs based on an open source framework *Java Universal Network/Graph Framework* [13]. A net prepared in the application can be stored in an XML file containing all the details related to the net.

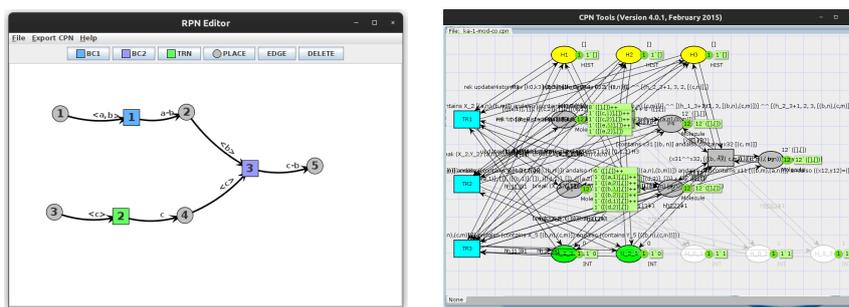


Fig. 7. The RPN created using RPNEditor (left) and its translation to CPN opened in CPN-Tools software (right).

The primary functionality of RPNEditor is the translation from RPNs to CPNs. An RPN prepared in the application (alternatively loaded from a XML file) may be transformed into a CPN and stored in the format required by CPN-Tools software [7].

CPN-Tools is a software with a well-established reputation in the community of Coloured Petri Nets. It contains many useful tools allowing investigation of properties and behaviour of CPNs. Therefore, we decided to produce the output in the CPN-Tools readable format. On the other hand, low-level RPNs are not widely known. Moreover, construction of a correct RPN requires a few essential conditions to be satisfied. Hence, providing an RPN editor equipped with the net correctness validation was necessary.

The translation from RPNs to CPNs is possible for all three reversing semantics, the desired one has to be chosen from the menu. The resulting CPN behaviour corresponds to the one of the original RPN according to the semantics chosen and may be directly simulated using CPN-Tools. During the transformation, several new structural elements are introduced: transition and connection history places, reversing transitions, etc. Moreover, the simulation of the original RPN behaviour requires the usage of a number of variables representing bases, bonds, molecules, etc. Therefore, it is necessary to define specialised colours. They include, among other, tuples stored at transition history places and containing the log of transition execution, representation of bases and bonds, lists of bases and bonds comprising a molecule, etc.

In the case of a transition with no dependence (i.e. it does not process effects of other transitions and its effect is not processed by any other transition) a transition history place would remain empty. As a consequence, the reversing of such a transition would not be possible. Avoiding such problems was one of the reasons for introducing the initial transition (denoted as t_0). It is not a part of the original net and may be executed only once just before the net computation starts. Its sole purpose is to produce the initial marking. After that neither its execution nor reversal are possible. Since t_0 produces the initial content of all the places, it is dependent with all other transitions, namely $t_0 \in dpc(t)$ for each transition $t \in T_R$. Therefore, as a result of the translation, the connection history places for each transition paired with t_0 are produced. To avoid creation of nodes and edges, which are not used during the actual computation, and losing the readability we decided to make t_0 virtual and not to put it directly in the resulting net. Instead, the initial marking of the CPN is

set to the one just after execution of t_0 . However, the traces of its initial execution may be observed at the history places of all other transitions.

The arrangement and location of all elements of the input RPN may be freely edited by the user resulting in the unpredictability of the processed reversing Petri net shape. Therefore, to facilitate the readability of the resulting CPN all the newly introduced objects are located around the RPN area. The reversing transitions are placed to the right, transition history places above, and connection history places below the original net. Moreover, we decided to distinguish all types of objects with different colours: the original net structure - grey, transition history places - yellow, connection history places - green, reversing transitions - blue, and connection history places related to the initial transition t_0 - light grey.

The newly-created RPN objects (transitions and places) are automatically assigned with unique identifiers (starting from 1). Since one is allowed to create, move, and delete them in any order, the resulting RPN may not preserve the topological order of transition identifiers. Therefore, before the transformation, all transitions are renamed according to their topological order. Moreover, the translation is not possible before RPN is completed (i.e. all transitions have the required number of input and output places, and all the edges have correct labels).

8 Conclusions

In this paper we have presented and formally proved the correctness of a translation from reversing Petri nets with multiple tokens to coloured Petri nets. Building upon previous work, we have enhanced the translation by lifting restrictions on token uniqueness and refining the transformation process. The resulting transformation accommodates all three semantics of backtracking, causal-order, and out-of-causal-order reversibility, providing a unified approach. Additionally, an automated transformation algorithm for RPNs to CPNs has been introduced along with the accompanied modelling tool providing the potential for analysing RPN models using CPN tools.

Our primary objectives for future work include optimizing the transformation process from reversing Petri nets to coloured Petri nets, and extending it to capture cycles as well as towards controlled reversibility [23,14]. Furthermore, we aim to assess the scalability of the approach and the developed tool for handling complex systems, whereby we foresee the application of the framework to model and analyse case studies arising within computer science and beyond.

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