

Do We Really Need to Use Constraint Violation in Constrained Evolutionary Multi-Objective Optimization? *

Shuang Li¹, Ke Li² and Wei Li¹

¹Control and Simulation Center, Harbin Institute of Technology, China

²Department of Computer Science, University of Exeter, EX4 4QF, Exeter, UK

*Email: k.li@exeter.ac.uk

Abstract: Constraint violation has been a building block to design evolutionary multi-objective optimization algorithms for solving constrained multi-objective optimization problems. However, it is not uncommon that the constraint violation is hardly approachable in real-world black-box optimization scenarios. It is unclear that whether the existing constrained evolutionary multi-objective optimization algorithms, whose environmental selection mechanism are built upon the constraint violation, can still work or not when the formulations of the constraint functions are unknown. Bearing this consideration in mind, this paper picks up four widely used constrained evolutionary multi-objective optimization algorithms as the baseline and develop the corresponding variants that replace the constraint violation by a crisp value. From our experiments on both synthetic and real-world benchmark test problems, we find that the performance of the selected algorithms have not been significantly influenced when the constraint violation is not used to guide the environmental selection.

Keywords: Constrained multi-objective optimization Constraint handling techniques Evolutionary multi-objective optimization.

1 Introduction

Real-world optimization problems in science [1], engineering [2] and economics [3] usually involve multiple conflicting objectives under a number of equality and inequality constraints, a.k.a. constrained multi-objective optimization problems (CMOPs). In this paper, we consider the CMOP defined as follows:

$$\begin{aligned} & \text{minimize} && \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}))^T \\ & \text{subject to} && \mathbf{g}(\mathbf{x}) \leq 0 \\ & && \mathbf{h}(\mathbf{x}) = 0 \\ & && \mathbf{x} = (x_1, \dots, x_n)^T \in \Omega \end{aligned} \quad (1)$$

where $\Omega = [x_i^L, x_i^U]_{i=1}^n \subseteq \mathbb{R}^n$ defines the search (or decision variable) space and \mathbf{x} is an n -dimensional vector therein. $\mathbf{F} : \Omega \rightarrow \mathbb{R}^m$ constitutes m conflicting objective functions, and \mathbb{R}^m is the objective space. $\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}), \dots, g_p(\mathbf{x}))^T$ and $\mathbf{h}(\mathbf{x}) = (h_1(\mathbf{x}), \dots, h_q(\mathbf{x}))^T$ are vectors of inequality and equality constraints respectively. Given a CMOP, the degree of constraint violation of a solution \mathbf{x} at the j -th constraint is calculated as:

$$c_i(\mathbf{x}) = \begin{cases} \langle g_j(\mathbf{x})/a_j - 1 \rangle, & j = 1, \dots, q, i = j, \\ \langle |h_k(\mathbf{x})/b_k - 1| - \epsilon \rangle, & k = 1, \dots, p, i = k + q, \end{cases} \quad (2)$$

where ϵ is a small tolerance term (e.g., $\epsilon = 10^{-6}$) that relaxes the equality constraints to the inequality constraints. a_j and b_k where $j \in \{1, \dots, q\}$ and $k \in \{1, \dots, p\}$ are normalization factors of the

*This manuscript is submitted for potential publication. Reviewers can use this version in peer review.

corresponding constraints. $\langle \alpha \rangle$ returns 0 if $\alpha \geq 0$ otherwise it returns the negative of α . Given a CMOP, the constraint violation (CV) value of a solution \mathbf{x} is calculated as:

$$CV(\mathbf{x}) = \sum_{i=1}^{\ell} c_i(\mathbf{x}), \quad (3)$$

where $\ell = p + q$. \mathbf{x} is feasible in case $CV(\mathbf{x}) = 0$; otherwise \mathbf{x} is infeasible. Given two feasible solutions \mathbf{x}^1 and \mathbf{x}^2 , \mathbf{x}^1 is said to *Pareto dominate* \mathbf{x}^2 (denoted as $\mathbf{x}^1 \preceq \mathbf{x}^2$) if and only if $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$, $\forall i \in \{1, \dots, m\}$ and $\exists j \in \{1, \dots, m\}$ such that $f_j(\mathbf{x}^1) < f_j(\mathbf{x}^2)$. A solution $\mathbf{x}^* \in \Omega$ is *Pareto-optimal* with respect to (1) if $\nexists \mathbf{x} \in \Omega$ such that $\mathbf{x} \preceq \mathbf{x}^*$. The set of all Pareto-optimal solutions is called the *Pareto-optimal set* (PS). Accordingly, $PF = \{\mathbf{F}(\mathbf{x}) | \mathbf{x} \in PS\}$ is called the *Pareto-optimal front* (PF).

Due to the population-based property, evolutionary algorithms (EAs) have been widely recognized as an effective approach for multi-objective optimization. Over the past three decades, much effort have been devoted to developing evolutionary multi-objective optimization (EMO) algorithms, e.g. elitist non-dominated sorting genetic algorithm (NSGA-II) [4], indicator-based EA (IBEA) [5] and multi-objective EA based on decomposition (MOEA/D) [6]. However, they cannot be directly applied to CMOPs without the assistance of a constraint handling technique (CHT), which can be seen as a selection mechanism to deal with constraints. In the 90s, some early endeavors to the development of EAs for solving CMOPs (e.g., [7] and [8]) are simply driven by a prioritization of the search for feasible solutions over ‘optimal’ one. However, such methods are notorious for the loss of selection pressure in case the population is filled with infeasible solutions.

After the development of the constrained dominance relation [4], most, if not all, prevalent CHTs in the EMO community directly or indirectly depend on the CV defined in equation (3). Specifically, a solution \mathbf{x}^1 is said to *constraint-dominate* \mathbf{x}^2 , if: 1) \mathbf{x}^1 is feasible while \mathbf{x}^2 is not; 2) both of them are infeasible and $CV(\mathbf{x}^1) < CV(\mathbf{x}^2)$; or 3) both of them are feasible and $\mathbf{x}^1 \prec \mathbf{x}^2$. By replacing the Pareto dominance relation with this constrained dominance relation, the state-of-the-art NSGA-II and NSGA-III [9] can be readily used to tackle CMOPs. Borrowing this idea, several MOEA/D variants (e.g., [9–12]) use the CV as an alternative criterion in the subproblem update procedure. Moreover, the constrained dominance relation is augmented with terms such as the number of violated constraints [13], ϵ -constraint [14–16] and angle between each other [17] to provide an additional selection pressure to infeasible solutions whose CV values have a marginal difference.

In addition to the above feasibility-driven CHTs, the second category aims at balancing the trade-off between convergence and feasibility during the search process. For example, Jiménez et al. [18] proposed a min-max formulation that drives feasible and infeasible solutions evolve towards optimality and feasibility, respectively. In [19], a Ray-Tai-Seow algorithm was proposed to simultaneously take the objective values, the CV along with the combination of them into consideration to compare and rank non-dominated solutions. Based on the similar rigour, some modified ranking mechanisms (e.g., [20–22]) were developed by leveraging the information from both the objective and constraint spaces. Instead of prioritizing feasible solutions, some researchers (e.g., [23–25]) proposed to exploit information from infeasible solutions in case they can provide additional diversity to the current evolutionary population.

As a step further, the third category seeks to strike a balance among convergence, diversity and feasibility simultaneously. As a pioneer along this line, Li et al. proposed a two-archive EA that maintains two co-evolving and complementary populations to solve CMOPs [26]. Specifically, one archive, denoted as the convergence-oriented archive (CA), pushes the population towards the PF; while the other one, denoted as the diversity-oriented archive, provides additional diversity. To complement the behavior of the CA, the DA explores the areas under-exploited by the CA including the infeasible region(s). In addition, to take advantage of the complementary effects of both CA and DA, a restricted mating selection mechanism was proposed to adaptively choose appropriate mating parents according to the evolution status of the CA and DA respectively. After [26], there have been a spike of efforts on the development of multi-population strategies (e.g., [27–32]) to leverage some complementary effects of both feasible and infeasible solutions simultaneously for solving CMOPs.

Instead of the environmental selection, the last category tries to repair the infeasible solutions in order to drives them towards the feasible region(s). For example, a so-called Pareto descent repair

operator [33] was proposed to explore possible feasible solutions along the gradient information around infeasible solutions in the constraint space. In [34], a feasible-guided strategy was developed to guide infeasible solutions towards the feasible region along the ‘feasible direction’, i.e., a vector starting from an infeasible solution and ending up with its nearest feasible solution. In [35], a simulated annealing was applied to accelerate the progress of movements from infeasible solutions toward feasible ones.

Remark 1. *As discussed at the outset of this subsection, all these prevalent CHTs require the access of the CV. This applies to the last category, since it needs to access the gradient information of the CV. The implicit assumption behind the prevalent CHTs is the access of the closed form of the constraint function(s). However, this is not practical in the real world as problems are usually as a black box. In other words, we can only know that whether a solution is feasible or not.*

Bearing this consideration in mind, we come up with the overarching research question of this paper: *do the prevalent CHTs in the EMO literature still work when we do not have an access to the CV?*

The rest of this paper is organized as follows. The experimental settings are summarized in Section 2 and the results are presented and analyzed in Section 3. Finally, Section 4 concludes this paper and sheds some lights on future directions.

2 Experimental Settings

In this section, we introduce the experimental settings of our empirical study including the benchmark test problems, the peer algorithms, the performance metrics and statistical tests.

2.1 Benchmark Test Problems

In our empirical study, we pick up 45 benchmark test problems widely studied in the literature to constitute our benchmark suite. More specifically, it consists of C1-DTLZ1, C1-DTLZ3, C2-DTLZ2 and C3-DTLZ4 from the C-DTLZ benchmark suite [9]; DC1-DTLZ1, DC1-DTLZ3, DC2-DTLZ1, DC2-DTLZ3, DC3-DTLZ1, DC3-DTLZ3 chosen from the DC-DTLZ benchmark suite [26]; and other 35 problems picked up from the real-world constrained multi-objective problems (RWCMPs) benchmark suite [36]. In particular, the RWCMPs are derived from the mechanical design problems (denoted as RCM1 to RCM21), the chemical engineering problems (denoted as RCM22 to RCM24), the process design and synthesis problems (denoted as RCM25 to RCM29), and the power electronics problems (denoted as RCM30 to RCM35), respectively. All these benchmark test problems are scalable to any number of objectives while we consider $m \in \{2, 3, 5, 10\}$ for C-DTLZ, DC-DTLZ problems and $m \in \{2, 3, 4, 5\}$ for RWCMPs in our experiments. The mathematical definitions of these benchmark test problems along with settings of the number of variables and the number of constraints can be found in the supplemental document of this paper¹.

2.2 Peer Algorithms and Parameter Settings

In our empirical study, we choose to investigate the performance of four widely studied EMO algorithms for CMOPs, including C-NSGA-II [4], C-NSGA-III [9], C-MOEA/D [9], and C-TAEA [26]. To address our overarching research question stated at the end of Section 1, we design a variant for each of these peer algorithms (dubbed *vC-NSGA-II*, *vC-NSGA-III*, *vC-MOEA/D*, and *vC-TAEA*, respectively) by replacing the CV with a crisp value. Specifically, if a solution \mathbf{x} is feasible, we have $CV(\mathbf{x}) = 1$; otherwise we set $CV(\mathbf{x}) = -1$. The settings of population size and the maximum number of function evaluations are detailed in the supplemental document of this paper [37–74].

2.3 Performance Metrics and Statistical Tests

This paper applies the widely used inverted generational distance (IGD) [75], IGD⁺ [76], and hypervolume (HV) [77] as the performance metrics to evaluate the performance of different peer algorithms.

¹The supplemental document can be downloaded from here.

Table 1: Summary of the Wilcoxon signed-rank test results of four selected EMO algorithms against their corresponding variants on IGD, IGD⁺, and HV.

Problems	Metrics	C-NSGA-II	C-NSGA-III	C-MOEA/D	C-TAEA
		+ / - / =	+ / - / =	+ / - / =	+ / - / =
C-DTLZ and DC-DTLZ	IGD	1/7/32	1/13/26	3/1/36	2/0/38
	IGD ⁺	1/7/32	0/15/25	3/7/36	1/0/39
	HV	0/8/32	0/15/25	4/1/35	0/0/40
RWCMOPs	HV	0/8/27	0/11/24	4/7/24	1/4/30

+, -, and = denote the performance of the selected algorithm is significantly better, worse, and equivalent to the corresponding variant, respectively.

In our empirical study, each experiment is independently repeated 31 times with a different random seed. To have a statistical interpretation of the significance of comparison results, we use the following two statistical measures in our empirical study.

- Wilcoxon signed-rank test [78]: This is a non-parametric statistical test that makes no assumption about the underlying distribution of the data and has been recommended in many empirical studies in the EA community [79]. In particular, the significance level is set to $p = 0.05$ in our experiments.
- A_{12} effect size [80]: To ensure the resulted differences are not generated from a trivial effect, we apply A_{12} as the effect size measure to evaluate the probability that one algorithm is better than another. Specifically, given a pair of peer algorithms, $A_{12} = 0.5$ means they are *equal*. $A_{12} > 0.5$ denotes that one is better for more than 50% of the times. $0.56 \leq A_{12} < 0.64$ indicates a *small* effect size while $0.64 \leq A_{12} < 0.71$ and $A_{12} \geq 0.71$ mean a *medium* and a *large* effect size, respectively.

3 Experimental Results

The PFs and the feasible regions of the synthetic problems are relatively simple whereas those of RWCMOPs are complex. In this section, we plan to separate the discussion on the synthetic problems (i.e., C-DTLZ and DC-DTLZ) from the RWCMOPs in view of their distinctive characteristics.

3.1 Performance Analysis on Synthetic Benchmark Test Problems

Due to page limit, we leave the complete comparison results of IGD, IGD⁺, and HV in Table 3 to Table 8 of supplemental document. Instead, we summarize the Wilcoxon signed-rank test results in the Table 1. From this table, it is clear to see that most comparison results (at least 62.5% and it even goes to 100% for the HV comparisons between C-TAEA and *v*C-TAEA) do not have any statistical significance. In other words, replacing the CV with a crisp value does not significantly influence the performance on C-DTLZ and DC-DTLZ problems. In addition to the pairwise comparisons, we apply the A_{12} effect size to have a better understanding of the performance difference between the selected EMO algorithm and its corresponding variant. From the collected comparison results ($50 \times 2 = 100$ in total) shown in Fig. 1, we can see that most of the comparison results are classified as *equal* (ranging from 38% to 58%). As reflected in Table 1, it is surprising to see that the corresponding variants (i.e., without using the CV to guide the evolutionary search process) have achieved better performance in many cases. In particular, up to 32% comparison results are classified to be *large*. In the following paragraphs, we plan to analyse some remarkable findings collected from the results.

- Let us first look into the performance of C-NSGA-II and C-NSGA-III w.r.t. their variants *v*C-NSGA-II and *v*C-NSGA-III. As shown in Table 1, the performance of C-NSGA-II and C-NSGA-III have been deteriorated (ranging from 17.5% to 37.5%) when replacing the CV with a crisp value in their corresponding CHTs, especially on C1-DTLZ1 and C2-DTLZ2.

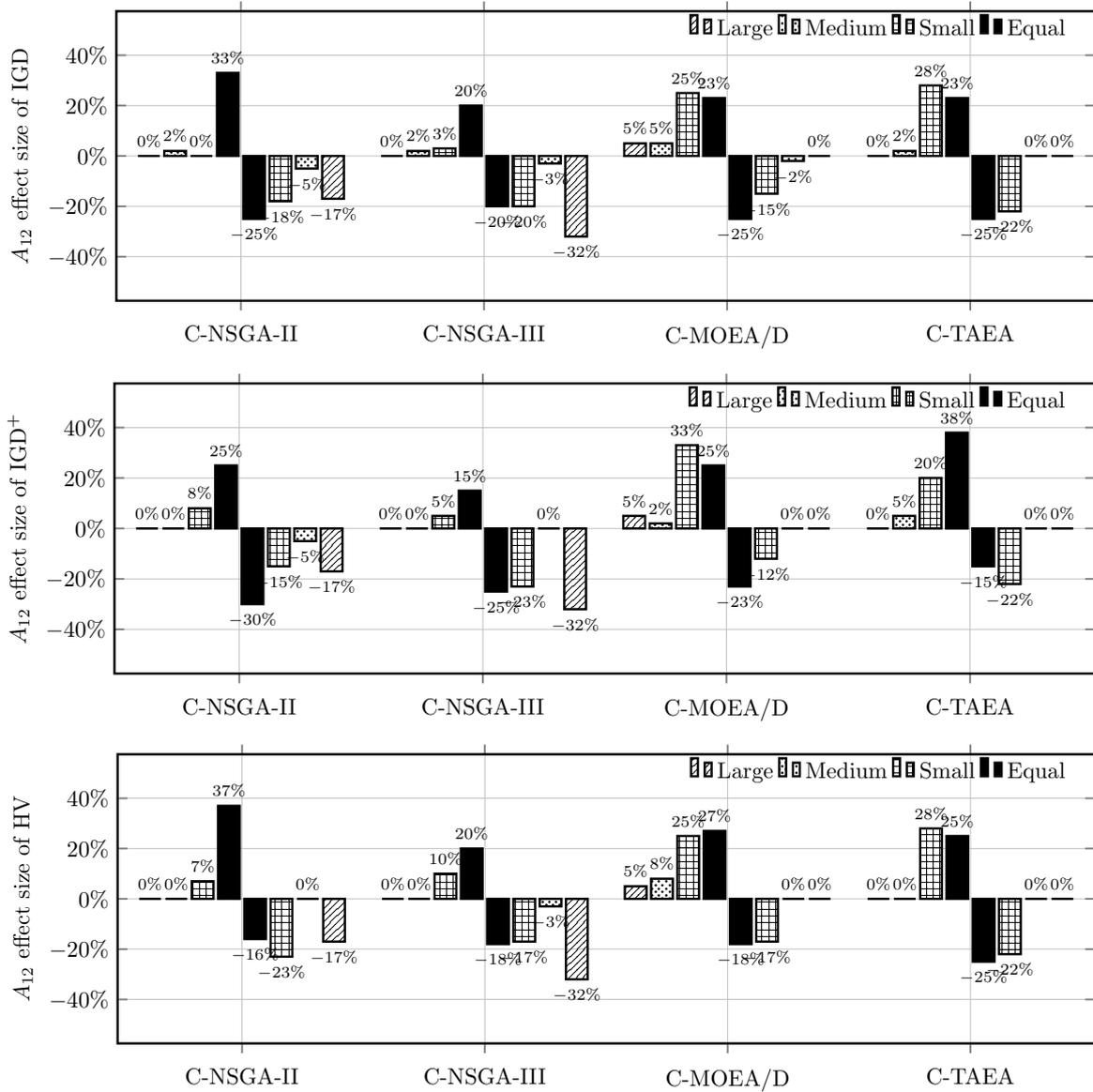


Figure 1: Percentage of the large, medium, small, and equal A12 effect size of metrics for C-DTLZ and DC-DTLZ problems. + means that the variant that replaces the CV with a crisp value can obtain a better result; - means the opposite case.

- As the illustrative example shown in Fig. 2, the feasible region of C1-DTLZ1 is a narrow wedge arrow right above the PF. Without the guidance of the CV, both C-NSGA-II and C-NSGA-III become struggling in the large infeasible region. In particular, there is no sufficient selection pressure to guide the population to move forward.
- C2-DTLZ2 has several disparately distributed feasible regions as the illustrative example shown in Fig. 3. Since the CHTs of both C-NSGA-II and C-NSGA-III do not have a dedicated diversity maintenance mechanism, the evolutionary population can be guided towards some, but not all, local feasible region(s) as the examples shown in Fig. 3(c).
- As for the other test problems, we find that the replacement of CV with a crisp value dose not make a significant impact to the performance of both C-NSGA-II and C-NSGA-III. This can be explained as a large feasible region that makes the Pareto dominance alone can provide sufficient selection pressure towards the PF.
- It is interesting to note that C-MOEA/D uses the same CHT as C-NSGA-II and C-NSGA-III, but its performance does not deteriorate significantly when replacing the CV with a crisp value as shown in Table 1. This can be understood as the baseline MOEA/D that provides a better

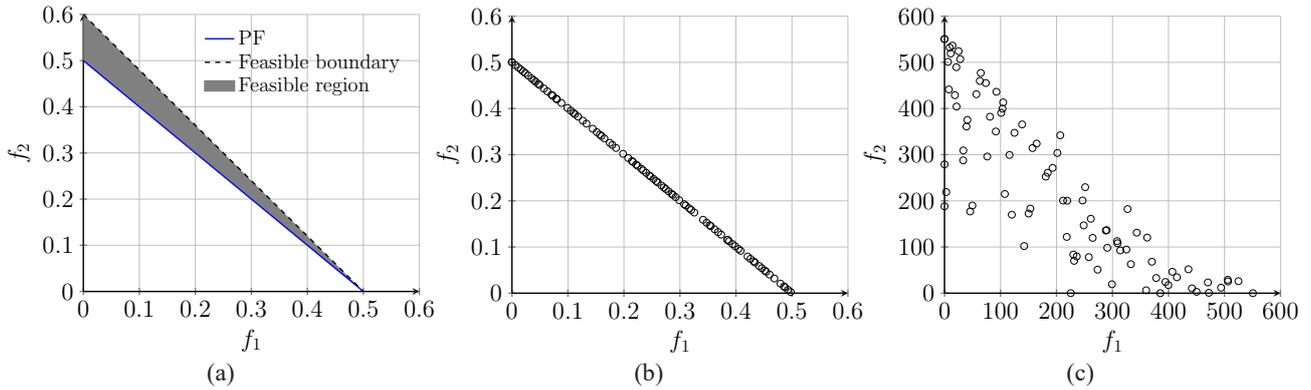


Figure 2: (a) The illustration of the feasible region of C1-DTLZ1; (b) and (c) are the scatter plots of the non-dominated solutions (with the median IGD value) obtained by C-NSGA-II and vC-NSGA-II, respectively.

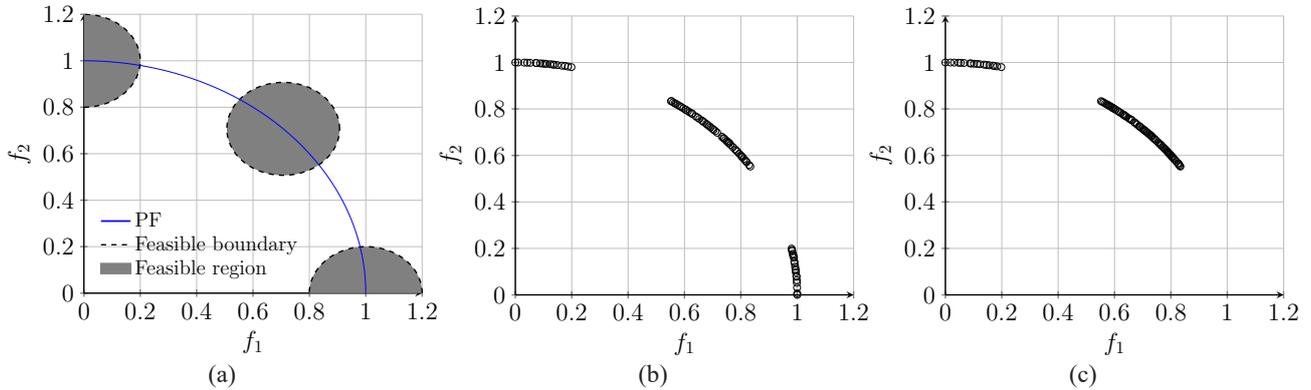


Figure 3: (a) The illustration of the feasible region of C2-DTLZ2; (b) and (c) are the scatter plots of the non-dominated solutions (with the median IGD value) obtained by C-NSGA-II and vC-NSGA-II, respectively.

mechanism to preserve the population diversity during the environmental selection. Thus, the evolutionary population can overcome the infeasible regions towards the PF.

- As for C-TAEA, it is surprising to note that the consideration of the CV does not pose any impact to its performance as evidenced in Table 1 (nearly all comparison results have no statistical significance). This can be explained as the use of the diversity-oriented archive in C-TAEA which does not consider the CV but just relies on the Pareto dominance along to drive the evolutionary population.

3.2 Performance Analysis on Real-World Benchmark Test Problems

Since the PFs of the RWCMOPs are unknown a priori, we only apply the HV as the performance metric in this study. As in Section 3.1, the complete comparison results of HV are given in Tables 9 and 10 of the supplemental document while the Wilcoxon signed-rank test results are summarized in Table 1. From these results, we can see that most of the comparisons (around 68.5% to 85.7%) do not have statistical significance. In other words, there is a marginal difference when replacing the CV with a crisp value. To have a better understanding of the difference, we again apply the A_{12} effect size to complement the results of the Wilcoxon signed-rank test. From the bar charts shown in Fig. 4, it is clear to see that most comparison results (ranging from 46% to 69%) are classified to be equal while only up to 14% comparison results are classified to have a large difference. In the following paragraphs, we plan to elaborate some selected results on problems with an equal and a large effect size, respectively.

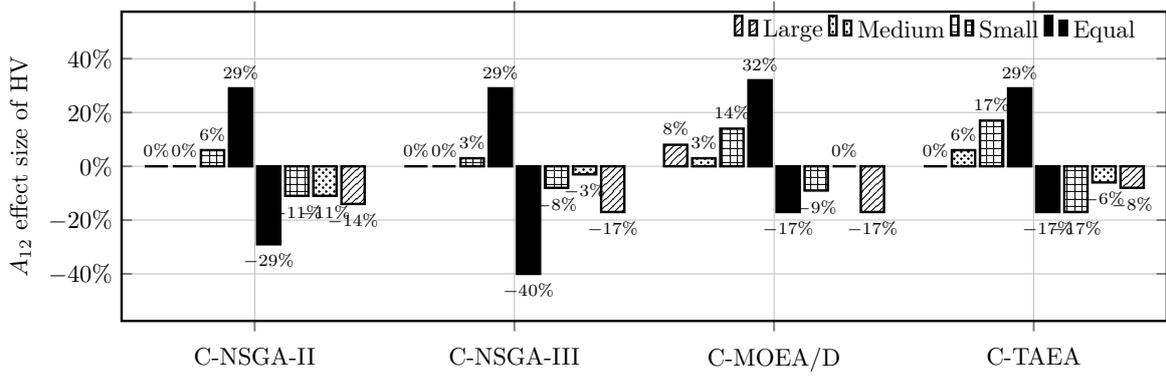


Figure 4: Percentage of the large, medium, small, and equal A_{12} effect size of metrics for RWCMOPs. + means that the variant that replaces the CV with a crisp value can obtain a better result; – means the opposite case.

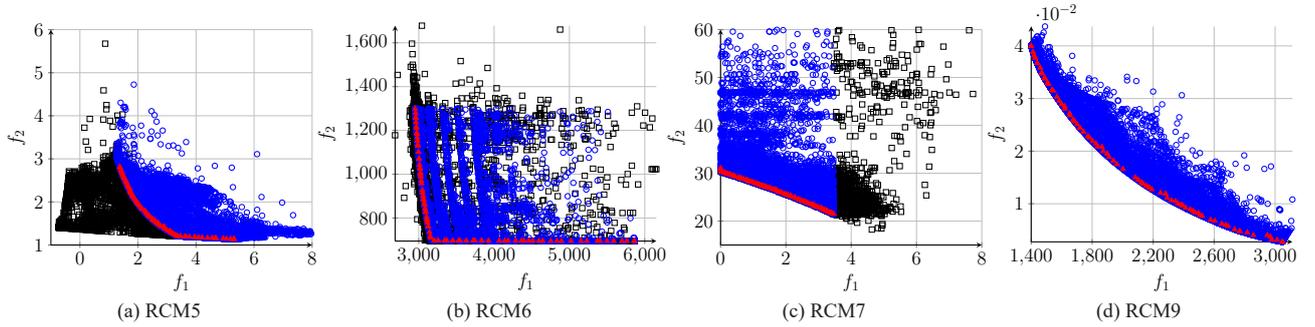


Figure 5: Distribution of feasible solutions (denoted as the blue circle), infeasible solutions (denoted as the black square), and non-dominated solutions (denoted as the red triangle) obtained by C-NSGA-II on RCM5, RCM6, RCM7 and RCM9.

As for the RWCMOPs whose A_{12} effect size comparison results are classified as equal, we consider the following four test problems in our analysis.

- Let us start from the RCM5 problem. As shown in Fig 5(a), the feasible and infeasible regions have almost the same size while the PF is located in the intersection between them. In this case, it is natural that the environmental selection can provide necessary selection pressure without using the CV.
- As for the RCM6 problem shown in Fig. 5(b), the feasible and infeasible regions are intertwined with each other. Therefore, the infeasible region does not really provide an obstacle to the evolutionary population. Accordingly, the CV plays a marginal role for constraint handling.
- Similar to the RCM5 problem, the RCM7 problem has a large and opening feasible region as shown in Fig. 5(c). In addition, the feasible and infeasible regions are hardly overlapped with each. In this case, the evolutionary population can have a large chance to explore in the feasible region without any interference from the infeasible solutions.
- At the end, as shown in Fig. 5(d), it is hardly to treat the RCM9 problem as a CMOP since the feasible region is overtaking the infeasible region. In other words, the feasible region is too large to find an infeasible solution. Accordingly, it is not difficult to understand that the CV becomes useless.

As for the other RWCMOPs, of which the comparison results are classified to be large according to the A_{12} effect size, we pick up two remarkable cases and make some analysis as follows.

- Let us first consider the RCM30 problem. As shown in Fig. 6(a), the feasible region of the RCM30 problem is very narrow and is squeezed towards the PF. Therefore, it is not difficult to

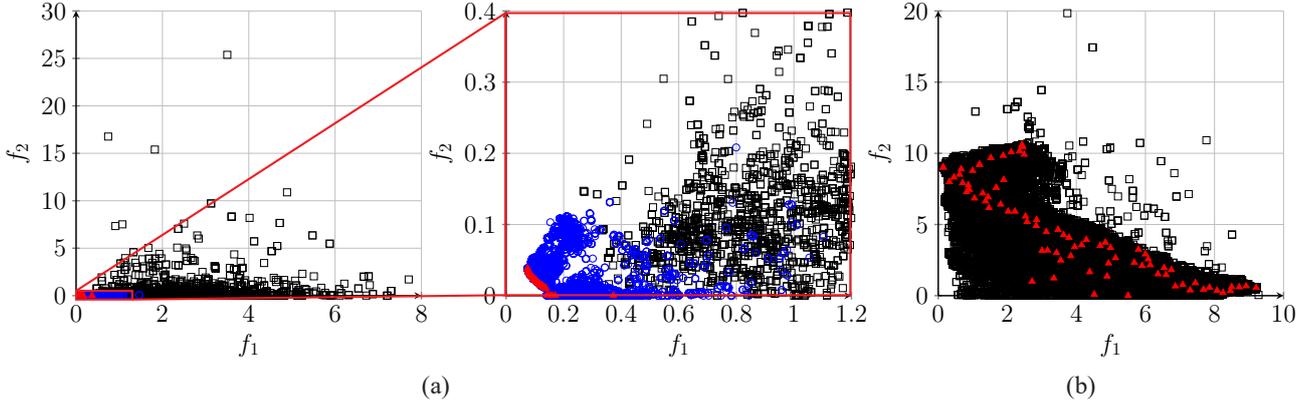


Figure 6: (a) Distribution of feasible solutions (denoted as the blue circle), infeasible solutions (denoted as the black square), and non-dominated solutions (denoted as red triangle) obtained by C-NSGA-II on RCM30. (b) Distribution of infeasible solutions (denoted as the black square) and non-dominated solutions (denoted as red triangle) obtained by vC-NSGA-II on RCM30.

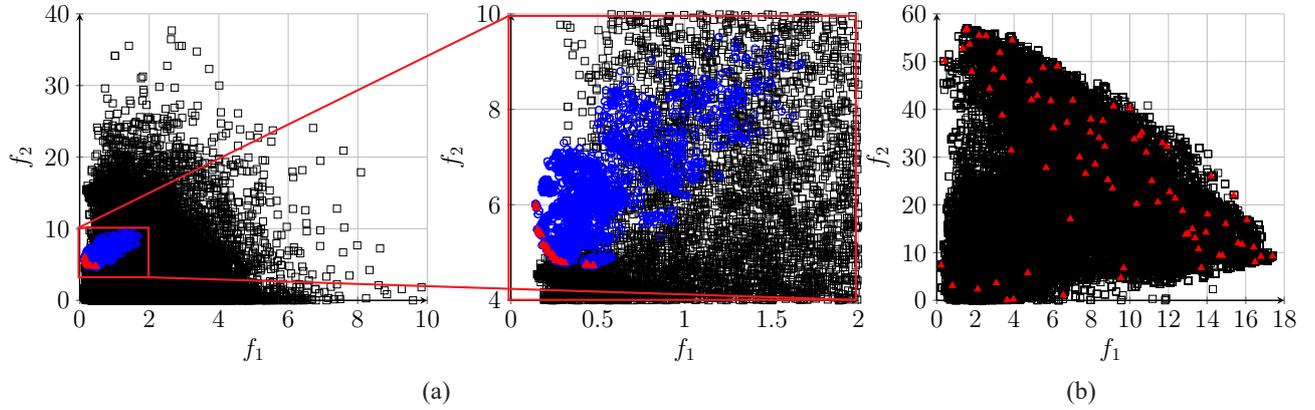


Figure 7: (a) Distribution of feasible solutions (denoted as the blue circle), infeasible solutions (denoted as the black square), and non-dominated solutions (denoted as red triangle) obtained by C-NSGA-II on RCM35. (b) Distribution of infeasible solutions (denoted as the black square) and non-dominated solutions (denoted as red triangle) obtained by vC-NSGA-II on RCM35.

understand that the evolutionary population can hardly be navigated without the guidance of the CV. As shown in Fig. 6(b), the solutions obtained by vC-NSGA-II are far away from the PF.

- As shown in Fig. 7(a), comparing to the RCM30 problem, the size of the feasible region of the RCM35 problem is much wider. However, it is still largely surrounded by the infeasible region. In this case, as shown in Fig. 7(b), without the guidance of the CV, the evolutionary population can not only have sufficient selection pressure to move forward, but also can be misled to the infeasible region that dominates the feasible region.

4 Conclusion

Most, if not all, existing CHT in EMO implicitly assume that the formulation of the constraint function(s) of a CMOP is well defined a priori. Therefore, the CV has been widely used as the building block for designing CHTs to provide an extra selection pressure in the environmental selection. However, this assumption is arguably viable for real-world optimization scenarios of which the problems are treated as a black box. In this case, it is hardly to derive the CV in practice. Bearing this consideration in mind, this paper empirically investigate the impact of replacing the CV with a crisp value in the CHTs of four prevalent EMO algorithms for CMOPs. From our empirical results on both synthetic and real-world benchmark test problems, it is surprising to see that the performance is

not significantly deteriorated when the CV is not used to guide the evolutionary population. One of the potential reasons is that the feasible is large enough to attract the evolutionary population thus leading to a marginal obstacle for an EMO algorithm to overcome the infeasible region. This directly comes up to the requirement of new benchmark test problems with more challenging infeasible regions. In addition, this also inspires new research opportunity to develop new CHT(s) to handle the CMOP with unknown constraint in the near future.

Acknowledgment

This work was supported by UKRI Future Leaders Fellowship (MR/S017062/1), EPSRC (2404317), NSFC (62076056), Royal Society (IES/R2/212077) and Amazon Research Award.

References

- [1] D. L. Thurston and S. Srinivasan, “Constrained optimization for green engineering decision-making,” *Environmental science & technology*, vol. 37, no. 23, pp. 5389–5397, 2003.
- [2] J. Andersson, “Applications of a multi-objective genetic algorithm to engineering design problems,” in *EMO’03: Proc. of the Second International Conference on Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, vol. 2632. Springer, 2003, pp. 737–751.
- [3] A. Ponsich, A. L. Jaimes, and C. A. C. Coello, “A survey on multiobjective evolutionary algorithms for the solution of the portfolio optimization problem and other finance and economics applications,” *IEEE Trans. Evol. Comput.*, vol. 17, no. 3, pp. 321–344, 2013.
- [4] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, 2002.
- [5] E. Zitzler and S. Künzli, “Indicator-based selection in multiobjective search,” in *PPSN’04: Proc. of 8th International Conference on Parallel Problem Solving from Nature*, 2004, pp. 832–842.
- [6] Q. Zhang and H. Li, “MOEA/D: A multiobjective evolutionary algorithm based on decomposition,” *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 712–731, 2007.
- [7] C. M. Fonseca and P. J. Fleming, “Multiobjective optimization and multiple constraint handling with evolutionary algorithms. i. A unified formulation,” *IEEE Trans. Syst. Man Cybern., Part A*, vol. 28, no. 1, pp. 26–37, 1998.
- [8] C. A. C. Coello and A. D. Christiansen, “MOSES: A multiobjective optimization tool for engineering design,” *Eng. Opt.*, vol. 31, no. 3, pp. 337–368, 1999.
- [9] H. Jain and K. Deb, “An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, part II: handling constraints and extending to an adaptive approach,” *IEEE Trans. Evol. Comput.*, vol. 18, no. 4, pp. 602–622, 2014.
- [10] M. A. Jan and Q. Zhang, “MOEA/D for constrained multiobjective optimization: Some preliminary experimental results,” in *UKCI’10: Proc. of the 2010 UK Workshop on Computational Intelligence*, 2010, pp. 1–6.
- [11] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, “A reference vector guided evolutionary algorithm for many-objective optimization,” *IEEE Trans. Evol. Comput.*, vol. 20, no. 5, pp. 773–791, 2016.
- [12] Z. Liu, Y. Wang, and P. Huang, “And: A many-objective evolutionary algorithm with angle-based selection and shift-based density estimation,” *Inf. Sci.*, vol. 509, pp. 400–419, 2020.
- [13] A. Oyama, K. Shimoyama, and K. Fujii, “New constraint-handling method for multi-objective and multi-constraint evolutionary optimization,” *Japan Society of Aeronautical Space Sciences Transactions*, vol. 50, pp. 56–62, 2007.

- [14] T. Takahama and S. Sakai, “Efficient constrained optimization by the ϵ constrained rank-based differential evolution,” in *CEC’12: Proc. of the 2012 IEEE Congress on Evolutionary Computation*, 2012, pp. 1–8.
- [15] S. Z. Martínez and C. A. C. Coello, “A multi-objective evolutionary algorithm based on decomposition for constrained multi-objective optimization,” in *CEC’14: Proc. of the 2014 IEEE Congress on Evolutionary Computation*, 2014, pp. 429–436.
- [16] M. Asafuddoula, T. Ray, and R. A. Sarker, “A decomposition-based evolutionary algorithm for many objective optimization,” *IEEE Trans. Evol. Comput.*, vol. 19, no. 3, pp. 445–460, 2015.
- [17] Z. Fan, W. Li, X. Cai, K. Hu, H. Lin, and H. Li, “Angle-based constrained dominance principle in MOEA/D for constrained multi-objective optimization problems,” in *CEC’16: Proc. of the 2016 IEEE Congress on Evolutionary Computation*. IEEE, 2016, pp. 460–467.
- [18] F. Jiménez, A. F. Gómez-Skarmeta, G. Sánchez, and K. Deb, “An evolutionary algorithm for constrained multi-objective optimization,” in *CEC’02: Proc. of the 2002 IEEE Congress on Evolutionary Computation*, 2002, pp. 1133–1138.
- [19] T. Ray, K. Tai, and K.-C. Seow, “Multiobjective design optimization by an evolutionary algorithm,” *Eng. Opt.*, vol. 33, no. 4, pp. 399–424, 2001.
- [20] N. Young, “Blended ranking to cross infeasible regions in constrained multi-objective problems,” in *CIMCA’05: Proc. of the 2005 International Conference on Computational Intelligence Modeling, Control and Automation*, 2005, pp. 191–196.
- [21] A. Angantyr, J. Andersson, and J.-O. Aidanpaa, “Constrained optimization based on a multiobjective evolutionary algorithm,” in *CEC’03: Proc. of the 2003 IEEE Congress on Evolutionary Computation*, 2003, pp. 1560–1567.
- [22] Y. G. Woldesenbet, G. G. Yen, and B. G. Tessema, “Constraint handling in multiobjective evolutionary optimization,” *IEEE Trans. Evol. Comput.*, vol. 13, no. 3, pp. 514–525, 2009.
- [23] K. Li, K. Deb, Q. Zhang, and S. Kwong, “An evolutionary many-objective optimization algorithm based on dominance and decomposition,” *IEEE Trans. Evol. Comput.*, vol. 19, no. 5, pp. 694–716, 2015.
- [24] C. Peng, H. Liu, and F. Gu, “An evolutionary algorithm with directed weights for constrained multi-objective optimization,” *Appl. Soft Comput.*, vol. 60, pp. 613–622, 2017.
- [25] A. E. Sorkhabi, M. D. Amiri, and A. R. Khanteymooiri, “Duality evolution: an efficient approach to constraint handling in multi-objective particle swarm optimization,” *Soft Comput.*, vol. 21, no. 24, pp. 7251–7267, 2017.
- [26] K. Li, R. Chen, G. Fu, and X. Yao, “Two-archive evolutionary algorithm for constrained multi-objective optimization,” *IEEE Trans. Evol. Comput.*, vol. 23, no. 2, pp. 303–315, 2019.
- [27] X. Shan and K. Li, “An improved two-archive evolutionary algorithm for constrained multi-objective optimization,” in *EMO’21: Proc. of the 11th International Conference on Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, vol. 12654. Springer, 2021, pp. 235–247.
- [28] Y. Tian, T. Zhang, J. Xiao, X. Zhang, and Y. Jin, “A coevolutionary framework for constrained multiobjective optimization problems,” *IEEE Trans. Evol. Comput.*, vol. 25, no. 1, pp. 102–116, 2021.
- [29] Z.-Z. Liu, B.-C. Wang, and K. Tang, “Handling constrained multiobjective optimization problems via bidirectional coevolution,” *IEEE Trans. Cybern.*, pp. 1–14, 2021, accepted for publication.

- [30] J. Wang, Y. Li, Q. Zhang, Z. Zhang, and S. Gao, “Cooperative multiobjective evolutionary algorithm with propulsive population for constrained multiobjective optimization,” *IEEE Trans. Syst. Man Cybern.: Syst.*, pp. 1–16, 2021, accepted for publication.
- [31] F. Ming, W. Gong, L. Wang, and L. Gao, “A constrained many-objective optimization evolutionary algorithm with enhanced mating and environmental selections,” *IEEE Trans. Cybern.*, pp. 1–13, 2022, accepted for publication.
- [32] F. Ming, W. Gong, L. Wang, and C. Lu, “A tri-population based co-evolutionary framework for constrained multi-objective optimization problems,” *Swarm Evol. Comput.*, vol. 70, p. 101055, 2022.
- [33] K. Harada, J. Sakuma, I. Ono, and S. Kobayashi, “Constraint-handling method for multi-objective function optimization: Pareto descent repair operator,” in *EMO’06: Proc. of the 4th International Conference on Evolutionary Multi-Criterion Optimization*, 2006, pp. 156–170.
- [34] L. Jiao, J. Luo, R. Shang, and F. Liu, “A modified objective function method with feasible-guiding strategy to solve constrained multi-objective optimization problems,” *Appl. Soft Comput.*, vol. 14, pp. 363–380, 2014.
- [35] H. K. Singh, T. Ray, and W. Smith, “C-PSA: constrained Pareto simulated annealing for constrained multi-objective optimization,” *Inf. Sci.*, vol. 180, no. 13, pp. 2499–2513, 2010.
- [36] A. Kumar, G. Wu, M. Z. Ali, Q. Luo, R. Mallipeddi, P. N. Suganthan, and S. Das, “A benchmark-suite of real-world constrained multi-objective optimization problems and some baseline results,” *Swarm Evol. Comput.*, vol. 67, p. 100961, 2021.
- [37] K. Li, J. Zheng, C. Zhou, and H. Lv, “An improved differential evolution for multi-objective optimization,” in *CSIE’09: Proc. of 2009 WRI World Congress on Computer Science and Information Engineering*, 2009, pp. 825–830.
- [38] K. Li, J. Zheng, M. Li, C. Zhou, and H. Lv, “A novel algorithm for non-dominated hypervolume-based multiobjective optimization,” in *SMC’09: Proc. of 2009 the IEEE International Conference on Systems, Man and Cybernetics*, 2009, pp. 5220–5226.
- [39] J. Cao, H. Wang, S. Kwong, and K. Li, “Combining interpretable fuzzy rule-based classifiers via multi-objective hierarchical evolutionary algorithm,” in *SMC’11: Proc. of the 2011 IEEE International Conference on Systems, Man and Cybernetics*. IEEE, 2011, pp. 1771–1776.
- [40] K. Li, S. Kwong, R. Wang, J. Cao, and I. J. Rudas, “Multi-objective differential evolution with self-navigation,” in *SMC’12: Proc. of the 2012 IEEE International Conference on Systems, Man, and Cybernetics*, 2012, pp. 508–513.
- [41] K. Li, S. Kwong, J. Cao, M. Li, J. Zheng, and R. Shen, “Achieving balance between proximity and diversity in multi-objective evolutionary algorithm,” *Inf. Sci.*, vol. 182, no. 1, pp. 220–242, 2012.
- [42] K. Li, S. Kwong, R. Wang, K. Tang, and K. Man, “Learning paradigm based on jumping genes: A general framework for enhancing exploration in evolutionary multiobjective optimization,” *Inf. Sci.*, vol. 226, pp. 1–22, 2013.
- [43] K. Li and S. Kwong, “A general framework for evolutionary multiobjective optimization via manifold learning,” *Neurocomputing*, vol. 146, pp. 65–74, 2014.
- [44] J. Cao, S. Kwong, R. Wang, and K. Li, “AN indicator-based selection multi-objective evolutionary algorithm with preference for multi-class ensemble,” in *ICMLC’14: Proc. of the 2014 International Conference on Machine Learning and Cybernetics*, 2014, pp. 147–152.

- [45] K. Li, Á. Fialho, S. Kwong, and Q. Zhang, “Adaptive operator selection with bandits for a multiobjective evolutionary algorithm based on decomposition,” *IEEE Trans. Evolutionary Computation*, vol. 18, no. 1, pp. 114–130, 2014.
- [46] K. Li, Q. Zhang, S. Kwong, M. Li, and R. Wang, “Stable matching-based selection in evolutionary multiobjective optimization,” *IEEE Trans. Evol. Comput.*, vol. 18, no. 6, pp. 909–923, 2014.
- [47] M. Wu, S. Kwong, Q. Zhang, K. Li, R. Wang, and B. Liu, “Two-level stable matching-based selection in MOEA/D,” in *SMC’15: Proc. of the 2015 IEEE International Conference on Systems, Man, and Cybernetics*, 2015, pp. 1720–1725.
- [48] K. Li, S. Kwong, Q. Zhang, and K. Deb, “Interrelationship-based selection for decomposition multiobjective optimization,” *IEEE Trans. Cybernetics*, vol. 45, no. 10, pp. 2076–2088, 2015.
- [49] K. Li, S. Kwong, and K. Deb, “A dual-population paradigm for evolutionary multiobjective optimization,” *Inf. Sci.*, vol. 309, pp. 50–72, 2015.
- [50] K. Li, K. Deb, and Q. Zhang, “Evolutionary multiobjective optimization with hybrid selection principles,” in *CEC’15: Proc. of the 2015 IEEE Congress on Evolutionary Computation*, 2015, pp. 900–907.
- [51] K. Li, K. Deb, Q. Zhang, and Q. Zhang, “Efficient nondomination level update method for steady-state evolutionary multiobjective optimization,” *IEEE Trans. Cybernetics*, vol. 47, no. 9, pp. 2838–2849, 2017.
- [52] M. Wu, S. Kwong, Y. Jia, K. Li, and Q. Zhang, “Adaptive weights generation for decomposition-based multi-objective optimization using gaussian process regression,” in *GECCO’17: Proc. of the 2017 Genetic and Evolutionary Computation Conference*. ACM, 2017, pp. 641–648.
- [53] M. Wu, K. Li, S. Kwong, Y. Zhou, and Q. Zhang, “Matching-based selection with incomplete lists for decomposition multiobjective optimization,” *IEEE Trans. Evolutionary Computation*, vol. 21, no. 4, pp. 554–568, 2017.
- [54] K. Li, K. Deb, and X. Yao, “R-metric: Evaluating the performance of preference-based evolutionary multiobjective optimization using reference points,” *IEEE Trans. Evolutionary Computation*, vol. 22, no. 6, pp. 821–835, 2018.
- [55] R. Chen, K. Li, and X. Yao, “Dynamic multiobjectives optimization with a changing number of objectives,” *IEEE Trans. Evol. Comput.*, vol. 22, no. 1, pp. 157–171, 2018.
- [56] T. Chen, K. Li, R. Bahsoon, and X. Yao, “FEMOSAA: feature-guided and knee-driven multi-objective optimization for self-adaptive software,” *ACM Trans. Softw. Eng. Methodol.*, vol. 27, no. 2, pp. 5:1–5:50, 2018.
- [57] M. Wu, K. Li, S. Kwong, Q. Zhang, and J. Zhang, “Learning to decompose: A paradigm for decomposition-based multiobjective optimization,” *IEEE Trans. Evolutionary Computation*, vol. 23, no. 3, pp. 376–390, 2019.
- [58] K. Li, R. Chen, D. A. Savic, and X. Yao, “Interactive decomposition multiobjective optimization via progressively learned value functions,” *IEEE Trans. Fuzzy Systems*, vol. 27, no. 5, pp. 849–860, 2019.
- [59] K. Li, “Progressive preference learning: Proof-of-principle results in MOEA/D,” in *EMO’19: Proc. of the 10th International Conference Evolutionary Multi-Criterion Optimization*, 2019, pp. 631–643.
- [60] H. Gao, H. Nie, and K. Li, “Visualisation of pareto front approximation: A short survey and empirical comparisons,” in *CEC’19: Proc. of the 2019 IEEE Congress on Evolutionary Computation*, 2019, pp. 1750–1757.

- [61] K. Li, Z. Xiang, and K. C. Tan, “Which surrogate works for empirical performance modelling? A case study with differential evolution,” in *CEC’19: Proc. of the 2019 IEEE Congress on Evolutionary Computation*, 2019, pp. 1988–1995.
- [62] J. Zou, C. Ji, S. Yang, Y. Zhang, J. Zheng, and K. Li, “A knee-point-based evolutionary algorithm using weighted subpopulation for many-objective optimization,” *Swarm and Evolutionary Computation*, vol. 47, pp. 33–43, 2019.
- [63] M. Liu, K. Li, and T. Chen, “DeepSQLi: deep semantic learning for testing SQL injection,” in *ISSTA’20: Proc. of the 29th ACM SIGSOFT International Symposium on Software Testing and Analysis*. ACM, 2020, pp. 286–297.
- [64] K. Li, Z. Xiang, T. Chen, and K. C. Tan, “BiLO-CPDP: Bi-level programming for automated model discovery in cross-project defect prediction,” in *ASE’20: Proc. of the 35th IEEE/ACM International Conference on Automated Software Engineering*. IEEE, 2020, pp. 573–584.
- [65] K. Li, M. Liao, K. Deb, G. Min, and X. Yao, “Does preference always help? A holistic study on preference-based evolutionary multiobjective optimization using reference points,” *IEEE Trans. Evol. Comput.*, vol. 24, no. 6, pp. 1078–1096, 2020.
- [66] M. Wu, K. Li, S. Kwong, and Q. Zhang, “Evolutionary many-objective optimization based on adversarial decomposition,” *IEEE Trans. Cybern.*, vol. 50, no. 2, pp. 753–764, 2020.
- [67] J. Billingsley, K. Li, W. Miao, G. Min, and N. Georgalas, “A formal model for multi-objective optimisation of network function virtualisation placement,” in *EMO’19: Proc. of the 10th International Conference Evolutionary Multi-Criterion Optimization*, 2019, pp. 529–540.
- [68] K. Li, Z. Xiang, T. Chen, S. Wang, and K. C. Tan, “Understanding the automated parameter optimization on transfer learning for cross-project defect prediction: an empirical study,” in *ICSE’20: Proc. of the 42nd International Conference on Software Engineering*. ACM, 2020, pp. 566–577.
- [69] R. Wang, S. Ye, K. Li, and S. Kwong, “Bayesian network based label correlation analysis for multi-label classifier chain,” *Inf. Sci.*, vol. 554, pp. 256–275, 2021.
- [70] J. Billingsley, K. Li, W. Miao, G. Min, and N. Georgalas, “Parallel algorithms for the multiobjective virtual network function placement problem,” in *EMO’21: Proc. of the 11th International Conference on Evolutionary Multi-Criterion Optimization*, ser. Lecture Notes in Computer Science, vol. 12654. Springer, 2021, pp. 708–720.
- [71] L. Yang, X. Hu, and K. Li, “A vector angles-based many-objective particle swarm optimization algorithm using archive,” *Appl. Soft Comput.*, vol. 106, p. 107299, 2021.
- [72] L. Chen, H. Liu, K. C. Tan, and K. Li, “Transfer learning-based parallel evolutionary algorithm framework for bilevel optimization,” *IEEE Trans. Evol. Comput.*, vol. 26, no. 1, pp. 115–129, 2022.
- [73] L. Li, Q. Lin, K. Li, and Z. Ming, “Vertical distance-based clonal selection mechanism for the multiobjective immune algorithm,” *Swarm Evol. Comput.*, vol. 63, p. 100886, 2021.
- [74] G. Lai, M. Liao, and K. Li, “Empirical studies on the role of the decision maker in interactive evolutionary multi-objective optimization,” in *CEC’21: Proc. of the 2021 IEEE Congress on Evolutionary Computation*. IEEE, 2021, pp. 185–192.
- [75] P. A. N. Bosman and D. Thierens, “The balance between proximity and diversity in multiobjective evolutionary algorithms,” *IEEE Trans. Evol. Comput.*, vol. 7, no. 2, pp. 174–188, 2003.

- [76] H. Ishibuchi, H. Masuda, Y. Tanigaki, and Y. Nojima, “Modified distance calculation in generational distance and inverted generational distance,” in *EMO’15: Proc. of the 8th International Conference on Evolutionary Multi-Criterion Optimization*, vol. 9019. Springer, 2015, pp. 110–125.
- [77] E. Zitzler and L. Thiele, “Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach,” *IEEE Trans. Evol. Comput.*, vol. 3, no. 4, pp. 257–271, 1999.
- [78] F. Wilcoxon, “Individual comparisons by ranking methods,” 1945.
- [79] J. Derrac, S. García, D. Molina, and F. Herrera, “A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms,” *Swarm Evol. Comput.*, vol. 1, no. 1, pp. 3–18, 2011.
- [80] A. Vargha and H. D. Delaney, “A critique and improvement of the cl common language effect size statistics of mcgraw and wong,” *J. Educ. Behav. Stat.*, vol. 25, no. 2, pp. 101–132, 2000.