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Maximum traffic flow patterns in interacting autonomous vehicles

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Abstract. We consider the auto-organization of a set of autonomous vehicles following each other on either an infinite or circular road. The behavior of each car is specified by its "speed regulator", a device that decides to increase or decrease the speed of the car as a function of the head-tail distance to its predecessor and the speed of both cars. A collective behavior emerges that corresponds to previously proposed cellular automata traffic models. We further analyze the traffic patterns of the system in the long term, as governed by the speed regulator and we study under which conditions traffic patterns of maximum flow can or cannot be reach. We show the existence of suboptimal flow conditions that require external coordination mechanisms (that we don not consider in this paper) in order to reach the optimal flow achievable with the given density.

1 Introduction

The collective behavior of interacting autonomous vehicles is an interesting question in view of its impact on traffic conditions, such as security or reduced congestion. Our goal is to investigate the capability of autonomous cars, following each other, to reach a state of global maximum flow, by only interacting deterministically with the preceding car.

Our approach follows the work initiated in [14] of modeling traffic using cellular automaton. The road is seen as a collection of cells and cars are moving from one cell to another following some rules that we refer to as the *speed regulator*. Among the questions of interest is the determination of the maximum flow as a function of the number N of cars. This is described as the fundamental diagram, a relation between the traffic flow and the car density $\rho = N/L$ [cars per unit of length], where L is the length of the road section. This fundamental diagram usually shows two distinct dynamics, a first where increasing the density increases the flow and a second one where, due to high density, the cars interact and traffic jams occur.

Classical analysis of traffic models amount to classify the different dynamics and identify the conditions for transition, for instance the existence of on/off-ramp [7, 6, 4], traffic lights [2], lane changing [15], mixed-traffic [13], and combinations [5].

Real Traffic have been subject to empirical studies where sensors located on the road provide measurements of various parameters like speeds and head-tail distances between cars. These works lead to an understanding of the various dynamics depending on the traffic conditions. In particular to the three-phase traffic theory [10, 8, 11]. Roughly, this theory distinguishes the free-flow phase where cars update their speeds independently, the synchronized flow where speeds and head-tail distances decrease and tend to synchronize and, lastly the jammed flow where we observe cars with null speed.

The large amount of data collected from real traffic conditions and the accurate analysis, foster speed regulator designers to devise models that reproduce the observed dynamics [9, 12, 19, 17, 20]. These models introduce new parameters, such as probabilities, that are wisely tuned to reproduced the observed dynamics.

This paper follows a different approach. We consider a simple collision-free, deterministic speed regulator, that mimics the behaviour of an autonomous car. This speed regulator the behaviour of each car. Collectively, it leads to various possible flow patterns that we want to compute rigorously and check whether the resulting traffic flow is maximum. We do not intend to reproduce real traffic patterns like those mentioned in the literature. Rather, our long-term goal is the search for an efficient speed regulator to equip autonomous car whose performance are better that human drivers.

We start here to consider the simplest speed regulator, which could be used by autonomous cars, ensuring that: (i) no collision occurs and, (ii) speed is maximized. We analyze the dynamic of a pool of cars obeying the speed regulator.

In particular, we find a bound for the maximum flow as a function of the density that leads to the fundamental diagram. We identify the traffic patterns of maximum flow - reaching the bound. We show as well that in some non optimal traffic patterns the cars involved cannot increase their speed due to the no-collision constraint. In this situation, some external mechanism, that we do not consider in this paper, is required to allow the cars to switch to a more efficient traffic pattern.

In Section 2 we define the speed regulator. It maximizes the speed v_a of a car a according to the head-tail distance d with a leading car b, ensuring that no-collision occurs. We call viable a configuration where no-collision occurs and shows that for viable configurations the property $d < v_a$ is transient, see Proposition 1. Hence, the long-term traffic patterns show $d \geq v_a$, see Proposition 2. This leads in Section 3 to bound the maximum flow, see Proposition 4. The fundamental diagram and related maximum flow traffic patterns are presented in Section 4.

Interestingly, the condition $d \geq v_a$ plays a similar role as the synchronization distance in the KKW-model [9, 20].

The evaluation of the speed regulator performance requires to understand the traffic patterns generated. We show traffic patterns where the car's speeds are locked to non-optimal values, and the general form of flow-optimal traffic patterns in Proposition 2. We have also found other traffic patterns that are metastable in the sense of [16] not included due to the lack of space.

Unsurprisingly, our speed regulator is similar to several CA traffic models proposed in the literature. For instance in [12] where it is complemented with parameters that are tuned to reproduce some traffic patterns and human behavior. Our use of the regulator is different.

2 The speed regulator

A (one-lane) road section consists in L cells that can be occupied by only one car at a time. Cars are moving from cell to cell. Time is discrete $t \in \mathbb{N}$. We use letters $a, b, \ldots$ to denote cars, $v_a, v_b, \ldots$ to denote the speeds of the cars and $x_a, x_b, \ldots$ to denote the positions of the cars along the section of the road. If the speed of a, is v_a at time t the position update of the car a is $x_a + v_a$ at time $t+1$, we do not write the factor $\Delta_t = 1$, i.e. $x_a + v_a \Delta_t$. Usually, the distance of a cell is 7.5 meters and speeds belong to $\{0, \ldots, v_{max}\}$. For our numerical experiments we use $v_{max} = 5$. Similarly, acceleration is bounded and belongs to $\{-1, 0, 1\}$. Once the position is updated, the velocity is adjusted as explained below. The updated quantities are indicated with a tilde on top (e.g. $\tilde{v}_a$ and $\tilde{d}$). Distances, positions and speeds are all natural numbers.

Because the speed is bounded we use the following definition for the bounded operators (adding a dot on top of the plus and minus signs).

Definition 1. *The bounded addition and subtraction are defined by:*

$$
\begin{aligned}
v \dot{+} 1 &= \min(v + 1, v_{max}), \\
v \dot{-} 1 &= \max(0, v - 1).
\end{aligned}
$$

To denote that the speed regulator updates the speed of a we use the notation $\tilde{v}_a$ and similarly for the updated head-tail distance $\tilde{d}$. This shorten the notation $v_a = v_a(t)$ and $v_a(t+1) = \tilde{v}_a$. Distances, positions and speeds are all natural numbers.

We first focus on the dynamics of two cars a following b and the state of the dynamical system is (v_a, v_b, d), where d is the head-tail distance of cars a and b (i.e. the number of empty cells).

In one step $(t \mapsto t+1)$ two operations are done: (1.) move the cars according to their speed, i.e. $\tilde{x}_a = x_a + v_a$, $\tilde{x}_b = x_b + v_b$ hence $\tilde{d} = d - v_a + v_b$ and, (2.) cars revise their speed following the speed regulator. We consider the succession of configurations of the form

$$
(v_a, v_b, d) \quad \underbrace{\Longrightarrow}_{\text{1. position updates}} \quad (v_a, v_b, \tilde{d} = d - v_a + v_b) \quad \underbrace{\Longrightarrow}_{\text{2. speed updates}} \quad (\tilde{v}_a, \tilde{v}_b, \tilde{d}) \tag{1}
$$

For the simulation where many cars are present the updates are done synchronously.

Definition 2. *We define the function $df : \mathbb{N} \to \mathbb{N}$ as $df(v) = v+(v-1)+\ldots+1 = \frac{v(v+1)}{2}, \ v \geq 0$.*

The function df gives the braking distance for a car moving at speed v and decelerating constantly such that $\Delta v = 1$ each time step. The speed regulator shown in Algorithm 1 sets the speed $\tilde{v}_a$ in such a way that no-collision occurs even if the leading car b brakes constantly to 0.

Algorithm 1 Basic speed regulator.

if $\tilde{d} + df(v_b \dot{-} 1) >= df(v_a + 1)$ **then**
  $\tilde{v}_a = v_a \dot{+} 1$
else if $\tilde{d} + df(v_b \dot{-} 1)) >= df(v_a)$ **then**
  $\tilde{v}_a = v_a$
else
  $\tilde{v}_a = v_a \dot{-} 1$
end if

For a configuration (v_a, v_b, d) we define the viability condition ensuring that the traffic dynamic avoids collisions.

Definition 3. *We say that a configuration (v_a, v_b, d) is viable if*

$$d \geq df(v_a) - df(v_b),$$

or equivalently $\tilde{d} \geq df(v_a \dot{-} 1) - df(v_b \dot{-} 1)$.

It can be shown that the regulator preserves the viability condition hence, the no-collision condition. In the next sections it is implicit that the viability property holds for the initial states and hence at any time. **This ensures that the speed regulator can at any time sets the maximum speed that avoids collision.**

Next, we show that the configurations with $d < v_a$ are transient and the speed regulator eventually leads to $d \geq v_a$.

Proposition 1. *If (v_a, v_b, d) is viable and $d < v_a$ then $v_b \geq v_a$. Moreover, such configuration is transient in the sense that eventually $d \geq v_a$ holds.*

Proof. $v_a + df(v_b) > d + df(v_b) \geq df(v_a)$ results from viability and $d < v_a$. Hence, $df(v_b) > df(v_a) - v_a = df(v_a \dot{-} 1)$, which implies $v_b \geq v_a$. For the second statement, the argument rests on $v_b > v_a$ except when b is decelerating. Each time $v_b > v_a$ the head-tail distance increases and when b decelerates $\tilde{v}_a$ decreases. The first case leads to $d \geq v_a$ because v_a is bounded and the second as well because v_a decreases to 0.

To exemplify Proposition 1 consider a configuration $(3, 3, 2)$, the two cars move at speed 3 and distance 2. This condition is viable (no-collision occurs in the future). Indeed, after updating the position, the trailing car reduces its speed to 2 and then, $d \geq 2 \geq v_a$. In the long term the configuration $(3, 3, 2)$ cannot be observed since it is transient and the property $d \geq v_a$ is preserved by the speed regulator as shown in Proposition 2. Such a configuration could only be observed because of the initial condition.

Proposition 2. $d \geq v_a \Longrightarrow \tilde{d} \geq \tilde{v}_a$.

Proof. If $d \geq v_a$ then $\tilde{d} = d - v_a + v_b \geq v_b$ and the result is true if $v_b \geq \tilde{v}_a$. Let us assume that $v_b < \tilde{v}_a$.
Case 1. We assume that $\tilde{v}_a = v_a + 1$ which occurs if $\tilde{d} \geq df(v_a + 1) - df(v_b \dot{-} 1)$ from which we deduce $\tilde{d} \geq \tilde{v}_a$.
Case 2. We assume that $\tilde{v}_a = v_a$ which occurs if $\tilde{d} \geq df(v_a) - df(v_b \dot{-} 1)$ from which we deduce $\tilde{d} \geq \tilde{v}_a$.
Case 3. We assume that $\tilde{v}_a = v_a - 1$ which occurs if $\tilde{d} \geq df(v_a - 1) - df(v_b \dot{-} 1)$ from which we deduce $\tilde{d} \geq \tilde{v}_a$.

The next proposition shows a set of configurations where the car copies the behavior of the leader car. This behavior is referred to lag synchronization in the literature [1]. Interestingly, such configurations are flow optimal, see Proposition 4.

Proposition 3. *If* $\mid v_a - v_b \mid \leq 1$ *and* $d = v_a$ *then* $\tilde{v}_a = v_b$ *and* $\tilde{d} = \tilde{v}_a$, *in particular* $\mid \tilde{v}_a - \tilde{v}_b \mid \leq 1$.

Proof. $\tilde{d} = d - v_a + v_b$, hence $d = v_a \Longrightarrow \tilde{d} = v_b$, it remains to see that $\tilde{v}_a = v_b$.
$\mid v_a - v_b \mid \leq 1 \Longrightarrow v_a = v_b - 1$ or $v_a = v_b$ or $v_a = v_b + 1$.
If $\mathbf{v_a = v_b - 1}$: (in particular $v_b \neq 0$) $\tilde{d} + df(v_b - 1) = df(v_b) = df(v_a + 1)$. The speed regulator follows line 1 $\tilde{v}_a = v_a + 1 = v_b$.
If $\mathbf{v_a = v_b}$: $\tilde{d} + df(v_b \dot{-} 1) = df(v_b) = df(v_a)$. The speed regulator follows line 3 and $\tilde{v}_a = v_a = v_b$ (condition $\tilde{d} + df(v_b \dot{-} 1) \geq df(v_a + 1)$ is not fulfilled).
If $\mathbf{v_a = v_b + 1}$: $\tilde{d} + df(v_b \dot{-} 1) = df(v_b) = df(v_a - 1)$, the regulator follows line 6 and $\tilde{v}_a = v_a - 1 = vb$ (conditions $\tilde{d} + df(v_b \dot{-} 1) \geq df(v_a + 1)$ and $\tilde{d} + df(v_b \dot{-} 1) \geq df(v_a)$ are not fulfilled).

3 Analysis of the flow

In the previous sections we analyzed the speed regulator by considering configurations of the form (v_a, v_b, d). Here, we consider a flow of cars. Recall that if N cars are on a road section of length L, we define the density $\rho = N/L$ [cars per length units] and the flow $j(N, L) = \rho \bar{v}$ [cars per time unit] where $\bar{v}$ is the average speed of the N cars.

Propositions 1 and 2 show that in the long term configurations satisfy $d \geq v_a$. This leads to the next proposition.

Proposition 4. *In the long term, for N cars are on a road of length L the maximum flow $j(N, L)$ is bounded by*

$$j(N, L) \leq 1 - \frac{N}{L}.$$

In particular, the flow is maximal if all the cars configurations (v_a, v_b, d) belong to the invariant set defined by Proposition 3.

Proof. Because in the long term configurations satisfy $d \geq v$ by Propositions 1 and 2 we have $N + \sum_{i=1}^{N} v_i \leq N + \sum_{i=1}^{N} d_i = L$, from which we get $j(N, L) = \frac{\sum v_i}{L} \leq 1 - \frac{N}{L}$. Proposition 3 defines a flow-optimal invariant set since the condition $v = d$ ($v_a = d$ in the notation of the proposition) is fulfilled.

It is common to express the flow as a function of the density $\rho = N/L$, i.e. $j(N, L) = j(\rho)$.

We represent flows with a typewriter style using `.` to denote an empty cell of the road and a number to indicate that the cell is occupied by a car and the speed of the car. For instance a configuration $(v_a = 3, v_b = 4, d = 3)$ is represented as $\underbrace{\texttt{3}}_{v_a}\underbrace{\texttt{...}}_{d}\underbrace{\texttt{4}}_{v_b}$.

We start with a counterexample of Proposition 4. The traffic pattern `3..3..3.. etc.` is viable (does not lead to collision), of density $\frac{1}{3}$ and of flow $\rho = 1$, hence it seems to contradict the statement of Proposition 4. However, this traffic pattern is transient (notice $d < v_a$) because at the next step cars decrease their speed to 2 and we get the traffic pattern `2..2..2..etc.` which satisfies the bound of Proposition 4 and $d \geq v_a$. In general, our analysis of the flow is in the long term and transient traffic patterns are ignored. It is not stated systematically that only long term traffic patterns are considered although everywhere assumed in the following.

A (long term) traffic pattern of maximal flow is

`.....5.....5.....5.....5.....5.....5..... etc.` (2)

This flow is maximal because the head-tail distance d equals the speed as proved in Proposition 4. Notice, that the flow is regular if measured on a road section of length L with $L = 6N$ where N is the number of cars, i.e. the configuration maximizes the function $j(N, L) = j(N, 6N)$. Hence for density $\rho = 0.16\bar{6}$. The reader can imagine the same pattern repeating infinitely often.

Another example of maximal flow is

`...3...3...3...3...3...3...3...3... etc.` (3)

which is maximal for $L = 4N$ hence for density $\rho = 0.25$.

Maximal flow can be obtained by other regular patterns for different values of N and L. For instance, the flow of `.1.2..3...2..1 etc.` is maximal for $N = 5k$ and $L = 14k$ for any $k > 1$, hence for density $\rho = 5/14 = 0.36$.

In general, for traffic patterns with $v = d$ and N_i cars at speed i satisfy

$$\sum_{i=0}^{v_{max}} iN_i = L, \text{ and } \sum_{i=0}^{v_{max}} N_i = N. \tag{4}$$

Equations (4) are useful to generate flow-equivalent traffic patterns. For instance, if we assume that there exists a configuration with $N_2, N_3, N_4 > 0$ we can find a traffic pattern with one less car in N_2 and N_4 and two more in N_3. Consider a car $a \in N_4$ at speed 4, it can be turned to speed 3. To ensure flow-optimality ($v_a = d$) the next cars are moved one cell back until the speed of the car moved back is 2. The speed of this last car is turned to 3. We obtain a new configuration still satisfying (4). Actually this process can be repeated until N_2 or N_4 is empty. For instance, `.1.2..`**`3`**`...2..`*`1`*`. etc.` can be turned to `.1.2..`**`2`**`..2..`*`2`*`.. etc.` without modifying the flow-optimality of the traffic pattern.

The following traffic patterns are not optimal and of same density than (3):
`.....4.....4.....4.....4..... etc.` and
`.....3.....3.....3.....3..... etc..`

Indeed, in both configurations the density is $\rho = 0.16\bar{6}$ and no car changes his speed due to the regulator speed in Algorithm 1. Indeed, for the traffic pattern `4.....4.....` to increase the speed to 5 the condition $df(5) \leq d + df(3)$ (the trailing car pays attention to the fact that the leading car can break) must be satisfied which is not the case, i.e. $25 \not\leq 5 + 6$. The same argument holds for the second traffic pattern, i.e. $10 \not\leq 5 + 3$.

Another non flow-optimal traffic pattern is given by

`3...4.....5......4.....3...4.....5...... etc.` (5)

This configuration is not optimal since the following one with same density has a higher flow `5.....5.....5.....5.....5..... etc`. Indeed, both configuration have density $\rho = 1/6$ but the flows are $16/24$ and $20/24$ respectively.

In summary, all these examples show that some traffic patterns are permanent but not optimal. This means that the flow of cars can be trapped in a sub-optimal state. Escaping such a state requires coordination. For instance, all the cars must agree to accelerate at the same time. Otherwise, an accelerating car would violate the viability condition.

Jam formation. A classical traffic pattern is the appearance of a jam without bottleneck, see [18] for real traffic experiment and [3] for a recent review. For instance, cars are following a traffic pattern of the form of `3...3...3...etc`. For a reason a car slow down to speed 2 then 1 at some time and for a given period. The trailing cars are following the speed changes and the cars are platooning at speed 1. When the braking car restarts following the speed regulator we observe that the flow increases. **The relevant observation is that no trailing car is slowing below speed 1.** Notice that the constant speed pattern seems hard to restore and we observe a regular pattern of the form of (5).

The point is that, this is not what may be observed in real traffic conditions where some cars are going to stop (at speed zero) [18]. Such an observation is then not compatible with the respect of the viability condition.

4 Fundamental diagrams

In this section we evaluate the average speed of cars $\bar{v}$ and the corresponding flow j for a traffic pattern where at most one car does not satisfy $v = d$. Consider N cars separated by a distance d on a circular road of length L, $d = \max\{x \mid n(x+1) \leq L\}$ except for one car where the distance between it and the next can be greater than d. Therefore, we can set the speed $v = min(d, v_{max})$ of the cars without having a risk of collision and the flow is maximal by proposition 4.
Case $\mathbf{d \geq v_{max}}$. In this part, the density of the road is low enough so that the cars can be at $v = v_{max}$ and the flow $\mathbf{j = v_{max}} \boldsymbol{\rho}$.
As the number of cars increases, we reach a critical density ρ_{crit} where their distance $d = v_{max}$. We denote the critical number of cars $N_{crit} = \frac{L}{(v_{max}+1)}$. If this value is an integer, we can reach the maximal flow $j_{max} = \frac{v_{max}}{(v_{max}+1)}$.
Case $\mathbf{d < v_{max}}$, i.e. $N > N_{crit}$. It is still possible that some cars reach the maximum speed but overall the cars will have their speed $v = d = \lfloor \frac{L}{N} \rfloor - 1$. Let $M + d$ be the remaining distance between the last car and the first car. This distance can be written as: $M = L - N(d+1) \geq 0$.
Subcase $\mathbf{M = 0}$. Every car have exactly a head-tail of d, no more, no less. The cars are in a synchronized state as their speed will never change i.e. $v = \bar{v} = d$ and the flow $\mathbf{j = 1} - \boldsymbol{\rho}$. This is the maximal achievable flow (see Proposition 4). Using $\rho_{crit} = \frac{N_{crit}}{L}$ we can substitute into the equation to get the maximal flow:

$$j_{max} = 1 - \rho_{crit} = 1 - \frac{N_{crit}}{L} = 1 - \frac{L}{L(v_{max}+1)} = 1 - \frac{1}{v_{max}} = \frac{v_{max}}{v_{max}+1}$$

This is the value where the two flow functions intersect (see Figure 1). Below shows such a configuration.

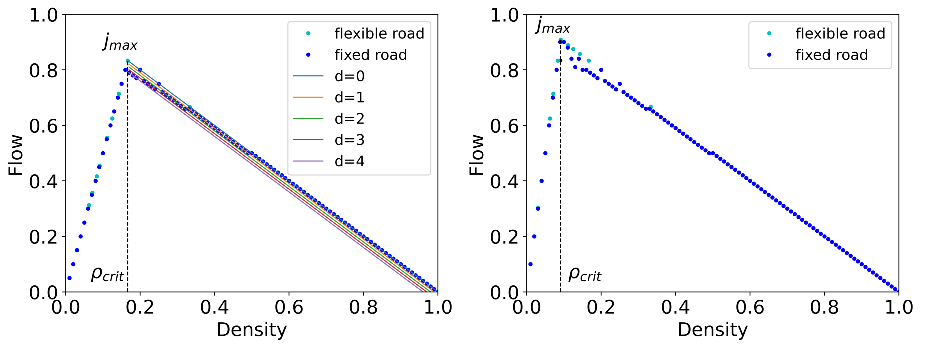
```
4....4....4....4....4....4....4....4....4....
```

Subcase $\mathbf{0 < M < d+1}$. The last car cannot increase its speed, therefore all cars are driving at the same speed $v = \bar{v} = d$. But the last car has some extra space which it will never catch up. The flow is given by $\mathbf{j} = (\lfloor \frac{\mathbf{1}}{\boldsymbol{\rho}} \rfloor - \mathbf{1})\boldsymbol{\rho}$. Below shows a configuration where all cars have a head-tail of 4 except the last one who has 7 but cannot increase its speed.

```
4....4....4....4....4....4....4....4....4.......
```

Subcase $\mathbf{M \geq d+1}$. The last car l has more head-tail and will increase its speed to $v = d+1$. This happens only when $M \geq d+1$, see the speed regulator. Such speed updates are going on for all trailing car successively. Eventually l reaches a head-tail of $2d$ to the next car and reduces its speed to $v = d$. The global flow follows the dynamic of Proposition 3. In the example below, most of the cars are at speed 3 except the ones that take advantage of $M \geq d+1$ to accelerate. In this case, we have $\bar{v} = \frac{L-d}{N} - 1$, since one car has head-tail $2d$ and cannot accelerate, and the corresponding flow is $\mathbf{j = 1 - \frac{d}{L}} - \boldsymbol{\rho}$. Notice that $d < v_{max}$.

```
3...3....4....4....4....4...3...3...3...3......
```



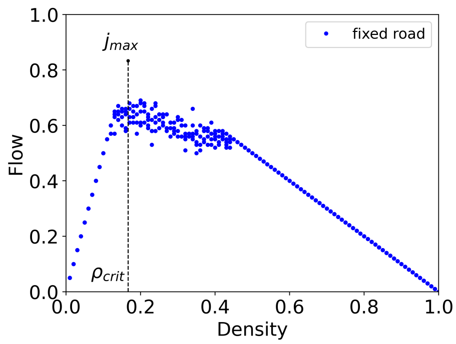
(a) $v_{max} = 5$ (b) $v_{max} = 10$

Fig. 1: Fundamental diagrams of our simulation with $L = 100$ and going through all the possible densities ($\rho = [0.01, 1]$). Starting initial position as described in the text.

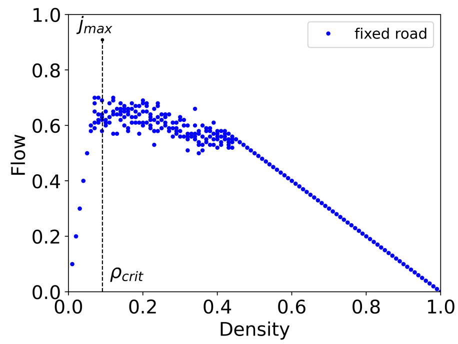
In figure 2a, we see the results of our simulation in dark blue dots. The plain lines are the theoretical solutions of $j = 1 - \frac{d}{L} - \rho$. The dashed vertical line shows the value of the maximal flow j_{max}. As stated, this value is not reachable since $N_{crit} = 16.6\bar{7}$ is not an integer. For a density N/L the car reaches a traffic pattern where $d \in \{0, 1, 2, 3, 4\}$.

In cyan we simulated a flexible road which changes its length $L_f = \lfloor \frac{L}{N} \rfloor \cdot N$ in order to always satisfy the condition $M = 0$. This implies that the flow is always maximal and that we can reach the maximum flow. In figure 2b, we see the same diagram with a higher speed. We observe the same pattern but with a j_{max} bigger and pushed to the left. The what seems to be random points after j_{max} are density who follows the flow $j = (\lfloor \frac{1}{\rho} \rfloor - 1)\rho$. From these observations, we can conclude that there are configurations where the flow is optimal. This happens only when the cars use all the space available, i.e. $M = 0 \implies v_i = d_i$ for all cars i. A configuration not optimal cannot go to an optimal one with our regulator conditions. It would require one car to violate the condition $\tilde{d} + df(v_b \dot{-} 1) >= df(v_a + 1)$ thereby risking a collision. Finally, the state depends on the initial conditions. If we start the cars at $v = d = 0$, like at a traffic light, they will reach the maximum flow since our regulator assure $v_i = d_i$. Nevertheless it only happens if the last car reaches the first car before it starts closing the gap M to zero. Otherwise, $M > 0$ and the flow will not be optimal.

To conclude the presentation of these experiments, we show the fundamental diagram obtained with random starting initial conditions, the positions of the cars are random and the speed is 0. We observe various flow value that are due to more general traffic patterns. These traffic patterns are not optimal and can be explained with an extended version of Proposition 3. In particular, it is always the case that $\tilde{v}_a = v_b$ meaning that the trailing cars copy the speed of their leading car, this correspond to lag synchronization.



(a) $v_{max} = 5$



(b) $v_{max} = 10$

Fig. 2: Fundamental diagrams of our simulation with $L = 100$ and going through all the possible densities ($\rho = [0.01, 1]$). Starting with random initial positions.

5 Conclusions

In this paper we analyzed theoretically and numerically the traffic patterns that are accessible for autonomous vehicles equipped with a simple, local and deterministic speed regulator. We consider a simple traffic situations (cars following each other) and identified the potential strength and weakness of an automatic driving system, such as stop and go waves or non-optimal flow conditions. Of course, more complex situations need to be investigated, such as junctions, merging or lane changing to better understand the emergent collective behavior of autonomous cars.

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