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Ekaterina Komendantskaya (Ed.)

Mathematics of Program Construction

14th International Conference, MPC 2022
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Proceedings

Editor

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Preface

Welcome to the proceedings of the 14th International Conference on Mathematics of Program Construction—MPC 2022!

The International Conference on Mathematics of Program Construction (MPC) aims to promote the development of mathematical principles and techniques that are demonstrably practical and effective in the process of constructing computer programs. Topics of interest range from algorithmics to support for program construction in programming languages and systems. Typical areas include type systems, program analysis and transformation, programming language semantics, security, and program logics. The notion of a ‘program’ is interpreted broadly, ranging from algorithms to hardware.

MPC 2022 welcomes theoretical contributions with relevance to the methods of program construction, reports on applications with solid mathematical basis, and programming pearls that present elegant and instructive examples of the mathematics of program construction.

The MPC series is a bi-annual conference; previous editions took place in Porto, Portugal (2019); Königswinter, Germany (2015); Madrid, Spain (2012); Québec City, Canada (2010); Marseille, France (2008); Kuressaare, Estonia (2006); Stirling, UK (2004); Dagstuhl, Germany (2002); Ponte de Lima, Portugal (2000); Marstrand, Sweden (1998); Kloster Irsee, Germany (1995); Oxford, UK (1992); and Twente, The Netherlands (1989).

The 14th installment of the conference was colocated with the Computational Logic Autumn Summit of 2022 (CLAS 2022), held during September 19–30, 2022, in Tbilisi, Georgia.

MPC 2022 featured invited talks, presentations of original research papers, and a discussion panel.

The program included three invited talks:

- Conor McBride, Strathclyde University, “Picking your way through Pascal’s triangle”.
- Daniela Petrisan, Université Paris Cité, “The semifree monad”.
- Fabio Zanasi, University College London, “Lens Theoretic Foundations for Learning: from Semantics to Verification”.

Research papers include nine original research papers, selected from 14 submissions including 13 complete submissions. Each submission was reviewed by at least three Program Committee members and went through an online discussion period undertaken by the Program Committee before a final decision was made. The selection was based only on the merit of each submission and regardless of scheduling or space constraints.

Research papers are grouped into three topics: (1) Semantics of Program Construction, (2) Programming Methods, and (3) Data Structures and Proofs.

The discussion panel “Mathematics of Program Construction Grand Challenge: Machine Learning” featured invited presenters speaking on

- Semantics for machine learning (Fabio Zanasi, University College London),
- Types and functional programming for machine learning (Nicholas Wu, Imperial College London), and
- Formalising machine learning in theorem provers (Reynold Affeldt, National Institute of Advanced Industrial Science and Technology, Japan).

We thank Besik Dundua and the local organizers of CLAS for hosting MPC 2022, and Springer for the longstanding, successful cooperation with the MPC series. We are grateful to the 19 members of the MPC 2022 Program Committee and the external reviewers for their timely and invaluable work. Many thanks to Matthew Daggitt, Heriot-Watt University, for serving as publicity chair of MPC 2022 and Alasdair Hill for designing the first version of the MPC 2022 webpage.

We are happy to note that the conference paper evaluation was successfully managed with the help of EasyChair.

August 2022

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Abstracts of Invited Talks

Picking Your Way Through Pascal's Triangle

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Abstract. Every place in Pascal's triangle enumerates the paths to that place, zig-zagging downwards from the top, but we can do more than count them. Binomial coefficients make for remarkable dependent types: bit vectors indexed by their length and one-count can document selections from data structures, or the embedding of a term's support into its scope. They become all the more discriminating when you recognize that numerical indices are but the erasures of richer, individuating information. They compose in sequence categorically, and in parallel monoidally. Being made of bits, you can do Boolean logic with them, but with your eyes open as to their meaning. Working with Pascal's Triangle, rendered into types, has changed the way I see and organise data in general, and syntactic data in particular. I shall recount my path to this place, and seek to find the meaning in the turns of my journey.

The Semifree Monad

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Abstract. Weak distributive laws were considered recently in the work of Garner as a means of composing monads for which there is no strong distributive law. For example, the canonical weak distributive law exhibited in Garner’s work between the powerset and the ultrafilter monad can be used to exhibit the Vietoris monad on compact Hausdorff spaces as a weak lifting of the powerset monad. Other weak distributive laws were considered in our work with Alexandre Goy. In particular we obtained a weak distributive law between the powerset monad and the distribution monad, which exhibits the convex powerset monad on barycentric algebras as a weak lifting of the powerset monad.

One essential ingredient in the theory of weak distributive laws is the notion of semialgebras for a monad, that is, algebras for the underlying functor of the monad subject to the associativity axiom alone. In this talk I will discuss the algebraic nature of the semialgebras of a monad. If the underlying category has coproducts then semialgebras for a monad “ M ” are in fact the Eilenberg-Moore algebras for a suitable monad structure on the functor “ $\text{id} + M$ ”, which we call the semifree monad. Then I discuss how weak distributive laws between a monad “ M ” and “ T ” can be seen as strong distributive laws between the semifree monad on “ M ” and “ T ”, subject to an additional condition. This is joint work with Ralph Sarkis.

Lens Theoretic Foundations for Learning: From Semantics to Verification

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Abstract. I will present recent and ongoing work on giving a semantic foundation to training algorithms in machine learning using the categorical formalisms of lenses. Lenses provide a much needed unifying perspective on various classes of such algorithms, as well as offering a different style of specifying and proving properties of training protocols. They also enable the study of machine learning for new classes of models such as Boolean circuits and polynomial circuits. In the last part of the talk I will also discuss how this foundation informs the development of new tools for the formal verification of machine learning algorithms.

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