Semantic preservation for a type directed translation scheme of Featherweight Go

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Abstract. Featherweight Go (FG) is a minimal core calculus that includes essential Go features such as overloaded methods and interface types. The most straightforward semantic description of the dynamic behavior of FG programs is to resolve method calls based on run-time type information. A more efficient approach is to apply a type-directed translation scheme where interface-values are replaced by dictionaries that contain concrete method definitions. Thus, method calls can be resolved by a simple lookup of the method definition in the dictionary. Establishing that the target program obtained via the type-directed translation scheme preserves the semantics of the original FG program is an important task.

To establish this property we employ logical relations that are indexed by types to relate source and target programs. We provide rigorous proofs and give a detailed discussion of the many subtle corners that we have encountered including the need for a step index due to recursive interfaces and method definitions.

1 Introduction

Type directed translation is the process of elaborating a source into some target program by making use of type information available in the source program. Source and target may be from the same language as found in the case of compiler transformations, e.g. consider [16]. The target may be a more elementary language compared to the source, e.g. consider [7,13]. In all cases it is essential to establish that the target program resulting from the translation preserves the meaning of the source program.

Here, we consider a type directed translation method applied to Feather-weight Go. Featherweight Go (FG) is a minimal core calculus that includes the essential features of Go such as method overloading and interfaces. Earlier work by Griesmer and co-authors [6] specifies static typing rules and a run-time method lookup semantics for FG. In our own prior work [26], we give a type directed translation that elaborates methods calls to method lookup in dictionaries that will be passed around in place of interfaces. We could establish correctness of our translation but the result was somewhat limited as semantic preservation only holds under the assumption that source and target programs terminate.

In this work, we significantly extend our earlier semantic preservation result and establish the following properties.

- If the source program terminates so will the target program and the resulting values are equivalent.
- If the source program diverges so will the target program.
- If the source program panics due to a failed run-time type check, the target program will panic as well.

These results require non-trivial extensions and adaptations of our earlier proof method and type-indexed relation to connect source to target values. The upcoming Section 2 gives an overview and highlights the changes from [26] to achieve the above results.

In summary, we make the following contributions.

- We introduce a family of syntactic, step-indexed logical relations to establish semantic preservation for terminating, diverging and panicking source programs (Section 5)
- We provide for rigorous proofs of our results (Section 5 and Appendix).

Section 3 specifies Featherweight Go (FG). The type directed translation of FG including a description of the target language is given in Section 4. Both sections are adopted from our earlier work [26]. Related work is covered in Section 6. Section 7 concludes.

2 Overview

Translation by example. We consider a type directed translation scheme that transforms a FG program into some target program. The target language is an untyped lambda-calculus extended with recursive let-bindings, constructors, and pattern matching. Here, we use Haskell-style notation.

For example, the FG program on the left translates to the target program on the right. For simplicity, we leave out some details (marked by ...).

The FG program on the left contains a struct Int, an interface Eq, and a definition for method eq for receiver type Int (line 3). Our example only contains one definition of the eq method. In general, FG methods can be overloaded on the receiver type. Hence, in the translation on the right, the function name eqInt uniquely identifies the definition of eq for receiver type Int.

Interfaces give a name to a set of method signatures. They are types, and so interface type Eq effectively describes all receiver types implementing the eq

method. Type Int implements this eq method and therefore $Int\{1\}$ is (also) of type Eq. Hence, the method call i.eq(i) type checks.

The FG semantics performs a run-time type lookup to resolve method calls such as i.eq(i). In the translation, an interface is represented as a pair that consists of the value that implements the interface and a dictionary of method definitions for this specific value. For example, i at type Eq translates to the pair (1,eqInt). Assuming that we represent the FG variable i as j in the target, the method call i.eq(i) translates to **case** j **of** (x,eq) -> eq x j, where we only require a pattern match to access the underlying value and the concrete method definition. See Section 4 for details of the type-directed translation.

Semantic preservation via logical relations. To establish that the translation is meaning preserving we need to relate source to target expressions. One challenge is that evaluation steps are not in sync. For example, FG method calls reduce in a single step whereas the translated code first performs a pattern match to obtain the method definition followed by another step to execute the call.

Fig. 1. Improvements compared to earlier work

In our earlier work [26], we introduce a logical relation $e \approx_{APLAS} E \in \llbracket t \rrbracket_k$ to express that source and target expressions behave the same. See top of Figure 1. The relation is indexed by type t and a step index k where we assume that step indices are natural numbers starting with 0. Source expression e and target expression e are in a relation: if the sum of evaluation steps to reduce e to some source value e and e to some target value e is less than e, then the values must

be in a relation.³ Thus, the number of reduction steps in the source and target do not need to be in sync. If we need more than k steps, or if only the source expression yields a value within k steps, or if one of the expressions diverges or panics, the relation $e \approx_{APLAS} E \in [\![t]\!]_k$ holds vacuously and does not give us any information.

In this work, we consider three cases: source terminates, diverges or panics. See Figure 1 that sketches a logical relation for each case. TERMINATE: if the source e terminates within less than k steps, then the target E terminates as well where the number of evaluation steps do not matter. DIVERGE / PANIC: if the source e evaluates to e' in less than k steps and e' diverges/panics, then so does target E. The detour via e' in the last two cases is required to prove that source e and its translation E are related. Taken together, the three cases yield a much stronger characterization of the semantic relation between source and target programs compared to our earlier work.

We use the convention that \equiv relates values whereas \approx relates expressions. The step index in the relation for values, see $v \equiv V \in [\![t]\!]_{k-k'}$, seems unnecessary but is important to guarantee that the definition of logical relations is well-founded (c.f. Section 5)

Ill-founded without step index. Consider the example from above where $Int\{1\}$ of type Eq translates to (1,eqInt). We expect $Int\{1\} \equiv (1,eqInt) \in [Eq]$ to hold; that is, FG value $Int\{1\}$ is equivalent to target value (1,eqInt) when viewed at FG type Eq.

The following reasoning steps try to verify this claim. We deliberately ignore the step index to show that without a step index we run into some issue.

```
(1) \operatorname{Int}\{1\} \equiv (1,\operatorname{eqInt}) \in \llbracket \operatorname{Eq} \rrbracket

if (2) \operatorname{Int}\{1\} \equiv 1 \in \llbracket \operatorname{Int} \rrbracket

and (3) func (x Int) \operatorname{eq}(y \operatorname{Eq}) bool \{e'\} \approx \operatorname{eqInt} \in \llbracket \operatorname{eq}(y \operatorname{Eq}) \operatorname{bool} \rrbracket
```

Statement (1) reduces to statements (2) and (3). (2) states that the underlying values are related at struct type Int. This clearly holds, we omit the details. (3) requires a bit more thought. Function eqInt is part of the dictionary. Hence, eqInt must have the same behavior as method eq defined on receiver type Int. This is the intention of statement (3). Compared to the earlier notation of the example, the function body has been replaced by some expression e'.

How can we establish (3)? It must hold that when applied to related argument values, eqInt and method eq defined on receiver type Int behave the same. Thus, establishing (3) requires (4):

(4)
$$\forall v \equiv V \in \llbracket \mathsf{Int} \rrbracket, v' \equiv V' \in \llbracket \mathsf{Eq} \rrbracket. \langle \mathsf{x} \mapsto v, \mathsf{y} \mapsto v' \rangle \mathsf{e'} \approx \mathsf{eqInt} \ V \ V' \in \llbracket \mathsf{bool} \rrbracket$$

where we write $\langle \mathsf{x} \mapsto v, \mathsf{y} \mapsto v' \rangle \mathsf{e}'$ to denote the substitution of arguments by values in the function body.

³ We defer all formal and missing definitions to Sections 3, 4 and 5. For now, we appeal

There is an issue. One of the arguments is of interface type Eq. Hence, for any values v', V' we require $v' \equiv V' \in \llbracket \mathsf{Eq} \rrbracket$. This leads to a cyclic dependency as a statement of the form $\cdot \equiv \cdot \in \llbracket \mathsf{Eq} \rrbracket$ relies on a statement $\cdot \equiv \cdot \in \llbracket \mathsf{Eq} \rrbracket$. See (1) and (4). This would mean that the definition of our logical relation is ill-founded.

```
Rule schemes parameterized by some binary ordering relation \blacktriangleleft:

\begin{array}{c} \text{METHOD-}\blacktriangleleft\\ \forall k', v, v', V, V'. k' \blacktriangleleft k \wedge v \equiv V \in \llbracket \text{Int} \rrbracket_{k'} \wedge v' \equiv V' \in \llbracket \text{Eq} \rrbracket_{k'}\\ & \Longrightarrow \langle \mathbf{x} \mapsto v, \mathbf{y} \mapsto v' \rangle \mathbf{e}^* \approx \text{eq } V \ V' \in \llbracket \text{bool} \rrbracket_{k'}\\ \hline \textbf{func (x Int) eq(y Eq) bool } \{\mathbf{e}^*\} \approx \mathbf{eq} \in \llbracket \text{eq(y Eq) bool} \rrbracket_{k} \\ \\ \text{IFACE-}\blacktriangleleft\\ & \forall k_1.k_1 \blacktriangleleft k \implies v \equiv V \in \llbracket \text{Int} \rrbracket_{k_1}\\ & \forall k_2.k_2 \blacktriangleleft k \implies \textbf{func (x Int) eq(y Eq) bool } \{\mathbf{e}^*\} \approx V' \in \llbracket \text{eq(y Eq) bool} \rrbracket_{k_2}\\ & v \equiv (V,V') \in \llbracket \text{Eq} \rrbracket_{k} \\ \\ \text{Logical relation properties:} \\ & \text{LR-STEP} \quad e \approx E \in \llbracket t \rrbracket_{k} \wedge e' \longrightarrow^1 e \wedge E' \longrightarrow^* E \implies e' \approx E' \in \llbracket t \rrbracket_{k+1}\\ & \text{LR-MONO} \quad e \approx E \in \llbracket t \rrbracket_{k} \wedge k' \leq k \implies e \approx E \in \llbracket t \rrbracket_{k'} \end{array}
```

Fig. 2. Getting the step index right

Step indices to the rescue. FG interfaces can have cyclic dependencies similar to recursive types, see interface Eq. To guarantee well-foundedness we include a step index in the definition of our logical relations. There is in fact a second reason for a step index. Method definitions may be recursive similar to recursive functions. There is no well-foundedness issue here. But to apply an inductive proof method where semantic preservation for expressions is lifted to method definitions require a step-index.

Step indices in case of recursive types and recursive functions have been studied before [2]. What makes our setting interesting is a subtle interaction between (recursive) interfaces and (recursive) methods. Figure 2 specifies the logical relation rules for methods and interfaces that we have used in the above reasoning steps (1-4). The rule for interfaces relies on the rule for methods and the rule for methods relies on the rule for interfaces (in case of recursive interfaces). For brevity, we omit rules for struct types such as Int.

Rules Method-◀ and IFACE-◀ are parameterized by some binary ordering relation ◀. Why not simply replace ◀ by <, the less than relation? Rule instances Method-< and IFACE-< are clearly well-founded.

Failed proof attempt in case of METHOD-< and IFACE-<. Via our running example we illustrate that the proof of semantic preservation for expressions will not go

through. Recall that

i.eq(i) of type bool translates to case j of
$$(x,eq) \rightarrow eq x j$$

i = Int{1} of type Eq j = (1, eqInt)

For values i and j, we may assume (1) $Int\{1\} \equiv (1, eqInt) \in [Eq]_k$ for some k. To verify that the translation yields related expressions, we must show

(2) i.eq(i)
$$\approx$$
 case j of (x,eq) \rightarrow eq x j \in [bool]_k

From (1), via reverse application of rule IFACE-<, we can derive

(3) **func** (x Int) eq(y Eq) bool
$$\{e'\} \approx \text{eqInt} \in [eq(y Eq) bool]_{k-1}$$

From (3) we get the implication in the premise of rule METHOD-<. The left-hand side of this implication can be satisfied for k-2 < k-1 via LR-MONO from Figure 2 and (1) and the fact that $\text{Int}\{1\} \equiv 1 \in [\![\text{Int}]\!]_k$. Thus, we can derive the right-hand side $\langle x \mapsto i, y \mapsto i \rangle e^* \approx \text{eqInt 1 } j \in [\![\text{bool}]\!]_{k-2}$. From this we get

(5) i.eq(i)
$$\approx$$
 case j of (x,eq) \rightarrow eq x j \in $[bool]_{k-1}$

via property LR-STEP in Figure 2 and the following evaluation steps:

$$\begin{array}{c} i.\, eq(i) \longrightarrow^1 \langle x \mapsto i, y \mapsto i \rangle e \text{'} \\ \text{case j of (x,eq) -> eq x j} \longrightarrow^* eqInt 1 j \end{array}$$

The issue is that from (5) we cannot deduce (2). Property LR-STEP allows us to bump up the step index in case of source reduction steps. Target reductions have no impact. Hence, we end up being one step short.

Fixing the proof by turning < into \le . The solution is to turn one < into \le . Then, we can derive (5) i.eq(i) \approx case j of (x,eq) -> eq x j \in [bool] $_k$ and the proof of semantic equivalence for expressions goes through. It seems that we have a choice between (A) rule instances METHOD- \le and IFACE-< and (B) rule instances METHOD-< and IFACE- \le . We pick choice (B) because under (A) the proof of semantic preservation for (possibly recursive) method definitions will not go through. See the proof of the upcoming Lemma 2 in Section 5.

Comparison to our earlier work [26]. The logical relation introduced in our earlier work [26] is more limited in that semantic preservation is only stated under the assumption that both expressions, source and target programs, terminate. Recall Figure 1 that shows that the logical relation $\approx_{APLAS} \in []$ also takes into account source as well as target steps. Under this stronger assumption it is easier to get the step index right as we can derive the following more general variant of property LR-STEP

$$e \approx_{APLAS} E \in [\![t]\!]_k \land e' \longrightarrow^{k_1} e \land E \longrightarrow^{k_2} E' \implies e' \approx_{APLAS} E' \in [\![t]\!]_{k+k_1+k_2}$$

That is, we can bump up the step index based on target reductions as well. This provides for more flexibility, even with rule instances METHOD-< and IFACE-<, the proofs go through. As highlighted above, more care is needed for the logical relations that we introduce in this work.

Next, we introduce the semantics of Featherweight Go and give the details of the typed-directed translation scheme followed by our semantic preservation results.

3 Featherweight Go

Featherweight Go (FG) [6] is a tiny fragment of Go containing only structs, methods and interfaces. Figure 3 gives its syntax and the dynamic semantics. Overbar notation $\overline{\mathfrak{s}}^n$ denotes the sequence $\mathfrak{s}_1 \dots \mathfrak{s}_n$ for some syntactic construct \mathfrak{s} , where in some places commas separate the sequence items. If irrelevant, we omit the n and simply write $\overline{\mathfrak{s}}$. Using the index variable i under an overbar marks the parts that vary from sequence item to sequence item; for example, $\overline{\mathfrak{s}'\mathfrak{s}_i}^n$ abbreviates $\mathfrak{s}'\mathfrak{s}_1 \dots \mathfrak{s}'\mathfrak{s}_n$ and $\overline{\mathfrak{s}_j}^q$ abbreviates $\mathfrak{s}_{j1} \dots \mathfrak{s}_{jq}$.

A FG program P consists of declarations \overline{D} and a main function. A declaration is either a type declaration for a struct or an interface, or a method declaration. Such a method declaration func $(x \ t_S) \ mM$ {return e} makes method of name m and signature M available for receiver type t_S , where the body e may refer to the receiver as x. Expressions e consist of variables x, method calls $e.m(\overline{e})$, struct literals $t_S\{\overline{e}\}$ with field values \overline{e} , access to a struct's field e.f, and dynamic type assertions e.(t). For convenience, we use disjoint sets of identifiers for structs t_S and interfaces t_I .

FG is a statically typed language. For brevity, we omit a detailed description of the FG typing rules as they appear in [6] and will show up in slightly different form in the type-directed translation in Section 4. However, we state the following conditions that must be satisfied by a FG program:

FG1: Structs must be non-recursive.

FG2: For each struct, field names must be distinct.

FG3: For each interface, method names must be distinct.

FG4: Each method declaration is uniquely identified by the receiver type and method name.

The execution of dynamic type assertions in FG relies on structural subtyping. The relation $\overline{D} \vdash_{\mathsf{FG}} t <: u$ denotes that under declarations \overline{D} type t is a subtype of type u (see Figure 3). A struct t_S is a subtype of an interface t_I if t_S implements all the methods specified by t_I . An interface t_I is a subtype of another interface u_I if the methods specified by t_I are a superset of the methods specified by u_I .

The dynamic semantics of FG is given in the bottom part of Figure 3 as structural operational semantics rules. The relation $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e'$ denotes that expression e reduces to expression e' under the sequence \overline{D} of declarations. Rule FG-CONTEXT makes use of evaluation contexts \mathcal{E} with holes \square to apply

```
Field name
                                                         f
                  Method name
                                                                                      Expression
                                                                                                                       d, e ::=
                                                         m
                  Variable name
                                                                                           Variable
                                                         x, y
                                                                                                                           \boldsymbol{x}
                                                                                          Method call
                  Struct type name
                                                         t_S, u_S
                                                                                                                           e.m(\overline{e})
                                                                                          Struct literal
                                                                                                                           t_S\{\overline{e}\}
                  Interface type name t_I, u_I
                  Type name
                                                         t, u ::= t_S \mid t_I
                                                                                          Select
                                                                                                                           e.f
                  Method signature M := (\overline{x_i \ t_i}) \ t
                                                                                          Type assertion
                                                                                                                           e.(t)
                  Method specification R, S := mM
        Type literal L ::=
                                                                    Declaration D ::=
                                   struct \{\overline{f}\ t\}
                                                                        Type
                                                                                              \mathbf{type}\ t\ L
            Struct
            Interface
                                                                        Method
                                                                                              func (x \ t_S) \ mM \ \{ return \ e \}
                                   interface \{\overline{S}\}
                                           Program P := \overline{D} func main(){\bot = e}
 |\overline{D} \vdash_{\mathsf{FG}} t <: u|
                                                                                                                                          Subtyping
                        METHODS-STRUCT
                        \mathsf{methods}(\overline{D}, t_S) = \{ mM \mid \mathsf{func}\ (x\ t_S)\ mM\ \{\mathsf{return}\ e\} \in \overline{D} \}
                                                                                                \frac{\text{SUB-IFACE}}{\text{methods}(\overline{D},t)} \supseteq \text{methods}(\overline{D},u_I)
METHODS-IFACE
                                                       {\tt SUB\text{-}STRUCT\text{-}REFL}
type t_I interface \{\overline{S}\} \in \overline{D}
   \mathsf{methods}(\overline{D},t_I) = \{\overline{S}\}
                                                                                                                 \overline{D} \vdash_{\mathsf{FG}} t <: u_I
  \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e
                                                                                                                                        Reductions
                                                 v ::= t_S\{\overline{v}\}\
\mathcal{E} ::= \Box \mid t_S\{\overline{v}, \mathcal{E}, \overline{e}\} \mid \mathcal{E}.f \mid \mathcal{E}.(t) \mid \mathcal{E}.m(\overline{e}) \mid v.m(\overline{v}, \mathcal{E}, \overline{e})
     Evaluation context
    Substitution (FG values) \Phi_{\mathsf{v}} ::= \langle \overline{x_i \mapsto v_i} \rangle
                                                                 \frac{\textbf{type}\ t_S\ \textbf{struct}\ \{\overline{f}\ t\}\in \overline{D}}{\overline{D}\vdash_{\mathsf{FG}}\ t_S\{\overline{v}\}.f_i\longrightarrow v_i}
                        FG-CALL
                                                                                                                           \overline{D} \vdash_{\mathsf{FG}} t_S <: t
                                                                                                      v = t_S\{\overline{v}\}
                       func (x t_S) m(\overline{x t}) t \{ return e \} \in \overline{D}
 v = t_S\{\overline{v}\}
             \overline{D} \vdash_{\mathsf{FG}} v.m(\overline{v}) \longrightarrow \langle x \mapsto v, \overline{x_i \mapsto v_i} \rangle e
```

Fig. 3. Featherweight Go (FG)

a reduction inside an expression. Values, ranged over by v, are struct literals whose components are all values. A capture-avoiding substitution $\Phi_{\mathsf{v}} = \langle \overline{x_i \mapsto v_i} \rangle$ replaces variables x_i with values v_i , applying a substitution Φ_{v} to an expression e is written $\Phi_{\mathsf{v}}e$.

Rule FG-FIELD deals with field access. Condition FG2 guarantees that field name lookup is unambiguous. Rule FG-CALL reduces method calls. Condition FG4 guarantees that method lookup is unambiguous. The method call is reduced to the method body e where we map the receiver argument to a concrete value v and method arguments x_i to concrete values v_i . Rule FG-ASSERT covers type assertions. We need to check that the type t_S of value v is consistent with the type t asserted in the program text. This check can be carried out by checking that t_S and t are in a structural subtype relation.

We write $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^k e'$ to denote that e reduces to e' in exactly k steps. We write $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^* e'$ to denote that there exists some $k \in \mathbb{N}$ with $e \longrightarrow^k e'$. We assume that \mathbb{N} includes zero. If e reduces ad infinitum, we say that e diverges, written $\overline{D} \uparrow_{\mathsf{FG}} e$. Formally, $\overline{D} \uparrow_{\mathsf{FG}} e$ if $\forall k \in \mathbb{N}. \exists e'. \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^k e'$.

FG enjoys type soundness [6]. A well-typed program either reduces to a value, diverges, or it panics by getting stuck on a failed type assertion. The predicate $\mathsf{panic}_{\mathsf{FG}}(\overline{D},e)$ formalizes panicking:

$$\frac{\neg \overline{D} \vdash_{\mathsf{FG}} t_S <: t}{\mathsf{panic}_{\mathsf{FG}}(\overline{D}, \mathcal{E}[t_S\{\overline{v}\}.(t)])} \,\, ^{\mathsf{FG-PANIC}}$$

4 Type Directed Translation

We specify a type-directed translation from FG to an untyped lambda-calculus extended with recursive let-bindings, constructors, and pattern matching. The translation itself has already been specified elsewhere [26], but there only a weak form of semantic equivalence between source and target programs was given. The goal of the article at hand is to prove a much stronger form of semantic equivalence (see Section 5).

4.1 Target Language

Figure 4 specifies the syntax and dynamic semantics of our target language (TL). We use capital letters for constructs of the target language. Target expressions E include variables X, Y, data constructors K, function application, lambda abstraction and case expressions to pattern match against constructors. In a case expression with only one pattern clause, we often omit the brackets. If a case expressions has more than one clause $[Pat \to E]$, we assume that the constructors in Pat are distinct. A program consists of a sequence of (mutually recursive) function definitions and a (main) expression. The function definitions are the result of translating FG method definitions.

We assume data constructors for tuples up to some fixed but arbitrary size. The syntax (\overline{E}^n) constructs an *n*-tuple when used as an expression, and deconstructs it when used in a pattern context. At some places, we use nested patterns

```
Expression
                                                E ::=
                                                      X \mid Y
                                                                                                                     Pattern clause Cls ::= Pat \rightarrow E
           Variable
                                                                                                                                                     Pat ::= K \overline{X}
                                                      K
                                                                                                                     Pattern
           Constructor
                                                                                                                                                              Prog ::= let \overline{Y_i = \lambda X_i.E_i}
           Application
                                                       E E
                                                                                                                     Program
            Abstraction
                                                      \lambda X.E
                                                      case E of [\overline{\mathit{Cls}}]
           Pattern case
                                                                                              \begin{array}{ll} V & ::= K \ \overline{V} \mid \lambda X.E \mid X \\ \mathcal{R} & ::= \square \mid \mathbf{case} \ \mathcal{R} \ \mathbf{of} \ [\overline{Pat \to E}] \mid \mathcal{R} \ E \mid V \ \mathcal{R} \end{array}
                TL values
                TL evaluation context
                Substitution (TL values) \Phi_{V} ::= \langle \overline{X} \mapsto \overline{V} \rangle
Substitution (TL methods) \Phi_{m} ::= \langle \overline{Y} \mapsto \lambda X.\overline{E} \rangle
\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E'
                                                                                                                                                                       TL\ expression\ reductions
                                                                                                                TL-LAMBDA
                      \frac{\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E'}{\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \mathcal{R}[E] \longrightarrow \mathcal{R}[E']}
                                                                                                          TL-LAMBDA
\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (\lambda X.E) \ V \longrightarrow \langle X \mapsto V \rangle E
    TL-CASE
     \frac{K \ \overline{X_i}^n \to E' \in \overline{Cls}}{\varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} \mathsf{case} \ K \ \overline{V_i}^n \ \mathsf{of} \ [\overline{Cls}] \longrightarrow \langle \overline{X_i} \mapsto \overline{V_i}^n \rangle E'}
                                                                                                                                                         \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} Y V \longrightarrow \Phi_{\mathsf{m}}(Y) V
  \vdash_{\mathsf{TL}} \mathit{Prog} \longrightarrow \mathit{Prog}'
                                                                                                                                                                                                      TL reductions
                                            \frac{\langle \overline{Y_i} \mapsto \lambda X_i.\overline{E_i} \rangle \vdash_{\mathsf{TL}} E \longrightarrow E'}{\vdash_{\mathsf{TL}} \mathsf{let} \ \overline{Y_i} = \lambda X_i.\overline{E_i} \mathsf{ in } E \longrightarrow \mathsf{let} \ \overline{Y_i} = \lambda X_i.\overline{E_i} \mathsf{ in } E'}
```

Fig. 4. Target Language (TL)

as an abbreviation for nested case expressions. The notation $\lambda Pat.E$ stands for $\lambda X.\mathbf{case}\ X$ of $Pat \to E$, where X is fresh.

Target values V consist of constructors, lambda expressions, and variables. A variable may be a value if it is bound in a **let** at the top-level; that is, it refers to a method from FG. A constructor value $K \overline{V}^n$ is short for $(\ldots (K V_1) \ldots) V_n$.

The structural operational semantics employs two types of substitutions. Value substitutions Φ_V records the bindings resulting from pattern matching and function applications. Method substitutions Φ_m records the bindings for translated method definitions (i.e. for top-level let-bindings). Reduction of programs is mapped to reduction of expressions under a method substitution, see rule TL-PROG. A variable Y applied to a value V reduces to $\Phi_m(Y)$ V via TL-METHOD. The remaining reduction rules are standard.

We write $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^k E'$ to denote that E reduces to E' with exactly k steps, and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^* E'$ for some finite number of steps. If E reduces an arbitrary number of steps, we say that E diverges, written $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E$. Formally, $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E$ iff $\forall k \in \mathbb{N}. \exists E'. \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^k E'$.

In the source language FG, evaluation might panic by getting stuck on a failed type assertion. The translation to the target language preserves panicking, so we need to formalize panic. A target language expression panics if it is stuck on a **case**-expression and there is no matching clause.

$$\frac{K \ \overline{X_i}^n \to E' \not\in [\overline{\mathit{Cls}}]}{\mathsf{panic}_{\mathsf{TL}}(\varPhi_{\mathsf{m}}, \mathcal{R}[\mathbf{case} \ K \ \overline{V_i}^n \ \mathbf{of} \ [\overline{\mathit{Cls}}]])} \ ^{\mathsf{TL-PANIC}}$$

4.2 Translation

The specification of the translation spreads out over three figures. Figure 5 gives the translation of expressions, relying on Figure 6 to define auxiliary relations for translating structural subtyping and type assertions. Finally, Figure 7 translates method declarations and programs.

Before explaining the translation rules, we establish the following conventions (see also the top of Figure 5). We assume that each FG variable x translates to the TL variable X. FG variables introduced in method declarations are assumed to be distinct. This guarantees that there are no name clashes in environment Γ . For each struct t_S we introduce a TL constructor K_{t_S} , and for each interface t_I we introduce a TL constructor K_{t_I} . For each method declaration func $(x t_S) mM$ {return e} we introduce a TL variable X_{m,t_S} , thereby relying on condition FG4 which guarantees that m and t_S uniquely identify this declaration. We write Γ to denote typing environments where we record the types of FG variables. The notation [n] is a short-hand for the set $\{1, \ldots, n\}$.

The overall idea of the translation is to choose the TL-representation of an FG-value $v = t_S\{\overline{v}\}$ based on the type t that v is used at:

- If t is a struct type t_S , then the representation of v is $K_{t_S}(\overline{V})$, where each V_i is the representation of v_i , so (\overline{V}) is a tuple for the struct fields.

- If t is an interface type, then the representation of v is an interface-value $K_{t_I}(V, \overline{X_{m_i,t_S}})$, where V is the representation of v at struct type t_S and $\overline{X_{m_i,t_S}}$ is a dictionary [7] containing all methods $\overline{m_i}$ of interface t_I . The translation makes each method func $(x \ t_S) \ m_i M$ {return e} available as a top-level binding let $X_{m_i,t_S} = E$. An interface value $K_{t_I}(V, \overline{X_{m_i,t_S}})$ bears close resemblance to an existential type [12], as it hides the concrete representation of the value V.

$$\begin{array}{c} \text{Convention for mapping source to target terms} \\ x \sim X \quad t_S \sim K_{t_S} \quad t_I \sim K_{t_I} \quad \text{func } (x \, t_S) \, mM \, \{\text{return } e\} \sim X_{m,t_S} \\ \text{FG environment } \Gamma ::= \{\} \mid \{x : t\} \mid \Gamma \cup \Gamma \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t \sim E \\ \hline \\ TD-VAR \quad type \, t_S \, \text{struct } \{\overline{f} \, t^n\} \in \overline{D} \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} x : t \sim X \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim X \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e : t_S \sim E \\ \hline \\ \overline{\langle D, \Gamma \rangle} \vdash_{\text{exp}} e :$$

Fig. 5. Translation of expressions

The translation rules for expressions (Figure 5) are of the form $\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t \leadsto E$ where \overline{D} refers to the sequence of FG declarations, Γ refers to type binding of local variables, e is the to be translated FG expression, t its type and E the resulting target term. Rule TD-VAR translates variables and follows our convention that x translates to X. Rule TD-STRUCT translates a struct creation.

The translated field elements E_i are collected in a tuple and tagged via the constructor K_{t_S} . Rule TD-ACCESS uses pattern matching to capture field access in the translation.

Method calls are dealt with by rules TD-CALL-STRUCT and TD-CALL-IFACE. Rule TD-CALL-STRUCT covers the case that the receiver e is of the struct type t_S . The first precondition guarantees that an implementation for this specific method call exists. (See Figure 3 for the auxiliary methods.) Hence, we can assume that we have available a corresponding definition for X_{m,t_S} in our translation. The method call then translates to applying X_{m,t_S} first on the translated receiver E, followed by the translated arguments collected in a tuple (\overline{E}^n) .

Rule TD-CALL-IFACE assumes that receiver e is of interface type t_I , so e translates to interface-value E. Hence, we pattern match on E to access the underlying value and the desired method in the dictionary. We assume that the order of methods in the dictionary corresponds to the order of method declarations in the interface. The preconditions guarantee that t_I provides a method m as demanded by the method call, where j denotes the index of m in interface t_I .

To explain the two remaining rules for expressions (TD-ASSERT and TD-SUB), we first introduce the two auxiliary relations defined in Figure 6. The relation $\overline{D} \vdash_{\mathsf{iCons}} t <: u_I \leadsto E$ constructs an interface-value for u_I . Thus, the resulting expression E is a λ -expression taking the representation of a value at type t and yields its representation at type u_I .

Fig. 6. Translation of structural subtyping and type assertions

The preconditions in rule TD-CONS-STRUCT-IFACE check that struct t_S implements the interface. This guarantees the existence of method definitions X_{m_i,t_S} . Hence, we can construct the desired interface-value. The preconditions in rule TD-CONS-IFACE-IFACE check that t_I 's methods are a superset of u_I 's methods. This is done via the total function $\pi:\{1,\ldots,q\}\to\{1,\ldots,n\}$ that matches each (wanted) method in u_I against a (given) method in t_I . We use pattern matching over the t_I 's interface-value to extract the wanted methods. Recall that dictionaries maintain the order of method as specified by the interface.

The relation $\overline{D} \vdash_{\mathsf{iDestr}} t_I \searrow u \leadsto E$ destructs an interface-value. The λ -expression E takes a representation at type t_I and converts it to the representation at type u. This conversion might fail, resulting in a pattern-match error.

Rule TD-DESTR-IFACE-STRUCT deals with the case that the target type u is a struct type t_S . Hence, we find the precondition $\overline{D} \vdash_{\mathsf{FG}} t_S <: t_I$. We pattern match over the interface-value that represents t_I to check that the underlying value matches t_S and extract the value. It is possible that some other value has been used to implement the interface-value that represents t_I . In such a case, the pattern match fails and we experience run-time failure.

Rule TD-DESTR-IFACE-IFACE deals with the case that the target type u is an interface type u_I . The outer case expression extracts the value Y underlying interface-value t_I (this case never fails). We then check if we can construct an interface-value for u_I via Y. This is done via an inner case expression. For each struct t_{Sj} implementing u_I , we have a pattern clause Cls_j that matches against the constructor $K_{t_{Sj}}$ of the struct and then constructs an interface-value for u_I . There are two reasons for run-time failure here. First, Y (used to implement t_I) might not implement u_I ; that is, none of the pattern clauses Cls_j match. Second, \overline{Cls} might be empty because no receiver at all implements u_I . This case is rather unlikely and could be caught statically.

We now come back explaining the two remaining translation rules for expressions (Figure 5). Rule TD-ASSERT translates a type assertion e.(u) by destructing e's interface-value, potentially yielding a representation at type u. The type of e must be an interface type because only conversions from an interface type to some other type must be checked dynamically. Rule TD-SUB translates structural subtyping by constructing an appropriate interface-value. This rule could be integrated as part of the other rules to make the translation more syntax-directed. For clarity, we prefer to have a stand-alone subtyping rule.

The translation of programs and methods (Figure 7) boils down to the translation of expressions involved. Rule TD-METHOD translates a specific method declaration, rule TD-PROG collects all method declarations and also translates the main expression. The type system induced by the translation rules is equivalent to the original type system of Featherweight Go. See the Appendix.

5 Semantic preservation

We establish correctness of the type-directed translation scheme by showing that source and target behave the same. Figure 8 introduces the details of the logical

Fig. 7. Translation of methods and programs

relations that are discussed in the earlier Section 2. We assume that step indices k are natural numbers starting with 0.

The relation $e \approx E \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ specifies how an FG expression e and TL expression E are related at FG type t. The three cases terminate, diverge and panic from Figure 1 are combined in one rule RED-REL-EXP.

The relation $v \equiv V \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ specifies when FG value v and TL value V are equivalent at FG type t. Rule Red-rel-struct covers struct values by ensuring that the constructor tag matches and all field values are equivalent. Rule Red-rel-iface generalizes iface- \leq from Figure 2. The auxiliary methodLookup retrieves the method declaration for some method name and receiver type:

$$\frac{\text{func }(x\;t_S)\;mM\;\{\text{return }e\}\in\overline{D}}{\text{methodLookup}(\overline{D},(m,t_S))=\text{func }(x\;t_S)\;mM\;\{\text{return }e\}}$$

Rule RED-REL-METHOD relates methods, generalizing METHOD-< from Section 2. Rules RED-REL-VB and RED-REL-DECLS lift the logical relation to environments and declarations.

Thanks to the step index our logical relations are well-founded. In the definitions in Figure 8, the relations $e \approx E \in [\![t]\!]_k^{\langle \overline{D}, \varPhi_m \rangle}$ and $v \equiv V \in [\![t]\!]_k^{\langle \overline{D}, \varPhi_m \rangle}$ form a cycle. But either the step index k decreases or it stays constant but the size of the target value V decreases in recursive calls. Several other basic properties hold, such as LR-STEP and LR-MONO from the earlier Figure 2. Details are given in the appendix. These properties are vital to establish the following results.

We can prove that target expressions resulting from FG expressions are semantically equivalent to the source.

Lemma 1 (Expression Equivalence). Let $\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t \leadsto E \text{ and } \Phi_{\mathsf{v}}, \Phi_{\mathsf{v}}, \Phi_{\mathsf{m}} \text{ such that } \langle \overline{D}, \Phi_{\mathsf{m}}, \Gamma \rangle \vdash_{\mathsf{rr}}^k \Phi_{\mathsf{v}} \approx \Phi_{\mathsf{v}} \text{ and } \vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}} \text{ for some } k. \text{ Then,} we find that } \Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{v}}(E) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$

$$\begin{array}{c} e \approx E \in \llbracket t \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \\ \text{RED-REL-EXP} \\ \left(\forall k' < k, v \cdot \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} v \implies \exists V. \Phi_m \vdash_{\mathsf{TL}} E \longrightarrow^* V \land v \equiv V \in \llbracket t \rrbracket_{k-k'}^{(\overline{D}, \Phi_m)} \right) \\ & \wedge \\ & \wedge \\ & \left(\forall k' < k, e' \cdot \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e' \land \overline{D} \pitchfork_{\mathsf{FG}} e' \implies \Phi_m \pitchfork_{\mathsf{TL}} E \right) \\ & \wedge \\ & \left(\forall k' < k, e' \cdot \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e' \land \overline{D} \pitchfork_{\mathsf{FG}} (\overline{D}, e') \implies \operatorname{panic}_{\mathsf{TL}} (\Phi_m, E) \right) \\ & e \approx E \in \llbracket t \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \\ \hline v \equiv V \in \llbracket t \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \hline v \equiv V \in \llbracket t \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \hline v \equiv V \in \llbracket t \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \hline v \equiv K_{ts} \operatorname{ctr} \left\{ \overline{f} \ t^* \right\} \in \overline{D} \qquad \forall i \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \hline v \in K_{us} \ \overline{V}^{u} \quad \forall k_1 \leq k. v \equiv V \in \llbracket u_s \rrbracket_{k_1}^{(\overline{D}, \Phi_m)} \\ \hline v \equiv K_{t_1} \left(V, \overline{Y}^{u} \right) \in \llbracket t_1 \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \hline v \equiv K_{t_1} \left(V, \overline{Y}^{u} \right) \in \llbracket t_1 \rrbracket_k^{(\overline{D}, \Phi_m)} \\ \hline \text{func} \left(x t_S \right) mM \left\{ \text{return } e \right\} \approx Y \in \llbracket mM \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\forall i \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\forall i \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{D}, \Phi_m) \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in \llbracket t_i \rrbracket_k^{(\overline{D}, \Phi_m)} \land (\overline{V}^{u} \in [n]. v_i \equiv V_i \in [n]. v_i \equiv V$$

Fig. 8. Relating FG to TL Reduction

As motivated in Section 2, for the proof to go through, one of the rules RED-REL-IFACE and RED-REL-METHOD must use < and the other <. In our case, we use ≤ in rule RED-REL-IFACE and < in rule RED-REL-METHOD. The lengthy proof is given in the appendix.

Based on the above result, we can establish semantic equivalence for method definitions. For this proof to go through it is essential that we find \leq in rule RED-REL-IFACE and < in rule RED-REL-METHOD.

Lemma 2 (Method Equivalence).

Let \overline{D} and Φ_{m} such that for each func $(x \ t_S) \ m(\overline{x} \ t^n) \ t \ \{\mathsf{return} \ e\}$ in \overline{D} we have $\Phi_{\mathsf{m}}(X_{m,t_S}) = \lambda X.\lambda(\overline{X}^n).E \text{ where } \overline{D} \vdash_{\mathsf{meth}} \mathsf{func} (x \ t_S) \ m(\overline{x} \ \overline{t}^n) \ t \ \{\mathsf{return} \ e\} \leadsto$ $\lambda X.\lambda(\overline{X}^n).E.$ Then, we find that $\vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ for any k.

Proof. To verify (1) $\vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ for each func $(x \ t_S) \ m(\overline{x_i \ t_i}^n) \ t \ \{\mathsf{return} \ e\}$ in \overline{D} we have to show based on rules RED-REL-DECLS and RED-REL-METHOD that

$$\forall k' < k, v, V, \overline{v}^n, \overline{V}^n. (v \approx V \in \llbracket t_S \rrbracket_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle} \wedge (\forall i \in [n]. v_i \approx V_i \in \llbracket t_i \rrbracket_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}))$$

$$\implies (2) \ \langle x \mapsto v, \overline{x_i \mapsto v_i}^n \rangle e \approx (X_{m, t_S} \ V) \ (\overline{V}^n) \in \llbracket t \rrbracket_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

We verify the result by induction on k.

- Case k = 0 or k = 1: Holds immediately. See rule RED-REL-EXP.
- Case $k \implies k+1$: Suppose k' < k+1 and (3) $v \approx V \in \llbracket t_S \rrbracket_{k'}^{\langle \overline{D}, \Phi_m \rangle}$ and (4) $v_i \approx V_i \in \llbracket t_i \rrbracket_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for some v, V, v_i, V_i for $i \in [n]$. Define $\Phi_{\mathsf{v}} = \langle x \mapsto v, \overline{x_i \mapsto v_i}^n \rangle$ and $\Phi_{\mathsf{v}} = \langle X \mapsto V, \overline{X_i \mapsto V_i}^n \rangle$ and $\Gamma = \{x : t_S, \overline{x_i : t_i}^n \}$.
 - (5) $\langle \overline{D}, \Phi_{\mathsf{m}}, \Gamma \rangle \vdash_{\mathsf{rr}}^{k'} \Phi_{\mathsf{v}} \approx \Phi_{\mathsf{V}} \text{ via (3) and (4)}.$
 - (6) $\vdash_{\mathsf{rr}}^{k'} \overline{D} \approx \Phi_{\mathsf{m}}$ by induction.
 - (7) $\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t \leadsto E$ from the assumption and rule TD-METHOD.
 - (8) $\Phi_{\mathsf{v}}e \approx \Phi_{\mathsf{V}}E \in [\![t]\!]_{k'}^{\langle \overline{D},\Phi_{\mathsf{m}}\rangle}$ via (5), (6), (7), and Lemma 1.
 - $(9) \ \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (X_{m,t_S} \ V) \ (\overline{V}^n) \longrightarrow^* \Phi_{\mathsf{V}} E$
 - via the assumption that $\Phi_{\mathsf{m}}(X_{m,t_S}) = \lambda X.\lambda(\overline{X}^n).E.$ (10) $\Phi_{\mathsf{v}}e \approx (X_{m,t_S} \ V) \ (\overline{V}^n) \in \llbracket t \rrbracket_{k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ via (8), (9) and because target reductions do not affect the step index (Lemma 10 in the Appendix).

Statement (10) corresponds to (2). Thus, we can establish (1).

If we would find ≤ instead of < in rule RED-REL-METHOD, the proof would not go through. We would then need to establish the implication at the beginning of the proof for $k' \leq k$, but the induction hypothesis gives us only $\vdash_{rr}^{k'-1} \overline{D} \approx \Phi_{m}$ in (6).

We state our main result that the dictionary-passing translation preserves the dynamic behavior of FG programs.

Theorem 1 (Program Equivalence). Let $\vdash_{prog} \overline{D}$ func main() $\{ \bot = e \} \leadsto$ let $\overline{X_{m_i,t_{S_i}}} = \overline{E_i}^n$ in E where we assume that e has type t. Let $\Phi_m = \langle \overline{X_{m_i,t_{S_i}}} \mapsto \overline{E_i}^n \rangle$. Then, we find that $e \approx E \in [t]_{L}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for any k.

6 Related Work

Logical relations have a long tradition of proving properties of typed programming languages. Such properties include termination [28,25], type safety [24], and program equivalence [18, Chapters 6, 7]. A logical relation (LR) is often defined inductively, indexed by type. If its definition is based on an operational semantics, the LR is called syntactic [20,5]. With recursive types, a step-index [3,2] provides a decreasing measure to keep the definition well-founded. See [15, Chapter 8] and [24] for introductions to the topic.

LRs are often used to relate two terms of the same language. For our translation, the two terms are from different languages, related at a type from the source language. Benton and Hur [4] prove correctness of compiler transformations. They used a step-index LR to relate a denotational semantics of the λ -calculus with recursion to configurations of a SECD-machine. The setup relies on biorthogonality [21,14,22,11] to allow for compositionality and extensionality of equivalences.

Hur and Dreyer [9] build on this idea to show equivalence between an expressive source language (polymorphic λ -calculus with references, existentials, and recursive types) and assembly language. Their biorthogonal, step-indexed Kripke LR does not directly relate the two languages but relies on abstract language specifications. The Kripke part of the LR [19] allows reasoning about the shape of the heap.

Our setting is different in that we consider a source language with support for overloading. Besides structured data and functions, we need to cover interface values. This then leads to some challenges to get the step index right. Recall Figure 2 and the discussion in Section 2.

Simulation or bisimulation (see e.g. [27]) is another common technique for showing program equivalences. In our setting, using this technique amounts to proving that reduction and translation commutes: if source term e reduces to e' and translates to target term E, then e' translates to E' such that E reduces to E'' (potentially in several steps) with E' = E''. One challenge is that two target terms E' and E'' are not necessarily syntactically equal but only semantically. With LR, we abstract away certain details of single step reductions, as we only compare values not intermediate results. A downside of the LR is that getting the step index right is sometimes not trivial.

Paraskevopoulou and Grover [17] combine simulation and an untyped, stepindexed LR [1] to relate the translation of a reduced expression (the E' from the preceding paragraph) with the reduction result of the translated expression (the E''). They use this technique to prove correctness of CPS transformations using small-step and big-step operational semantics. Resource invariants connect the number of steps a term and its translation might take, allowing them to prove that divergence and asymptotic runtime is preserved by the transformation. Our LR does not support resource invariants but includes a case for divergence directly.

Hur and coworkers [10] as well as Hermida and coworkers [8] also blend bisimulation with LRs, building on previous results [9].

7 Conclusion

In this work, we established a strong semantic preservation result for a typedirected translation scheme of Featherweight Go. To achieve this result, we rely on syntactic, step-indexed logical relations. There are some subtle corners and we gave a detailed discussion of how to get the definition of logical relations right so that the proofs will go through. The proofs are still hand-written where all cases are worked out in detail. To formalize the proofs in a proof assistant we yet need to mechanize the source and target semantics. This is something we plan to pursue in future work.

We believe that the methods developed in this work will be useful in other language settings that employ a type-directed translation scheme for a form of overloading, e.g. consider Haskell type classes [7] and traits in Scala [29] and Rust [23]. This is another topic for future work. In another direction, we plan to adapt our translation scheme and proof method to cover Featherweight Go extended with generics [6].

Acknowledgments. We thank the MPC'22 reviewers for their comments.

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Properties of FG and TL

The translation rules from FG to the target language also induce a set of typing rules for FG by simply omitting the translation part. These typing rules are slightly different from FG's original typing rules [6], because the translation rules are not syntax-directed due the structural subtyping rule TD-SUB defined in Figure 5. We could integrate this rule via the other rules but this would make all the rules harder to read. Hence, we prefer to have a separate rule TD-SUB. Nevertheless, the typing rules induced by the translation are equivalent to FG's original typing rules.

Lemma 3. Assume P is an FG program. Then we have that P is well-typed according to FG's original typing rules [6, Figure 13] if, and only if, $\vdash_{prog} P \rightsquigarrow$ Prog for some Prog.

Proof. We write $\Gamma \vdash_{\mathsf{FG}} e : t$ for FG's original typing relation on expressions, and $\vdash_{\mathsf{FG}} P$ ok for FG's original typing relation on programs. These relation are defined in [6, Figure 13], the sequence of declaration \overline{D} is implicit there. Further, we need to prepend **package main**; to program P. FG's original subtyping relation, written $\overline{D} \vdash_{\mathsf{FG}} t <: u \text{ is identical to the one defined in Figure 3, expect}$ that in FG's original definition \overline{D} is implicit.

We then prove the following facts, where (e) is the claim of the lemma.

- (a) If $\overline{D} \vdash_{\sf FG} t <: u$ then either $\overline{D} \vdash_{\sf iCons} t <: u \leadsto E$ for some E or t = u and tis a struct type.
- (b) If $\overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E \text{ then } \overline{D} \vdash_{\mathsf{FG}} t <: u$.
- (c) If $\Gamma \vdash_{\mathsf{FG}} e : t \text{ then } \langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t \leadsto E \text{ for some } E$. (d) If $\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t \leadsto E \text{ then } \Gamma \vdash_{\mathsf{FG}} e : t' \text{ for some } t' \text{ with } \overline{D} \vdash_{\mathsf{FG}} t' <: t$.
- (e) $\vdash_{\mathsf{FG}} P$ ok iff $\vdash_{\mathsf{prog}} P \leadsto Prog$ for some Prog.

We prove (a) and (b) by case distinctions on the last rule of the given derivations; (c) and (d) follow by induction on the derivations, using (a) and (b). Claim (e) then follows by examining the type rules, using (c), (d), and conditions FG2, FG3, FG4.

Lemma 4 (FG reductions are deterministic). If $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e'$ and $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e''$ then e = e'.

Proof. We first state and prove three sublemmas:

- (a) If $e = \mathcal{E}_1[\mathcal{E}_2[e']]$ then there exists \mathcal{E}_3 with $e = \mathcal{E}_3[e']$. The proof is by induction
- (b) If $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e'$ then there exists a derivation of $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e'$ that ends with at most one consecutive application of rule FG-CONTEXT. The proof is by induction on the derivation of $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e'$. From the IH, we know that this derivation ends with at most two consecutive applications of rule FG-CONTEXT. If there are two such consecutive applications, (a) allow us to merge the two evaluation contexts involved, so that we need only one consecutive application of FG-CONTEXT.

(c) We call an FGG expression directly reducible if it reduces but not by rule FG-CONTEXT. If e_1 and e_2 are now directly reducible and $\mathcal{E}_1[e_1] = \mathcal{E}_2[e_2]$ then $\mathcal{E}_1 = \mathcal{E}_2$ and $e_1 = e_2$. For the proof, we first note that $\mathcal{E}_1 = \square$ iff $\mathcal{E}_2 = \square$. This holds because directly reducible expressions have no inner redexes. The rest of the proof is then a straightforward induction on \mathcal{E}_1 .

Now assume $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e'$ and $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow e''$. By (b) we may assume that both derivations ends with at most one consecutive application of rule FG-CONTEXT. It is easy to see (as values do not reduce) that both derivations must end with the same rule. If this rule is not FG-CONTEXT, then obviously e' = e'' (note condition FG4 for rule FG-CALL). Otherwise, we have the following situation with $R_1 \neq \mathsf{FG-CONTEXT}$ and $R_2 \neq \mathsf{FG-CONTEXT}$:

FG-CONTEXT
$$\frac{R_1}{\underbrace{\mathcal{E}_1[e_1]}} \xrightarrow{e_1} \underbrace{\underbrace{\mathcal{E}_1[e'_1]}}_{=e} \qquad \underbrace{\underbrace{\frac{e_2 \longrightarrow e'_2}{E_2[e_2]}}}_{=e} \xrightarrow{\mathcal{E}_2[e'_2]} \text{FG-CONTEXT}$$

As neither R_1 nor R_2 are FG-CONTEXT, we know that e_1 and e_2 are directly reducible. Thus, with $\mathcal{E}_1[e_1] = \mathcal{E}_2[e_2]$ and (c) we get $\mathcal{E}_1 = \mathcal{E}_2$ and $e_1 = e_2$. With R_1 and R_2 not being FG-CONTEXT, we have $e'_1 = e'_2$, so e' = e'' as required. \square

Lemma 5 (Target reductions are deterministic). If $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E'$ and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E''$ then E' = E''. Further, if $\vdash_{\mathsf{TL}} Prog \longrightarrow Prog'$ and $\vdash_{\mathsf{TL}} Prog \longrightarrow Prog''$ then Prog = Prog'.

Proof. Assume $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E'$ and $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow E''$. The claim that E' = E'' follows analogously to the proof of Lemma 4. If both derivations end with rule TL-CASE, we get deterministic evaluation by the syntactic restriction that case clauses have distinct constructors.

Deterministic evaluation for programs is a simple consequence of deterministic evaluation of expressions. $\hfill\Box$

B Logical Relation Properties

A fundamental property of step indexed logical relations is that if two expressions are in a relation for k steps then they are also in a relation for any smaller number of steps. (See LR-MONO from Figure 2.)

Lemma 6 (Monotonicity). (1) Let $e \approx E \in [\![t]\!]_k^{\langle \overline{D}, \Phi_m \rangle}$ and $k' \leq k$. Then, we find that $e \approx E \in [\![t]\!]_{k'}^{\langle \overline{D}, \Phi_m \rangle}$. (2) Let $v \equiv V \in [\![t]\!]_k^{\langle \overline{D}, \Phi_m \rangle}$ and $k' \leq k$. Then, we find that $v \equiv V \in [\![t]\!]_{k'}^{\langle \overline{D}, \Phi_m \rangle}$.

Proof. By mutual induction over the derivations $e \approx E \in [t]_k^{\overline{D},\Phi_{\mathsf{m}}}$ and $v \equiv V \in [t]_k^{\overline{D},\Phi_{\mathsf{m}}}$.

Case RED-REL-EXP: Follows immediately.

Case RED-REL-STRUCT:

$$\frac{\text{type }t_S \text{ struct } \{\overline{f\ t}^n\} \in \overline{D} \qquad \forall i \in [n].v_i \equiv V_i \in [\![t_i]\!]_k^{\langle \overline{D}, \varPhi_m \rangle}}{t_S \{\overline{v}^n\} \equiv K_{t_S} \ (\overline{V}^n) \in [\![t_S]\!]_k^{\langle \overline{D}, \varPhi_m \rangle}}$$

Follows immediately by induction.

Case RED-REL-IFACE:

$$V = K_{u_S} \ \overline{V'}$$

$$(1) \ \forall k_1 \leq k.v \equiv V \in \llbracket u_S \rrbracket_{k_1}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle} \quad \mathsf{methods}(\overline{D}, t_I) = \{\overline{mM}^n\}$$

$$(2) \ \forall k_2 \leq k, i \in [n]. \mathsf{methodLookup}(\overline{D}, (m_i, u_S)) \approx Y_i \in \llbracket m_i M_i \rrbracket_{k_2}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$$

$$v \equiv K_{t_I} \ (V, \overline{Y}^n) \in \llbracket t_I \rrbracket_k^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$$

Consider the first premise (1). If there exists $k_1 \leq k'$ then $v \equiv V \in [u_S]_{k_1}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$. Otherwise, this premise holds vacuously.

The same argument for $k_2 \leq k'$ applies to the second premise (2) by unfolding methodLookup $(\overline{D},(m_i,u_S)) \approx Y_i \in \llbracket m_i M_i \rrbracket_{k_2}^{\langle \overline{D},\Phi_{\mathsf{m}} \rangle}$ via rule RED-REL-METHOD. Hence, $v \equiv K_{t_I} (V, \overline{Y}^n) \in \llbracket t_I \rrbracket_{k'}^{\langle \overline{D},\Phi_{\mathsf{m}} \rangle}$.

Hence,
$$v \equiv K_{t_I}(V, \overline{Y}^n) \in [\![t_I]\!]_{k'}^{\langle D, \Phi_{\mathsf{m}} \rangle}$$
.

A similar monotonicity result applies to method definitions and declarations.

Lemma 7 (Monotonicity 2). Let func $(x t_S) mM$ {return e} $\approx V \in [mM]_k^{\langle \overline{D}, \Phi_m \rangle}$ and $k' \leq k$. Then, we find that func $(x \ t_S) \ mM$ {return e} $\approx V \in [\![mM]\!]_{k'}^{\langle \overline{D}, \Phi_m \rangle}$.

Proof. Follows immediately by observing the premise of rule RED-REL-METHOD.

Lemma 8 (Monotonicity 3). Let $\vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ and $k' \leq k$. Then, we find that $\vdash_{\mathsf{rr}}^{k'} \overline{D} \approx \Phi_{\mathsf{m}}$.

Monotonicity is an essential property that is exploited frequently in our proofs. Another useful property is LR-STEP from Figure 2. We also need several variations of this property in our proofs.

Lemma 9. Let $e \approx E \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for some $k, e, E, \overline{\underline{D}}$ and Φ_{m} . Let $\overline{D} \vdash_{\mathsf{FG}}$ $e_2 \longrightarrow^1 e$ for some e_2 . Then, we have that $e_2 \approx E \in [t]_{k+1}^{\langle \overline{D}, \Phi_m \rangle}$.

Proof. Based on our assumption we find that RED-REL-EXP

$$(1) \left(\forall k' < k, v : \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} v \implies \exists V . \varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^{*} V \land v \equiv V \in \llbracket t \rrbracket_{k-k'}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle} \right)$$

$$(2) \left(\forall k' < k, e' : \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e' \land \overline{D} \uparrow_{\mathsf{FG}} e' \implies \varPhi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E \right)$$

$$(3) \left(\forall k' < k, e' : \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e' \land \mathsf{panic}_{\mathsf{FG}}(\overline{D}, e') \implies \mathsf{panic}_{\mathsf{TL}}(\varPhi_{\mathsf{m}}, E) \right)$$

$$e \approx E \in \llbracket t \rrbracket_{k}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$$

$$e pprox E \in [t]_k^{\langle D, \Phi_{\mathsf{m}} \rangle}$$

From (1) and $\overline{D} \vdash_{\mathsf{FG}} e_2 \longrightarrow^1 e$ we conclude that

$$\forall k' < k+1, v \ . \ \overline{D} \vdash_{\mathsf{FG}} e_2 \longrightarrow^{k'+1} v \implies \exists V . \varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^* V \land v \equiv V \in \llbracket t \rrbracket_{k+1-k'}^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}$$

From (2) and $\overline{D} \vdash_{\mathsf{FG}} e_2 \longrightarrow^1 e$ we conclude that

$$\forall k' < k+1, e' : \overline{D} \vdash_{\mathsf{FG}} e_2 \longrightarrow^{k'} e' \land \overline{D} \uparrow_{\mathsf{FG}} e' \implies \Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E$$

From (3) and $\overline{D} \vdash_{\mathsf{FG}} e_2 \longrightarrow^1 e$ we conclude that

$$\forall k' < k+1, e'$$
. $\overline{D} \vdash_{\mathsf{FG}} e_2 \longrightarrow^{k'} e' \land \mathsf{panic}_{\mathsf{FG}}(\overline{D}, e') \implies \mathsf{panic}_{\mathsf{TI}}(\varPhi_{\mathsf{m}}, E)$

Thus,
$$e_2 \approx E \in \llbracket t \rrbracket_{k+1}^{\langle \overline{D}, \Phi_m \rangle}$$
 and we are done.

Lemma 10. Let $e \approx E \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for some k, e, E, \overline{D} and Φ_{m} . Let $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^{k_2} E$ for some E_2 and k_2 . Then, we have that $e \approx E_2 \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

Proof. By assumption we have the following

For each case we can argue that E_2 satisfies the requirements. Hence, we find that $e \approx E_2 \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

Lemma 11. Let $e \approx E \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for some k, e, E, \overline{D} and Φ_{m} . Let $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^* v$ for some value v. Let $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^* V$ for some value V. Let E' be a target expression such that $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E' \longrightarrow^* V$. Then, we have that $e \approx E' \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

Proof. Expression e reduces to a value. Hence, the statement $e \approx E \in [t]_k^{\langle \overline{D}, \Phi_m \rangle}$ implies

$$\forall k' < k, v \ . \ \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} v \implies \exists V. \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E \longrightarrow^{*} V \land v \equiv V \in \llbracket t \rrbracket_{k-k'}^{\sqrt{\overline{D}}, \Phi_{\mathsf{m}} \rangle}$$

We can simply replace E by E' in the above. Hence, we also find that $e \approx E' \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

\mathbf{C} Semantic Preservation for Interface-Value Constructors and Destructors

Interface-value constructors and destructors, see Figure 6, preserve equivalent expressions via logical relations as stated by the following results.

Lemma 12 (Structural Subtyping versus Interface-Value Constructors).

Let $\overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E_1 \text{ and } \vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}} \text{ and } e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}.$ Then, we find that $e \approx E_1 \ E_2 \in \llbracket u \rrbracket_k^{\overline{\langle \overline{D}}, \Phi_{\mathsf{m}} \rangle}$.

Proof. We perform a case analysis of the derivation for $\overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E_1$ and label the assumptions (1) $\vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ and (2) $e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ as well as the to be proven statement (3) $e \approx E_1 E_2 \in [t]_k^{\langle \overline{D}, \Phi_m \rangle}$ for later reference. Case TD-CONS-STRUCT-IFACE:

$$\frac{\text{type }t_I \text{ interface } \{\overline{S}\} \in \overline{D} \quad \text{methods}(\overline{D}, t_S) \supseteq \overline{S} \quad \overline{S} = \overline{mM}^n}{\overline{D} \vdash_{\mathsf{iCons}} t_S <: t_I \leadsto \lambda X. K_{t_I} \left(X, \overline{X_{m_i, t_S}}^n\right)}$$

We establish statement (3) by distinguishing among the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate:

Suppose (4) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} v$ for some k' and value v where k' < k.

- (5) $\Phi_{\mathbf{m}} \vdash_{\mathsf{TL}} E_2 \xrightarrow{}^* V$ for some V where (6) $v \equiv V \in \llbracket t_S \rrbracket_{k-k'}^{\langle \overline{D}, \Phi_{\mathbf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (4).

- $(7) \ \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (\lambda X. K_{t_I} (X, \overline{X_{m_i, t_S}}^n)) \ E_2 \longrightarrow^* K_{t_I} (V, \overline{X_{m_i, t_S}}^n)$ via reduction step (5).
- (8) func $(x\ t_S)\ m_i M_i\ \{\text{return}\ e\} \approx X_{m_i,t_S} \in [\![m_i M_i]\!]_k^{\langle\overline{D},\Phi_{\mathsf{m}}\rangle}\ \text{for}\ i=1,...,n$ via reverse application of rule RED-REL-DECLS on (1).
- (9) func $(x \ t_S) \ m_i M_i \ \{\text{return } e\} \approx X_{m_i,t_S} \in [\![m_i M_i]\!]_{k-k'}^{\langle \overline{D},\Phi_{\mathsf{m}}\rangle}$ via (7) and the Monotonicity Lemma.

(10) $v \equiv K_{t_I} (V, \overline{X_{m_i,t_S}}^n) \in \llbracket t_I \rrbracket_{k-k'}^{\langle \overline{D}, \Phi_m \rangle}$ via application of rule RED-REL-IFACE on (6) and (8). Statements (6) and (8) hold for any $k'' \leq k - k''$ via the Monotonicity Lemma.

Thus, this subcase in statement (3) holds.

Subcase-Diverge:

Suppose (4) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e'$ for some k' and e' where k' < k and $\overline{D} \uparrow_{\mathsf{FG}} e'$.

(5) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E_2$

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (4).

(6) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} (\lambda X. K_{t_I}(X, \overline{X_{m_i, t_S}}^n)) E_2 \text{ via (5)}.$

Thus, this subcase in statement (3) holds.

Subcase-Panic:

Suppose (4) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e'$ for some k' and e' where k' < k and $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e').$

(5) panic_{TI} (Φ_m, E_2)

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (4).

(6) $\operatorname{\mathsf{panic}}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, (\lambda X. K_{t_I} (X, \overline{X_{m_i, t_S}}^n)) E_2) \operatorname{via}(5).$

Thus, this subcase in statement (3) holds.

Case TD-CONS-IFACE-IFACE

$$\begin{array}{c} \text{type } t_I \text{ interface } \{\overline{R}^n\} \in \overline{D} \\ \text{type } u_I \text{ interface } \{\overline{S}^q\} \in \overline{D} \quad S_i = R_{\pi(i)} \quad (\forall i \in [q]) \\ \overline{D} \vdash_{\mathsf{iCons}} t_I <: u_I \leadsto \lambda X. \mathsf{case} \, X \, \mathsf{of} \, K_{t_I} \, (X, \overline{X}^n) \to K_{u_I} \, (X, X_{\pi(1)}, \dots, X_{\pi(q)}) \end{array}$$

Via similar reasoning as for the other upcast case, we establish statement (3) by distinguishing among the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate:

Suppose (4) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} v$ for some k' and value v where k' < k.

- (5) $\Phi_{\mathbf{m}} \vdash_{\mathsf{TL}} E_2 \xrightarrow{}^* V$ for some V where (6) $v \equiv V \in \llbracket t_I \rrbracket_{k-k'}^{\langle \overline{D}, \Phi_{\mathbf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (4).

- (7) for any $k'' \le k k'$
- (8) $v \equiv V_{val} \in \llbracket u_S \rrbracket_{k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and
- (9) **func** $(x \ u_S) \ R_j \approx Y_j \in \llbracket R_j \rrbracket_{k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for j = 1, ..., n where (10) $V_{val} = K_{u_S} \ (\overline{V'})$ and
- $(11) V = K_{t_I} (V_{val}, \overline{Y_j}^n)$

via reverse application of rule RED-REL-IFACE on (6).

 $(12) \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (\lambda X.\mathsf{case} \, X \, \mathsf{of} \, K_{t_I} \, (X, \overline{X}^n) \to K_{u_I} \, (X, X_{\pi(1)}, \dots, X_{\pi(q)})) \, E_2 \longrightarrow^*$ K_{u_I} $(V_{val}, Y_{\pi(1)}, \dots, Y_{\pi(q)})$ via

reduction steps (5), (10) and (11).

(8)
$$v \equiv K_{u_I} (V_{val}, Y_{\pi(1)}, \dots, Y_{\pi(q)}) \in \llbracket u_I \rrbracket_{k-k'}^{\langle \overline{D}, \Phi_m \rangle}$$
 via application of rule RED-REL-IFACE on (6), (7) and (9) in combination with

the Monotonicity Lemma.

Thus, the first subcase in statement (3) holds.

We omit the other subcases as the reasoning steps exactly correspond to case TD-CONS-STRUCT-IFACE.

Lemma 13 (Type Assertions versus Interface-Value Destructors). Let $\overline{D} \vdash_{\mathsf{iDestr}} t \searrow u \leadsto E_1 \ and \vdash_{\mathsf{rr}}^k \overline{D} \approx \varPhi_{\mathsf{m}} \ and \ e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \varPhi_{\mathsf{m}} \rangle}. \ Then, \ we \ find$ that $e.(u) \approx E_1 \ E_2 \in \llbracket u \rrbracket_{L}^{\langle \overline{D}, \Phi_m \rangle}$

Proof. We perform a case analysis of the derivation $\overline{D} \vdash_{\mathsf{iDestr}} t \searrow u \leadsto E_1$ and label the assumptions (1) $\vdash_{\mathsf{rr}}^k \overline{D} \approx \Phi_{\mathsf{m}}$ and (2) $e \approx E_2 \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ as well as the to be proven statement (3) $e.(u) \approx E_1 E_2 \in [\![t]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ for later reference. Case TD-DESTR-IFACE-STRUCT:

We establish statement (3) by distinguishing among the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate: Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} e.(t_S) \longrightarrow^{k'} v$ for some value v where

- (7) $v = t_S\{\overline{v}\}$ for some \overline{v}
- via (5), (6) and the FG reduction rules.
- $(8) \ \overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'-1} t_S\{\overline{v}\}$
- via (6) and (7).
- (9) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^* V$ for some V where
- (10) $v \equiv V \in [t_I]_{k-(k'-1)}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (2) where in the premise the right-hand side of the implication is satisfied via (8) and the fact that k'-1 < k.

- (11) $v \equiv V_{val} \in \llbracket t_S \rrbracket_{k''}^{\langle \overline{D}, \Phi_m \rangle}$ for any $k'' \leq k (k' 1)$ where (12) $V_{val} = K_{t_S}(\overline{V'})$ for some $\overline{V'}$ and (13) $V = K_{t_I}(V_{val}, \overline{Y_j}^n)$ for some $\overline{Y_j}^n$

via reverse application of rule RED-REL-IFACE on (10) where we make use of (7) and rule RED-REL-STRUCT to derive the shape of V_{val} and (4) to guarantee that there are n method variables Y_j .

- $(14) \ \varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} (\lambda X.\mathsf{case} \ X \ \mathsf{of} \ K_{t_I} \ (K_{t_S} \ \overline{Y}, \overline{X}^n) \to K_{t_S} \ \overline{Y}) \ E_2 \longrightarrow^* V_{val}$ via the reduction step (9) and the equations (12) and (13).
- (15) $v \equiv V_{val} \in [t_S]_{k-k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$
- via (11) and the Monotonicity Lemma.

Thus, we can establish this subcase in statement (3).

Subcase-Diverge: Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} e.(t_S) \longrightarrow^{k'} e'$ for some e' where k' < k

- (7) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e''$ for some e'' where $k'' \leq kA$ and $\overline{D} \uparrow_{\mathsf{FG}} e''$
- by observing the reduction (6).
- (8) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E_2$

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (8).

(9) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} (\lambda X.\mathsf{case}\, X\,\mathsf{of}\, K_{t_I}\, (K_{t_S}\, \overline{Y}, \overline{X}^n) \to K_{t_S}\, \overline{Y}) E_2 \,\mathrm{via}$ (7).

Thus, we can establish this subcase in statement (3).

Subcase-Panic: Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} e.(t_S) \longrightarrow^{k'} e'$ for some e' where k' < kand panic_{FG} (\overline{D}, e') .

We distinguish among the following two cases. Either (1) the expression panics or (2) the type assertion fails.

Subcase-Panic-1: (7) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k''} e''$ for some e'' where k'' < k and $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e').$

Subcase-Panic-2: (8) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k''} u_S\{\overline{v'}\} \text{ for some } u_S\{\overline{v'}\} \text{ where } k'' < k$ and $t_S \neq u_S$.

Consider Subcase-Panic-1.

(9) panic_{TI} (Φ_{m}, E_2)

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (7).

(10) $\mathsf{panic}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, (\lambda X. \mathsf{case} X \mathsf{ of } K_{t_I} (K_{t_S} \overline{Y}, \overline{X}^n) \to K_{t_S} \overline{Y}) E_2) \text{ via } (9) \text{ and }$ we are done here.

Consider Subcase-Panic-2.

(11) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^* V$ for some V where

$$(12) \ u_S\{\overline{v'}\} \equiv V \in [t_I]_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via reverse application of rule RED-REL-EXP on (2) where in the premise the right-hand side of the implication is satisfied via (8)

- (13) $V_{val} = K_{u_S} (\overline{V'})$ for some $\overline{V'}$ and (14) $V = K_{t_I} (V_{val}, \overline{Y})$ for some method variables \overline{Y}

via reverse application of rule RED-REL-IFACE on (12) where we make use of RED-REL-STRUCT to derive the shape of V_{val} .

(15)
$$\operatorname{panic}_{\operatorname{TL}}(\Phi_{\operatorname{m}}, (\lambda X. \operatorname{case} X \operatorname{of} K_{t_I} (K_{t_S} \overline{Y}, \overline{X}^n) \to K_{t_S} \overline{Y}) E_2)$$

via (11), (13) and (14) and the fact that $K_{t_S} \neq K_{u_S}$. Thus, we are done

Case TD-DESTR-IFACE-IFACE:

$$(4) \ \text{ type } t_I \ \text{ interface } \{\overline{R}^n\} \in \overline{D}$$

$$(5) \ \text{for all type } t_{Sj} \ \text{ struct } \{\overline{f} \ u\} \in \overline{D} \ \text{with } \overline{D} \vdash_{\mathsf{iCons}} t_{Sj} <: u_I \leadsto E_j :$$

$$Cls_j = K_{t_{Sj}} \ Y' \to (E_j \ (K_{t_{Sj}} \ Y'))$$

$$\overline{D} \vdash_{\mathsf{iDestr}} t_I \searrow u_I \leadsto \lambda X. \text{case } X \ \text{of } K_{t_I} \ (Y, \overline{X}^n) \to \text{case } Y \ \text{of } \overline{[Cls]}$$

We establish statement (3) by distinguishing among the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate: Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} e.(u_I) \longrightarrow^{k'} v$ for some value v where k' < k.

- (7) $v = t_S\{\overline{v}\}$ for some \overline{v} where
- (8) $\overline{D} \vdash_{\mathsf{FG}} t_S <: u_I$
- via (6) and the FG reduction rules. (9) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'-1} t_S\{\overline{v}\}$
- via (6) and (7).
- (10) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^* V$ for some V where

(11)
$$v \equiv V \in [t_I]_{k-(k'-1)}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via reverse application of rule RED-REL-EXP on (2) where in the premise the right-hand side of the implication is satisfied via (9) and the fact that k'-1 < k.

- (12) $v \equiv V_{val} \in \llbracket t_S \rrbracket_{k''}^{\langle \overline{D}, \Phi_m \rangle}$ for any $k'' \leq k (k' 1)$ where (12) $V_{val} = K_{t_S}(\overline{V'})$ for some $\overline{V'}$ and (13) $V = K_{t_I}(V_{val}, \overline{Y_j}^n)$ for some $\overline{Y_j}^n$

via reverse application of rule RED-REL-IFACE on (11) where we make use of (7) and rule RED-REL-STRUCT to derive the shape of V_{val} and (4) to guarantee that there are n method variables Y_j .

- $\begin{array}{l} (14) \; e \approx V_{val} \in [\![t_S]\!]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle} \\ \mathrm{via} \; (9), \, (12) \; \mathrm{and} \; \mathrm{rule} \; \mathrm{RED\text{-}REL\text{-}EXP}. \end{array}$
- (15) $e \approx E' \ V_{val} \in [u_I]_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ where
- $(16) \ \overline{D} \vdash_{\mathsf{iCons}} t_S <: u_I \leadsto E'$
- via (8), (14) and Lemma 12.
- (17) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E' V_{val} \longrightarrow^* V'$ and
- (18) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} (\lambda X.\mathsf{case}\ X\ \mathsf{of}\ K_{t_I}(Y,\overline{X}^n) \to \mathsf{case}\ Y\ \mathsf{of}\ [\overline{\mathit{Cls}}])\ E_2 \longrightarrow^* V'\ \mathrm{for}$ some V'
 - via (4), (10), (13) and (16).
 - $(19) \ e \approx (\lambda X.\mathsf{case} \ X \ \mathsf{of} \ K_{t_I} \left(Y, \overline{X}^n \right) \to \mathsf{case} \ Y \ \mathsf{of} \ [\overline{\mathit{Cls}}]) \ E_2 \in \llbracket u_I \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ via (9), (17), (18) and Lemma 11.
 - $(20) \ e.(u_I) \approx (\lambda X.\mathsf{case}\ X \ \mathsf{of}\ K_{t_I}(Y,\overline{X}^n) \to \mathsf{case}\ Y \ \mathsf{of}\ [\overline{\mathit{Cls}}]) \ E_2 \in \llbracket u_I \rrbracket_k^{\langle \overline{D},\Phi_{\mathsf{m}} \rangle}$ via (6), (7), (9), (19) and the Monotonicity Lemma.

Thus, this subcase in statement (3) holds.

Subcase-Diverge: Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} e.(u_I) \longrightarrow^{k'} e'$ for some e' where k' < kand $\overline{D} \uparrow_{\mathsf{FG}} e'$.

- (7) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k'} e''$ for some e'' where $k'' \leq kA$ and $\overline{D} \uparrow_{\mathsf{FG}} e''$
- by observing the reduction (6).
- (8) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E_2$

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (8).

(9) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} (\lambda X.\mathsf{case}\ X\ \mathsf{of}\ K_{t_I}(Y,\overline{X}^n) \to \mathsf{case}\ Y\ \mathsf{of}\ [\overline{\mathit{Cls}}])\ E_2\ \mathrm{via}\ (7).$

Thus, we can establish this subcase in statement (3).

Subcase-Panic: Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} e.(u_I) \longrightarrow^{k'} e'$ for some e' where k' < kand panic_{EG} (\overline{D}, e') .

We distinguish among the following two cases. Either (1) the expression panics or (2) the type assertion fails.

Subcase-Panic-1: (7) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k''} e''$ for some e'' where k'' < k and $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e').$

Subcase-Panic-2: (8) $\overline{D} \vdash_{\mathsf{FG}} e \longrightarrow^{k''} u_S\{\overline{v'}\} \text{ for some } u_S\{\overline{v'}\} \text{ where } k'' < k$ and $\overline{D} \vdash_{\mathsf{FG}} u_S <: u_I \text{ does not hold.}$

Consider Subcase-Panic-1.

(9) $\mathsf{panic}_{\mathsf{TI}}\left(\Phi_{\mathsf{m}}, E_2\right)$

via reverse application of rule RED-REL-EXP on (2) where in the premise the left-hand side of the implication is satisfied via (7).

(10) $\mathsf{panic}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, (\lambda X.\mathsf{case}\ X\ \mathsf{of}\ K_{t_I}(Y, \overline{X}^n) \to \mathsf{case}\ Y\ \mathsf{of}\ [\overline{\mathit{Cls}}])\ E_2)\ \mathrm{via}\ (9)$ and we are done here.

Consider Subcase-Panic-2.

(11) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} E_2 \longrightarrow^* V$ for some V where

$$(12) \ u_S\{\overline{v'}\} \equiv V \in [t_I]_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

(12) $u_S\{\overline{v'}\} \equiv V \in [\![t_I]\!]_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ via reverse application of rule RED-REL-EXP on (2) where in the premise the right-hand side of the implication is satisfied via (8)

(13) $V_{val} = K_{u_S}(\overline{V'})$ for some $\overline{V'}$ and (14) $V = K_{t_I}(V_{val}, \overline{Y})$ for some method variables \overline{Y}

via reverse application of rule RED-REL-IFACE on (12) where we make use of RED-REL-STRUCT to derive the shape of V_{val} .

(15) panic_{TL}(
$$\Phi_m$$
, (λX .case X of $K_{t_I}(Y, \overline{X}^n) \to \text{case } Y$ of $[\overline{Cls}]$) E_2)

via the assumption that $\overline{D} \vdash_{\mathsf{FG}} u_S <: u_I \text{ does not hold and therefore none}$ of the pattern clauses \overline{Cls} will yield a match. Thus, we are done here.

Proof of Lemma 1

Proof. By induction over the derivation $\langle \overline{D}, \Gamma \rangle \vdash_{\sf exp} \underline{e} : t \leadsto E$. We label the assumptions (1) $\langle \overline{D}, \varPhi_{\sf m}, \Gamma \rangle \vdash_{\sf rr}^k \varPhi_{\sf v} \approx \varPhi_{\sf V}$ and (2) $\vdash_{\sf rr}^k \overline{D} \approx \varPhi_{\sf m}$ as well as the to be proven statement (3) $\varPhi_{\sf v}(e) \approx \varPhi_{\sf V}(E) \in [\![t]\!]_k^{\langle \overline{D}, \varPhi_{\sf m} \rangle}$ for some later reference. Case TD-VAR:

$$\frac{(x:t) \in \varGamma}{\langle \overline{D}, \varGamma \rangle \, \vdash_{\mathsf{exp}} \, x:t \leadsto X}$$

(3) follows immediately from (1).

Case TD-STRUCT:

$$\frac{\text{type } t_S \text{ struct } \{\overline{f_i \ t_i}n\} \in \overline{D} \qquad \langle \overline{D}, \Gamma \rangle \ \vdash_{\text{exp}} \ e_i : t_i \leadsto E_i \quad (\forall \, i \in [n])}{\langle \overline{D}, \Gamma \rangle \ \vdash_{\text{exp}} \ t_S \{\overline{e_i}^n\} : t_S \leadsto K_{t_S} \ (\overline{E_i}^n)}$$

(4) $\Phi_{\mathsf{v}}(e_i) \approx \Phi_{\mathsf{V}}(E_i) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ by induction.

To establish (3) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{v}}(E) \in [t]_{k}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ we consider the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate:

Suppose (5) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(t_S\{\overline{e_i}^n\}) \longrightarrow^{k'} t_S\{\overline{v_i}^n\}$ for some values v_i where (6)

- (7) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{V}}(e_i) \longrightarrow^{k_i} v_i \text{ for some } k_i \text{ where}$
- (8) $k_i < k' < k$

by observing the reduction (5) and the number of reduction steps taken (6).

- (9) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i) \longrightarrow^* V_i$ for some V_i where
- $(10) \ v_i \equiv V_i \in [t_i]_{k-k_i}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (4) where in the premise the left-hand side of the implication is satisfied via (7) and (8).

- (11) $k k' \le k k_i$ via (8).

(12) $v_i \equiv V_i \in [\![t_i]\!]_{k-k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ via (10), (11) and the Monotonicity Lemma.

 $(10) \ \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} K_{t_S} (\overline{\Phi_{\mathsf{V}}(E_i)}^n) \longrightarrow^* K_{t_S} (\overline{V_i}^n)$

via the reduction step (7).

$$(11) \ t_S\{\overline{e_i'}^n\} \equiv K_{t_S} \ (\overline{V_i}^n) \in [\![t_S]\!]_{k-k'}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via application of rule RED-REL-STRUCT on (9).

Via (10) and (11) we can establish this subcase in statement (3).

Subcase-Diverge:

Suppose (5) $\overline{\overline{D}} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(t_S\{\overline{e_i}^n\}) \longrightarrow^{k'} t_S\{\overline{e_i'}^n\}$ for some expressions $e_1', ..., e_n'$ where k' < k and $\overline{D} \uparrow_{\mathsf{FG}} t_S \{\overline{e'_i}^n\}$.

- (6) $\overline{D} \uparrow_{\mathsf{FG}} e'_{j}$ for some j where $e'_{1}, ..., e'_{j-1}$ are values via (5) and FG reduction rules
- (7) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_l) \longrightarrow^* V_l$ for some values V_l for l = 1, ..., j 1 via (4), (6) and rule RED-REL-EXP.
 - (8) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} E_i$ via (4), (6) and rule RED-REL-EXP.
 - (9) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} \Phi_{\mathsf{V}}(K_{t_S}(\overline{E_i}^n))$ via (7) and (8).
 - Via (9) we can establish this subcase in statement (3).

Subcase-Panic:

Suppose (5) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(t_S\{\overline{e_i}^n\}) \longrightarrow^{k'} t_S\{\overline{e_i'}^n\}$ for some $e_1', ..., e_n'$ where k' < 1 $k \text{ and } \mathsf{panic}_{\mathsf{FG}}(\overline{\overline{D}}, t_S\{\overline{e'_i}^n\}).$

- (6) $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e'_j)$ for some j where $e'_1, ..., e'_{j-1}$ are values via (5) and evalu-
- ation context $t_S\{e'_i, \dots, e'_{j-1}, \mathcal{E}, e_{j+1}, \dots, e_n\}$. (7) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_l) \longrightarrow^* V_l$ for some values V_l for $l = 1, \dots, j-1$ via (4), (6) and rule RED-REL-EXP.
 - (8) $panic_{TL}(\Phi_m, E)$ via (4), (6) and rule RED-REL-EXP.
 - (9) $\operatorname{\mathsf{panic}}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, \Phi_{\mathsf{V}}(K_{t_S}(\overline{E_i}^n))) \text{ via } (7) \text{ and } (8).$

Via (9) we can establish this subcase in statement (3).

Case TD-ACCESS:

$$\frac{\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t_S \leadsto E \qquad \mathsf{type} \ t_S \ \mathsf{struct} \ \{ \overline{f_j \ t_j}^n \} \in \overline{D}}{\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e. f_i : t_i \leadsto \mathsf{case} \ E \ \mathsf{of} \ K_{t_S} \ (\overline{X_i}^n) \to X_i}$$

(4)
$$\Phi_{\mathsf{V}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t \rrbracket_{\iota}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$
 by induction.

To establish (3) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{v}}(E) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ we consider the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate:

Suppose (5) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.f_i) \longrightarrow^{k'} v'$ for some value v' where (6) k' < k. (7) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} v$ for some v and k'' where

- (8) k'' < k'

by observing the reduction (5) and the number of reduction steps taken (6).

- (9) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^* V$ for some V where
- (10) $v \equiv V \in [t_S]_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (4) where the left-hand side of the implication is satisfied via (7) and (8).

- (11) $v = t_S\{\overline{v_i}^n\}$ and
- (12) $V = K_{t_S}(\overline{V_j}^n)$ for some v_j and V_j where (13) $v_j \equiv V_j \in \llbracket t_j \rrbracket_{k-k''}^{\langle \overline{D}, \Phi_m \rangle}$ for j = 1, ..., n via reverse application of rule RED-REL-STRUCT on (10).

(14) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(\mathsf{case}\ E\ \mathsf{of}\ K_{t_S}\ (\overline{X_j}^n) \to X_i) \stackrel{\cdot}{\longrightarrow}^* V_i$

via reduction step (9) and (12).

 $(15) v' = v_i$

via reduction step (5) and (11).

(16) $v' \equiv V_i \in [t_j]_{k-k'}^{\langle \overline{D}, \Phi_m \rangle}$

via (13), (15) and the Monotonicity Lemma as we have that $k - k' \le k - k''$.

Via (14) and (16) we can establish this subcase in statement (3).

Subcase-Diverge:

Suppose (5) $\overline{\overline{D}} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.f_i) \longrightarrow^{k'} e'$ for some e' where k' < k and $\overline{D} \uparrow_{\mathsf{FG}} e'$.

- (6) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k'} e'$ and $\overline{D} \uparrow_{\mathsf{FG}} e'$ via (5) and the FG reduction rules.
- (7) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} \Phi_{\mathsf{V}}(E)$ via (4), (6) and rule RED-REL-EXP.
- (8) $\Phi_{\mathsf{m}} \Uparrow_{\mathsf{TL}} \Phi_{\mathsf{V}}(\mathsf{case}\ E\ \mathsf{of}\ K_{t_S}\ (\overline{X_j}^n) \to X_i) \ \mathrm{via}\ (7)$

Via (8) we can establish this subcase in statement (3).

Subcase-Panic:

Suppose (5) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.f_i) \longrightarrow^{k'} e'$ for some e' where k' < k and $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e')$. (6) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k'} e'$ and $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e')$ via (5) and the FG reduction

- and panic rules.
 - (7) $\mathsf{panic}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, \Phi_{\mathsf{V}}(E))$ via (4), (6) and rule RED-REL-EXP.
 - (8) $\operatorname{\mathsf{panic}}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, \Phi_{\mathsf{V}}(\mathsf{case}\ E\ \mathsf{of}\ K_{t_S}\ (\overline{X_j}^n) \to X_i))\ \mathrm{via}\ (7).$

Via (8) we can establish this subcase in statement (3).

Case TD-CALL-STRUCT:

$$\frac{m(\overline{x_i\ t_i}^n)\ t\in \mathsf{methods}(\overline{D},t_S)}{\langle \overline{D},\Gamma\rangle \vdash_{\mathsf{exp}} e: t_S \leadsto E \quad \langle \overline{D},\Gamma\rangle \vdash_{\mathsf{exp}} e_i: t_i \leadsto E_i \quad (\forall\ i\in[n])}{\langle \overline{D},\Gamma\rangle \vdash_{\mathsf{exp}} e.m(\overline{e_i}^n): t \leadsto X_{m,t_S}\ E\ (\overline{E_i}^n)}$$

(4) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t_S \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ and (5) $\Phi_{\mathsf{v}}(e_i) \approx \Phi_{\mathsf{V}}(E_i) \in \llbracket t_i \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ by induction.

To establish (3) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ we consider the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate:

Suppose (6) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^{k'} v'$ for some value v' where (7) k' < k.

(8)
$$\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} v \text{ for some } v, k'' \text{ where }$$

(9)
$$k'' < k$$

by observing the reduction (6) and the number of steps taken (7).

(10)
$$\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k_i} v_i \text{ for some } v_i, k_i \text{ where}$$

$$(11) \sum_{i} k_{i} < k'$$

by observing the reduction (6) and the number of steps taken (7).

(12)
$$\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^* V$$
 for some V where

(13)
$$v \equiv V \in [t_S]_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via reverse application of rule RED-REL-EXP on (4) where in the premise the left-hand side of the implication is satisfied via (8) and (9).

(14)
$$\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i) \longrightarrow^* V_i$$
 for some V_i where

$$(15) \ v_i \equiv V_i \in [t_i]_{k-k_i}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via reverse application of rule RED-REL-EXP on (5) where in the premise the left-hand side of the implication is satisfied via (10) and (11).

(16) Set
$$k''' = min(k - k'', k - \sum_{i} k_{i}) - 1$$
 where

(17)
$$0 \le k''' < k$$
 and

(18)
$$k''' < k - k''$$
 and $k''' < k - k_i$

via
$$(7)$$
, (9) and (11) .

(19)
$$v \equiv V \in [t_S]_{k'''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via (13), (18) and the Monotonicity Lemma.

(20)
$$v_i \equiv V_i \in [t_i]_{k'''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via (15), (18) and the Monotonicity Lemma.

$$(21) \ \mathbf{func} \ (x \ t_S) \ mM \{\mathbf{return} \ e''\} \approx X_{m,t_S} \in [\![mM]\!]_k^{\langle \overline{D},\Phi_{\mathsf{m}}\rangle}$$

via reverse application of rule RED-REL-DECLS on (2).

$$(22)\ \langle x\mapsto v, \overline{x_i\mapsto v_i}^n\rangle e''\approx X_{m,t_S}\ V\ (\overline{V_i}^n)\in [\![t]\!]_{k'''}^{\langle\overline{D},\Phi_{\mathsf{m}}\rangle}$$

via reverse application of rule RED-REL-METHOD on (21) where in the premise the left-hand side of the implication is satisfied via (18), (19) and (20).

$$(23) \ v.m(\overline{v_i}^n) \approx (X_{m,t_S} \ V) \ (\overline{V_i}^n) \in \llbracket t \rrbracket_{k^{\prime\prime\prime}+1}^{\sqrt{\overline{D}}, \varPhi_m \rangle}$$

via (22) and Lemma 9.

$$(24) \ \overline{D} \vdash_{\mathsf{FG}} v.m(\overline{v_i}^n) \longrightarrow^{k'-(k''+\sum_i k_i)} v'$$

via reduction steps (6), (8) and (10).

(25)
$$k''' + 1 > k' - (k'' + \sum_{i} k_i)$$

via the following reasoning.

$$k''' + 1$$

$$= min(k - k'', k - \sum_{i} k_{i})$$
 by definition
$$= k - max(k'', \sum_{i} k_{i})$$
 by min/max distributivity law
$$\geq k - (k'' + \sum_{i} k_{i})$$
 by max approximation
$$> k' - (k'' + \sum_{i} k_{i})$$
 by $k' < k$

(26) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} X_{m,t_S} V(\overline{V_i}^n) \longrightarrow^* V'$ for some V' where (27) $v' \equiv V' \in \llbracket t \rrbracket_{k'''+1-(k'-(k''+\sum_i k_i))}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

$$(27) \ v' \equiv V' \in \llbracket t \rrbracket_{k'''+1-(k'-(k''+\sum_i k_i))}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via reverse application of rule RED-REL-EXP on (23) where in the premise the left-hand side of the implication is satisfied via (24) and (25).

(28)
$$k - k' \le k''' + 1 - (k' - (k'' + \sum_i k_i))$$
 via the following reasoning.

$$k'''+1-(k'-(k''+\sum_i k_i))\\ = \min(k-k'',k-\sum_i k_i)-(k'-(k''+\sum_i k_i)) \qquad \text{by definition}\\ = k-\max(k'',\sum_i k_i)-(k'-(k''+\sum_i k_i)) \qquad \text{by } \min/\max \text{ distributivity law}\\ = k-k'+k''+\sum_i k_i-\max(k'',\sum_i k_i) \qquad \text{by equivalence}\\ \geq k-k' \qquad \text{by approximation}$$

(29) $v' \equiv V' \in \llbracket t \rrbracket_{k-k'}^{\langle \overline{D}, \Phi_m \rangle}$ via (27), (28) and the Monotonicity Lemma.

 $(30) \ \Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(X_{m,t_S} \ E \ (\overline{E_i}^n)) \longrightarrow^* V'$

via reduction steps (12), (14) and (24).

Via (29) and (30) we can establish this subcase in statement (3).

Subcase-Diverge: Suppose (5) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^{k'} e'$ for some e' where $k' < k \text{ and } \overline{D} \uparrow_{\mathsf{FG}} e'.$

We distinguish among the following three cases. Either the (1) expression on which the method call is performed diverges or (2) one of the arguments diverges or (3) reduction of the method call leads to some expression that diverges.

Subcase-Diverge-1:
$$e' = e''.m(...)$$
 where (6) $\overline{D} \uparrow_{\mathsf{FG}} e''$ and $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} e''$ and $k'' < k'$.

Subcase-Diverge-2: $e' = v.m(v_1, ..., v_{i-1}, e'', ...)$ for some j where

- (7) $\overline{D} \uparrow_{\mathsf{FG}} e''$ and
- (8) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} v \text{ and } k'' < k' \text{ and}$ (9) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_l) \longrightarrow^{k_j} v_j \text{ and } k_l < k' \text{ for } l = 1, ..., j 1 \text{ and}$ (10) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_j) \longrightarrow^{k'''} e'' \text{ and } k''' < k'.$

Subcase-Diverge-3: (11)
$$\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} v \text{ and } k'' < k' \text{ and } (12) \overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k_i} v_i \text{ and } k_i < k' \text{ and } (13) \overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^{k''+\sum_i k_i} v.m(\overline{v_i}^n) \longrightarrow^{k''''} e' \text{ where } (14) \ k' = k'' + \sum_i k_i + k''''.$$

(14)
$$k' = k'' + \sum_{i} k_i + k''''$$

Consider Subcase-Diverge-1.

- (15) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} \Phi_{\mathsf{V}}(E)$ via (4), (6) and rule RED-REL-EXP.
- (16) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} \Phi_{\mathsf{V}}(X_{m,t_S} E(\overline{E_i}^n))$ via (15) and we are done here.

Consider Subcase-Diverge-2.

- (17) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^* V$ for some V via (4), (8) and rule RED-REL-EXP.
- (18) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_l) \longrightarrow^* V_l$ for some V_l via (4), (9) and rule RED-REL-EXP for l = 1, ..., j - 1.
 - (19) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i)$ via (4), (7), (1) and rule RED-REL-EXP.
 - (20) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} X_{m,t_S} E(\overline{E_i}^n)$ via (17), (18) and (19) and we are done here.

Consider Subcase-Diverge-3.

- (20) $v.m(\overline{v_i}^n) \approx (X_{m,t_S} V) (\overline{V_i}^n) \in [\![t]\!]_{k'''+1}^{\langle \overline{D}, \Phi_m \rangle}$ via the exact same reasoning steps that lead to (23) as found in the first subcase. (21) Recall k''' = min(k 1) $k'', k - \sum_i k_i) - 1.$
 - (22) Recall $k''' + 1 > k' (k'' \sum_{i} k_i) = k''''$.
- (23) $\Phi_{\mathsf{m}} \uparrow_{\mathsf{TL}} X_{m,t_S} E(\overline{E_i}^n)$ via (13), (20), (22) and rule RED-REL-EXP and we are done here.

Subcase-Panic: Suppose (5) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^{k'} e'$ for some e' where k' < k and panic_{FG} (\overline{D}, e') .

We distinguish among the following three cases. Either (1) the expression on which the method call is performed panics or (2) one of the arguments panics or (3) reduction of the method call leads to some expression that panics.

Subcase-Panic-1:
$$e' = e''.m(...)$$
 where (6) $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e'')$ and $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} e''$ and $k'' < k'.$

Subcase-Panic-2: $e' = v.m(v_1, ..., v_{j-1}, e'', ...)$ for some j where

- (7) $\mathsf{panic}_{\mathsf{FG}}(\overline{D}, e'')$ and
- (8) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_l) \longrightarrow^{k''} v \text{ and } k'' < k' \text{ and}$ (9) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_l) \longrightarrow^{k_j} v_j \text{ and } k_l < k' \text{ for } l = 1, ..., j 1 \text{ and}$ (10) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_j) \longrightarrow^{k'''} e'' \text{ and } k''' < k'.$

- (12) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k_i} v_i \text{ and } k_i < k' \text{ and}$ (13) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^{k'' + \sum_i k_i} v.m(\overline{v_i}^n) \longrightarrow^{k'''} e' \text{ where}$ (14) $k' = k'' + \sum_i k_i + k''''$.

Consider Subcase-Panic-1.

- (15) $\mathsf{panic}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, \Phi_{\mathsf{V}}(E))$ via (4), (6) and rule RED-REL-EXP.
- (16) $\mathsf{panic}_{\mathsf{TI}}(\Phi_{\mathsf{m}}, \Phi_{\mathsf{V}}(X_{m,t_S} E(\overline{E_i}^n)))$ via (15) and we are done here.

Consider Subcase-Panic-2.

- (17) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^* V$ for some V via (4), (8) and rule RED-REL-EXP.
- (18) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_l) \longrightarrow^* V_l$ for some V_l via (4), (9) and rule RED-REL-EXP for l = 1, ..., j - 1.
 - (19) $\mathsf{panic}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, \Phi_{\mathsf{V}}(E_j))$ via (4), (7), (1) and rule RED-REL-EXP.
- (20) $\mathsf{panic}_{\mathsf{TI}}(\Phi_{\mathsf{m}}, X_{m,t_S} E(\overline{E_i}^n))$ via (17), (18) and (19) and we are done here.

- Consider Subcase-Panic-3. (20) $v.m(\overline{v_i}^n) \approx (X_{m,t_S} \ V) \ (\overline{V_i}^n) \in \llbracket t \rrbracket_{k'''+1}^{\langle \overline{D}, \Phi_m \rangle}$ via the exact same reasoning steps that lead to (23) as found in the first subcase. (21) Recall k''' = min(k-1) $k'', k - \sum_{i} k_{i} - 1.$
 - (22) Recall $k''' + 1 > k' (k'' \sum_{i} k_i) = k''''$.
- (23) $\operatorname{\mathsf{panic}}_{\mathsf{TL}}(\Phi_{\mathsf{m}}, X_{m,t_S} \ E \ (\overline{E_i}^n))$ via (13), (20), (22) and rule RED-REL-EXP and we are done here.

Case TD-CALL-IFACE:

$$\begin{array}{c} \langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e: t_I \leadsto E \\ (4) \ \mathsf{type} \ t_I \ \mathsf{interface} \ \{ \overline{S_i}^q \} \in \overline{D} \qquad (5) \ S_j = m(\overline{x_i \ t_i}^n) \ t \quad (\mathsf{for some} \ j \in [q]) \\ \hline \langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e_i : t_i \leadsto E_i \quad (\forall i \in [n]) \\ \hline \langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e.m(\overline{e_i}^n) : t \leadsto \mathsf{case} \quad E \ \mathsf{of} \ K_{t_I} \ (X_{val}, \overline{X_i}^q) \to X_j \ X_{val} \ (\overline{E_i}^n) \end{array}$$

(6)
$$\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t_I \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$
 and (7) $\Phi_{\mathsf{v}}(e_i) \approx \Phi_{\mathsf{V}}(E_i) \in \llbracket t_i \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ by induction.

To establish (3) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$ we consider the subcases that arise in rule RED-REL-EXP.

Subcase-Terminate:

Suppose (8) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e.m(\overline{e_i}^n)) \longrightarrow^{k'} v'$ for some value v' where (9) k' < k. (10) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e) \longrightarrow^{k''} v$ for some v, k'' where

- (11) k'' < k'

by observing the reduction (8) and the number of steps taken (9).

- (12) $\overline{D} \vdash_{\mathsf{FG}} \Phi_{\mathsf{v}}(e_i) \longrightarrow^{k_i} v_i \text{ for some } v_i, k_i \text{ where}$
- $(13) \sum_{i} k_{i} < k'$

by observing the reduction (8) and the number of steps taken (9).

- (14) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E) \longrightarrow^* V$ for some V where
- (15) $v \equiv V \in [t_I]_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (6) where in the premise the left-hand side of the implication is satisfied via (10) and (11).

- (16) $\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} \Phi_{\mathsf{V}}(E_i) \xrightarrow{*} V_i \text{ for some } V_i \text{ where}$ (17) $v_i \equiv V_i \in \llbracket t_i \rrbracket_{k-k_i}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

via reverse application of rule RED-REL-EXP on (7) where in the premise the left-hand side of the implication is satisfied via (12) and (13).

- (18) $\operatorname{methods}(\overline{D}, t_I) = \{\overline{S}^q\}$
- via (4).
- (19) $V_{val} = K_{u_S} (\overline{V'})$ and

(20)
$$V = K_{t_l} (V_{val}, \overline{Y_l}^q)$$
 for some $u_S, \overline{V'}, \overline{Y_l}^q$ and

(20)
$$V=K_{t_I}$$
 $(V_{val},\overline{Y_l}^q)$ for some $u_S,\overline{V'},\overline{Y_l}^q$ and (21) methodLookup $(\overline{D},(m,u_S))=$ func $(x\ u_S)\ m(\overline{x_i\ t_i}^n)\ t$ {return e' } and

(22)
$$v \equiv V_{val} \in \llbracket u_S \rrbracket_{k-k''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$
 and

(23) func
$$(x \ u_S) \ m(\overline{x_i \ t_i}^n) \ t \ \{\text{return } e'\} \approx Y_j \in [\![m(\overline{x_i \ t_i}^n) \ t]\!]_{k-k''}^{\langle \overline{D}, \Phi_m \rangle}$$

via reverse application of rule RED-REL-IFACE on (15) where we assume (18) and (5). Index k - k'' is the largest index that satisfies the logical relations in the premise of rule RED-REL-IFACE.

(24) Set
$$k''' = min(k - k'', k - \sum_{i} k_i) - 1$$
 where

$$(25) 0 \le k''' < k \text{ and }$$

(26)
$$k''' < k - k''$$
 and $k''' < k - k_i$

via
$$(9)$$
, (11) and (13) .

(27)
$$v_i \equiv V_i \in [t_i]_{k'''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via (9), (11) and (13). (27) $v_i \equiv V_i \in [t_i]_{k'''}^{\langle \overline{D}, \Phi_{\mathfrak{m}} \rangle}$ via (17), (26) and the Monotonicity Lemma.

(28)
$$v \equiv V_{val} \in [\![u_S]\!]_{k'''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via (22), (26) and the Monotonicity Lemma.

$$(29) \langle x \mapsto v, \overline{x_i} \mapsto \overline{v_i}^n \rangle \approx Y_j \ V_{val} \ (\overline{V_i}^n) \in \llbracket t \rrbracket_{k'''}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$$

via reverse application of rule RED-REL-METHOD on (23) where in the premise the left-hand side of the implication is satisfied via (26), (27) and (28).

$$\begin{array}{l} (30) \ v.m(\overline{v_i}^n) \approx V_j' \ V_{val} \ (\overline{V_i}^n) \in [\![t]\!]_{k^{\prime\prime\prime}+1}^{\langle \overline{D}, \varPhi_m \rangle} \\ \mathrm{via} \ (29) \ \mathrm{and} \ \mathrm{Lemma} \ 9. \end{array}$$

(31)
$$\overline{D} \vdash_{\mathsf{FG}} v.m(\overline{v_i}^n) \longrightarrow^{k'-(k''+\sum_i k_i)} v'$$

via reduction steps (8), (10) and (12). (32) $k' - (k'' + \sum_i k_i) < k''' + 1$

(32)
$$k' - (k'' + \sum_{i} k_i) < k''' + 1$$

via the following reasoning.

$$\begin{array}{ll} k'''+1 \\ = \min(k-k'',k-\sum_i k_i) & \text{by definition} \\ = k - \max(k'',\sum_i k_i) & \text{by } \min/\max \text{ distributivity law} \\ \geq k - (k''+\sum_i k_i) & \text{by } \max \text{ approximation} \\ > k' - (k''+\sum_i k_i) & \text{by } k' < k \end{array}$$

(33)
$$\Phi_{\mathsf{m}} \vdash_{\mathsf{TL}} Y_j \ V_{val} \left(\overline{V_i}^n \right) \longrightarrow^* V' \text{ for some } V' \text{ where}$$
(34) $v' \approx V' \in \llbracket t \rrbracket_{k'''+1-(k'-(k''+\sum_i k_i))}^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$

(34)
$$v' \approx V' \in [t]_{k'''+1-(k'-(k''+\sum_i k_i))}^{\langle \overline{D}, \Phi_m \rangle}$$

via reverse application of rule RED-REL-EXP on (30) where in the premise the left-hand side of the implication is satisfied via (31) and (32).

(35)
$$k - k' \le k''' + 1 - (k' - (k'' + \sum_{i} k_i))$$

via the following reasoning.

$$k'''+1-(k'-(k''+\sum_i k_i))\\ = \min(k-k'',k-\sum_i k_i)-(k'-(k''+\sum_i k_i)) \qquad \text{by definition}\\ = k-\max(k'',\sum_i k_i)-(k'-(k''+\sum_i k_i)) \qquad \text{by } \min/\max \text{ distributivity law}\\ = (k-k')+(k''+\sum_i k_i)-\max(k'',\sum_i k_i) \qquad \text{by equivalence}\\ \geq k-k' \qquad \text{by approximation}$$

(36) $v' \approx V' \in \llbracket t \rrbracket_{k-k'}^{\sqrt{\overline{D}},\Phi_m\rangle}$ via (34), (35) and the Monotonicity Lemma.

 $(37) \ \varPhi_{\mathsf{m}} \vdash_{\mathsf{TL}} \varPhi_{\mathsf{V}}(\mathsf{case} \ E \ \mathsf{of} \ K_{t_I} \ (X_{val}, \overline{X_i}^q) \to X_j \ X_{val} \ (\overline{E_i}^n)) \longrightarrow^* V'$ via reduction steps (14), (20) and (33).

Via (36) and (37) we can establish this subcase in statement (3).

Subcase-Diverge and Subcase-Panic are left out as the reasoning is very close to the reasoning for case TD-CALL-STRUCT.

Case TD-SUB:

$$\frac{\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : t \leadsto E_2 \qquad \overline{D} \vdash_{\mathsf{iCons}} t <: u \leadsto E_1}{\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : u \leadsto E_1 \ E_2}$$

By induction we obtain that (4) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E_2) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$. From (3), (4) and Lemma 12 we obtain that $\Phi_{\mathsf{v}}(e) \approx E_1 \; \Phi_{\mathsf{V}}(E_2) \in \llbracket u \rrbracket_{\mathsf{v}}^{2}(\overline{D}, \Phi_{\mathsf{m}})$.

We have that $\Phi_{V}(E_1) = E_1$ and thus we are done for this case.

Case TD-ASSERT:

$$\frac{\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e : u \leadsto E_2 \quad \overline{D} \vdash_{\mathsf{iDestr}} u \searrow t \leadsto E_1}{\langle \overline{D}, \Gamma \rangle \vdash_{\mathsf{exp}} e.(t) : t \leadsto E_1 \ E_2}$$

By induction we obtain that (4) $\Phi_{\mathsf{v}}(e) \approx \Phi_{\mathsf{V}}(E_2) \in \llbracket u \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$. From (3), (4) and Lemma 13 we obtain that $\Phi_{\mathsf{v}}(e).(t) \approx E_1 \Phi_{\mathsf{V}}(E_2) \in \llbracket t \rrbracket_k^{\langle \overline{D}, \Phi_{\mathsf{m}} \rangle}$.

We have that $\Phi_{V}(E_1) = E_1$ and thus we are done for this case.