Attaining Class-level Forgetting in Pretrained Model using Few Samples

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Abstract

In order to address real-world problems, deep learning models are jointly trained on many classes. However, in the future, some classes may become restricted due to privacy/ethical concerns, and the restricted class knowledge has to be removed from the models that have been trained on them. The available data may also be limited due to privacy/ethical concerns, and re-training the model will not be possible. We propose a novel approach to address this problem without affecting the model's prediction power for the remaining classes. Our approach identifies the model parameters that are highly relevant to the restricted classes and removes the knowledge regarding the restricted classes from them using the limited available training data. Our approach is significantly faster and performs similar to the model re-trained on the complete data of the remaining classes.

1. Introduction

There are several real-world problems in which deep learning models have exceeded human-level performance. This has led to a wide deployment of deep learning models. Deep learning models generally train jointly on a number of categories/classes of data. However, the use of some of these classes may get restricted in the future (restricted classes), and a model with the capability to identify these classes may violate legal/privacy concerns. Individuals and organizations are becoming increasingly aware of these issues leading to an increasing number of legal cases on privacy issues in recent years. In such situations, the model has to be stripped of its capability to identify these categories (Class-level Forgetting). Due to legal/privacy concerns, the available training data may also be limited. In such situations, the problem becomes even more difficult to solve in the absence of the full training data. Real world problems such as incremental and federated learning also suffer from this problem as discussed in Sec. 3. We present a "Restricted Category Removal from

Model Representations with Limited Data" (RCRMR-LD) problem setting that simulates the above problem. In this paper, we propose to solve this problem in a fast and efficient manner.

The objective of the RCRMR-LD problem is to remove the information regarding the restricted classes from the network representations of all layers using the limited training data available without affecting the ability of the model to identify the remaining classes. If we have access to the full training data, then we can simply exclude the restricted class examples from the training data and perform a full training of the model from scratch using the abundant data (FDR full data retraining). However, the RCRMR-LD problem setting is based on the scenario that the directive to exclude the restricted classes is received in the future after the model has already been trained on the full data and now only a limited amount of training data is available to carry out this process. Since only limited training data is available in our RCRMR-LD problem setting, the FDR model violates our problem setting and is therefore, not a solution to our RCRMR-LD problem setting. Simply training the network from scratch on only the limited training data of the remaining classes will result in severe overfitting and significantly affect the model performance (Baseline 2, as shown in Table 1).

Another possible solution to this problem is to remove the weights of the fully-connected classification layer of the network corresponding to the excluded classes such that it can no longer classify the excluded classes. However, this approach suffers from a serious problem. Since, in this approach, we only remove some of the weights of the classification layer and the rest of the model remains unchanged, the model still contains the information required for recognizing the excluded classes. This information can be easily accessed through the features that the model extracts from the images and, therefore, we can use these features for performing classification. In this paper, we use a nearest prototype-based classifier to demonstrate that the model features still contain information regarding the restricted classes. Specifically, we use the model features of the examples from the limited training data to compute the average class prototype for each class and create a nearest class prototype-based classifier using

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them. Next, for any given test image, we extract its features using the model and then find the class prototype closest to the given test image. This nearest class prototype-based classifier performs close to the original fully-connected classifier on the excluded classes as shown in Table 1 (Baseline 1). Therefore, even after using this approach, the resulting model still contains information regarding the restricted classes. Another possible approach can be to apply the standard fine-tuning approach to the model using the limited available training data of the remaining classes (Baseline 8). However, fine-tuning on such limited training data is not able to sufficiently remove the restricted class information from the model representations (see Table 1), and aggressive fine-tuning on the limited training data may result in overfitting.

Considering the problems faced by the naive approaches mentioned above, we propose a novel "Efficient Removal with Preservation" (ERwP) approach to address the RCRMR-LD problem. First, we propose a novel technique to identify the model parameters that are highly relevant to the restricted classes, and to the best of our knowledge, there are no existing prior works for finding such class-specific relevant parameters. Next, we propose a novel technique that optimizes the model on the limited available training data in such a way that the restricted class information is discarded from the restricted class relevant parameters, and these parameters are reused for the remaining classes.

To the best of our knowledge, this is the first work that addresses the RCRMR-LD problem. We also propose several baseline approaches for this problem (see Sec. 5.1). Our proposed approach significantly outperforms all the proposed baseline approaches. Our proposed approach requires very few epochs to address the RCRMR-LD problem and is, therefore, very fast ($\sim 200 \times$ on ImageNet) and efficient. The model obtained after applying our approach forgets the excluded classes to such an extent that it behaves as though it was never trained on examples from the excluded classes. The performance of our model is very similar to the full data retraining (FDR) model (see Sec. 7.1 in the main paper and Fig. 5 in the Appendix). We also propose the performance metrics needed to evaluate the performance of any approach for the RCRMR-LD problem.

2. Problem Setting

In this work, we present the restricted category removal from model representations with limited data (RCRMR-LD) problem setting, in which a deep learning model M_o trained on a specific dataset has to be modified to exclude information regarding a set of restricted/excluded classes from all layers of the deep learning model without affecting its identification power for the remaining classes (see Fig. 1). The classes that need to be excluded are referred to as the restricted/excluded classes. Let $\{C_1^e, C_2^e, ..., C_{N_e}^e\}$ be the restricted/excluded classes, where N_e refers to the number of excluded classes. The remaining classes of the dataset are the remaining/non-excluded classes. Let $\{C_1^{ne}, C_2^{ne}, ..., C_{N_{ne}}^{ne}\}$ be the non-excluded classes, where N_{ne} refers to the number of remaining/non-excluded classes. Additionally, we only have access to a limited amount of training data for the restricted classes and the remaining classes, for carrying out this process. Therefore, any approach for addressing this problem can only utilize this limited training data.

3. RCRMR-LD Problem in Real World Scenarios

A real-world scenario where our proposed RCRMR-LD problem can arise is the incremental learning setting [25, 17], where the model receives training data in the form of sequentially arriving tasks. Each task contains a new set of classes. During a training session t, the model receives the task t for training and cannot access the full data of the previous tasks. Instead, the model has access to very few exemplars of the classes in the previous tasks. Suppose before training a model on training session t, it is noticed that some classes from a previous task (< t) have to be removed from the model since those classes have become restricted due to privacy or ethical concerns. In this case, only a limited number of exemplars are available for all these previous classes (restricted and remaining). This demonstrates that the RCRMR-LD problem is present in the incremental learning setting. We experimentally demonstrate in Sec. 7.3, how our approach can address the RCRMR-LD problem in the incremental learning setting.

Let us consider another example. The EU GDPR laws require a data provider to remove information about an individual from a dataset upon that individual's request. In face recognition, this may lead to cases where the model has to be retrained from scratch, leaving out the training data for the restricted classes. In many such cases, it may be highly impractical and inefficient for the model creators to retrain the entire model from scratch. The RCRMR problem simulates this problem setting. Other examples of this problem include ethical AI concerns where protected classes (pregnant women, prisoners, children, etc.) need to be removed.

There can also be other real-world scenarios, such as federated learning [22], where our RCRMR-LD problem can arise. In the federated learning setting, there are multiple collaborators that have a part of the training data stored locally, and a model is trained collaboratively using these private data without sharing or collating the data due to privacy concerns. Suppose organization A has a part of the training data, and there are other collaborators that have other parts of the training data for the same classes. Organization A collaboratively trains a model with other collaborators using federated learning. After the model has been trained, a few classes may become restricted in the future due to some



Figure 1: The RCRMR-LD problem setting aims to remove the information regarding the restricted/excluded classes ({ $C_1^e, ..., C_{N_e}^e$ }) from all layers of a trained model M_o while preserving its predictive power for the remaining classes ({ $C_1^{ne}, ..., C_{N_{ne}}^{ne}$ }) using limited training data. The category removal (denoted by a red cross) has to take place at the classifier level (denoted as squares for each output logit) and at the feature/representation level (denoted as a circle)

Figure 2: ERwP identifies those parameters in the model that are highly relevant to the restricted classes. To obtain these parameters, ERwP modifies training images from a restricted class using a data augmentation f and performs backpropagation using the classification loss on these training images. ERwP then studies the gradient update that each parameter receives in this process in order to identify the highly relevant parameters for the restricted classes (denoted by dotted circles)

Figure 3: ERwP only optimizes the restricted class relevant parameters in the model (denoted by dotted circles). ERwP uses \mathcal{L}_c^e , \mathcal{L}_c^{ne} and \mathcal{L}_{kd} losses to remove the restricted class information from the model while preserving its performance on the remaining classes. \mathcal{L}_c^e and \mathcal{L}_c^{ne} denote the classification loss on the restricted class training examples and the remaining class training example, respectively. \mathcal{L}_{kd} denotes the knowledge distillation-based regularization loss that preserves the logits corresponding to only the remaining classes for all the training examples

ethical or privacy concerns, and these classes should be removed from the model. However, the other collaborators may not be available or may charge a huge amount of money for collaborating again to train a fresh model from scratch. In this case, organization A does not have access to the full training data of the non-excluded/remaining classes that it can use to re-train a model from scratch in order to exclude the restricted classes information. This clearly shows that the RCRMR-LD problem is present in federated learning.

4. Proposed Method

4.1. Method Description

Let, B refer to a mini-batch (of size S) from the available limited training data, and B contains training datapoints from the restricted/excluded classes $(\{(x_i^e, y_i^e) | (x_1^e, y_1^e), ..., (x_{S_e}^e, y_{S_e}^e)\})$ and remaining/non-excluded from the classes $(\{(x_j^{ne}, y_j^{ne}) | (x_1^{ne}, y_1^{ne}), ..., (x_{S_{ne}}^{ne}, y_{S_{ne}}^{ne})\})$. Here, (x_i^e, y_i^e) refers to a training datapoint from the excluded classes where x_i^e is an image, y_i^e is the corresponding label

and $y_i^e \in \{C_1^e, C_2^e, ..., C_{N_e}^e\}$. (x_j^{ne}, y_j^{ne}) refers to a training datapoint from the non-excluded classes where x_j^{ne} is an image, y_j^{ne} is the corresponding label and $y_j^{ne} \in \{C_1^{ne}, C_2^{ne}, ..., C_{N_{ne}}^{ne}\}$. Here, S_e and S_{ne} refer to the number of training examples in the mini-batch from the excluded and non-excluded classes, respectively, such that $S = S_e + S_{ne}$. N_e and N_{ne} refer to the number of excluded classes, respectively. Let M refer to the deep learning model being trained using our approach and M_o is the original trained deep learning model. In a trained model, some of the parameters may be highly

relevant to the restricted classes, and the performance of the model on the restricted classes is mainly dependent on such highly relevant parameters. Therefore, in our approach, we focus on removing the excluded class information from these restricted class relevant parameters. Since the model is trained on all the classes jointly, the parameters are shared across the different classes. Therefore identifying these classspecific relevant parameters is very difficult. Let us consider a model that is trained on color images of a class. If we now train it on grayscale images of the class, then the model has to learn to identify these new images. In order to do so, the parameters relevant to that class will receive large gradient updates as compared to the other parameters (see Sec. 9.3.1 in the Appendix). We propose a novel approach for identifying the relevant parameters for the restricted classes using this idea. For each restricted class, we choose the training images belonging to that class from the limited available training data. Next, we apply a grayscale data augmentation technique/transformation f to these images so that these images become different from the images that the original model was earlier trained on (assuming that the original model has not been trained on grayscale images). We can also use other data augmentation techniques that are not seen during the training process of the original model and that do not change the class of the image (refer to Sec. 9.3.6 in the Appendix). Next, we combine the predictions for each training image into a single average prediction and perform backpropagation. During the backpropagation, we study the gradients for all the parameters in each layer of the model. Accordingly, we select the parameters with the highest absolute gradient as the relevant parameters for the corresponding restricted class. Specifically, for a given restricted class, we choose all the parameters from each network layer such that pruning (zeroing out) these parameters will result in the maximum degradation of model performance on that restricted class. We provide a detailed description of the process for identifying the restricted class relevant parameters in Sec. 9.1 of the Appendix. The combined set of the relevant parameters for all the excluded classes is referred to as the restricted/excluded class relevant parameters Θ_{exrel} (see Fig. 2). Please note that we use this process only to identify Θ_{exrel} , and we do not update the model parameters during this step.

Pruning the relevant parameters for a restricted class can severely impact the performance of the model for that class (see Sec. 9.3.1 in the Appendix). However, this may also degrade the performance of the model on the non-excluded classes because the parameters are shared across multiple classes. Therefore, we cannot address the RCRMR-LD problem by pruning the relevant parameters of the excluded classes. Finetuning these parameters on the limited remaining class data will also not be able to sufficiently remove the restricted class information from the model. Based on this, we propose to address the RCRMR-LD problem by optimizing the relevant parameters of the restricted classes to remove the restricted class information from them and to reuse them for the remaining classes.

After identifying the restricted class relevant parameters, our ERwP approach uses a classification loss based on the cross-entropy loss function to optimize the restricted class relevant parameters of the model on each mini-batch (see Fig. 3). We know that the gradient ascent optimization algorithm can be used to maximize a loss function and encourage the model to perform badly on the given input. Therefore, we use the gradient ascent optimization on the classification loss for the limited restricted class training examples to remove the information regarding the restricted classes from Θ_{exrel} . We achieve this by multiplying the classification loss for the training examples from the excluded classes by a constant negative factor of -1. We also optimize Θ_{exrel} using the gradient descent optimization on the classification loss for the limited remaining class training example, in order to reuse these parameters for the remaining classes. We validate using this approach through various ablation experiments as shown in Sec. 9.3.2 in the Appendix. The classification loss for the examples from the excluded and non-excluded classes and the overall classification loss for each mini-batch are defined as follows.

$$\mathcal{L}_{c}^{e} = \sum_{i=1}^{S_{e}} -1 * \ell(y_{i}^{e}, y_{i}^{e*})$$
(1)

$$\mathcal{L}_{c}^{ne} = \sum_{j=1}^{S_{ne}} \ell(y_{j}^{ne}, y_{j}^{ne*})$$
(2)

$$\mathcal{L}_c = \frac{1}{S} (\mathcal{L}_c^e + \mathcal{L}_c^{ne}) \tag{3}$$

Where, y_i^{e*} and y_j^{ne*} refer to the predicted class labels for x_i^e and x_j^{ne} , respectively. $\ell(.,.)$ refers to the cross-entropy loss function. \mathcal{L}_c^e and \mathcal{L}_c^{ne} refer to the classification loss for the examples from the excluded and non-excluded classes in the mini-batch, respectively. \mathcal{L}_c refers to the overall classification loss for each mini-batch.

Since all the network parameters were jointly trained on all the classes (restricted and remaining), the restricted class relevant parameters also contain information relevant to the remaining classes. Applying the above process alone will still harm the model's predictive power for the non-excluded classes (as shown in Sec. 9.3.2, Table 5 in the Appendix). This is because the gradient ascent optimization strategy will also erase some of the relevant information regarding the remaining classes. Further, applying \mathcal{L}_{c}^{ne} on the limited training examples of the remaining classes will lead to overfitting and will not be effective enough to fully preserve the model performance on the remaining classes. In order to ensure that the model's predictive power for the non-excluded classes does not change, we use a knowledge distillation-based regularization loss. Knowledge distillation [15] ensures that the predictive power of the teacher network is replicated in the student network. In this problem setting, we want the final model to replicate the same predictive power of the original model for the remaining classes. Therefore, given any training example, we use the knowledge distillation-based regularization loss to ensure that the output logits produced by the model corresponding to only the non-excluded classes remain the same as that produced by the original model. We

apply the knowledge distillation loss to the limited training examples from both the excluded and remaining classes, to preserve the non-excluded class logits of the model for any input image. We validate this knowledge distillation-based regularization loss through ablation experiments as shown in Table 5 in the Appendix. We use the original model M_o (before applying ERwP) as the teacher network and the current model M being processed by ERwP as the student network, for the knowledge distillation process. Please note that the optimization for this loss is also carried out only for the restricted class relevant parameters of the model. Let KDrefer to the knowledge distillation loss function. It computes the Kullback-Liebler (KL) divergence between the soft predictions of the teacher and the student networks and can be defined as follows:

$$KD(p_s, p_t) = KL(\sigma(p_s), \sigma(p_t))$$
(4)

where, $\sigma(.)$ refers to the softmax activation function that converts logit a_i for each class i into a probability by comparing a_i with logits of other classes a_j , i.e., $\sigma(a_i) = \frac{exp^{a_i/\kappa}}{\sum_j exp^{a_j/\kappa}}$. κ refers to the temperature [15], KL refers to the KL-Divergence function. p_s, p_t refer to the logits produced by the student network and the teacher network, respectively.

The knowledge distillation-based regularization losses in our approach are defined as follows.

$$\mathcal{L}_{kd}^{e} = \sum_{i=1}^{S_{e}} KD(M(x_{i}^{e})[C^{ne}], M_{o}(x_{i}^{e})[C^{ne}])$$
(5)

$$\mathcal{L}_{kd}^{ne} = \sum_{j=1}^{S_{ne}} KD(M(x_j^{ne})[C^{ne}], M_o(x_j^{ne})[C^{ne}])$$
(6)

$$\mathcal{L}_{kd} = \frac{1}{S} \left(\mathcal{L}_{kd}^e + \mathcal{L}_{kd}^{ne} \right) \tag{7}$$

Where, $M(\#)[C^{ne}]$ and $M_o(\#)[C^{ne}]$ refer to the output logits corresponding to the remaining classes produced by M and M_o , respectively. # can be either x_i^e or x_j^{ne} . \mathcal{L}_{kd}^e and \mathcal{L}_{kd}^{ne} refer to knowledge distillation-based regularization loss for the examples from the excluded and non-excluded classes, respectively. \mathcal{L}_{kd} refers to the overall knowledge distillation-based regularization loss for each mini-batch. The \mathcal{L}_{kd}^{ne} loss helps in preserving the model performance for the non-excluded classes. If some of the restricted classes are similar to some of the remaining/non-excluded classes, the \mathcal{L}_{kd}^e loss ensures that the model performance on the remaining classes is not degraded due to this similarity. This is because the \mathcal{L}_{kd}^e loss preserves the logits corresponding to the non-excluded classes for the restricted class training examples. The total loss \mathcal{L}_{erwp} of our approach for each mini-batch is defined as follows.

$$\mathcal{L}_{erwp} = \mathcal{L}_c + \beta \mathcal{L}_{kd} \tag{8}$$

Where, β is a hyper-parameter that controls the contribution of the knowledge distillation-based regularization loss. We use this loss for fine-tuning the model for very few epochs.

5. Related Work

Pruning [1, 32, 11, 13] involves removing redundant and unimportant weights [2, 7, 9] or filters [12, 14, 19] from a deep learning model without affecting the model performance. Pruning approaches generally identify the important parameters in the network and remove the unimportant parameters. In the RCRMR-LD problem setting, the restricted class relevant parameters are also important parameters. However, we empirically observe that pruning the restricted class relevant parameters severely affects the model performance for the remaining classes since the parameters are shared among all the classes. Therefore, pruning approaches cannot be applied in the RCRMR-LD problem setting.

In the incremental learning setting [16, 31, 3, 20], the objective is to preserve the predictive power of the model for previously seen classes while learning a new set of classes. The work in [28] uses a topology-preserving loss to prevent catastrophic forgetting by maintaining the topology in feature space. In contrast to the incremental learning setting, our proposed RCRMR-LD problem setting involves removing the information regarding specific classes from the pre-trained model while preserving the predictive power of the model for the remaining classes.

There has been some research involving deleting individual data points from trained machine learning models such as [5, 6]. The work in [5] deals with data deletion in the context of a machine learning algorithm and model. It shows how to remove the influence of a data point from a k-means clustering model. Our work focuses on restricted category removal from deep learning models with limited data. Therefore, the approaches proposed in [5] cannot be applied to RCRMR-LD. Further, the objective of data deletion is to remove a data point without affecting the model performance on any classes, including the class of the deleted data point. This is in stark contrast to our RCRMR-LD problem, where the objective is to remove the knowledge of a set of classes or categories from the model. Further, data deletion methods will require access to the entire training data of a class in order to remove the entire knowledge of a class (refer to the appendix A.1. of [6]). This is because deep learning models have a high generalization power even on unseen examples of a class on which they have been trained, and simply deleting a few data points of a class from the knowledge base of the model will not be enough to forget that class. However, in our proposed problem setting, only a limited number of training examples are present for any class. Therefore, data-deletion approaches are not solutions to our proposed RCRMR-LD problem setting. This is why we have not applied these approaches in our problem setting.

Privacy-preserving deep learning [21, 4, 8] involves learning representations that incorporate features from the data relevant to the given task and ignore sensitive information (such as the identity of a person). The authors in [23] propose a simple variational approach for privacy-preserving representation learning. In contrast to existing privacy preservation works, the objective of the RCRMR-LD problem setting is to achieve class-level forgetting, i.e., if a class is declared as private/restricted, then all information about this class should be removed from the model trained on it, without affecting its ability to identify the remaining classes. To the best of our knowledge, this is the first work to address the class-level forgetting problem in the limited data regime, i.e., RCRMR-LD problem setting.

5.1. Baselines

We propose 9 baseline models for the RCRMR-LD problem and compare our proposed approach with them. The baseline 1 involves deleting the weights of the fully-connected classification layer corresponding to the excluded classes. Baselines 2, 3, 4, 5 involve training the model on the limited training data of the remaining classes. Baselines 6, 7, 8, 9 involve fine-tuning the model on the available limited training data. The details about the baselines are provided below:

Original model: It refers to the original model that is trained on the complete training set containing all the training examples from both the excluded and non-excluded classes. It represents the model that has not been modified by any technique to remove the excluded class information.

Baseline 1 - Weight Deletion (WD): It refers to the original model with a modified fully-connected classification layer. Specifically, we remove the weights corresponding to the excluded classes in the fully-connected classification layer so that it cannot classify the excluded classes.

Baseline 2 - Training from Scratch on Limited Non-Restricted Class data (TSLNRC): In this baseline, we train a new model from scratch using the limited training examples of only the non-excluded classes. It uses the complete training schedule as the original model and only uses the classification loss for training the model.

Baseline 3 - Training from Scratch on Limited Non-Restricted Class data with KD (TSLNRC-KD): This baseline is the same as baseline 2, but in addition to the classification loss, it also uses a knowledge distillation loss to ensure that the non-excluded class logits of the model (student) match that of the original model (teacher).

Baseline 4 - Training of Original model on Limited Non-Restricted Class data (TOLNRC): This baseline is the same as baseline 2, but the model is initialized with the weights of the original model instead of randomly initializing it.

Baseline 5 - Training of Original model on Limited Non-Restricted Class data with KD (TOLNRC-KD): This baseline is the same as baseline 4, but in addition to the classification loss, it also uses a knowledge distillation loss.

Baseline 6 - Fine-tuning of Original model on Limited data after Mapping Restricted Classes to a Single Class (FOLMRCSC): In this baseline approach, we first replace all the excluded class labels in the limited training data with a new single excluded class label and then fine-tune the original model for a few epochs on the limited training data of both the excluded and remaining classes. In the case of the examples from the excluded classes, the model is trained to predict the new single excluded class, the model is trained to predict the corresponding non-excluded classes.

Baseline 7 - Fine-tuning of Original model on Limited data after Mapping Restricted Classes to a Single Class with KD (FOLMRCSC-KD): This baseline is the same as baseline 6, but in addition to the classification loss, it also uses a knowledge distillation loss to ensure that the nonexcluded class logits of the model (student) match that of the original model (teacher).

Baseline 8 - Fine-tuning of Original model on Limited Non-Restricted Class data (FOLNRC): In this baseline approach, we fine-tune the original model for a few epochs on the limited training data of non-excluded/remaining classes. The model is trained to predict the corresponding non-excluded classes of the training examples.

Baseline 9 - Fine-tuning of Original model on Limited Non-Restricted Class data with KD (FOLNRC-KD): This baseline is the same as baseline 8, but in addition to the classification loss, it also uses a knowledge distillation loss.

6. Performance Metrics

In the RCRMR-LD problem setting, we propose three performance metrics to validate the performance of any method: forgetting accuracy (FA_e), forgetting prototype accuracy (FPA_e), and constraint accuracy (CA_{ne}). The forgetting accuracy refers to the fully-connected classification layer accuracy of the model for the excluded classes. The forgetting prototype accuracy refers to the nearest class prototypebased classifier accuracy of the model for the excluded classes. CA_{ne} refers to the fully-connected classification layer accuracy of the model for the non-excluded classes.

In order to judge any approach on the basis of these metrics, we follow the following sequence. First, we analyze the constraint accuracy (CA_{ne}) of the model produced by the

given approach to verify if the approach has preserved the prediction power of the model for the non-excluded classes. CA_{ne} of the model should be close to that of the original model. If this condition is not satisfied, then the approach is not suitable for this problem, and we need not analyze the other metrics. This is because if the constraint accuracy is not maintained, then the overall usability of the model is hurt significantly. Next, we analyze the forgetting accuracy (FA_{e}) of the model to verify if the excluded class information has been removed from the model at the classifier level. FA_e of the model should be as close to 0% as possible. Finally, we analyze the forgetting prototype accuracy (FPA_{e}) of the model to verify if the excluded class information has been removed from the model at the feature level. FPA_e of the model should be significantly less than that of the original model. However, the FPA_e will not become zero since any trained model will learn to extract meaningful features, which will help the nearest class prototype-based classifier to achieve some non-negligible accuracy even on the excluded classes. Therefore, for a better analysis of the level of forgetting of the excluded classes at the feature level, we compare the FPA_e of the model with the FPA_e of the FDR model. The FDR model is a good candidate for this analysis since it has not been trained on the excluded classes (only trained on the complete dataset of the remaining classes), and it still achieves a non-negligible performance of the excluded classes (see Sec 7.1). However, it should be noted that this comparison is only for analysis and the comparison is not fair since the FDR model needs to train on the entire dataset (except the excluded classes).

A naive approach for measuring the capability of any approach for removing the excluded class information in this problem setting is to only consider how low the forgetting accuracy (FAe) of the model for the excluded classes drops to after the excluded category removal process. However, using FAe alone may be misleading since zero or random forgetting accuracy (FAe) for a excluded class does not mean that the excluded class information has been removed from all layers of the model. In order to understand this point, let us consider the weight deletion (WD) baseline (baseline 1) that simply deletes the classification layer weights corresponding to the excluded classes and achieves a forgetting accuracy (FA_e) of 0% for the excluded classes. However, this does not mean that the excluded class information has been removed from all the layers of the network since the rest of the network remains intact. Therefore, using only (FA_e) metric is not enough. Now, if we consider the forgetting prototype accuracy (FPAe) of the WD model, we will observe that the ${\tt FPA}_{\rm e}$ of WD model is the same as that of the original model for the excluded classes. This clearly indicates that the excluded class information is still present in the layers of the network. Further, we also need to check whether the model performance for the remaining classes is

maintained. We use our proposed constraint accuracy (CA_{ne}) of the non-excluded classes for this purpose. Therefore, the above discussion clearly demonstrates that a single metric is not effective in this problem setting.

7. Experiments

7.0.1 Datasets

For the RCRMR-LD problem setting, we modify the CIFAR-100 [18], CUB-200 [29] and ImageNet-1k [26] datasets. In order to simulate the RCRMR-LD problem setting with limited training data, we choose the last 20 classes of the CIFAR-100 dataset as the excluded classes and take only 10% of the training images of each class. Similarly, we choose the last 20 classes of the CUB-200 dataset as the excluded classes with only 3 training images per class. For ImageNet-1K, we choose the last 100 classes as the excluded classes with 5% of the training images to simulate the limited data available for this problem setting.

7.0.2 Implementation Details

In this section, we provide all the details required to reproduce our experimental results. We use the ResNet-20 [10], ResNet-56, ResNet-164 architectures for the experiments on the CIFAR-100 dataset. We use the standard data augmentation methods of random cropping to a size of 32×32 (zero-padded on each side with four pixels before taking a random crop) and random horizontal flipping, which is a standard practice for training a model on CIFAR-100. In order to obtain the original and FDR models for the CIFAR-100 dataset, we train the network for 300 epochs with a mini-batch size of 64 using the stochastic gradient descent optimizer with momentum 0.9 and weight decay 1e - 4. We choose the initial learning rate as 0.1, and we decrease it by a factor of 5 after the 90, 150, 210, 240, and, 270 epochs. For the CIFAR-100 experiments with ERwP using the ResNet-20, ResNet-56, and ResNet-164 architectures, we use learning rate = 1e - 4, $\beta = 10$ and optimize the network for 10 epochs. Since the available limited training data is only 10% of the entire CIFAR-100 dataset, therefore, our ERwP approach is approximately $30 * 10 = 300 \times$ faster than the FDR method.

For the experiments on the ImageNet dataset, we use the ResNet-18, ResNet-50, and MobileNet-V2 architectures. We use the standard data augmentation methods of random cropping to a size of 224×224 and random horizontal flipping, which is a standard practice for training a model on ImageNet-1k. In order to obtain the original and FDR models for the ImageNet dataset, we train the network for 100 epochs with a mini-batch size of 256 using the stochastic gradient descent optimizer with momentum 0.9 and weight decay 1e - 4. We choose the initial learning rate as 0.1, and we decrease it by a factor of 10 after every 30 epochs. For evaluation, the validation images are subjected to center cropping of size 224×224 . For the ImageNet-1k experiments (5% training data) with ERwP using the ResNet-50 architecture, we optimize the network for 10 epochs with a learning rate of 9e - 5 using $\beta = 200$. For the ERwP experiments using the ResNet-18 architecture, we optimize the network for 10 epochs using $\beta = 200$ with an initial learning rate of 1.1e - 4 and a learning rate of 1.1e - 5 from the third epoch onward. In the case of the ERwP experiments with the MobileNet-V2 architecture, we optimize the network for 10 epochs using $\beta = 400$ with an initial learning rate of 1.5e - 4 and a learning rate of 1.5e - 5 from the third epoch onward. Since the available limited training data is only 5% of the entire ImageNet-1k dataset, therefore, our ERwP approach is approximately $20 * 10 = 200 \times$ faster than the FDR method. For the experiments on the CUB-200 dataset, we use the ResNet-50 architecture pre-trained on the ImageNet dataset. In order to obtain the original and FDR models for the CUB-200 dataset, we train the network for 50 epochs with a mini-batch size of 64 using the stochastic gradient descent optimizer with momentum 0.9 and weight decay 1e - 3. We choose the initial learning rate as 1e - 2, and we decrease it by a factor of 10 after epochs 30 and 40. For the CUB-200 experiments (3 images per class, i.e., 10%training data) with ERwP using the ResNet-50 architecture, we optimize the network for 10 epochs with a learning rate of 1e - 4 using $\beta = 10$. Since the available limited training data is only 10% of the entire CUB-200 dataset, therefore, our ERwP approach is approximately $5 * 10 = 50 \times$ faster than the FDR method.

In our proposed approach, we use $\kappa = 2$ for all the experiments (see Sec. 9.3.5 in the Appendix). We use a popular Pytorch implementation¹ for performing knowledge distillation. We run all the experiments 5 times and report the average accuracy. We perform all the experiments using the Pytorch [24] and Python 3.0. We use 4 GeForce GTX 1080 Ti graphics processing units for our experiments. We have discussed the experimental results for the CIFAR-100 and ImageNet-1k datasets in this page and the next page. We have also provided the results on the CUB-200 dataset in the Table 4 of the Appendix. We have provided the experimental results for the different components of our works in Sec. 9.3 of the Appendix.

7.1. CIFAR-100 Results

We report the performance of different baselines and our proposed ERwP method on the RCRMR-LD problem using the CIFAR-100 dataset with different architectures in Table 1. We observe that the baseline 1 (weight deletion) achieves high constraint accuracy CA_{ne} and 0% forgetting accuracy

FAe. But its forgetting prototype accuracy FPAe remains the same as the original model for all the three architectures, i.e., ResNet-20/56/164. Therefore, baseline 1 fails to remove the excluded class information from the model at the feature level. Baseline 2 is not able to preserve the constraint accuracy CAne even though it performs full training on the limited excluded class data. Baseline 3 achieves higher CAne than baseline 2, but the constraint accuracy is still too low. Baselines 4 and 5 demonstrate significantly better constraint accuracy than baseline 2 and 3, but their constraint accuracy is still significantly lower than the original model (except baseline 5 for ResNet-20). The baseline 5 with ResNet-20 maintains the constraint accuracy and achieves 0% forgetting accuracy FA_e but its FPA_e is still significantly high and, therefore, is unable to remove the excluded class information from the model at the feature level. The fine-tuning based baselines 6 and 7 are able to significantly reduce the forgetting accuracy FAe but their constraint accuracy CAne drops significantly. The fine-tuning based baselines 8 and 9 only finetune the model on the limited remaining class data and as a result they are not able to sufficiently reduce either the FA_{e} or the FPA_{e} .

Our proposed ERwP approach achieves a constraint accuracy CA_{ne} that is very close to the original model for all three architectures. It achieves close to 0% FA_e. Further, it achieves a significantly lower FPA_e than the original model. Specifically, the FPAe of our approach is significantly lower than that of the original model by absolute margins of 17.19%, 20.81%, and 20.17% for the ResNet-20, ResNet-56, and ResNet-164 architectures, respectively. The FPA_e for the FDR model is 44.20%, 45.40% and 51.85% for the ResNet-20, ResNet-56 and ResNet-164 architectures, respectively. Therefore, the FPA_e of our approach is close to that of the FDR model by absolute margins of 3.86%, 2.44% and 4.38% for the ResNet-20, ResNet-56 and ResNet-164 architectures, respectively. Therefore, our ERwP approach makes the model behave similar to the FDR model even though it was trained on only limited data from the excluded and remaining classes. Further, our ERwP requires only 10 epochs to remove the excluded class information from the model. Since the available limited training data is only 10% of the entire CIFAR-100 dataset, therefore, our ERwP approach is approximately $30 * 10 = 300 \times$ faster than the FDR method that is trained on the full training data for 300 epochs.

The FPA_e accuracy obtained using ERwP is significantly lower than the original model, e.g., for the ResNet-56 architecture FPA_e of ERwP is 47.84% compared to 68.65% of the original model for the CIFAR-100 dataset using the ResNet-56 model. However, this does not indicate the presence of much restricted category information. This is because the process for obtaining the FPA_e accuracy involves creating prototypes from the limited training data of the restricted

¹https://github.com/peterliht/knowledge-distillationpytorch/blob/master/model/net.py

Methods	ResNet-20		ResNet-56		ResNet-164				
	FAe	\mathtt{FPA}_{e}	$\operatorname{CA}_{\operatorname{ne}}$	FAe	\texttt{FPA}_{e}	$\operatorname{CA}_{\operatorname{ne}}$	FAe	FPAe	CA_{ne}
Original	70.15%	65.25%	67.06%	70.80%	68.65%	69.88%	79.00%	76.40%	76.30%
No Training									
Baseline 1 - WD	0.00%	65.25%	69.88%	0.00%	68.65%	72.44%	0.00%	76.40%	78.23%
Full Train Schedule									
Baseline 2 - TSLNRC	0.00%	22.20%	31.55%	0.00%	20.20%	30.21%	0.00%	33.05%	40.65%
Baseline 3 - TSLNRC-KD	0.00%	27.55%	40.81%	0.00%	22.50%	32.26%	0.00%	38.55%	45.74%
Baseline 4 - TOLNRC	0.00%	50.85%	58.01%	0.00%	48.60%	57.81%	0.00%	51.55%	63.78%
Baseline 5 - TOLNRC-KD	0.00%	60.25%	67.85%	0.00%	51.25%	61.14%	0.00%	52.80%	63.75%
Only Fine-Tuning									
Baseline 6 - FOLMRCSC	24.25%	59.55%	64.03%	13.35%	60.25%	65.23%	15.40%	59.20%	71.06%
Baseline 7 - FOLMRCSC-KD	13.50%	58.80%	63.79%	12.75%	64.95%	63.41%	16.75%	65.30%	68.61%
Baseline 8 - FOLNRC	59.05%	64.30%	68.34%	66.90%	68.45%	70.11%	77.35%	75.85%	75.95%
Baseline 9 - FOLNRC-KD	57.99%	64.40%	68.40%	65.95%	68.40%	70.01%	73.30%	73.55%	75.99%
ERwP (Ours)	0.00%	48.06%	66.84%	0.00%	47.84%	69.32%	0.74%	56.23%	75.65%

Table 1: Experimental results on the CIFAR-100 dataset for RCRMR-LD

classes and the remaining classes and finding the nearest neighbor class. Therefore, this process is dependent on the features generated by the deep learning model. Deep learning models generally produce highly discriminative features that can be used to create good prototype classifiers even for classes that the models were not trained on. For example, in the few-shot learning setting, the model is generally trained only on the base classes and then evaluated on novel class episodes using a prototype-based classifier. The prototype-based classifier of the few-shot learning setting is very effective in classifying the novel classes even though the deep model, which was used to obtain the features for the prototypes, was never trained on the novel classes. The discriminative nature of the features produced by deep learning models is the main reason why ImageNet pre-trained model features are used to train classifiers for other datasets and settings, such as in zero-shot learning. In order to better appreciate the effectiveness of our approach, we also consider the FDR model, which has not seen any training data of the restricted classes and still achieves a FPA_e accuracy close to that of our approach, e.g. FDR achieves a FPA_e accuracy of 45.40% for the CIFAR-100 dataset using the ResNet-56 model while our approach achieves an ${\tt FPA}_{\rm e}$ accuracy of 47.84%. We provide this result as a reference to demonstrate that the non-zero accuracy of ERwP is due to the generalization power of deep CNNs and not due to the restricted classes information in the model. However, comparing FDR with our approach is not fair since FDR requires the full training data of the remaining classes, which violates the RCRMR-LD problem setting. Therefore, we have not provided the FDR results in the tables to maintain fairness.

Table 2: Experimental results on ImageNet-1k

Model	Methods	Top-1		Top-5	
		FAe	CA_{ne}	FAe	CA_{ne}
Res-18	Original	69.76%	69.76%	89.58%	89.02%
	ERwP	0.28%	69.13%	1.01%	88.93%
Res-50	Original	76.30%	76.11%	93.04%	92.84%
	ERwP	0.25%	75.45%	2.55%	92.39%
Mob-V2	Original	72.38%	70.83%	91.28%	90.18%
	ERwP	0.17%	70.81%	0.81%	89.95%

7.2. ImageNet Results

Table 2 reports the experimental results for different approaches to RCRMR-LD problem over the ImageNet-1k dataset using the ResNet-18, ResNet-50 and MobileNet V2 architectures. Our proposed ERwP approach achieves a top-1 constraint accuracy CA_{ne} that is very close to that of the original model by absolute margins of 0.63%, 0.66% and 0.02% for the ResNet-18, ResNet-50 and MobileNet V2 architectures, respectively. It achieves close to 0% top-1 forgetting accuracy FAe for all the three architectures. Therefore, our approach performs well even on the large-scale ImageNet-1k dataset. Further, our ERwP requires only 10 epochs to remove the excluded class information from the model. Since the available limited training data is only 5% of the entire ImageNet-1k dataset, therefore, our ERwP approach is approximately $20 * 10 = 200 \times$ faster than the FDR method that is trained on the full data for 100 epochs.

Table 3: Performance of ERwP in incremental learning setting using ResNet-18

Model	FA _e	CA_{ne}
Original Model obtained after Session 4 [M4]	56.39%	58.32%
M4 modified with ERwP (Ours)	0.20%	59.93%

7.3. RCRMR-LD Problem in Incremental Learning

In this section, we experimentally demonstrate how the RCRMR-LD problem in the incremental learning setting is addressed using our proposed approach. We consider an incremental learning setting on the CIFAR-100 dataset in which each task contains 20 classes. We use the BIC [30] method for incremental learning on this dataset. The exemplar memory size is fixed at 2000 as per the setting in [30]. In this setting, there are 5 tasks. Let us assume that the model (M4) has already been trained on 4 tasks (80 classes), and we are in the fifth training session. Suppose, at this stage, it is noticed that all the classes in the first task (20 classes) have become restricted and need to be removed before the model is trained on task 5. However, we only have a limited number of exemplars of the 80 classes seen till now, i.e., 2000/80 = 25 per class. We apply our proposed approach to the model obtained after training session 4, and the results are reported in Table 3. The results indicate that our approach modified the model obtained after session 4, such that the forgetting accuracy of the restricted classes approaches 0% and the constraint accuracy of the remaining classes is not affected. In fact, the modified model behaves as if, it was never trained on the classes from task 1. We can now perform the incremental training of the modified model on task 5.

8. Conclusion

In this paper, we present a "Restricted Category Removal from Model Representations with Limited Data" problem in which the objective is to remove the information regarding a set of excluded/restricted classes from a trained deep learning model without hurting its predictive power for the remaining classes. We propose several baseline approaches and also the performance metrics for this setting. We propose a novel approach to identify the model parameters that are highly relevant to the restricted classes. We also propose a novel efficient approach that optimizes these model parameters in order to remove the restricted class information and re-use these parameters for the remaining classes.

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9. Appendix

9.1. Process for Selecting the Restricted Class Relevant Parameters

In this section, we provide a detailed description of our process for selecting the restricted class relevant parameters. First, we apply a data augmentation technique f, not used during training, to the images of the given restricted class. Next, we combine the predictions for these images and perform backpropagation. Finally, we select the parameters with the highest absolute gradient as the relevant parameters for the corresponding restricted class. Specifically, for a given restricted class, we choose all the parameters from each network layer such that pruning these parameters will result in the maximum degradation of model performance on that restricted class. We use a process similar to the binary search for automatically selecting the parameters with the highest absolute gradient.

We first create a list of parameters in each layer, sort them in descending order according to the absolute gradient values, and check if zeroing out the weights of the first 20% parameters from this list for a particular layer leads to low accuracy (less than 10% for ResNet-164 on CIFAR-100) for that class. If the accuracy is not low after zeroing out the chosen parameters, then we select double the number of parameters chosen earlier and repeat this process. However, if the accuracy is low after zeroing out the chosen parameters, we still need to check if a low accuracy can be achieved by zeroing out fewer parameters. To check this, we reduce the number of parameters by half the difference between the current and previously chosen number of parameters and observe the effect of zeroing out these parameters. If the accuracy is low for the reduced parameters, then we stop the process with the current set of parameters as the relevant parameters for the current restricted class. If the accuracy is not low for the reduced parameters, then we take the previously chosen higher number of parameters as the relevant parameters of the layer for the current restricted class. We repeat this process for all the restricted classes to obtain the relevant parameters for each restricted class in all the layers. The combined set of the relevant parameters for all the excluded classes is referred to as the restricted/excluded class relevant parameters. Please note that this process is just for identifying the parameters relevant to the restricted classes, and their weights are restored after this process.

Table 4: Experimental results on the CUB-200 dataset with ResNet-50 architecture for the RCRMR-LD problem with 20 excluded classes using only 3 training images per class

Methods	FA_{e}	FPA_{e}	CA_{ne}
Original	85.20%	84.69%	77.37%
No Training			
Baseline 1 - WD	0.00%	84.69%	77.64%
Full Train Schedule			
Baseline 2 - TSLNRC	0.00%	30.27%	27.56%
Baseline 3 - TSLNRC-KD	0.00%	35.54%	31.66%
Baseline 4 - TOLNRC	0.00%	60.37%	64.60%
Baseline 5 - TOLNRC-KD	0.00%	68.37%	70.48%
Only Fine-Tuning			
Baseline 6 - FOLMRCSC	53.40%	77.38%	74.39%
Baseline 7 - FOLMRCSC-KD	60.88%	81.12%	75.14%
Baseline 8 - FOLNRC	84.86%	84.18%	76.85%
Baseline 9 - FOLNRC-KD	84.35%	85.20%	77.70%
ERwP (Ours)	0.77%	48.89%	75.45%

9.2. Experiments

9.2.1 CUB-200 Results

Table 4 reports the experimental results for different approaches to the RCRMR-LD problem over the CUB-200 dataset using the ResNet-50 architecture. Our proposed ERwP approach achieves a constraint accuracy CAne that is very close to that of the original model even though we use only 3 images per class for optimizing the model. It achieves close to 0% forgetting accuracy ${\tt FA}_{\rm e}$ and achieves a ${\tt FPA}_{\rm e}$ that is significantly lower than that of the original model by an absolute margin of 35.80%. Our ERwP approach outperforms all the baseline approaches. Further, our ERwP requires only 10 epochs to remove the excluded class information from the model. Since the available limited training data is only 10% of the entire CUB dataset, therefore, our ERwP approach is approximately $5 * 10 = 50 \times$ faster than the FDR method that is trained on the full training data for 50 epochs.

9.3. Ablation Studies

9.3.1 Ablation on Our Approach of Identifying the Restricted Class Relevant Parameters

We perform ablation experiments to verify our approach of identifying the highly relevant parameters for any restricted class. We perform these experiments on the CIFAR-100 dataset with the ResNet-56 architecture and report the forgetting accuracy FA_e for the randomly chosen excluded class. Please note that in this case, only the chosen class of CIFAR-100 is the restricted class and all the remaining



Figure 4: Ablation to validate our approach for identifying relevant model parameters for a random restricted class of CIFAR-100

classes constitute the non-excluded classes. In order to show the effectiveness of our approach, we sort the absolute gradients of the parameters in the model (obtained through backpropagation for the excluded class augmented images) and choose a set of high relevance and low relevance parameters. We then prune/zero out these parameters and record the forgetting accuracy. Fig. 4 demonstrates that as we zero out the high relevance parameters, the forgetting accuracy of the excluded class drops by a huge margin. It also shows that as we zero out the low relevance parameters, there is only a minor change in the forgetting accuracy of the excluded class. Therefore, the parameters relevant to the excluded class receive large gradient updates as compared to the other parameters. This validates our approach for identifying the high relevant parameters for the restricted classes.

9.3.2 Significance of the Components of the Proposed ERwP Approach

We perform ablations on the CIFAR-100 dataset using the ResNet-56 model to study the significance of the \mathcal{L}_c^e , \mathcal{L}_c^{ne} and \mathcal{L}_{kd} components of our proposed ERwP approach. Table 5 indicates that optimizing the restricted class relevant parameters using only \mathcal{L}_c^{ne} cannot significantly remove the information regarding the restricted classes from the model. Applying \mathcal{L}_c^{ne} along with \mathcal{L}_c^e significantly reduces the forgetting accuracy FA_e and forgetting prototype accuracy FPA_e but also significantly reduces the constraint accuracy CA_{ne}.



Figure 5: Plots denoting the performance of our proposed ERwP during the optimization process for forgetting 20 excluded classes from CIFAR-100 using a) ResNet-20 and b) ResNet-56 architectures

Table 5: Significance of ERwP components

\mathcal{L}_{c}^{ne}	\mathcal{L}^e_c	\mathcal{L}_{kd}	FAe	FPA _e	CA_{ne}
√ √	× ✓	× ×	66.50% 0.00%	68.19% 24.40%	69.79% 6.45%
1	1	1	0.00%	47.84%	69.32%

Finally, applying the \mathcal{L}_{kd} loss along with \mathcal{L}_c^{ne} and \mathcal{L}_c^e significantly reduces FA_e and FPA_e while maintaining the constraint accuracy CA_{ne} very close to that of the original model.

9.3.3 Ablation on the Number of Excluded Classes

We report the experimental results for our approach for different splits of excluded and remaining classes of the CIFAR-100 dataset in Table 6. We observe that our ERwP performs well for all the splits for both the ResNet-20 and ResNet-56 architectures.

9.3.4 Performance of ERwP over Training Epochs

We analyze the change in the performance of the model after every epoch of our proposed ERwP approach in Fig. 5 for the CIFAR-100 dataset with 20 excluded classes using the ResNet-20 and ResNet-56 architectures. For both architectures, we observe that as the training progresses, ERwP

Table 6: Experimental results on the CIFAR-100 dataset using ResNet-56 for ERwP with different numbers of excluded classes. # $R/E \rightarrow$ no. of non-excluded classes / no. of excluded classes

# R/E	Methods	ResNet-20		ResNet-56	
		FAe	CA_{ne}	FAe	CA_{ne}
60/40	Original	68.18%	67.35%	69.98%	70.11%
	ERwP	0.00%	67.03%	0.00%	69.98%
70/30	Original	67.83%	67.61%	69.60%	70.26%
	ERwP	0.00%	67.25%	0.00%	69.81%
80/20	Original	70.15%	67.06%	70.80%	69.88%
	ERwP	0.00%	66.84%	0.00%	69.32%
90/10	Original	67.90%	67.66%	68.40%	70.24%
	ERwP	0.00%	67.26%	0.00%	69.69%
95/5	Original	66.20%	67.76%	67.00%	70.22%
	ERwP	0.00%	67.55%	0.00%	69.63%

maintains the constraint accuracy close to that of the original model and forces the forgetting accuracy to drop to 0%. ERwP also forces the forgetting prototype accuracy to keep dropping and makes it similar to the FDR model.

Table 7: Experimental results on the CIFAR-100 dataset with ResNet-20 architecture for the RCRMR-LD problem with 20 excluded classes using our proposed ERwP with different values of β

β	Methods	FAe	CA _{ne}
-	Original	70.15%	67.06%
8	ERwP	0.00%	66.03%
9	ERwP	0.00%	66.23%
10	ERwP	0.00%	66.84 %
11	ERwP	0.00%	66.58%
12	ERwP	0.00%	66.15%

Table 8: Experimental results on the CIFAR-100 dataset with ResNet-20 architecture for the RCRMR-LD problem with 20 excluded classes using our proposed ERwP with different values of κ

κ	Methods	FAe	CA _{ne}
-	Original	70.15%	67.06%
1.0	ERwP	0.00%	66.05%
1.5	ERwP	0.00%	66.08%
2.0	ERwP	0.00%	66.84 %
2.5	ERwP	0.00%	66.30%
3.0	ERwP	0.00%	66.23%

9.3.5 Ablation Experiments for β and κ

We perform ablation experiments to identify the most suitable values for the hyper-parameters β and κ for our proposed ERwP. The ablation results in Tables 7, 8, validate our choice of hyper-parameter values considering the forgetting accuracy and the constraint accuracy of the resulting model.

9.3.6 Effect of Different Data Augmentations on the Identification of Class Relevant Model Parameters

The purpose of applying any data augmentation (not used during training) in our approach is to study the gradient updates when the model performs backpropagation over slightly different versions of the training data of a class and use this information to identify the highly relevant parameters of the model with respect to that class. We have performed experiments using various data augmentation techniques (grayscale, vertical flip, rotation, random affine augmentations) and have provided these results in Fig-6. We chose the same restricted class of CIFAR-100 and use the ResNet-56 network for all the experiments. The results in Fig. 6 indicate that for all the compared data augmentations approaches, pruning/zeroing out the high relevance parameters obtained using our approach results in a huge drop in the forgetting accuracy of the excluded class. Further, zeroing out the low relevance parameters has a minor impact on the forgetting accuracy of the excluded class. Therefore, the data augmentation techniques are almost equally effective in our approach for finding the relevant parameters with respect to any restricted class.

9.3.7 Ablation Experiments on the Restricted Class Relevant Parameters

We perform ablation experiments with ERwP to check if only 25% and 50% of the restricted class relevant parameters of each layer identified using our proposed procedure can be used for ERwP. We run each of these experiments for the same number of epochs for the CIFAR-100 dataset and ResNet-56 network. However, we observed that the final FPA_e falls from 68.65% to 60.35% and 53.7%, respectively, for 25% and 50% of restricted class relevant parameters of each layer as compared to 47.84% when using all the restricted class relevant parameters per layer identified using our approach. The good performance of our approach is more evident in light of the performance of the FDR model that achieves a FPA_e accuracy of 45.40%. We provide this result as a reference to demonstrate that the 47.84% FPA_e accuracy is due to the generalization power of the model and not due to the restricted classes information in the model. This shows that our approach effectively identifies the classrelevant parameters of the model for a given class.

9.3.8 Effect of Using the Proposed ERwP Approach When the Entire Dataset is Available

We perform ablation experiments to demonstrate the performance of our proposed ERwP approach when the entire training data is available. We perform these experiments on the CIFAR-100 dataset using ResNet-20 and ResNet-56. We observe experimentally that for both the ResNet-20 and ResNet-56 experiments using ERwP, the forgetting accuracy FA_e accuracy is 0% and the constraint accuracy CA_{ne} matches that of the original model. Further, the gap between the forgetting prototype accuracy FPAe of ERwP and the FDR model reduces from 3.86% (for limited data) to 2.79% for ResNet-20. Similarly, the gap reduces from 2.44% (for limited data) to 1.65% for ResNet-56. However, ERwP requires only 2-3 epochs of optimization (\sim 100-150 \times faster than the FDR model) for achieving this performance when trained on the entire dataset. This makes it significantly faster than any approach that trains on the entire dataset.



Figure 6: Ablation to validate our approach for identifying restricted class relevant model parameters using different augmentation techniques w.r.t. the same randomly chosen restricted class of CIFAR-100. We use the ResNet-56 network for these experiments. The data augmentation techniques used are (a) grayscale augmentation, (b) vertical flip augmentation, (c) rotation augmentation, (d) random affine augmentation. In each case, the figure shows the model performance when the low relevance and high relevance parameters obtained using our approach are zeroed out



Figure 7: Class activation maps of ImageNet images from the excluded and non-excluded classes, for the original ResNet-50 (second row) and ResNet-50 after applying our proposed ERwP approach (third row). First row depicts the real images

9.3.9 Qualitative Analysis

In order to analyze the effect of removing the excluded class information from the model using our proposed ERwP approach, we study the class activation map visualizations [27] of the model before and after applying ERwP. We observe in Fig. 7 that for the images from the excluded classes, the model's region of attention gets scattered after applying ERwP, unlike the images from the remaining classes.