Tourism Stock Prices, Systemic Risk and Tourism Growth: A Kalman Filter with Prior Update DSGE-VAR Model

Abstract. Dynamic Stochastic General Equilibrium (DSGE) and Vector Autoregressive (VAR) models allow for probabilistic estimations to formulate macroeconomic policies and monitor them. One of the objectives of creating these models is to explain and understand financial fluctuations through a consistent theoretical framework. In the tourism sector, stock price and systemic risk are key financial variables in the international transmission of business cycles. Advances in Bayesian theory are providing an increasing range of tools that researchers can employ to estimate and evaluate DSGE and VAR models. One area of interest in previous literature has been to design a Bayesian robust filter, that performs well concerning an uncertainty class of possible models compatible with prior knowledge. In this study, we propose to apply the Bayesian Kalman Filter with Prior Update (BKPU) in a tourism field to increase the robustness of DSGE and VAR models built for small samples and with irregular data. Our results indicate that BKPU improves the estimation of these models in two aspects. Firstly, the accuracy levels of the computing of the Markov Chain Monte Carlo model are increased, and secondly, the cost of the resources used is reduced due to the need for a shorter run time. Our model can play an essential role in the monetary policy process, as central bankers could use it to investigate the relative importance of different macroeconomic shocks and the effects of tourism stock prices and achieve a country´s international competitiveness and trade balance for this sector.

Keywords: Dynamic Stochastic General Equilibrium, Bayesian Kalman Filter, Prior Update, Markov Chain Monte Carlo, Tourism Stock Prices, Systemic Risk, Volatility.
JEL Codes: C52,E44,G15
MSC Codes: 91B

1 Introduction

The relevance of Dynamic Stochastic General Equilibrium (DSGE) and Vector Autoregressive (VAR) models has recently become increasingly important for their application in the analysis of business cycles and the detection of recessions [1]. These models are often estimated using the Kalman filter built into the Markov Chain Monte Carlo algorithm (in its Metropolis-Hastings version) to predict the posterior distributions of the parameters considered in the model. The evaluation of these models is being carried out by the result obtained in the standard deviation, the sample size, and its complexity. For these models, the literature shows the different fitting results once various Kalman filter algorithms have been applied. For instance, models built with the classical Kalman filter have offered a fit of 0.38-1.43 standard deviation with small samples and irregular data [2], however, their fits vary to 0.27-0.98 in large and regular samples [3]. On the other hand, models using non-classical Kalman filters have achieved even better results than those described above. With small samples and irregular data, the results of these models have been 0.29-0.41 [4], in contrast to 0.14-0.38 with large samples and regular data [5]. Hence, it is noticed that the classical Kalman Filter presents an accuracy in the interval 0.82-0.43 after simulations with samples larger than 100 observations, not giving a deviation sufficiently low in a small sample (a deviation of 0.64 when using a sample size of under 100 observations) [6]. Other investigations have also used DSGE models to analyse the link between monetary policy and stock price and exchange rate volatility using the VAR method for that connection [7]. In this way, [8] analyzed the connection between American tourism companies for the period 2018-2020, including the scenario of the Covid-19 pandemic. The analysis of the risk of contagion showed a significant increase during the Covid-19 pandemic. Small businesses become more systemically important in the pandemic, while the level of serious risk contagion harms the stock performance of American tourism companies. Since the tourism sector, stock price, and systemic risk are financial variables that are crucial for the transmission of economic cycles across borders, and most of the existing research in tourism is based on the hedonic method or the traditional single regression econometric model, it is essential to analyse these macroeconomic variables in this sector with DSGE models. Recent research has applied the DSGE models in other different areas of the tourism field [8,9]. [9] incorporated the VAR model into a DSGE framework for the analysis of tourism development and sustainable economic growth. The results show that a 10% rise in tourism productivity can improve the value-added of the tourism sector by more than 4.11% and boost about 0.5% of GDP growth. [8] suggested a DSGE model to understand the effect of an infectious disease outbreak on tourism, They concluded the suitability of DSGE model to address the impact of the health crisis in this scenario given that the duration and severity of the outbreak are uncertain.

In summary, all this research has applied DSGE models, which have an infinite-order VAR representation. Hence, VARs have been widely used in the forecasting literature evaluating DSGE models. However, due to many parameters and short time series, classical estimates of the coefficients of the unconstrained VAR are often imprecise and forecasts are of low quality due to large estimation errors. A common method to address this problem is to apply Bayesian techniques. For addressing these precision problems of the current DSGE and VAR models, this paper builds on the Bayesian Kalman Filter with Prior Updating (BKPU), having already established its methodological supremacy in other domains for accurate sampling with only a few observations and with non-regular distributions of data [10]. Compared to the classical Kalman Filter used, our results show, in terms of accuracy, a more robust estimation, especially in out-of-sample estimations, a better performance with small and irregular samples. Therefore, the misspecification of previous literature is reduced, as our results also show a better estimation of posterior distributions [11]. These results can be very valuable when applied in DSGE and VAR models, as well as in other macroeconomic models that guide policymakers and other related interest groups in performing estimations. To fill the gap in the existing literature, our investigation evaluates the tourism stock prices volatility and systemic risk applying a BKPU DSGE model for Spain, increasing the robustness of DSGE and VAR models built for small samples and with irregular data.

2 Methods

2.1. Bayesian Kalman Filter with Prior Update

This algorithm framework involves similar recursive equations to those of the classical Kalman filter with the posterior effective noise statistics in place of the ordinary noise statistics. Posterior effective noise statistics represent the posterior distribution of the noise second-order statistics, namely, the covariance matrix, where the posterior distribution is obtained by incorporating observations into the prior distribution of unknown noise parameters. Now assume that the covariance matrices of the process and observation noise are not known and parameterized as $E\left[u_k^{\theta_1}(u_l^{\theta_1})^T\right] = Q^{\theta_1}\delta_{kl}$ and $E\left[v_k^{\theta_2}(v_l^{\theta_2})^T\right] = R^{\theta_2}\delta_{kl}$, $\theta = [\theta_1, \theta_2]$ being the set of unknown parameters governed by the prior distribution π(θ). The state-space model belongs to an uncertainty class Θ (θ ∈ Θ) of possible state-space models. If θ_1 and θ_2 are statistically independent, then the state-space model can be parameterized as $x_{k+1}^{\theta_1} = \Phi_k x_k^{\theta_1} + \Gamma_k u_k^{\theta_1}$ (3) and $y_k^{\theta} = H_k x_k^{\theta_1} + v_k^{\theta_2}$. The intrinsically the Bayesian robust Kalman filter that provides optimal performance on average concerning a prior distribution has been developed using the notions of Bayesian orthogonality principle and Bayesian innovation process in [10], and its structure is completely similar to that of the classical Kalman filtering with the noise covariances and the Kalman gain matrix replaced by the expected noise covariances and the effective Kalman gain matrix, respectively. Being $\psi_{IBR}(y;k) = arg \min_{\psi\in\Psi} E_\theta\,[C_\theta(x_k, \psi(y;k))]$ where the expectation is taken relative to the prior distribution π(θ) governing Θ, $C_\theta(.)$ characterizes the filter cost relative to θ, and ψ_{IBR} is called an intrinsically Bayesian robust filter [10]. Considering the state-space model in y_k^θ and $\psi_{IBR}(y;k)$, let $\Upsilon_{k-1} = \{y_0, \dots, y_{k-1}\}$ and $X_k = \{x_0, \dots, x_k\}$ be the sequences of observations and states up to times k−1 and k, respectively, with $f(\theta, \Upsilon_{k-1,} X_k)$ being the joint probability distribution of the uncertainty class Θ and observations and states. In the context of optimal Bayesian filtering theory, we seek a linear filter of the form $\hat{x}_k^\theta = \sum_{l\le k-1} G_{k,l}^{\Theta} y_l^\theta$ $G_{k,l}^{\Theta} = \arg\min_{G_{k,l\,\in\,G}} E_\theta\left[E\left[\left(x_k^{\theta_1} - \sum_{l\le k-1} G_{k,l} y_l^\theta\right)^T \times \left(x_k^{\theta_1} - \sum_{l\le k-1} G_{k,l} y_l^\theta\right)\right] |\Upsilon_{k-1}\right]$, where G is the vector space of all n × m matrix-valued functions, $G_{k,l\,\in\,G}$ is a mapping $G_{k,l}$: N×N→$R^{n\,x\,m}$ such that $\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\left\|G_{k,l}\right\|_2 < \infty$, $\|\bullet\|_2$ being the L_2 norm and $\hat{x}_k^\theta$ is called the optimal Bayesian least-squares estimate of x_k^θ. The following theorem, definition, and lemma are essential for the derivation of the OBKF framework and are restatements of their counterparts in [10] concerning the posterior distribution. The linear estimate $\hat{x}_k^\theta$ obtained in the last equation of the prior paragraph, is an optimal Bayesian least-squares estimate of x_k^θ, if and only if $E_\theta\left[E\left[(x_k^{\theta_1} - \hat{x}_k^\theta)\,(y_l^\theta)^T\right]|\Upsilon_{k-1}\right] = 0_{n\times m}\quad \forall\, l \le k-1$.

Consider this state-space model and let $\hat{x}_k^\theta$ be a linear estimate of x_k^θ that satisfies this space, then the random process $\tilde{z}_k^\theta = y_k^\theta - H_k\hat{x}_k^\theta$ is a zero-mean process, called the Bayesian innovation process, and ∀ l, l′ ≤ k − 1, we have $E_\theta\left[E\left[\tilde{z}_l^\theta(\tilde{z}_{l'}^\theta)^T\right]|\Upsilon_{k-1}\right] = E_\theta\left[H_l P_l^{x,\theta} H_l^T + R^{\theta_2}\,|\Upsilon_{k-1}\right]\delta_{ll'}$ where $P_l^{x,\theta} = E\left[(x_l^{\theta_1} - \hat{x}_l^\theta)\,(x_l^{\theta_1} - \hat{x}_l^\theta)^T\right]$ is the estimation error covariance matrix of the OBKF at time l relative to θ. Let $\check{x}_k^\theta = \sum_{l\le k-1} G_{k,l}\,\tilde{z}_l^\theta$

be an estimate of x_k^θ obtained using the information in $\tilde{z}_l^\theta = y_l^\theta - H_k \check{x}_l^\theta$, such that, $\mathrm{E}_\theta\left[\mathrm{E}\left[(\mathrm{x}_k^{\theta_1} - \check{\mathrm{x}}_k^\theta)\,(\tilde{\mathrm{z}}_l^\theta)^\mathrm{T}\right]|\mathrm{Y}_{k-1}\right] = 0_{n\times m}$. Then $\mathrm{E}_\theta\left[\mathrm{E}\left[(\mathrm{x}_k^{\theta_1} - \check{\mathrm{x}}_k^\theta)\,(\mathrm{y}_l^\theta)^\mathrm{T}\right]|\mathrm{Y}_{k-1}\right] = 0_{n\times m}$. Using the Bayesian orthogonality principle and the Bayesian innovation process, the recursive equations constituting the OBKF can be found similar to those for the Kalman filter in [10]. According to this, we can write $\hat{x}_k^\theta$ that satisfies the previous equations as: $\hat{x}_k^\theta = \sum_{l\le k-1} G_{k,l}^\Theta \tilde{z}_l^\theta$.

Using $\hat{x}_k^\theta = \sum_{l\le k-1} \mathrm{E}_\theta\left[\mathrm{E}\left[\mathrm{x}_k^{\theta_1}\,(\tilde{\mathrm{z}}_l^\theta)^\mathrm{T}\right]|\mathrm{Y}_{k-1}\right]\mathrm{E}_\theta^{-1}\left[\,\mathrm{H}_l\mathrm{P}_l^{x,\theta}\mathrm{H}_l^\mathrm{T} + \mathrm{R}^{\theta^2}|\mathrm{Y}_{k-1}\right]\tilde{z}_l^\theta$, an update equation for $\hat{x}_k^\theta$ can be found as $\hat{x}_{k+1}^\theta = \Phi_k \hat{x}_k^\theta + \Phi_k \mathrm{K}_k^{\Theta*}\tilde{z}_k^\theta$, where $\mathrm{K}_k^{\Theta*} = \mathrm{E}_\theta\left[\mathrm{P}_k^{x,\theta}|\mathrm{Y}_{k-1}\right]\mathrm{H}_k^\mathrm{T}\mathrm{E}_\theta^{-1}\left[\mathrm{H}_k\mathrm{P}_k^{x,\theta}\mathrm{H}_k^\mathrm{T} + \mathrm{R}^{\theta^2}|\mathrm{Y}_{k-1}\right]$ is the posterior effective Kalman gain matrix. Note that we use $\mathrm{K}_k^{\Theta*}$ and K_k^Θ to distinguish between the effective Kalman gain matrix obtained relative to the posterior distribution in this paper and the one obtained relative to the prior distribution in [10]. Letting $x_k^{e,\theta} = \hat{x}_k^{\theta_1} - \hat{x}_k^\theta$ be the Bayesian least-squares estimation error at time k, the update equation for $x_k^{e,\theta}$ is $x_{k+1}^{e,\theta} = \Phi_k\left(\mathrm{I} - \mathrm{K}_k^{\Theta*}\mathrm{H}_k\right)x_k^{e,\theta} + \Gamma_k u_k^{\theta_1} - \Phi_k \mathrm{K}_k^{\Theta*} v_k^{\theta_2}$. Letting $\mathrm{P}_{k+1}^{x,\theta} = \mathrm{E}\left[x_{k+1}^{e,\theta}\left(x_{k+1}^{e,\theta}\right)^T\right]$ and after some mathematical manipulations, $\mathrm{E}_\theta\left[\mathrm{P}_{k+1}^{x,\theta}|\mathrm{Y}_k\right] = \Phi_k\left(\mathrm{I} - \mathrm{K}_k^{\Theta*}\mathrm{H}_k\right)\mathrm{E}_\theta\left[\mathrm{P}_k^{x,\theta}|\mathrm{Y}_k\right]\Phi_k^T + \Gamma_k \mathrm{E}_\theta\left[Q^{\theta_1}|\mathrm{Y}_k\right]\Gamma_k^T$.

To implement an OBKF, we need to compute the conditional expectations $\mathrm{E}_\theta\left[Q^{\theta_1}|\mathrm{Y}_k\right]$ and $\mathrm{E}_\theta\left[\mathrm{R}^{\theta^2}|\mathrm{Y}_k\right]$ concerning the posterior distribution $\pi(\theta\ |\mathrm{Y}_k) \propto f(y_k|\theta)\pi(\theta)$, where $f(y_k|\theta)$ is the likelihood function of θ given the sequence of observations y_k. As there is no closed-form solution for $\pi(\theta|\mathrm{Y}_k)$ for many prior distributions, we employ a Markov Chain Monte Carlo (MCMC) method to generate samples from the posterior distribution $\pi(\theta|\mathrm{Y}_k)$ and then approximate $\mathrm{E}_\theta\left[Q^{\theta_1}|\mathrm{Y}_k\right]$ and $\mathrm{E}_\theta\left[\mathrm{R}^{\theta^2}|\mathrm{Y}_k\right]$ as sample means of the generated MCMC samples. First, we need to compute the likelihood function $f(y_k|\theta)$. Assume that node α_i has received message $\mu_{\beta_i\to\alpha_i} = (S_i, M_i, \mathcal{E}_i)$. Now we aim to compute the outgoing message $\mu_{\beta_{i+1}\to\alpha_{i+1}}$ from node β_{i+1} to node α_{i+1} . Computing $\mu_{\beta_{i+1}\to\alpha_{i+1}}$ corresponds to the computation of the following integral: $\int_{x_i}\mathcal{N}\left(x_{i+1};\Phi_i x_i,\tilde{Q}_i^{\theta_1}\right)\mathcal{N}\left(y_i, H_i x_i, R^{\theta_2}\right)\times S_i\mathcal{N}(x_i; M_i,\mathcal{E}_i)\,dx_i$. The solution of the integral given in (20) is a scaled multivariate Gaussian function $S_{i+1}\mathcal{N}(x_{i+1}, M_{i+1}, \mathcal{E}_{i+1})$, whose parameters S_{i+1}, M_{i+1}, and $\mathcal{E}_{i+1}$ are given by $\mathcal{E}_{i+1}^{-1} = (\tilde{Q}_i^{\theta_1})^{-1} - (\tilde{Q}_i^{\theta_1})^{-1}\Phi_i\Lambda_i\Phi_i^T(\tilde{Q}_i^{\theta_1})^{-1}$, $M_{i+1} = \mathcal{E}_{i+1}\left(\tilde{Q}_i^{\theta_1}\right)^{-1}\Phi_i\Lambda_i(H_i^T\left(R^{\theta_2}\right)^{-1}y_i + \mathcal{E}_i^{-1}M_i)$ and $S_{i+1} = S_i\sqrt{\frac{|\Lambda_i||\mathcal{E}_{i+1}|}{\left|\tilde{Q}_i^{\theta_1}|\mathcal{E}_i|\right|}}\,\mathcal{N}\left(y_i; O_{m\times 1}, R^{\theta_2}\right)\times \exp\left(\frac{\mathrm{M}_{i+1}^\mathrm{T}\mathcal{E}_{i+1}^{-1}M_{i+1} + W_i^T\Lambda_i W_i - \mathrm{M}_i^\mathrm{T}\mathcal{E}_i^{-1}M_i}{2}\right)$, where $W_i = H_i^T\left(R^{\theta_2}\right)^{-1}y_i + \mathcal{E}_i^{-1}M_i$ and $\Lambda_i = (\Phi_i^T(\tilde{Q}_i^{\theta_1})^{-1}\Phi_i + H_i^T\left(R^{\theta_2}\right)^{-1}H_i + \mathcal{E}_i^{-1})^{-1}$.

The update rules given should be iterated for $0 \le \mathrm{i} \le \mathrm{k} - 1$ to finally obtain the message $\mu_{\beta_k\to\alpha_k} = (S_k, M_k, \mathcal{E}_k)$. Then the likelihood function is obtained as $f(y_k|\theta) = \int_{x_k}\mathcal{N}\left(y_k; H_k x_k, R^{\theta_2}\right)S_k\,\mathcal{N}(x_k, M_k, \mathcal{E}_k)dx_k = \int_{x_k}\frac{S_k}{\sqrt{(2\pi)^m|R^{\theta_2}|}\sqrt{(2\pi)^n|\mathcal{E}_k|}}\times$

$exp\left(\frac{-1}{2}(y_k - H_k x_k)^T \left(R^{\theta_2}\right)^{-1}(y_k - H_k x_k) + (x_k - M_k)^T \mathcal{E}_k^{-1}(x_k - M_k)\right) dx_k$. To estimate the posterior effective noise statistics $E_\theta\left[Q^{\theta_1}|Y_k\right]$ and $E_\theta\left[R^{\theta_2}|Y_k\right]$, we employ the Metropolis-Hastings MCMC [4]. Let the last accepted MCMC sample in the sequence of samples be $\theta^{(j)}$ generated at the j-th iteration. A candidate MCMC sample θ^{candid} will be drawn according to a proposal distribution $f(\theta^{candid}|\theta^{(j)})$. The candidate MCMC sample θ^{candid} will be either accepted or rejected according to an acceptance ratio r defined as $r = min\left\{1, \frac{f(\theta^{(j)}|\theta^{candid}) f(y_k|\theta^{candid})\pi(\theta^{candid})}{f(\theta^{candid}|\theta^{(j)}) f(y_k|\theta^{(j)})\pi(\theta^{(j)})}\right\} = min\left\{1, \frac{f(y_k|\theta^{candid})\pi(\theta^{candid})}{f(y_k|\theta^{(j)})\pi(\theta^{(j)})}\right\}$, where the second formula is used when the proposal distribution is symmetric, $f(\theta^{candid}|\theta^{(j)}) = f(\theta^{(j)}|\theta^{candid})$. The (j + 1)-th MCMC sample is: $\theta^{(j+1)} = \begin{cases} \theta^{candid} \text{with probability r} \\ \theta^{(j)} \text{ otherwise} \end{cases}$. The positivity of the proposal distribution ($f(\theta^{candid}|\theta^{(j)}) > 0$ for any $\theta^{(j)}$) is a sufficient condition for having an ergodic Markov chain of MCMC samples whose steady-state distribution is the target distribution $\pi(\theta|Y_k)$[12,13].

2.2. DSGE Model

According to [13], a DSGE model can be used to examine tourism with the general balance of the economy (see Table 1).

Table 1. Dynamic Stochastic General Equilibrium (DSGE)

Functions	Variables
The utility function of households	
$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(C_t - hC_{t-1}) + \frac{u_t^{1+v_1}}{1+v_1} + \frac{(La_{l,t}\varsigma_{la,t})^{1+v_2}}{1+v_2}\right]^{1-\sigma}}{1-\sigma}$	*E_0:* expected utility function hypothesis *β:* discounted rate *h:* typifies the habit persistence of consumption *Ct: (*using a CES function*)* is composed by: ▪ *$C_{T,t}$: T*ourism goods ▪ *$C_{NT,t}$:* Non-tourism goods ▪ *C_P, t:* Public services *ut:* Unemployment rate *$La_{l,t}$:* Private land supply shock *$\varsigma_{la,t}$* : The exogenous variable that is estimated by an autoregression process to represent the result of private land inputs on the economy *σ, v_1*, and *v_2:* the parameters of the constant elasticity of substitution (CES) *$C_{M,t}$:* is composed by: ▪ *$C_{MT,t}$:* Imports of tourism products ▪ *$C_{MNT,t}$:* Non-tourism products
The production functions of the tourism and non-tourism activities	
$Y_{T,t} = \Omega_{T,t} K_{T,t}^{\alpha_1} N_{T,t}^{\alpha_2} La_{T,t}^{1-\alpha_1-\alpha_2}$ $Y_{NT,t} = \Omega_{NT,t} K_{NT,t}^{\alpha_3} N_{NT,t}^{1-\alpha_3}$	*$Y_{i,t}$ (i = T, NT)*: The value-added of the given sector *$\Omega_{i,t}$ (i = T, NT)*: The productivity function connected to the effects of physical capital and public sector *$K_{i,t}$ (i = T, NT)*: The physical capital and is calculated by the process: *$K_{i,t+1} = I_{i,t} + (1-\delta)K_{i,t}$* (i= T, NT) ▪ *$I_{i,t}$* : The physical capital investment in every sector

- δ: The depreciation rate

$N_{i,t}$: Human capital enhancement:

$N_{i,t} = H_t n_{i,t}$ $(i = T, NT, P)$

- $n_{i,t}$: points out the labor force for the sectors
- H_t: The spill-over effects of capital and the accumulation of human capital$La_{T,t}$: The private land rentals to the tourism sector

The productivity function connected to the effects of physical capital and public sector

$$\Omega_{i,t}= A_t A_{i,t}(\zeta_{P,t} Y_{P,t})^{\varphi_{P,i}} K_{i,t}^{\varphi_i}\left(\frac{K_{P,t}}{K_{T,t}+K_{NT,t}}\right)^{\varphi_{c,i}} \quad i=(T, NT)$$

A_t: The auto-regression processes of the total productivity shocks

$A_{i,t}$: The auto-regression processes of the sector that point out sector and total productivity shocks

$\zeta_{P,t}$: The exogenous shock to the spill-over effects of public sector

$Y_{P,t}$ the effect of public sector

$K_{i,t}$: *The* effect of physical capital

$\left(\frac{K_{P,t}}{K_{T,t}+K_{NT,t}}\right)^{\varphi_{c,i}}$: The spillover effect of $K_{p,t}$

$\varphi_{P,i}$: The effect of the public sector

$\varphi_{c,i}$:The effect of the private sector

($i = T, NT$): The parameters

The spill-over effects of capital and the accumulation of human capital

$$H_t= \frac{EX_{T,t}^{\alpha_T}\left(Y_{T,t}\text{-}EX_{T,t}\right)^{b_T}\zeta_{H,t}}{H_t^{\pi_T}} + \frac{EX_{NT,t}^{\alpha_{NT}}\left(Y_{NT,t}\text{-}EX_{NT,t}\right)^{b_{NT}}}{H_t^{\pi_{NT}}}$$

$\delta_H H_{t-1}$

$EX_{T,t}$: The exports of tourism

$\zeta_{H,t}$: The shock to human capital accumulation

$EX_{NT,t}$: The non-tourism products

$E_{i,t}^{a_i}$ and $\left(Y_{i,t}\text{-}EX_{i,t}\right)^{b_i}$: The effect of the tourism product on human capital

a_i, b_i and π_i: The parameters

δ_H: The depreciation rate of human capital

$H_t^{\pi_i}$: The externality of experience

Systemic Risk (Risk Contagion)

$$X_{j,t}=g\left(\beta_{J|R_j}^T R_{j,t}\right)+\epsilon_{j,t}$$

$$\widehat{CoVaR}_{J|\tilde{R}_{j,t,\tau}}^{TENET} \equiv \hat{g}\,(\hat{\beta}_{j|\tilde{R}_j}^T \tilde{R}_{j,t})$$

$$\hat{D}_{j|\tilde{R}_j} \equiv \frac{\partial \hat{g}\left(\hat{\beta}_{j|R_j}^T R_{j,t}\right)}{\partial R_{j,t}}\Bigg|_{R_{j,t}=\tilde{R}_{j,t}} = \hat{g}'\left(\hat{\beta}_{j|\tilde{R}_j}^T \tilde{R}_{j,t}\right)\hat{\beta}_{j|\tilde{R}_j}$$

$R_{j,t} \equiv \{X_{-j,t}, M_{t-1}, B_{j,t-1}\}$

$X_{-j,t} \equiv \{X_{1,t}, X_{2,t}, \dots, X_{k,t}\}$ is the set of (k-1) independent variables such as the log-returns of tourism stocks, except tourism stock j, and k number o tourism stocks which is 95 in our case.

$\beta_{j|R_j} \equiv \left\{\beta_{j|-j}, \beta_{j|M}, \beta_{j|B_j}\right\}^T$

$\widehat{CoVaR}_{J|\tilde{R}_{j,t,\tau}}^{TENET}$ *is the TENET risk that contains* other tourism stocks on tourism stock j and integrates the non-linearity displayed in the shape of a link function (g.).

The exports

$$EX_{i,t}=\left(\frac{P_{i,t}}{RER_t}\right)^{\theta_{EX_i}} Y_{ROW,t}^{\omega_i} \quad (i=T, NT)$$

$\left(\frac{P_{i,t}}{RER_t}\right)$: The real exchange rate in USD

RER_t: The exchange rate

$Y_{ROW,t}$: The world income level

$$P_{H,t} = \overline{P_{H,t}} + \eta P_{N,t}$$

$$P_{T,t} = \left[a_H P_{H,t}^{\frac{\rho}{\rho-1}} + a_F P_{F,t}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho}{\rho-1}}$$

$P_{H,t}$: Consumer price if the Home traded goods

$\overline{P_{H,t}}$: Price of Home traded goods at producer level

$P_t = \left[a_T P_{T,t}^{\frac{\phi}{\phi-1}} + a_N P_{N,t}^{\frac{\phi}{\phi-1}}\right]^{\frac{\phi}{\phi-1}}$	$\eta P_{N,t}$: Value of the nontraded goods that are necessary to distribute to consumers
$AC^{p}_{H,t}(h) = \frac{k^{p}_{H}}{2}\left(\frac{\bar{p}_t(h)}{\bar{p}_{t-1(h)}} - \pi\right)^2 D_{H,t}$ $AC^{p*}_{H,t}(h) = \frac{k^{*p}_{H}}{2}\left(\frac{\bar{p}^{*}_t(h)}{\bar{p}^{*}_{t-1}(h)} - \pi\right)^2 D_{H,t}$	$AC^{p}_{H,t}(h)$ and $AC^{p*}_{H,t}(h)$: The price adjustment costs faced by firms in the traded goods according to the destination market.
$AC^{p}_{t}(n) = \frac{k^{p}_{N}}{2}\left(\frac{p_t(n)}{p_{t-1}(n)} - \pi\right)^2 D_{N,t}$	$AC^{p}_{t}(n)$: The price adjustment costs faced by firms in the non-traded goods.
The Government and the Equilibrium	
$G_t = \left(\frac{g_t}{1+g_t}\right) Y_t = T_t$	G_t: Government purchases. We assume a public sector that consumes a fraction T_t of the output of each good, being $g_{t=-\log(1-T_t)}$ Y_t: $Aggregate\ output$
$E_t\{\mathcal{F}_{t,t+1}\prod_{t+1}\Omega_{t+1}\} = Q_t$	The present discounted real value of future financial wealth equals the current level of the real stock-price index
$(\Sigma_t - 1)(C_t - hC_{t-1}) = \xi Q_t + (1 - \xi)E_t\{\mathcal{F}_{t,t+1}\prod_{t+1}\Sigma_{t+1}(C_{t+1}hC_t)\}$	State equation for aggregate consumption
$Q_t = E_t\{\mathcal{F}_{t,t+1}\prod_{t+1}[Q_{t+1} + D_{t+1}]\}$	Standard pricing equation is micro-founded on the consumer´s optimal behavior.

Against this backdrop, we develop a DSGE-VAR model (see Table 2). First, we have determined a vector of endogenous variables to express the model VAR. Then, we have defined the vector of VAR variables, where is it established the trade-weighted nominal exchange rate in the United States. Therefore, growth in the trade-weighted nominal exchange rate causes the U.S. dollar to depreciate. Nevertheless, the DSGE-VAR estimation requires a hierarchical prior, for this reason, we have carried out the DSGE parameter vector. First, we use the DSGE model to generate artificial data according to the prior distributions of the DSGE-VAR estimation. Second, these data are subsequently taken as priors for the Bayesian VAR estimation [13]. Finally, is necessary to stipulate a posterior distribution: $p\ (\Phi,\sum_u,\theta/Y)=p(\Phi,\sum_u/\theta,\ Y)p(\theta|Y)$ for correctly estimating the model

Table 2: **DSGE-VAR Model**

Functions	Variables
The model VAR	
$y^{v}_{t}=c+B_1y^{v}_{t-1}+\ldots+B_py^{v}_{t-p}+u_t$	y^{v}_{t} : represent an nH × 1 vector corresponding to endogenous variables for t = 1…, T *c:* Group of terms *p:* The VAR lag length *[B1, . . ., Bp]:* Parameter matrices *ut:* The vector of forecast errors defined by the multivariate normal distribution *N (0; ∑u)*
Vector of VAR variables	

$y_t^{v'}$=100×[$\Delta log(Y_{T,t})$,$\Delta log(Y_{NT,t})$,$\Delta log(C_t)$,$\Delta log(GDP_t)$,4$\Delta log(P_t)$,R $\Delta log(TSP_t)$, $\Delta log(\widehat{VaR}_{i,t,\tau})$]	*YT,t:* The production in the tourism sector *YNT,t:* The production in the non-tourism sector *Ct:* Per capita real consumption *GDPt:* Per capita real GDP *Pt:* Applies the GDP deflator *Rt:* The federal funds rate adjusted at the annual rate *EXt:* The trade-weighted nominal exchange rate in Spain *TSP_t:* The tourism stock prices index for Spanish companies $\widehat{\mathrm{VaR}}_{i,t,\tau}$: Estimation of the systemic interdependence among Spanish tourism stocks
The DSGE-VAR estimation	
Y^v=$X^v$$\Phi$+$u_t$	Y_t^v be a *T×nH* matrix with each row consisting of $\boldsymbol{y}_t^{v\prime}$ *Xv* be a *T×k* matrix with the *t-th* row containing in $x_t^{v'} \equiv \left[1, y_{t-1}^{v'}, \ldots, y_{t-p}^{v'}\right]$ where *k ≡ 1 + p× nH.* *ϕ:* The maximum-likelihood estimator is calculated according to DSGE parameters vector
DSGE parameters vector	
$\widetilde{\Phi}(\theta) = \left(\lambda T \Gamma_{X^v X^v}(\theta) + X^{v\prime} X^v\right)^{-1} \left(\lambda T \Gamma_{X^v X^v}(\theta) + X^{v\prime} Y^v\right)$	*ϑ:* Vector consisting of the DSGE parameters *EDh:* The expectation operator conditional on the DSGE parameter vector *ϑ*

3. Empirical Results

The sample period in the valuation of the model has been from 1992Q1 to 2021Q3, during which data from the Spanish economy have been used. These data have been extracted from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis, Eurostat, and SABI (Iberian Balance Sheet Analysis System of Bureau Van Dijk). Once the posterior distribution has been estimated, it is also useful to perform an estimation of the so-called Marginal Data Density (MDD) for DSGE models $p(Y) = \int p(Y/\theta)\, p(\theta)\, d\theta$. In this study, the posterior moments are estimated with the three models proposed (DSGE, VAR, and DSGE-VAR). We employ the Metropolis algorithm to simulate the posterior distribution to evaluate the accuracy of the models, running these algorithms 10,000 times and calculating the means and standard deviations of the posterior moment estimates in all runs. Tables 4 report the results of the estimates obtained by the different models with the prior distribution previously inserted and the posterior distribution from the estimation. To guarantee greater robustness in the estimates, three stages of the configuration of the DSGE model described above have been carried out. Table 4 shows the MDD estimates after the estimation of the models developed. These results demonstrate the greater stability offered by the DSGE-VAR model compared to the rest, especially in light of the deviations obtained for three settings. The results of the new Kalman filter with Prior Update improve the results of the Classical Kalman Filter, just as it improves the precision results shown in previous works [1,5,6], even if it is small and irregular samples like the one used in the present study.

Table 4. Log MDD Estimates (Base Model)

		Classical Kalman Filter		Bayesian Kalman Filter with Prior Update	
N	Model	MEAN (Log MDD)	STD (Log MDD)	MEAN (Log MDD)	STD (Log MDD)
Prior Distribution					
100	DSGE	-1728.812	0.81	-1357.738	0.49
	VAR	-1754.258	0.87	-1384.483	0.55
	DSGE-VAR	-1711.593	0.76	-1274.524	0.41
Posterior Distribution					
500	DSGE	-1625.851	0.74	-1236.104	0.39
	VAR	-1664.593	0.82	-1286.342	0.44
	DSGE-VAR	-1572.294	0.71	-1238.592	0.32

Note: N is the sample size; STD is the standard deviation

Table 5 provides the mean and standard deviation of the prior distributions of each parameter for Spain. The mean of the posterior distributions and the range of the 90% interval estimated by the Bayesian approach are presented. The estimation results of some of the structural parameters, such as β, δ, α_3, α_4, and h, work as the prior means according to the structure of the optimal equations used in some previous works [13]. The parameter α_1 increases from 0.41 to 0.58 but α_2 decreases from 0.50 to 0.11. By comparison with the non-tourism and public services sectors, the output of the tourism sector continues to be more labor-dependent, which is consistent with the realities of the tourism sector. On the other side, the coefficients of leisure (v_1), private land (v_2), and intertemporal substitution (σ) are estimated as 2.06, 1.97, and 1.98, which after dividing the unity by these results shows us the following elasticities 0.485, 0.508 and 0.505 respectively. In the three cases, the elasticities are less than 1, which is in line with previous works [13]. The substitute elasticity between tourism and non-tourism goods (θ_1) is 0.42.

Table 5. Estimations results for Spain (Main components)

		Prior Distribution	Posterior Distribution	90% Interval	
				Low	High
Physical Capital Depreciation Rate	δ	Beta (0.03,0.00)	0.03	0.01	0.04
Output Elasticity of Physical Capital in the Tourism Sector	$\alpha 1$	Beta (0.41,0.10)	0.58	0.56	0.63
Output Elasticity of Human Capital in the Tourism Sector	$\alpha 2$	Beta (0.50,0.10)	0.11	0.07	0.16
Habit Persistent	h	Beta (0.81,0.01)	0.79	0.73	0.84

Elasticity of Leisure	v_1	Gamma (2.00,0.10)	2.06	2.03	2.11
Elasticity of Private Land	v_2	Gamma (2.00,0.10)	1.97	1.96	2.01
Elasticity of Intertemporal Substitution	σ	Gamma (2.00,0.10)	1.98	1.97	2.00
Substitute Elasticity between Tourism, Non-tourism Goods and Public Services	θ_1	Gamma (0.42,0.10)	0.42	0.36	0.47
Substitute Elasticity between FDI and Domestic Investment	θ_2	Gamma (1.45,0.10)	1.34	1.28	1.40
Substitute Elasticity between Tourism and Non-tourism Imports	θ_3	Gamma (0.40,0.10)	0.43	0.39	0.44
Price Elasticity of Tourism Exports (Absolute)	$\theta_{EX,T}$	Gamma (0.40,0.10)	0.45	0.42	0.47
Price Elasticity of Non-tourism Exports (Absolute)	$\theta E,$	Gamma (0.20,0.10)	0.28	0.24	0.33
Income Elasticity of Tourism Exports	ωT	Gamma (0.75,0.10)	1.08	1.05	1.15
Income Elasticity of Non-tourism Exports	ωNT	Gamma (0.30,0.10)	0.07	0.02	0.09
Autoregressive Coefficient of Return Rate	θtr	Beta (0.80,0.10)	0.84	0.80	0.90
Elasticity of Price in the Taylor Rule	θ_p	Gamma (1.70,0.10)	1.83	1.81	1.85
Elasticity of GDP in the Taylor Rule	θy	Gamma (0.15,0.05)	0.17	0.16	0.24
Elasticity of Tourism Exports in Human Capital Accumulation	a_T	Gamma (0.25,0.10)	0.51	0.47	0.56
Elasticity of Non-exports of the Tourism Sector in Human Capital Accumulation	b_T	Gamma (0.05,0.01)	0.06	0.01	0.11
Scale Effect of Human Capital Accumulated by the Tourism Sector	πT	Gamma (0.30,0.10)	0.24	0.23	0.26
Elasticity of Non-tourism Exports in Human Capital Accumulation	$a\ T$	Gamma (0.30,0.10)	0.47	0.46	0.51
Elasticity of Non-exports in the Non-tourism sector of Human Capital Accumulation	$B\ T$	Gamma (0.05,0.01)	0.05	0.01	0.07
Scale Effect of Human Capital Accumulated by the Non-tourism Sector	$\Pi\ T$	Gamma (0.30,0.10)	0.42	0.41	0.45
Depreciation Rate of Human Capital	δ_H	Gamma (0.05,0.01)	0.08	0.02	0.12
Spill-over Effect of Public Service on Tourism Productivity	$\varphi P,$	Gamma (0.10,0.01)	0.14	0.09	0.17
Spill-over Effect of Tourism Physical Capital on its Productivity	φ_T	Gamma (0.05,0.01)	0.03	0.01	0.04
Congestion Effect of Physical Capital on Tourism Productivity	$\varphi C,$	Gamma (0.06,0.01)	0.05	0.01	0.09
Spill-over Effect of Public Service on Non-tourism Productivity	$\varphi P,$	Gamma (0.10,0.01)	0.14	0.10	0.17
Spill-over Effect of Non-tourism Physical Capital on its Productivity	$\varphi\ T$	Gamma (0.05,0.01)	0.08	0.03	0.12

Finally, to carry out a forecast evaluation, the three versions of the model (DSGE, VAR, DSGE-VAR) are estimated with the final configuration of the model used by [3] with out-of-sample data, with a horizon of one year. For this, the root-mean-square error (RMSE) is estimated to analyze the deviation obtained outside the sample by the BKPU filter. Table 6 shows the RMSE results obtained from the posterior distributions by the different models estimated by the Classical Kalman Filter and Bayesian Kalman Filter with Prior Update, respectively. These results also show greater precision of the new proposed Kalman filter with out-of-sample data, and also improve the precision shown in previous works that performed a forecast evaluation with out-of-sample data [4]. These simulations with out-of-sample data obtain robust and stable precision results, which would rule out possible parameter misspecification estimated by the new Kalman filter used (BKPU), a concern shown by previous works [3]. It also shows an improvement in precision results compared to other filters used by recent works such as the central difference Kalman filter [1,3] and Quadratic Kalman filter [6]. Finally, the average run time of the Classical Kalman Filter for this estimate with data outside the sample is 0.23 minutes, while the same estimate is made by the BKPU method in a time of 0.11 minutes.

Table 6. Prior and Posterior Distributions

	Classical Kalman Filter			
	2018Q1	2018Q2	2018Q3	2018Q4
DSGE	0.78	0.82	0.84	0.88
VAR	0.85	0.88	0.92	0.96
DSGE -VAR	0.74	0.75	0.78	0.82
	Bayesian Kalman Filter with Prior Update			
	2018Q1	2018Q2	2018Q3	2018Q4
DSGE	0.43	0.44	0.47	0.48
VAR	0.52	0.52	0.53	0.55
DSGE -VAR	0.38	0.41	0.41	0.43

4. Conclusions

This research provides an additional simulation procedure for the estimation of DSGE and VAR models. It is demonstrated that, when properly adjusted to DSGE and VAR models, the BKPU technique is more robust than other commonly used algorithms, such as the Classical Kalman Filter. After a comparison of simulations with these two Kalman filters carried out successfully on three scenarios of a medium-scale Keynesian DSGE model, our results reveal high robustness of the BKPU algorithm for small samples with irregular data and possible cases of statistical misspecification, which has been a matter of concern shown by the literature in the DSGE and VAR model estimation. The results obtained in our research are valid both for the RMSE results as a criterion for out-of-sample data and for the marginal data density as a criterion to measure the fit of the in-sample models. Given the high accuracy shown by this new algorithm, this study also implies an improvement in the optimization of the calculation of macroeconomic forecasts, since it is not necessary to use any available resources or to carry out an extensive specification of the DSGE models. In addition, our research provides an important contribution to the literature on the tourism sector through a DSGE model, both in the estimation of the prior and posterior distribution. In this model, we analyse the effect of stock price volatility, systemic risk, and tourism productivity in the tourism economy. The estimation results reveal that a 10% increase in tourism productivity can improve the value-added of the tourism sector by 1.15% and increase GDP growth by about 0.74%. Given that Spain is an important tourism country, any increase in tourism development will increase GDP by a considerable proportion. Likewise, whereas an increase in tourism productivity leads to a rise in tourism prices, an increase in tourism consumption, and, in theory, a drop in tourism investment, the positive effect on other sectors produces different consequences. Furthermore, the estimation results also reveal that a 10% increase in systemic risk decreases the value-added of the tourism sector by 1.04%, in turn declining GDP growth by approximately 1.06%. Considering that one of the main sources of income in Spain is derived from the tourism sector, it is very important to consider the systemic risk, which is caused by the failure in payments by one or more members of the market system. This can lead to a generalized market collapse, particularly affecting companies in the tourism industry. Lastly, we observe a slight increase in non-tourist exports, but a small fall in consumption of these non-tourist products and a more persistent decline in investment in our Tourism productivity model.

Furthermore, the accuracy results show how the extended DSGE-VAR model is better than the previous DSGE model in the analysed country, both in the estimation of the prior and posterior distribution. These results show the higher stability provided by the DSGE-VAR model compared to the others, especially in comparison to the deviations obtained for three settings. The results of the new Kalman filter with prior updates improve the results of the classical Kalman filter, as well as improve the accuracy results shown in previous works, even when dealing with small and irregular samples like the one used in the present study. It would be an interesting idea as future research to compare this new Kalman application to another economic and finance in order to check the superiority shown in DSGE models.

In summary, this study offers a significant opportunity to contribute to the field of macroeconomic analysis, since the results obtained have important implications for public institutions and other interest groups. A policymaker is usually only really interested in a restricted number of available resources when making a macroeconomic forecast, based only on the variables of interest, an issue that our model has introduced by improving the optimization of macroeconomic forecast calculations. In addition, the BKPU technique can be extended to a wide variety of problems for which the Classical Kalman Filter has been previously applied. For instance, the study of the transmission of monetary policy shocks across economic areas and the tourism sector, the construction of measures of core inflation, and the natural rate of unemployment in country settings. Therefore, our study has relevant implications for monetary policy since the exchange rate, tourism stock prices and tourism productivity have a significant impact on the business cycle.

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