

HSIC-InfoGAN: Learning Unsupervised Disentangled Representations by Maximising Approximated Mutual Information

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Abstract. Learning disentangled representations requires either supervision or the introduction of specific model designs and learning constraints as biases. InfoGAN is a popular disentanglement framework that learns unsupervised disentangled representations by maximising the mutual information between latent representations and their corresponding generated images. Maximisation of mutual information is achieved by introducing an auxiliary network and training with a latent regression loss. In this short exploratory paper, we study the use of the Hilbert-Schmidt Independence Criterion (HSIC) to approximate mutual information between latent representation and image, termed HSIC-InfoGAN. Directly optimising the HSIC loss avoids the need for an additional auxiliary network. We qualitatively compare the level of disentanglement in each model, suggest a strategy to tune the hyperparameters of HSIC-InfoGAN, and discuss the potential of HSIC-InfoGAN for medical applications.

Keywords: Disentangled representation learning · HSIC · InfoGAN.

1 Introduction

Recently, machine learning (ML) and deep learning (DL) have achieved significant success in many computer science areas, for instance vision and natural language processing. [12]. However, traditional fully supervised approaches cannot always be applied in specific domains such as medical imaging analysis, as the available annotations are limited due to the labeling process being tedious and costly. Thus, significant effort has been placed on alternative training methods such as unsupervised and semi-supervised learning. In particular, recent works [4,11,15] show that the typical unsupervised approach of disentangled representation learning without labeled data significantly boosts the performance of ML/DL models.

The widely agreed definition of a disentangled representation is one in which “*single latent units are sensitive to changes in single generative factors, while being relatively invariant to changes in other factors*” [2]. This definition is based on an implicit assumption that there is a generation process in the real world that translates independent generative factors to images. Hence, the overall goal in disentangled representation learning is to discover this generation process and the constituent generative factors from images. However, a comprehensive study [18] recently showed that it is impossible to learn a disentangled representation in an unsupervised setting, and all previous methods use various inductive biases or assumptions on either model design or learning process. Then, model performance largely depends on the introduced inductive biases that are tailored for specific tasks. In other words, different tasks require domain-specific expert knowledge to devise suitable inductive biases and assumptions.

As an unsupervised disentanglement method, InfoGAN [5] considers that the latent representations consist of categorical (we assume that we know the number of classes) and continuous latents. With this inductive bias, InfoGAN solves the information-regularised minimax game by jointly training a generator, a discriminator and an auxiliary network. In particular, the auxiliary network takes the generated image as input and is trained to correctly predict the corresponding latent representations of this image i.e. latent regression [26]. This forces the generated image to be highly dependent on the latent representations i.e. maximising the mutual information. Despite the cost of introducing an additional network/module, this strategy has been used in many disentanglement models such as [9, 20, 26].

In this paper, we examine the question of whether we can approximate mutual information between the latent representations and the generated image without the need for such an auxiliary network. We consider the Hilbert-Schmidt Independence Criterion (HSIC) [19] to approximate the mutual information between the latent representations and the generated image, termed HSIC-InfoGAN. HSIC is a kernel-based independence measurement. By projecting the inputs into kernel space, HSIC allows inputs to have different dimensionality. Directly optimising the HSIC loss removes the need for an auxiliary network, which could reduce the model training time and the memory load for saving model weights. As shown in our experiments, achieves satisfactory levels of disentanglement compared to InfoGAN. We further discuss a strategy to effectively tune the hyperparameters of HSIC-InfoGAN, and discuss its potential impact on medical applications.

2 Methodology

2.1 InfoGAN

Generative adversarial networks [8] train the generator G and discriminator D using a minimax game by optimising the following objective:

$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{X}}[\log D(\mathbf{X})] + \mathbb{E}_{\mathbf{z}}[\log(1 - D(G(\mathbf{z})))] \quad (1)$$

where \mathbf{X} denotes an image sample and \mathbf{z} is the noise vector. InfoGAN [5] proposes that the latent space contains the noise \mathbf{z} as well as the disentangled latent code \mathbf{c} . To learn the disentangled representations, InfoGAN solves the information-regularised minimax game:

$$\min_G \max_D V_I(D, G) = V(D, G) - \lambda_I I(\mathbf{c}; G(\mathbf{z}, \mathbf{c})), \quad (2)$$

where $I(\mathbf{c}; G(\mathbf{z}, \mathbf{c}))$ denotes the mutual information between the latent code \mathbf{c} and the generated image $G(\mathbf{z}, \mathbf{c})$. However, we can only compute the exact and tractable mutual information for discrete variables or for specific problems that we know the probability distributions [1]. Due to the difficulty of directly maximising the mutual information term, InfoGAN introduces an auxiliary network Q to derive the lower bound of the mutual information:

$$I(\mathbf{c}; G(\mathbf{z}, \mathbf{c})) \geq L_I(G, Q) = \mathbb{E}_{\mathbf{X}}[\mathbb{E}_{\mathbf{c}'}[\log Q(\mathbf{c}' | \mathbf{X})]]. \quad (3)$$

Overall, the objective of InfoGAN is defined as:

$$\min_{G, Q} \max_D V_{\text{InfoGAN}}(D, G, Q) = V(D, G) - \lambda_{\text{InfoGAN}} L_I(G, Q). \quad (4)$$

Note that most of the network weights of Q and D can be shared. Separate final (head) layers are used for the Q and D networks in InfoGAN.

2.2 Hilbert-Schmidt Independence Criterion (HSIC)

Considering the kernel function $k(\cdot, \cdot)$, the HSIC loss is defined in [19] as:

$$\text{HSIC}(\mathbf{X}, \mathbf{z}) = (m - 1)^{-2} \text{trace}(K_{\mathbf{X}} H K_{\mathbf{z}} H), \quad (5)$$

where m is the batch size in our case. $K_{\mathbf{X}_{i,j}} = k(\mathbf{X}_i, \mathbf{X}_j)$ and $K_{\mathbf{z}_{i,j}} = k(\mathbf{z}_i, \mathbf{z}_j)$ are the entries of $K_{\mathbf{X}} \in R^{m \times m}$ and $K_{\mathbf{z}} \in R^{m \times m}$. H is the centering matrix $H = I_m - \frac{1}{m} \mathbf{1}_m \mathbf{1}_m^T$. Following [19], we choose the Gaussian kernel $k(\mathbf{X}_i, \mathbf{X}_j) \sim \exp(-\frac{1}{2} \|\mathbf{X}_i - \mathbf{X}_j\|^2 / \sigma^2)$, where σ is a hyperparameter. Here, HSIC values are always positive and lower HSIC means higher independence (i.e. lower mutual information). Note that in Eq. 5, \mathbf{X} and \mathbf{z} can have different dimensionality i.e. they can be a tensor and a vector. We refer the readers to the section ‘‘Relating HSIC to Entropy’’ in [19] for a informal discussion about the relationship between HSIC and mutual information. Overall, mutual information is defined in terms of entropy that is related to volume [6], which can be considered as the product of the eigenvalues of the covariance matrix. HSIC is related to Frobenius norm that is a sum of the eigenvalues.

2.3 HSIC-InfoGAN

As discussed in [19], the mutual information can be approximated with HSIC. We propose to replace the mutual information term in InfoGAN with HSIC as an alternative. Using the HSIC loss we drop the need for an auxiliary network.

It could potentially contribute to the stabilisation of InfoGAN training as there is no need for sharing the network weights of discriminator with the auxiliary network. Overall, HSIC-InfoGAN can be represented as:

$$\min_G \max_D V_{\text{HSIC}}(D, G) = \mathbb{E}_{\mathbf{X}}[\log D(\mathbf{X})] + \mathbb{E}_{\mathbf{z}, \mathbf{c}}[\log(1 - D(G(\mathbf{z}, \mathbf{c})))] - \lambda \text{HSIC}(\mathbf{X}, \mathbf{c}), \quad (6)$$

where λ is the weight of the HSIC loss. For HSIC-InfoGAN, the tunable hyperparameters are the kernel variance σ and loss weight λ . We will discuss the strategy to tune the two hyperparameters in detail in Section 3.

3 Experiments

3.1 Implementation details

We perform experiments using the MNIST dataset [13] that contains 60,000 images of 10 digits with image size 28×28 . All models are trained using the Adam optimiser [10] with a learning rate of $2 \times e^{-4}$ for the discriminator and $1 \times e^{-3}$ for the generator. Batch size is 100. We train the models for 100 epochs. Following [5], we set the dimension of \mathbf{z} to 62 and the dimension of \mathbf{c} to 12, where 10 dimensions of \mathbf{c} represent categorical information (a 10-dimensional one-hot vector) and 2 dimensions of \mathbf{c} represent continuous information (sampled from a uniform distribution $U(0, 1)$). All models are implemented in PyTorch [22] and are trained using an NVIDIA 2080 Ti GPU. The code for calculating the HSIC loss can be found in <https://github.com/choasma/HSIC-Bottleneck>.

3.2 Results

In Fig. 1, we share qualitative results for InfoGAN and HSIC-InfoGAN. c_1 and c_2 are the two continuous latent codes. For each row, the categorical latent codes are the same for all the 10 images. For each column, the continuous latent codes are the same for the 10 images. For each row, we traverse/vary c_1 and c_2 from -1 to 1 . We observe that InfoGAN disentangles nicely the discrete latent codes whilst HSIC-InfoGAN achieves a satisfactory level of disentanglement. When varying c_2 , HSIC-InfoGAN mixes digit 0 and 8 as well as digit 3 and 5. Considering c_1 mostly captures rotation information and c_2 mostly captures thickness information, we observe that HSIC-InfoGAN learns better c_1 and similar c_2 compared to InfoGAN. Overall, HSIC-InfoGAN achieves satisfactory performance on unsupervised learning of disentangled representations. Considering the benefits of avoiding introducing auxiliary networks, HSIC-InfoGAN offers a good alternative to InfoGAN.

3.3 Strategy for hyperparameter tuning

As we discussed in Section 2, the tunable hyperparameters are the HSIC loss weight λ and the kernel variance σ . We observe that it is important to ensure

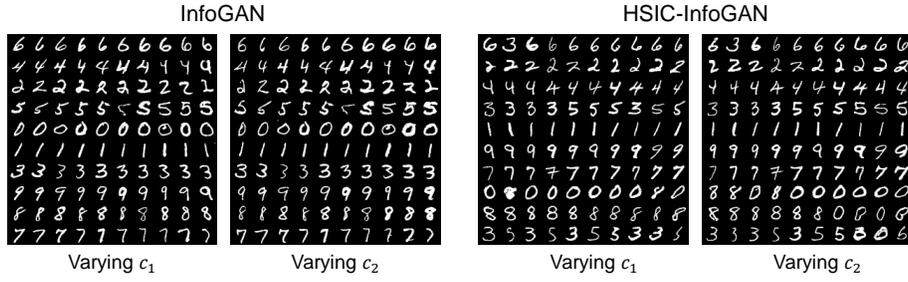


Fig. 1. Visual results from InfoGAN and HSIC-InfoGAN.

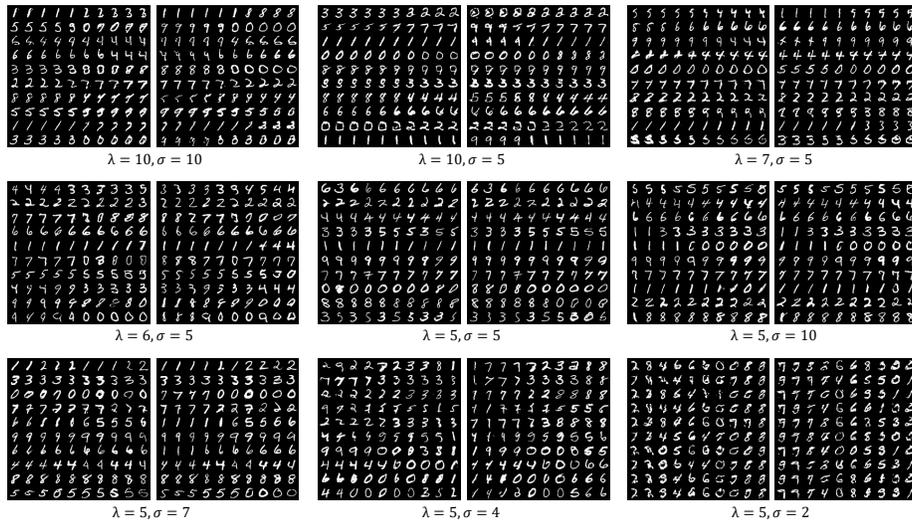


Fig. 2. Qualitative results of finding the optimal λ and σ for HSIC-InfoGAN.

the generator loss and the HSIC loss have the same order of magnitude. Note that increasing λ or decreasing σ increase the HSIC loss. As shown in Fig. 2, we show the results of varying λ and σ . Changing σ causes more significant changes to the HSIC loss (roughly, we can consider that $\text{HSIC} \sim \exp(-\frac{1}{\sigma^2})$). Hence, the strategy is to find a good σ first (search from $\sigma = 2$ to $\sigma = 10$ in our case) and then fine tune by finding the optimal λ .

4 Discussion

HSIC-InfoGAN could be widely used in many medical applications. As discussed in [14], many disentanglement methods in the medical domain take advantage of content-style disentanglement [4, 15, 16, 24, 25]. In this case, the content and style representations are usually a tensor and a vector [17]. HSIC-InfoGAN can be

applied as an alternative to replace the (InfoGAN-style) latent regression losses to save training time and decrease the memory requirements for model weights. In addition, considering other generative models such as normalising flows [21], energy-based models [7] and denoising diffusion models [23], the HSIC loss (approximating the mutual information) could be used as an unsupervised objective to learn the disentangled latent representations for medical applications. Finally, we envision that HSIC-InfoGAN can be applied in the context of contrastive learning for medical applications [3], where one can maximise and minimise the mutual information between features of (different) images as contrastive losses.

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