# Incorporating preference information interactively in NSGA-III by the adaptation of reference vectors

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**Abstract.** Real-world multiobjective optimization problems involve decision makers interested in a subset of solutions that meet their preferences. Decomposition-based multiobjective evolutionary algorithms (or MOEAs) have gained the research community's attention because of their good performance in problems with many objectives. Some efforts have been made to propose variants of these methods that incorporate the decision maker's preferences, directing the search toward regions of interest. Typically, such variants adapt the reference vectors according to the decision maker's preferences. However, most of them can consider a single type of preference, the most common being reference points. Interactive MOEAs aim to let decision-makers provide preference information progressively, allowing them to learn about the trade-offs between objectives in each iteration. In such methods, decision makers can provide preferences in multiple ways, and it is desirable to allow them to select the type of preference for each iteration according to their knowledge. This article compares three interactive versions of NSGA-III utilizing multiple types of preferences. The first version incorporates a mechanism that adapts the reference vectors differently according to the type of preferences. The other two versions convert the preferences from the type selected by the decision maker to reference points, which are then utilized in two different reference vector adaptation techniques that have been used in *a priori* MOEAs. According to the results, we identify the advantages and drawbacks of the compared methods.

**Keywords:** Multiobjective optimization · Interactive methods · Decision making · Multiobjective evolutionary algorithms · Decomposition-based MOEAs · NSGA-III.

## 1 Introduction

Real-world problems typically involve multiple conflicting objective functions to be optimized simultaneously. Because of such conflict among the objectives, the so-called multiobjective optimization problems (MOPs) do not have a single

solution, but a set of trade-off solutions called Pareto front. However, solving these problems aims to help a DM, which is a domain expert, to find their most preferred solution.

In the operations research field, multiobjective optimization methods have been classified according to the role of the DM during the solution process [25]. *No preference* methods do not involve the DM at all, being suitable when the DM does not have clear expectations of the final solution. *A priori* methods only ask for preference information from the DM at the beginning of the solution process. These methods are mainly utilized when the DM already knows the trade-offs between the objectives and their preferences are clear. *A posteriori* methods consider the DM's preferences after approximating the Pareto optimal set. Finally, *interactive* methods allow the DM to provide preference information iteratively during the solution process. In each iteration, the DM can learn about the trade-offs among the objectives and utilize this new insight for updating the preference information.

DMs can express their preferences in multiple ways, e.g., by providing desirable values for each objective function, specifying desirable ranges for each objective function, comparing pairs of solutions, etc. The type of preferences utilized in the solution process should depend on the DM, as according to their experience, they may feel more comfortable using a specific type of preference. In interactive methods, there should also be possible to allow the DM to change the type of preferences according to their needs [1].

MOEAs have been successfully applied to approximate the Pareto front of multiobjective optimization problems. There are mainly three types of MOEAs [35]: dominance-based, indicator-based, and decomposition-based. Decomposition-based MOEAs have received much attention from the research community in recent years for maintaining a good performance even when the number of objectives is increased, as opposed to domination-based methods [18]. These methods decompose an MOP into multiple single-objective optimization problems or multiple simpler MOPs, which are then optimized collaboratively. Such decomposition is performed using a set of reference vectors (RVs)[1] and a scalarization function. To find a representative set of near-Pareto optimal solutions, the RVs are generated uniformly in a simplex. The most popular decomposition-based MOEAs are MOEA/D [36], RVEA [5], and NSGA-III [10]. These methods typically do not consider the preferences of the DM during the solution process. However, multiple variants of such methods have been proposed that utilize preference information a priori or interactively (e.g., [14, 15, 19, 33]).

Decomposition-based MOEAs with preference incorporation generally utilize the preferences to adapt the RVs, directing the search toward the region of interest. However, most of them work with a single type of preference, reference points being the most common. Interactive RVEA [15] is the only decomposition-based MOEA that can handle multiple types of preferences. This method allows DM to choose among four types of preferences: reference points, preferred ranges,

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[1] RVs also known as weight vectors or reference points in different MOEAs. To avoid confusion, we will continue to use the term "RVs" only.

and preferred and non-preferred solutions. Then, the RVs are adapted according to the selected type of preference. As this approach does not need any other modification in the structure of the MOEA, the RV adaptation technique can be used in other decomposition-based MOEAs (e.g., MOEA/D [17]).

The performance of interactive RVEA using multiple types of preferences has been analyzed in various articles [2, 17]. From the results, it has been shown that the method's performance decreases when preferred ranges are utilized in most of the test instances. However, it is difficult to identify the reason behind this behavior, as the method has not been compared with other MOEAs using different types of preferences.

In this article, we compare three interactive versions of NSGA-III. All of them work by adapting the RVs using the preference information. However, the adaptation technique is performed differently in all versions. The first version utilizes the RV adaptation techniques employed in interactive RVEA. In this case, the RVs are adapted differently depending on the type of preference. The other two versions incorporate an intermediate step that converts the preferences from the type given by the DM to reference points. We do this, as reference points are the most common type of preference utilized by the RV adaptation techniques. This preference conversion allows us to adopt the existing RV adaptation techniques allowing the DM to select between multiple types of preferences. The main contributions of this article are the following:

- We incorporate the RVs adaptation technique from interactive RVEA into NSGA-III. Both methods are compared utilizing reference points and preferred ranges in different benchmark problems to identify their main advantages and disadvantages.
- We propose a simple conversion technique for transforming preferred ranges and preferred solutions into reference points. The resulting reference points are utilized as input for two different RV adaptation techniques: the one employed by R-NSGA-III [33] and NUMS [19]. In addition, we utilize these techniques in an interactive method for the first time.
- We compare three interactive versions of NSGA-III utilizing multiple types of preference information. These methods are tested with various benchmark problems considering 5, 7, and 9 objective functions. As involving real DMs will be very time consuming, the experimentation was conducted using an artificial DM [2], which enabled inexpensive comparison of interactive methods without real DMs.

The rest of the article is organized as follows. Section 2 discusses the background of multiobjective optimization and evolutionary algorithms. A brief overview of the a priori and interactive versions of NSGA-III available in the literature is presented in Section 3. Interactive RVEA, RNSGA-III, and NUMS are also described in the same section. The proposed interactive versions of NSGA-III and the mechanism to convert preferred ranges and preferred solutions into reference points are described in section 4. Section 5 compares the three interactive versions of NSGA-III presented in this article using various benchmark problems. Finally, we conclude this article in Section 6.

## 2    Background

### 2.1    Multiobjective Optimization

A multiobjective optimization problem can be mathematically formulated as follows:

$$\begin{aligned} \text{minimize} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_k(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{x} \in S, \end{aligned} \tag{1}$$

where $f_i$ $(i = 1, \dots, k)$ are the $k$ conflicting objective functions (with $k \geq 2$). $S \subset \mathbb{R}^n$ is the feasible set of decision vectors $\mathbf{x} = (x_1, \dots, x_n)^T$ with $n$ decision variables. There is a corresponding objective vector $\mathbf{F}(\mathbf{x})$ for every feasible decision vector $\mathbf{x}$. The problem can involve equality and inequality constraints that must be satisfied by the decision vectors for them to be feasible. It is not possible for all of the objective functions in (1) to reach their optimal values simultaneously due to the conflicts between them. Given two solutions $\mathbf{x}^* \in S$ and $\mathbf{x}' \in S$, $\mathbf{x}^*$ dominates $\mathbf{x}'$ if and only if $f_i(\mathbf{x}^*) \leq f_i(\mathbf{x}')$ for all $i = 1, \dots k$, and $f_j(\mathbf{x}^*) < f_j(\mathbf{x}')$ for at least one index $j = 1, \dots, k$. If there is no solution $\mathbf{x} \in S$ that dominates solution $\mathbf{x}^* \in S$, then $\mathbf{x}^*$ is Pareto optimal. The set of all Pareto optimal solutions is known as Pareto optimal set, and the corresponding objective vectors compose the Pareto front.

The best and worst objective function values in the Pareto front are known as ideal $\mathbf{z}^*$ and a nadir $\mathbf{z}^{nad}$ points, respectively. The ideal point can be computed by minimizing each objective function separately. Usually, calculating the nadir point requires computing the whole Pareto set. However, it can be approximated using a pay-off table [25], or other means [11].

In real-world problems, a DM is usually involved in the solution process. A DM is typically interested in a part of the Pareto front close to their preferences, named region of interest. DM can express their preference information in multiple ways [3, 23]. In this article, we are interested in the following types of preferences:

- Giving a reference point $\mathbf{r} = (r_1, \dots, r_k)$, where each $r_i$ is a desirable value (also known as aspiration level) for the objective function $f_i$ $(i = 1, \dots, k)$.
- Selecting $t$ (with $t \geq 1$) most preferred solutions from a solution set. We denote them by $PS = [\mathbf{ps}_1, \dots, \mathbf{ps}_t]$.
- Specifying preferred ranges for the objective functions. The preferred range for an objective $f_i$ $(i = 1, \dots, k)$ is denoted by $[f_i^l, f_i^u]$, being $f_i^l$ and $f_i^u$ the lower and upper bounds, respectively. Then, the preferred ranges for all the objectives is a $k$-dimensional hyper-box $PR = [f_1^l, f_1^u] \times, \dots, \times [f_k^l, f_k^u]$.

In interactive methods, the DM provides preference information progressively. The intervals in which MOEAs ask for preference information from the DM are known as iterations. In most cases, iterations take place every $N_{gen}$ generations, where $N_{gen}$ is a parameter that is set before the method begins. Typically, two stages can be observed in an interactive solution process: the learning and decision phases [24]. During the learning phase, the DM explores different parts of the Pareto front until they find a region of interest. Then, in the decision phase, the DM tries to fine-tune the solutions in the region of interest until finding the most preferred solution.

## 2.2   Evolutionary Algorithms

MOEAs are capable of generating an approximation of the Pareto optimal front in a single run [9]. They can be classified [35] according to their structure into: *dominance-based MOEAs*, which compare solutions utilizing Pareto dominance-based mechanisms; *indicator-based MOEAs* that use quality indicators as selection criteria; and *decomposition-based MOEAs*, which decompose the MOP into multiple single objective optimization problems or a set of simpler MOPs, which are optimized collaboratively.

In this article, we are interested in decomposition-based MOEAs. These methods need two main components: a set of RVs (typically uniformly distributed in the objective space) and a scalarization function. RVs can be generated using a simplex lattice design [7]. The number of vectors generated by such a method is given by $\binom{q+k-1}{k-1}$, where $q$ is a parameter to control the density of the solutions and $k$ is the number of objective functions. Scalarizing functions map objective vectors to real-value scalars and are used to evaluate the solutions of a section of the objective space. Such sections evolve in the direction of the RV associated with them. Among the most well-known decomposition-based MOEAs are MOEA/D [36], RVEA [5], and NSGA-III [10].

## 3   Related Work

Multiple versions of NSGA-III with a priori and interactive preference incorporation have been proposed in the literature. Such methods typically modify the distribution of the RVs according to the preferences provided by the DM. As a result, the method does not provide an approximation of the complete Pareto front, but the obtained solutions focus on a region of interest. In this section, we describe the existing versions of NSGA-III that consider the preferences of the DM. We classify these methods according to the type of preference incorporation: a priori and interactive. We also describe some methods utilized in the rest of the article: interactive RVEA, R-NSGA-III, and NUMS.

**A priori methods:** In the article where NSGA-III was proposed [10], the authors suggested a mechanism to incorporate preferences a priori by filtering the RVs utilized by the method. A similar approach was used by Yan et al. [34] and da Silva et al. [8, 31, 30]. Cheng et al. [6] proposed an a priori version of NSGA-III that requires the DM to specify a central RV, which is utilized to adapt the complete set of RVs. P-NSGA-III [4] also modifies the RVs according to the preference information, but it asks the DM for information about the importance of objectives. Finally, R-NSGA-III [33] receives one or multiple reference points from the DM, which are used to adapt the RVs toward one or multiple regions of interest. The RVs are updated at each generation, as the process involves normalizing the reference points utilizing an approximation of the ideal and nadir points, which change depending on the obtained solutions.

**Interactive methods**: Mnasri et al. [27] proposed PI-NSGAIII-VF, the first interactive version of NSGA-III. This method is a hybrid of NSGA-III and a

strategy for incorporating DM's preferences on any MOEA: PI-EMO-VF [12]. PI-NSGAIII-VF asks the DM to classify a set of solutions during each iteration. Such information is utilized to approximate a value function progressively. Then, a stopping condition is set up according to this value function. T-NSGA-III [22] modifies the selection procedure of NSGA-III to incorporate the DM's preferences in the form of preferred ranges. Such ranges are utilized to transform the objective values into a new coordinate system defined by the upper and lower bounds for each objective function specified by the DM. I-NSGA-III-PLVF [20] asks the DM to score a set of solutions. The method learns a value function using the provided scores, which is then employed to model the DM's preferences. IOPIS-NSGA-III [29] asks the DM for a reference point, which is utilized to create a (typically lower-dimensional) preference-incorporated space (consisting of a set of scalarization functions) to reformulate the optimization problem. Finally, PI-NSGA-III-PC-INK [26] modifies the dominance relation using polyhedral cones constructed using the preference information. This method shows a set of solutions in each iteration, from which the DM can select the preferred one.

**Interactive RVEA.** Interactive RVEA [15] is the only decomposition-based MOEA that provides the DM with multiple options to give preferences. The method initializes a set of uniformly distributed RVs, which are adapted differently according to the type of preferences selected by the DM. For reference points and preferred solutions, the RVs are redirected toward the preference. For non-preferred solutions, the RVs closer to such solutions are removed, while the rest are kept. A Latin hyper-cube sampling is utilized for preferred ranges. Then, the obtained vectors are normalized into unit vectors, which replace the initial RVs. This method needs a parameter $v \in (0,1)$ to control the spread of the RVs. A small value of $v$ results in RVs close to the preference information, while a value close to 1 will produce more sparse RVs. In the rest of this article, we will refer to this RV adaptation technique as *IRA*.

**R-NSGA-III.** R-NSGA-III [33] is an a priori method that requires the DM to provide one or multiple reference points. As a result, the method will provide one set of solutions close to each of the provided reference points. R-NSGA-III adapts the RVs at each generation to incorporate the preferences of the DM. First, the reference points provided by the DM are normalized using the normalization procedure of NSGA-III. Then, a set of uniformly distributed RVs is generated and shrunk using a spread parameter that controls the size of the region of interest. The intercepts of the unit hyperplane and the vectors from the ideal point to each normalized reference point are computed. Finally, the obtained values are utilized to shift the shrunken RVs to the unit hyperplane. The same procedure is performed for each reference point provided by the DM. In addition, the extreme points of the hyperplane are also added to the set of RVs. In the rest of this article, we will refer to this technique as *RPA*.

**NUMS.** Li et al. [19] proposed a nonuniform mapping scheme (NUMS) to map a set of uniformly distributed RVs on a canonical simplex to new positions close to a reference point provided by the DM. In this case, the mapping function is nonlinear and is determined by an RV's position in relation to the pivot point.

A pivot point represents the region of interest on a simplex and determines its position. The nonuniform mapping is utilized to bias the RVs toward the pivot point. The method gives the option of keeping the boundary of the simplex or not. In addition, a spread parameter is needed for this method, which is a value between 0 and 1, representing the relative ratio of the size of the region of interest with respect to the Pareto front. This method has been applied to MOEA/D both a priori and interactively.

## 4   Proposal

In this article, we propose three interactive versions of NSGA-III that can utilize multiple types of preference information: reference points, preferred ranges, and preferred solutions. The proposed methods do not change the structure of NSGA-III but only modify the distribution of the RVs to obtain solutions in a region of interest.

The first interactive NSGA-III proposed on this article incorporates the IRA technique to handle multiple types of preferences. We will refer to this method as *iNSGA-III-IRA*. It is worth noting that the possibility of having this type of method has been mentioned in [29]. However, the authors compared this method with interactive RVEA using only reference points. Although some experiments with interactive RVEA have shown a lower performance when using preferred ranges [2], it is difficult to identify if this behavior is related to the performance of RVEA or the IRA technique. In this article, we compare iNSGA-III-IRA with interactive RVEA utilizing reference points, preferred ranges, and preferred solutions to identify the potential and drawbacks of the IRA technique in problems with different features.

The second and third methods proposed in this article adopt a preference conversion layer that converts preferred ranges and preferred solutions into reference points. The preference conversion layer allows us to utilize reference-point-based RV adaptation techniques from the literature but with other types of preference. In a real-world scenario, the DM is unaware of the preferences conversion, as it is an intermediate layer between the user interface and the method. The conversion of the preference information is performed as follows:

- *Preferred ranges:* Let $PR = [[f_i^l, f_i^u], \ldots, [f_k^l, f_k^u]]$ be a preferred range given by the DM. We can obtain a reference point $\boldsymbol{r} = [f_1^l, f_2^l, \ldots, f_k^l]$, or alternatively $\boldsymbol{r} = [f_1^u, f_2^u, \ldots, f_k^u]$.
- *Preferred solutions:* If the DM selects a single solution ($\boldsymbol{ps}$) in an iteration, it can be directly utilized as a reference point. If more than one solution is selected ($PS$), a reference point can be computed as $\boldsymbol{r} = [max(PS_1), max(PS_2), \ldots, max(PS_k)]$.

It is worth noting that when selecting a set of preferred solutions, the DM can provide preferences with different ideologies in mind. The case we considered here is the simplest one, where the DM chooses solutions close to each other, indicating that they are interested in a specific region of interest. However, this

is not always the case, as it is also possible that the DM selects solutions on different parts of the Pareto front. The interpretation in such a case can be divided into two. The first option is that the DM is interested in multiple regions of interest and wants to obtain more solutions for all of them. In such a case, the conversion would not lead to a single reference point but a set of them. This can easily be handled by the RPA technique, as it can distribute the RVs among multiple regions of interest. However, if a technique such as NUMS is utilized, it would be necessary to make separate runs for each reference point. The second cause is related to the learning process of the DM. Suppose the solutions are too sparse in the Pareto front. In that case, it can also mean that the DM still does not have a clear idea of their preferences and that the method should keep providing solutions in a region of interest that cover most of the Pareto front. Both types of interpretations together with different mechanisms to compute a suitable reference point are subject to further research.

In this article, we utilize two reference point-based RV adaptation techniques with NSGA-III: NUMS, and RPA (we refer to these algorithms as iNSGA-III-NUMS and iNSGA-III-RPA, respectively). To make these methods interactive, we run them multiple times changing the preference information. As the experiments only involve benchmark problems, no further changes are needed. However, it is worth noting that when utilizing interactive methods in real-world problems, some general properties need to be considered to reduce the cognitive load of the DM [1, 32, 16].

Figure 1 illustrates the structure of one iteration for each proposed method. The yellow line represents the stages involved in iNSGA-III-IRA. In this case, the preferences of the DM are received directly by the IRA method, as it can already handle multiple types of preference. Then, both iNSGA-III-NUMS (blue line) and iNSGA-III-RPA (red line) convert the preferences into reference points, which are utilized as input for the RV adaptation technique. For all the proposed methods, the adapted reference vectors are used by NSGA-III to direct the search toward a region of interest.
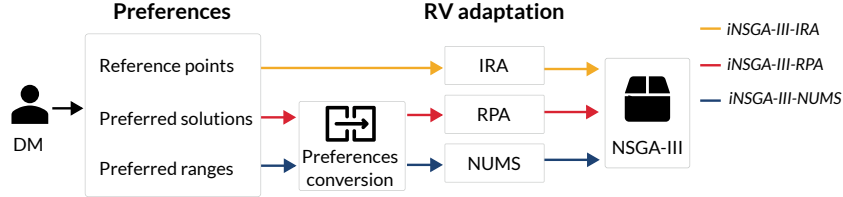


**Fig. 1.** Structure of an iteration of each of the methods proposed in this article.

Our proposal is the first attempt to unify preference information provided by the DM, which is an important research topic for both a priori and interactive methods [28]. In addition, although we only consider two RV adaptation techniques, a similar structure can be utilized with other methods.

## 5   Algorithmic Comparison

In this section, we compare the three interactive versions of NSGA-III presented in this article utilizing ADM-II [2], an artificial DM capable of comparing interactive methods using reference points, preferred ranges, and preferred and non-preferred solutions. The performance evaluation using ADM-II is divided into two stages: one for the learning phase and another for the decision phase. The main difference between the two stages is how ADM-II computes the preference information. For the learning phase, the preferences are distributed on the Pareto front, indicating that the DM is still exploring multiple regions of interest. For the decision phase, the preferences are closer to a specific region of interest, fine-tuning the solutions belonging to it. Each iteration, ADM-II generates a reference point that is utilized in the method and when applying some of the performance indicators (e.g., R-IGD). If the method requires preferred ranges, they are obtained by perturbing the reference point. This feature of ADM-II will allow us to evaluate if the solutions provided by the proposed methods are in the same region of interest after changing the type of preferences.

For the experimentation, we considered four iterations for the learning phase and three for the decision phase. After each iteration, ADM-II computes the performance indicators for the solutions obtained by the methods. Then, a cumulative indicator value is calculated for each phase as suggested in [2]. We run the methods 15 times for each problem with a given number of objectives. We considered two problems of the DTLZ benchmark [13] in the experiments: DTLZ1 and DTLZ3, with 5, 7, and 9 objectives. The number of variables is given by $10 + k - 1$. The maximum number of generations for each iteration was set to 200 for each method. The spread parameter was set to 0.2 for all the methods.

We utilized two performance indicators with ADM-II: R-IGD [21] and the composite front contribution (CFC) [2]. R-IGD measures the convergence and diversity of the obtained solutions. On the other hand, CFC measures the number of non-dominated solutions provided by each method when constructing a composite front. The more non-dominated solutions a method has contributed to constructing the composite front, the better the performance of this method is.

First, we compared interactive RVEA (referred as iRVEA in Table 1 and 2) and iNSGA-III-IRA using reference points and preferred ranges. Table 1 and 2 show the obtained cumulative R-IGD and CFC, respectively. The best results are highlighted in **boldface**. When utilizing reference points, the iRVEA outperforms iNSGA-III-IRA in most of the test instances, both in the learning and decision phase, according to the cumulative R-IGD and CFC. Such values mean that the solutions obtained by iRVEA are closer to the preference information and that this method produces more non-dominated solutions than iNSGA-III-IRA. When using preferred ranges, the R-IGD values indicate a better performance of iNSGA-III-IRA in DTLZ1, while iRVEA obtained better results for DTLZ3. However, according to the cumulative CFC, iNSGA-III-IRA obtained more non-dominated solutions in most test instances.

Then we compared iNSGA-III-IRA, iNSGA-III-NUMS, and iNSGA-III-RPA using reference points, preferred ranges, and preferred solutions. It is worth not-

**Table 1.** Cumulative R-IGD for iRVEA and iNSGA-III using reference points and preferred ranges.

| Problem | k | Phase | Reference Points | | | | Preferred Ranges | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | iRVEA | | iNSGA-III-IRA | | iRVEA | | iNSGA-III-IRA | |
| | | | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| DTLZ1 | 5 | Learning | **2.5333** | 0.1772 | 2.5910 | 0.2128 | 2.7213 | 0.3402 | **2.6263** | 0.3677 |
| | | Decision | 1.9421 | 0.2115 | **1.8894** | 0.2673 | 2.1656 | 0.2291 | **1.9068** | 0.4622 |
| | 7 | Learning | **2.7599** | 0.1845 | 3.1748 | 0.4511 | 3.1315 | 0.6090 | **3.0019** | 0.8780 |
| | | Decision | **2.0264** | 0.3609 | 3.0600 | 1.1414 | 3.0205 | 0.5322 | **2.5845** | 1.1048 |
| | 9 | Learning | **2.6071** | 0.1974 | 2.7505 | 0.2968 | 3.2852 | 0.5054 | **2.9947** | 0.5919 |
| | | Decision | **1.8139** | 0.1492 | 2.5000 | 0.6562 | 3.1749 | 0.6887 | **2.3387** | 1.0481 |
| DTLZ3 | 5 | Learning | **0.2460** | 0.2358 | 0.3289 | 0.3686 | **0.5332** | 0.1669 | 1.0590 | 0.7600 |
| | | Decision | **0.3913** | 0.6630 | 0.4887 | 1.1339 | **0.2312** | 0.2518 | 0.9985 | 0.9803 |
| | 7 | Learning | **0.6357** | 0.5161 | 0.7592 | 0.2355 | **1.9082** | 0.4623 | 3.6905 | 1.4945 |
| | | Decision | **0.6928** | 0.7482 | 0.7321 | 0.7137 | **0.7134** | 0.4282 | 3.3938 | 1.4959 |
| | 9 | Learning | **0.6371** | 0.3334 | 0.8224 | 0.4365 | **0.2361** | 0.7682 | 3.6308 | 1.3574 |
| | | Decision | **0.4576** | 0.3578 | 0.8221 | 0.7066 | **0.4497** | 0.3196 | 3.4340 | 1.9351 |

ing that R-IGD cannot be utilized to compare methods using preferred solutions. For this reason, we only considered the cumulative CFC for that type of preference. The obtained results are shown in Table 3. The best results are highlighted in **boldface**. According to the cumulative R-IGD and CFC, iNSGA-III-IRA showed a better performance than the compared methods in most test instances when utilizing reference points. For preferred ranges, the solutions obtained by iNSGA-III-RPA are closer to the preference information and provide more non-dominated solutions than the compared methods. For preferred solutions, we selected five solutions as preferences for each iteration. According to the results, iNSGA-III-IRA obtained many more non-dominated solutions. However, this is due to how the IRA technique handles sets of preferred solutions. It computes a region of interest for each solution the DM selects without considering if they overlap. The other compared methods convert the preferred solutions into a reference point, making it easier to compare their performance. Among them, iNSGA-III-RPA obtained more non-dominated solutions in most test instances.

### 5.1   Discussion

When comparing iNSGA-III-IRA and interactive RVEA we could notice that the latter obtained better results when using reference points. Although iNSGA-III-IRA obtained better R-IGD values only on DTLZ1 utilizing preferred ranges, we can notice that it produces more non-dominated solutions than the compared method in both DTLZ1 and DTLZ3. A more extensive experimentation is needed to have a clearer idea of the features of the problems in which iNSGA-III-IRA outperforms interactive RVEA. Also, at the moment it is not possible to compare the performance of these methods using preferred and non-preferred solutions due to the lack of performance indicators utilizing these types of preferences.

After comparing iNSGA-III-NUMS and iNSGA-III-RPA with iNSGA-III-IRA we could notice that the preference conversion did not affect the quality of the solutions. According to the obtained results, the IRA method performs best using reference points. However, utilizing the preference conversion mechanism helps get better results with preferred ranges and preferred solutions. It is worth

**Table 2.** Cumulative CFC for iRVEA and iNSGA-III using reference points and preferred ranges.

| Problem | k | Phase | Reference Points | | | | Preferred Ranges | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | iRVEA | | iNSGA-III-IRA | | iRVEA | | iNSGA-III-IRA | |
| | | | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| DTLZ1 | 5 | Learning | 443.2 | 68.0835 | **454.8** | 53.7788 | 111.6 | 112.6634 | **485.6** | 31.7055 |
| | | Decision | **387.6** | 2.2000 | 369.1 | 8.1664 | 283.5 | 206.3745 | **357.9** | 34.0307 |
| | 7 | Learning | **343.5** | 18.7150 | 138.5 | 57.1896 | 116.2 | 99.8637 | **261.4** | 60.8805 |
| | | Decision | **269.7** | 0.6403 | 199.1 | 79.6485 | 168.0 | 183.7145 | **235.4** | 22.0191 |
| | 9 | Learning | **671.1** | 40.2006 | 257.8 | 149.4957 | 218.4 | 187.6205 | **574.7** | 60.7339 |
| | | Decision | **517.4** | 1.6248 | 465.0 | 71.8053 | 249.4 | 293.6914 | **443.5** | 56.3777 |
| DTLZ3 | 5 | Learning | 367.4 | 111.3240 | **441.2** | 85.5871 | 413.3 | 128.8682 | **477.0** | 43.0976 |
| | | Decision | 309.8 | 148.9495 | **334.6** | 72.7010 | 202.3 | 89.5255 | **358.0** | 40.9072 |
| | 7 | Learning | **292.7** | 62.9842 | 169.6 | 95.9252 | **346.1** | 109.1040 | 221.0 | 91.3893 |
| | | Decision | **214.5** | 98.5792 | 192.7 | 52.2323 | 131.1 | 172.7637 | **176.4** | 66.6381 |
| | 9 | Learning | **611.2** | 75.5299 | 415.0 | 130.3580 | **698.8** | 96.7831 | 532.0 | 102.3426 |
| | | Decision | **407.2** | 180.1426 | 367.8 | 132.7764 | 301.2 | 222.4984 | **409.1** | 92.2339 |

noting that iNSGA-III-RPA performed better than iNSGA-III-NUMS in most test instances. Considering other types of preference conversions and more RV adaptation techniques are interesting future research directions.

Using ADM-II enabled inexpensive comparison of interactive methods without real DMs. However, it needs performance indicators to evaluate the solutions provided by each method. There are only a few performance indicators for MOEAs with preference incorporation. In addition, most of them can only compare methods that utilize reference points. ADM-II can use R-IGD for preferred ranges due to the mechanism utilized to generate the preference information. When generating the preferred ranges, ADM-II employs an equivalent reference point, which is then used in the performance indicator. Comparing methods that utilize preferred solutions is still an open problem. In this case, although the cumulative CFC gives us an idea of the number of non-dominated solutions provided by each method, it is difficult to measure the quality of such solutions without an additional quality indicator.

## 6 Conclusions

In this article, we proposed three interactive versions of NSGA-III. These methods incorporate the preferences of the DM by adapting the set of RVs. iNSGA-III-IRA utilizes the RV adaptation technique used by interactive RVEA. However, when comparing both methods, we could identify that the performance of interactive RVEA is better when utilizing reference points. However, when using preferred ranges, iNSGA-III-IRA obtained better results for one of the test problems with 5, 7, and 9 objectives. Extended experimentation considering more realistic problems is needed to identify the types of problems in which each method performs better. We also proposed a mechanism to convert preferred ranges and preferred solutions into reference points. The obtained reference points were utilized as input for the NUMS and RPA techniques. The performance of iNSGA-III-NUMS and iNSGA-III-RPA compared with iNSGA-III-IRA suggest that the obtained results still reflect the preferences after the conversion. In addition,

**Table 3.** Cumulative R-IGD and CFC for iNSGA-III-IRA, iNSGA-III-NUMS, and iNSGA-III-RPA.Column k indicates the number of objectives, P is the phase of the solution process (L: learning, D: decision), and the method names are shorten as IRA, NUMS, and RPA.

| Problem | k | P | R-IGD IRA Mean | Std | NUMS Mean | Std | RPA Mean | Std | CFC IRA Mean | Std | NUMS Mean | Std | RPA Mean | Std |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Reference points | | | | | | | |
| DTLZ1 | 5 | L | **2.5171** | 0.1017 | 3.5837 | 0.6634 | 2.5746 | 0.1360 | 425.0 | 81.0086 | **452.2** | 65.0443 | 409.9 | 113.7580 |
| | | D | 1.9301 | 0.1754 | 2.0761 | 0.2814 | **1.9064** | 0.1693 | **387.1** | 3.4482 | 360.2 | 7.3185 | 367.6 | 7.5657 |
| | 7 | L | **2.5484** | 0.1501 | 9.2395 | 1.0619 | 2.7300 | 0.3176 | **329.4** | 39.0620 | 108.8 | 66.5219 | 132.3 | 49.3053 |
| | | D | **1.8378** | 0.1492 | 6.0133 | 2.2759 | 1.9784 | 0.3551 | **269.0** | 1.5492 | 47.8 | 38.6000 | 190.5 | 79.6382 |
| | 9 | L | **2.5164** | 0.1352 | 5.6678 | 1.1235 | 2.5593 | 0.1840 | **655.8** | 63.3574 | 1.9 | 3.2696 | 422.4 | 133.4820 |
| | | D | **1.8940** | 0.3560 | 4.7159 | 1.2630 | 1.9412 | 0.4530 | **517.6** | 2.1541 | 2.2 | 4.9759 | 493.3 | 4.4508 |
| DTLZ3 | 5 | L | 0.3032 | 0.1339 | 0.4785 | 0.0955 | **0.2405** | 0.0782 | 336.6 | 117.5884 | **444.7** | 66.8581 | 412.4 | 79.1052 |
| | | D | 0.4480 | 0.6420 | 0.2555 | 0.1301 | **0.1335** | 0.0482 | 293.5 | 146.2315 | 363.4 | 3.2311 | **370.2** | 6.9397 |
| | 7 | L | **0.6406** | 0.2752 | 6.7688 | 2.5678 | 0.6795 | 0.6611 | **257.4** | 80.5409 | 67.7 | 27.7995 | 157.5 | 72.7451 |
| | | D | **0.4848** | 0.4233 | 4.4562 | 3.2389 | 1.3165 | 1.9153 | **199.3** | 96.5578 | 13.5 | 10.1316 | 178.5 | 60.7260 |
| | 9 | L | **0.5366** | 0.2089 | 9.0865 | 1.2953 | 0.6724 | 0.3866 | **629.7** | 60.9230 | 94.6 | 46.7979 | 497.1 | 63.9022 |
| | | D | **0.3523** | 0.2050 | 7.4008 | 0.6856 | 0.8317 | 1.1832 | **514.5** | 8.9917 | 24.2 | 16.7021 | 435.7 | 69.2272 |
| | | | | | | | Preferred ranges | | | | | | | |
| DTLZ1 | 5 | L | 2.9547 | 0.3119 | 2.9987 | 0.4754 | **2.4959** | 0.1699 | 91.5 | 118.4924 | **482.6** | 18.3314 | 443.9 | 66.3151 |
| | | D | 1.9964 | 0.1886 | 2.1821 | 0.5939 | **1.8669** | 0.2768 | 101.0 | 135.5699 | **363.1** | 10.2806 | 362.0 | 20.7605 |
| | 7 | L | 3.0137 | 0.3383 | 8.2075 | 1.6523 | **2.4159** | 0.1714 | 49.3 | 51.7012 | 76.1 | 57.9473 | **312.6** | 48.8512 |
| | | D | 2.7146 | 0.9704 | 4.4261 | 2.8460 | **1.7150** | 0.0142 | 35.6 | 68.3728 | 12.4 | 21.7127 | **251.6** | 1.3565 |
| | 9 | L | 3.2866 | 0.3085 | 6.1723 | 1.6570 | **2.5955** | 0.2913 | 294.8 | 141.4205 | 25.2 | 42.2583 | **644.0** | 46.4909 |
| | | D | 2.1963 | 0.5379 | 4.4676 | 1.2529 | **1.7571** | 0.1213 | 202.8 | 276.1850 | 17.5 | 42.3019 | **492.9** | 4.7634 |
| DTLZ3 | 5 | L | 0.3937 | 0.1353 | 1.6153 | 0.2997 | **0.1137** | 0.0433 | 215.7 | 135.9059 | 446.4 | 70.3210 | **464.3** | 57.3882 |
| | | D | 0.3835 | 0.3890 | 1.0444 | 0.1871 | **0.1062** | 0.0416 | 122.7 | 95.7831 | 365.0 | 4.5826 | **368.2** | 13.3626 |
| | 7 | L | 1.7638 | 0.4380 | 8.2372 | 2.0041 | **1.0111** | 0.6701 | **331.8** | 122.8184 | 73.2 | 25.4982 | 284.8 | 32.3691 |
| | | D | 0.6916 | 0.8202 | 5.9940 | 2.4421 | **0.2821** | 0.1333 | 113.1 | 124.3627 | 20.7 | 18.9634 | **192.7** | 51.9077 |
| | 9 | L | 1.7413 | 0.3917 | 9.8321 | 0.6294 | **1.1918** | 0.7182 | **695.1** | 104.4581 | 152.9 | 74.4613 | 595.2 | 50.4892 |
| | | D | 0.5013 | 0.5152 | 6.8605 | 2.0710 | **0.2970** | 0.1104 | 261.0 | 217.8454 | 31.9 | 26.4290 | **422.9** | 76.1294 |
| | | | | | | | Preferred solutions | | | | | | | |
| DTLZ1 | 5 | L | - | - | - | - | - | - | **1205.2** | 407.4918 | 398.3 | 96.2310 | 440.6 | 80.3308 |
| | | D | - | - | - | - | - | - | **1866.3** | 37.4514 | 349.6 | 36.9681 | 377.5 | 2.2472 |
| | 7 | L | - | - | - | - | - | - | **1237.9** | 307.3807 | 101.1 | 59.6899 | 300.3 | 76.0514 |
| | | D | - | - | - | - | - | - | **1267.5** | 12.3713 | 48.4 | 39.4416 | 254.0 | 1.5492 |
| | 9 | L | - | - | - | - | - | - | **4053.6** | 753.0172 | 16.7 | 47.4406 | 560.6 | 103.1079 |
| | | D | - | - | - | - | - | - | **2492.2** | 5.4000 | 11.9 | 34.3874 | 485.7 | 21.8406 |
| DTLZ3 | 5 | L | - | - | - | - | - | - | **1599.5** | 345.5605 | 404.2 | 107.6362 | 463.5 | 54.9877 |
| | | D | - | - | - | - | - | - | **1895.4** | 3.3226 | 348.7 | 21.6474 | 378.6 | 1.4967 |
| | 7 | L | - | - | - | - | - | - | **1508.1** | 409.6194 | 65.6 | 33.4610 | 194.3 | 91.0066 |
| | | D | - | - | - | - | - | - | **1263.8** | 24.1031 | 28.8 | 20.4392 | 245.7 | 21.9456 |
| | 9 | L | - | - | - | - | - | - | **4075.1** | 704.5970 | 136.7 | 60.7833 | 583.7 | 96.0532 |
| | | D | - | - | - | - | - | - | **2472.6** | 25.2198 | 51.2 | 42.5977 | 485.4 | 15.6729 |

iNSGA-III-RPA obtained better results than the compared methods using preferred ranges. Converting between more types of preferences is subject to further research and also considering other types of RV adaptation techniques.

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