Calculus for Computer Graphics

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This book is dedicated to my best friend, Heidi.

Preface

Calculus is one of those subjects that appears to have no boundaries, which is why some calculus books are so large and heavy! So when I started writing this book, I knew that it would not fall into this category: it would be around 200 pages long and take the reader on a gentle journey through the subject, without placing too many demands on their knowledge of mathematics.

The objective of the book is to inform the reader about functions and their derivatives, and the inverse process: integration, which can be used for computing area and volume. The emphasis on geometry gives the book relevance to the computer graphics community, and hopefully will provide the mathematical background for professionals working in computer animation, games and allied disciplines to read and understand other books and technical papers where differential and integral notation is found.

The book divides into 13 chapters, with the obligatory Introduction and Conclusion chapters. Chapter 2 reviews the ideas of functions, their notation and the different types encountered in every-day mathematics. This can be skipped by readers already familiar with the subject.

Chapter 3 introduces the idea of limits and derivatives, and how mathematicians have adopted limits in preference to infinitesimals. Most authors introduce integration as a separate subject, but I have included it in this chapter so that it is seen as an antiderivative, rather than something independent.

Chapter 4 looks at derivatives and antiderivatives for a wide range of functions such as polynomial, trigonometric, exponential and logarithmic. It also shows how function sums, products, quotients and function of a function are differentiated.

Chapter 5 covers higher derivatives and how they are used to detect a local maximum and minimum.

Chapter 6 covers partial derivatives, which although are easy to understand, have a reputation for being difficult. This is possibly due to the symbols used, rather than the underlying mathematics. The total derivative is introduced here as it is required in a later chapter.

Chapter 7 introduces the standard techniques for integrating different types of functions. This can be a large subject, and I have deliberately kept the examples simple in order to keep the reader interested and on top of the subject.

Chapter 8 shows how integration reveals the area under a graph and the concept of the Riemann Sum. The idea of representing and area or a volume as the limiting sum of some fundamental unit, is central to understanding calculus.

Chapter 9 deals with arc length, and uses a variety of worked examples to compute the length of different curves.

Chapter 10 shows how single and double integrals are used to compute the surface area for different objects. It is also a convenient point to introduce Jacobians, which hopefully I have managed to explain convincingly.

Chapter 11 shows how single, double and triple integrals are used to compute the volume of familiar objects. It also shows how the choice of a coordinate system influences a solution's complexity.

Finally, Chap. 12 covers vector-valued functions, and provides a short introduction to this very large subject.

The book contains over one hundred illustrations to provide a strong visual interpretation for derivatives, antiderivatives and the calculation of area and volume.

There is no way I could have written this book without the internet and several excellent books on calculus. One only has to Google "What is a Jacobian" to receive over one million entries in about 0.25 seconds! YouTube also contains some highly informative presentations on virtually every aspect of calculus one could want. So I have spent many hours watching, absorbing and disseminating videos, looking for vital pieces of information that are key to understanding a topic.

The books I have referred to include: *Teach Yourself Calculus*, by Hugh Neil, *Calculus of One Variable*, by Keith Hirst, *Inside Calculus*, by George Exner, *Short Calculus*, by Serge Lang, and my all time favourite: *Mathematics from the Birth of Numbers*, by Jan Gullberg. I acknowledge and thank all these authors for the influence they have had on this book. One other book that has helped me is *Digital Typography Using* LaTeX by Apostolos Syropoulos, Antonis Tsolomitis and Nick Sofroniou.

I would also like to thank Professor Wordsworth Price and Professor Patrick Riley for their valuable feedback on early versions of the manuscript. However, I take full responsibility for any mistakes that may have found their way into this publication.

Finally, I would like to thank Beverley Ford, Editorial Director for Computer Science, and Helen Desmond, Editor for Computer Science, Springer UK, for their continuing professional support.

Ashtead, UK John Vince

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