# Automatic Textual Explanations of Concept Lattices

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Abstract Lattices and their order diagrams are an essential tool for communicating knowledge and insights about data. This is in particular true when applying Formal Concept Analysis. Such representations, however, are difficult to comprehend by untrained users and in general in cases where lattices are large. We tackle this problem by automatically generating textual explanations for lattices using standard scales. Our method is based on the general notion of ordinal motifs in lattices for the special case of standard scales. We show the computational complexity of identifying a small number of standard scales that cover most of the lattice structure. For these, we provide textual explanation templates, which can be applied to any occurrence of a scale in any data domain. These templates are derived using principles from human-computer interaction and allow for a comprehensive textual explanation of lattices. We demonstrate our approach on the spices planner data set, which is a medium sized formal context comprised of fifty-six meals (objects) and thirty-seven spices (attributes). The resulting 531 formal concepts can be covered by means of about 100 standard scales.

**Keywords:** Ordered Sets, Explanations, Formal Concept Analysis, Closure System, Conceptual Structures

#### 1 Introduction

There are several methods for the analysis of relational data. One such method is Formal Concept Analysis [4] (FCA). The standard procedure in the realm of FCA is to compute the concept lattice, i.e., a data representation on the ordinal level of measurement [16]. Ordered data structures are comparatively more comprehensible for users than, e.g., Euclidean embeddings. Nevertheless, untrained users may have difficulties in grasping knowledge from lattices (and lattice diagrams). Moreover, even trained users cannot cope with lattice structures of large size. In addition, there are up until now only rudimentary methods to derive basic meaning of lattices that are of standard scale [4, Figure 1.26].

A meaningful approach to cope with both issues is to employ more complex ordinal patterns, e.g., scales composed from standard scales. A recent result by Hirth et al. [9] allows for the efficient recognition of such patterns, there called as *ordinal motifs*. Based on these we propose a method to automatically generate textual explanations of concept lattices. For the recognition of ordinal motifs we employ *scale-measures*, i.e., continuous maps between closure spaces. These are able to analyze parts of a conceptual structure with respect to a given set of scale contexts. While this approach is very expressive there may be exponentially many scalemeasures. Therefore we introduce an importance measure of ordinal motifs based on the proportion of the conceptual structure that they reflect. With this our method can identify a small number of ordinal motifs that covers most of the concept lattice.

An advantage of employing sets of standard scales is their well-known structural semantic, cf. *basic meaning* Figure 1.26 [4]. Based on this we constructed for every standard scale textual templates based on principle from human computer interaction. In detail we applied the five goodness criteria [11] for explainability in machine learning to ensure that the textual templates are human comprehensible.

Besides our theoretical investigations we provide an experimental example on a real world data set of medium size. All proposed methods are implemented in conexp-clj [5], a research tool for Formal Concept Analysis. Our approach is not only beneficial for untrained users but also provides explanations of readable size for concept lattices that are too large even for experienced users.

# 2 Formal Concept Analysis

Throughout this paper we presume that the reader is familiar with standard FCA notation [4]. In addition to that, for a formal context  $\mathbb{K} := (G, M, I)$  we denote by  $\mathbb{K}[H,N] := (H,N,I \cap H \times N)$  the induced sub-context for a given set of objects  $H \subseteq G$  and attributes  $N \subseteq M$ . If not specified differently the lift of a map  $\sigma: G_1 \to G_2$  on  $\mathcal{P}(G_1) \to \mathcal{P}(G_2)$  is defined as  $\sigma(A) := \{\sigma(a) \mid a \in A\}$  where  $A \subseteq G_1$ . The second lift to  $\mathcal{P}(\mathcal{P}(G_1)) \to \mathcal{P}(\mathcal{P}(G_2))$  is defined as  $\sigma(A) := \{\sigma(A) \mid A \in A\}$  for  $A \subseteq \mathcal{P}(G_1)$ . For a closure system  $\mathcal{A}$  on G we call  $\mathcal{D}$  a finer closure system, denoted  $\mathcal{A} \leq \mathcal{D}$ , iff  $\mathcal{D}$  is a closure system of  $\mathcal{A}$  iff  $\mathcal{D}$  is a closure system on  $H \subseteq G$  and  $\mathcal{D} = \{H \cap A \mid A \in \mathcal{A}\}$ . Note that there are other definitions for sub-closure systems in the literature [7].

3 Recognizing Ordinal Motifs of Standard Scale

For the generation of textual explanations we recognize parts of the concept lattice that match an ordinal motif, i.e., are isomorphic to a standard scale. For this task we employ (full) scale-measures as introduced in the following. **Definition 3.1 (Scale-Measure (Definition 91 [4])).** For two formal contexts  $\mathbb{K}$ ,  $\mathbb{S}$  a map  $\sigma: G_{\mathbb{K}} \to G_{\mathbb{S}}$  is a scale-measure iff for all  $A \in \text{Ext}(\mathbb{S})$  the pre-image  $\sigma^{-1}(A) \in \text{Ext}(\mathbb{K})$ . A scale-measure is full iff  $\text{Ext}(\mathbb{K}) = \sigma^{-1}(\text{Ext}(\mathbb{S}))$ .

We may note that we use a characterization for full scale-measures (Definition 91 [4]) which can easily be deduced. The existence of a scale-measure from a context  $\mathbb{K}$  into a scale context  $\mathbb{S}$  implies that the conceptual structure of the image of  $\sigma$  in  $\mathbb{S}$  is entailed in  $\mathfrak{B}(\mathbb{K})$ . Thus, if we are able to explain  $\mathbb{S}$  we can derive a *partial* explanation of  $\mathbb{K}$ . In contrast, for full scale-measures we can derive an *exact* explanation (up to context isomorphism) of  $\mathbb{K}$ . Obviously, both scale-measures and full scale-measures differ in their *coverage* of  $\text{Ext}(\mathbb{K})$ , i.e., partial and exact. However, both morphisms are defined on the entire set of objects G of  $\mathbb{K}$  and are therefore global scope.

Even though global explanations are the gold standard for explainable artificial intelligence, they often elude from human comprehensibility due to their size. Therefore we divide the problem of deriving a single global explanation into multiple local explanations. To *locally* describe a part of context  $\mathbb{K}$  a generalization of scale-measures is introduced in Hirth et al. [9].

**Definition 3.2 (Local Scale-Measures [9]).** For two contexts  $\mathbb{K}$ ,  $\mathbb{S}$  a map  $\sigma : H \to G_{\mathbb{S}}$  is a local scale-measure iff  $H \subseteq G_{\mathbb{K}}$  and  $\sigma$  is a scale-measure from  $\mathbb{K}[H,M]$  to  $\mathbb{S}$ . We say  $\sigma$  is full iff  $\sigma$  is a full scale-measure from  $\mathbb{K}[H,M]$  to  $\mathbb{S}$ .

Based on this we want to construct in the following templates for textual explanations. The basis for these templates are standard scales. Given a context  $\mathbb{K}$ , a local (full) scale-measure  $\sigma$  of  $\mathbb{K}$  into  $\mathbb{S}$  we can replace every instance of an object  $g \in G_{\mathbb{S}}$  in a textual explanation template of  $\mathbb{S}$  by its pre-image  $\sigma^{-1}(g) \subseteq G$ . This yields a textual explanation of  $\mathbb{K}$  with respect to  $\sigma$ .

For the standard scales, i.e., *nominal*, *ordinal*, *interordinal*, *crown* and *contranominal*, we show textual templates in Section 5. These are designed such that they can be universally applied in all settings. Prior to discussing the textual templates we have to discuss how to recognize standard scales in a given formal context. The general ordinal motif recognition problem was introduced in Hirth et al. [9]. In this work the authors are only concerned with the recognition of ordinal motifs based on scale-measures into standard scales. Nonetheless, we recall the general problem for enumerating scale-measures.

**Problem 1** (Recognizing Ordinal Motifs [9]). Given a formal context  $\mathbb{K}$  and an ordinal motif  $\mathbb{S}$  find a surjective map from  $\mathbb{K}$  into  $\mathbb{S}$  that is:

	global	local	
partial	scale-measure	local scale-measure	
full	full scale-measure	local full scale-measure	

The underlying decision problem of Problem 1 has been proven to be NP-complete [9]. In a moment we will investigate a particular instance of this problem for standard scales. But first we want to give the idea of how the recognition of standard scales relates to the overall explanation task.

In practice we consider families of standard scales for investigating a given formal context  $\mathbb{K}$  such that we have explanation templates for each scale. Thus we have to solve the above problem for a family of scale contexts  $\mathcal{O}$ . Moreover, usually we are not only interested in a single scale-measure into a scale context  $\mathbb{S}$  but all occurrences of them.

Fortunately, for all standard scales but the crown scales is the existence of local full scale-measures hereditary with respect to subsets of  $H \subseteq G$ . For example a context for which there exists a local full scale-measure  $\sigma: H \to G_{\mathbb{S}}$  into the ordinal scale of size three does also allows for a local full scale-measure into the ordinal scale of size two by restricting  $\sigma$  to two elements of H. Thus when enumerating all local full scale-measures a large number of candidates do not need to be considered.

Another meaningful restriction for the rest of this work is to consider local full scale-measures only. Thus our methods focus on local full explanations (cf. Problem 1). Moreover, this choice allows to mitigate the enumeration of all scale-measures. For a family of standard scales of a particular type, e.g., the family of all ordinal scales, let  $\mathbb{S}_n$  be the scale context of size n. We thus consider only the local full scale-measures  $\sigma: H \to G_{\mathbb{S}_n}$  of  $\mathbb{K}$  where there is no local full scale-measure  $H \cup \{g\} \to G_{\mathbb{S}_{n+1}}$  from  $\mathbb{K}$  to  $\mathbb{S}_{n+1}$  with  $g \in G, g \notin H$ . For example, in case that H is of ordinal scale with respect to  $\sigma$  we can infer that all proper subsets of H are of ordinal scale. We remind the reader that we only consider surjective maps (cf. Problem 1).

**Proposition 3.1 (Recognizing Standard Scales).** Deciding whether for a given formal context  $\mathbb{K}$  there is a full scale-measure into either standard scale  $\mathbb{N}_n$ ,  $\mathbb{O}_n$ ,  $\mathbb{I}_n$ ,  $\mathbb{C}_n$  or  $\mathbb{B}_n$  is in P.

#### *Proof.* WLoG we assume that $\mathbb{K}$ is clarified.

For a contranominal scale  $\mathbb{B}_n := ([n], [n], \neq)$  every pair of bijective maps  $(\alpha : [n] \to [n], \beta : [n] \to [n])$  is a context automorphism of  $\mathbb{B}_n$  ([4]). Thus we can select an arbitrary mapping from G into [n] and check if it is a full scale-measure from  $\mathbb{K}$  into the contranominal scale  $\mathbb{B}_n$ . The verification of full scale-measures is in P [9]. The same reasoning applies to nominal scales  $\mathbb{N}_n := ([n], [n], =)$ .

For ordinal scales we need to verify that for each pair of objects their object concepts are comparable. Hence, the recognition for ordinal scales is in P.

For an interordinal scale  $\mathbb{I}_n := ([n], [n], \leq) | ([n], [n], \geq)$  we can infer from the extents of K of cardinality two two possible mappings  $\sigma_{\leq}, \sigma_{\geq}$  that are the only candidates to be a full scale-measure. For interordinal scales the extents of cardinality two overlap on one object each and form a chain. From said chain we can infer two order relations of the objects G given by position in which they occur in the chain. From the total order on G we can infer a mapping  $\sigma_{\leq}: G \to [n]$  where the objects are mapped according to their position. We can construct  $\sigma_{\geq}$  analogously by reversing the positions. All maps other than  $\sigma_{\leq}$  and  $\sigma_{\geq}$  would violate the extent structure of the chain. For  $\sigma_{\leq}$  and  $\sigma_{\geq}$  we can verify in P either is a full scale-measure. Moreover, the extents of cardinality two can be computed in polynomial time using TITANIC or next\_closure. Hence, the recognition for interordinal scales is in P.

For crown scales  $\mathbb{C}_n := ([n], [n], J)$  where  $(a, b) \in J \iff a = b$  or (a, b) = (n, 1) or b = a + 1 we can select an arbitrary object  $g \in [n]$  and select repeatedly

without putting back a different  $h \in [n]$  with  $\{g\}' \cap \{h\}' \neq \{\}$ . Starting from g there is a unique drawing order. In order to find a full scale-measure we have to find an isomorphic drawing order for the elements of G in the same manner. From this we can derive a map  $G \rightarrow [n]$  with respect to the drawing order and verify if it is a full scale-measure. The computational cost of the drawing procedure as well as the verification is in P.

We may note that our problem setting in the contranominal case is related but different to the question by Dürrschnabel, Koyda, and Stumme [2] for the largest contranominal scale of a context  $\mathbb{K}$ .

Once we can recognize standard scales we are able to provide contextual explanations that are based on them. One may extend the set of scale to non-standard scales, yet this may be computationally intractable if they cannot be recognized in polynomial time.

While we are able to decide if a context  $\mathbb{K}$  is of crown scale, it is NP-hard to decide if it allows for a surjective scale-measure into a crown scale of size |G|.

**Proposition 3.2.** Deciding for a context  $\mathbb{K}$  if there is a surjective scale-measure into a crown scale of size |G| is NP-hard.

*Proof.* To show the NP-hardness of this problem we reduce the Hamilton cycle (HC) problem for undirected graphs to it, i.e., for a graph G is there a circle(-path) visiting every node of G exactly ones. This problem is known to be NP-complete.

For the reduction, we map the graph G := (V, E) (WLoG  $|V| \ge 2$ ) to a formal context  $\mathbb{K} := (V, \hat{V} \cup E, \in)$  where  $\hat{V} := \{\{v\} | v \in V\}$ . This map is polynomial in the size of the input. The set of extents of  $\mathbb{K}$  is equal to  $\hat{V} \cup E \cup \{V, \{\}\}$ . The context  $\mathbb{K}$  accepts a surjective scale-measure into the crown scale of size |G| iff there is a sequence of extents of cardinality two  $A_1, \dots, A_n$  of  $\mathbb{K}$  such that  $(V, \{A_1, \dots, A_n\}) \le G$  is a cycle visiting each object  $v \in V$  exactly ones. This is the case iff G has a Hamilton cycle.  $\Box$ 

First experiments [9] on a real world data set with 531 formal concepts revealed that the number of local full scale-measures into standard scales is too large to be humanly comprehended. Thus we propose in the following section two importance measures for selection approaches.

## 4 Important Ordinal Motifs

Our goal is to cover large proportions of a concept lattice  $\underline{\mathfrak{B}}(\mathbb{K})$  using a small set of scale-measures  $\mathcal{S}$  into a given set of ordinal motifs. We say a concept  $(A,B) \in \underline{\mathfrak{B}}(\mathbb{K})$  is covered by  $(\sigma, \mathbb{S}) \in \mathcal{S}$  iff it is reflected by  $(\sigma, \mathbb{S})$ , i.e., there exists an extent  $D \in \mathbb{S}$  with  $\sigma^{-1}(D) = A$ .

The above leads to the formulation of the general ordinal motif covering problem.

**Problem 2** (Ordinal Motif Covering Problem). For a context  $\mathbb{K}$ , a family of ordinal motifs  $\mathcal{O}$  and  $k \in \mathbb{N}$ , what is the largest number  $c \in \mathbb{N}$  such that there are

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surjective local full scale-measures  $(\sigma_1, \mathbb{O}_1), ..., (\sigma_k, \mathbb{O}_k)$  of  $\mathbb{K}$  with  $\mathbb{O}_1, ..., \mathbb{O}_k \in \mathcal{O}$  and

$$\left| \bigcup_{1 \le i \le k} (\varphi_{\mathbb{K}} \circ \sigma_i^{-1}) (\operatorname{Ext}(\mathbb{O}_i)) \right| = c$$

where  $\varphi_{\mathbb{K}}$  denotes the object closure operator of  $\mathbb{K}$ . If  $\mathbb{K}$  does not allow for any scale-measure into an ordinal motif from  $\mathcal{O}$  the value of c is 0.

We call the set  $\{(\sigma_1, \mathbb{O}_1), ..., (\sigma_k, \mathbb{O}_k)\}$  an ordinal motif covering of K.

If one is able to find an ordinal motif covering that reflects all formal concepts of  $\mathbb{K}$  we can construct a formal context  $\mathbb{O}$  which accepts a scale-measure  $(\sigma, \mathbb{S})$  if and only if  $(\sigma, \mathbb{S})$  is a scale-measure of  $\mathbb{K}$ .

**Proposition 4.1 (Ordinal Motif Basis of** K). Let K be a formal context with object closure operator  $\varphi_{\mathbb{K}}$  and ordinal motif covering  $\{(\sigma_1, \mathbb{O}_1), ..., (\sigma_k, \mathbb{O}_k)\}$  that covers all concepts of K, i.e.,  $c = |\underline{\mathfrak{B}}(\mathbb{K})|$ . Let

$$\mathbb{O} \coloneqq |_{1 \leq i \leq k} (G, M_{\mathbb{O}_i}, I_{\mathbb{O}_i, \varphi_{\mathbb{K}}}), with (g, m) \in I_{\mathbb{O}_i, \varphi_{\mathbb{K}}} \Longleftrightarrow g \in \varphi_{\mathbb{K}} (\sigma_i^{-1}(\{m\}^{I_{\mathbb{O}_i}}))$$

where | is the context apposition. Then is a pair  $(\sigma, \mathbb{S})$  a local full scale-measure from  $\mathbb{K}[H,M]$  to  $\mathbb{S}$  iff  $\sigma$  is a local full scale-measure from  $\mathbb{O}[H,M_{\mathbb{O}}]$  to  $\mathbb{S}$ . In this case we call  $\mathbb{O}$  an ordinal motif basis of  $\mathbb{K}$ .

*Proof.* We have to show that the identity map is a full scale-measure from  $\mathbb{K}$  to  $\mathbb{O}$ . Hence, we need to prove that all attribute extents of  $\mathbb{O}$  are extents in  $\mathbb{K}$  [7, Proposition 20] and each extent of  $\mathbb{K}$  is an extent of  $\mathbb{O}$ . For an attribute  $m \in M_{\mathbb{S}_i}$  is  $\{m\}^{I_{\mathbb{S}_i,\varphi}} \in \text{Ext}(\mathbb{K})$  per definition. The second requirement follows from the fact that  $c = |\mathfrak{B}(\mathbb{K})|$ .

The just introduced basis is a useful tool when investigating scale-measures of a context  $\mathbb{K}$  given a set of ordinal motifs  $\mathcal{O}$ . One can perceive  $\mathcal{O}$  as a set of analytical tools and the existence of  $\mathbb{O}$  implies that a found ordinal motif covering  $\{(\sigma_1, \mathbb{O}_1), ..., (\sigma_k, \mathbb{O}_k)\}$  is complete with respect to scale-measures of  $\mathbb{K}$ .

#### 4.1 Scaling Dimension Complexity

An interesting problem based on the ordinal motif covering for (non-local) scalemeasures is to determine the smallest number k such that  $c = |\underline{\mathfrak{B}}(\mathbb{K})|$ . This number is also the *scaling dimension* [3] of  $\mathbb{K}$  with respect to the family of scale contexts  $\mathcal{O}$ . Note that the scaling dimension for a given context  $\mathbb{K}$  and family of scales  $\mathcal{O}$ does not need to exist. In the following we recall the scaling dimension problem in the language scale-measures.

**Problem 3** (Scaling Dimension Problem [3]). For a context  $\mathbb{K}$  and a family of scale contexts  $\mathcal{O}$ , what is the smallest number  $d \in \mathbb{N}$  of scale contexts  $\mathbb{S}_1, ..., \mathbb{S}_d \in \mathcal{S}$ , if existent, such that  $\mathbb{K}$  accepts a full scale-measure into the semi-product

$$\underset{1 \leq i \leq d}{\mathbb{S}_i}$$

The scaling dimension can be understood as a measurement for the complexity of deriving explanations for a formal context based on scale-measures and a set of ordinal motifs. However, determining the scaling dimension is a combinatorial problem whose related decision problem is NP-complete, as can be seen in the following.

**Theorem 4.1 (Scaling Dimension Complexity).** Deciding for a context  $\mathbb{K}$  and a set of ordinal motifs  $\mathcal{O}$  if the scaling dimension is at most  $d \in \mathbb{N}$  is NP-complete.

*Proof.* To show NP-hardness we reduce the recognizing full scale-measure problem (RfSM) [9] to it.

For two input contexts  $\hat{\mathbb{K}}$  and  $\hat{\mathbb{S}}$  of the RfSM let context  $\mathbb{K} := \hat{\mathbb{K}}$ . We map  $\hat{c}ontext$  to  $\mathbb{K}$  and  $\hat{\mathbb{K}}[S]$  to the set of ordinal motifs  $\mathcal{O} := \{\hat{\mathbb{S}}\}$  and set d = 1. This map is polynomial in the size of the input.

If there is a full scale-measure from  $\hat{\mathbb{K}}$  into  $\hat{\mathbb{S}}$  we can deduce that there is a full scale-measure of  $\mathbb{K}$  into the semi-product that has only one operand and is thus just one element of  $\mathcal{O}$ . Hence, this element is  $\hat{\mathbb{S}}$  and therefore the scaling dimension is at most one. The inverse can be followed analogously.

An algorithm to decide the scaling dimension problem can be given by nondeterministically guessing d scale contexts  $\mathbb{S}_1, \ldots, \mathbb{S}_d \in \mathcal{O}$  and d mappings from  $\sigma_i = G_{\mathbb{K}} \to G_{\mathbb{S}_i}$ . These are polynomial in the size of the input. The verification for full scale-measures in P [9].

#### 4.2 Ordinal Motif Covering with Standard Scales

The ordinal motif covering problem is a combinatorial problem which is computationally costly even for standard scales. Thus, we propose in the following a greedy approach which has two essential steps. First, we compute all full scale-measures S which is computationally tame due to the heredity of local full scale-measures for standard scales. Our goal is now to identify in a greedy manner elements of Sthat increase c of the ordinal motif covering the most. Thus, secondly, we select k full scale-measures where at each selection step i with  $1 \le i \le k$  we select a scale-measure  $(\sigma, \mathbb{Q}) \in S$  that maximizes Equation (1).

$$\left| (\varphi_{\mathbb{K}} \circ \sigma^{-1}) (\operatorname{Ext}(\mathbb{O})) \setminus \bigcup_{1 \le j < i} (\varphi_{\mathbb{K}} \circ \sigma_j^{-1}) (\operatorname{Ext}(\mathbb{O}_j)) \right|$$
(1)

In the above equation  $(\sigma_j, \mathbb{O}_j)$  denotes the scale-measure that was selected at step  $j \leq i$ . The union is the covering number c of the ordinal motif covering  $(\sigma_1, \mathbb{O}_1), ..., (\sigma_{i-1}, \mathbb{O}_{i-1})$ . Overall, the computed cardinality is equal to the number of concepts reflected by  $(\sigma, \mathbb{O})$  that are not already reflected by  $(\sigma_1, \mathbb{O}_1), ..., (\sigma_{i-1}, \mathbb{O}_{i-1})$ .

For obvious reasons this approach results in the selection of scale-measures that have the largest number of (so far) uncovered concepts. A downside of this heuristic is that it favors ordinal motifs that have in general more concepts, e.g., contranominal scales over ordinal scales. To compensate for this we propose to normalize the heuristic by the number of concepts of the ordinal motif, i.e.,  $|\sigma^{-1}(\text{Ext}(\mathbb{O}))|$ .

In the first step, the normalized heuristic does not account for the total size of the ordinal motif. The first selected scale-measure covers at least the top extent, i.e., G, and thus the scores for all following ordinal motifs are at most |Ext(S)|-1/|Ext(S)|.

## 5 Human-Centered Textual Explanations

We want to elaborate on textual explanations of concept lattices based on principles drawn from human-computer interaction for state of the art human-centered explanations. One of the currently most applied fields of these explanations in computer science is Explainable AI (XAI) [15]. Developing explainable systems commonly begins with "an assertion about what makes for a good explanation" [13], which are not seldomly based on guidelines or collections of principles. Those principles aim to derive human-centered textual explanations that impart complex concepts in a manner that is accessible, relevant, and understandable. They are designed to cater to the individual cognitive and emotional needs of readers, anticipating their concerns and queries. Thereby they aim at fostering the understanding of the reader by exposing reasoning and additional information to accompany data structures they rely on [17]. Moreover, textual explanations based on goodness criteria in the context of computer-generated knowledge and information help to strengthen trust in the computed reasoning results [11].

Mamun et al. [11] proposed five goodness criteria for explainability in the context of machine learning models. We identify them as adaptable to our task for textual explanations of concept lattices. The first criterion is *accuracy*, which requires that an explanation is a valid reflection of the underlying data. [14]. The second criterion is *scope*, which refers to the level of detail in the explanation, which can vary from explaining a single action to a global description of a system, depending on the tasks and needs of the reader. The third criterion relates to the type of question the explanation answers, which is called the *explanation form criterion*. The questions can be of type "what...", "why...", "why not...", "what if...", or "how to...". This is related to the so-called explanation triggers identified by Mueller et al. [12]. In their study, Mamun et al. [11] found that many explanations in Explainable AI contexts were "what" statements. The fourth criterion is *simplicity*, which emphasizes the importance of making an explanation easy to read and understand (e.g., Kulesza et al. [10]). Mamun et al. [11] suggested testing the appropriate readability level by comparing the grade level of other related content with one's explanations. Finally, the fifth criterion is the knowledge base criterion, which emphasizes the importance of providing workable knowledge in the explanation. Thus, explanations should predominantly be written as factual statements [11]. In the following, we first propose our textual explanation templates for standard scales and afterwards discuss how the principles above are implemented in their design.

- **Nominal Scale:** The elements  $n_1, \ldots, n_{k-1}$  and  $n_k$  are incomparable, i.e., all elements have at least one property that the other elements do not have.
- **Ordinal Scale:** There is a ranking of elements  $n_1, ..., n_{k-1}$  and  $n_k$  such that an element has all the properties its successors has.

- **Interordinal Scale:** The elements  $n_1, ..., n_{k-1}$  and  $n_k$  are ordered in such a way that each interval of elements has a unique set of properties they have in common.
- **Contranominal Scale:** Each combination of the elements  $n_1, \ldots, n_{k-1}$  and  $n_k$  has a unique set of properties they have in common.
- **Crown Scale:** The elements  $n_1, ..., n_{k-1}$  and  $n_k$  are incomparable. Furthermore, there is a closed cycle from  $n_1$ , over  $n_2, ..., n_{k-1}$  and  $n_k$  back to  $n_1$  by pairwise shared properties.

We motivate how our setup based on scale-measures relates to the goodness criteria above.

- Accuracy The generation of textual explanations are based on ordinal motif coverings with scale-measures, i.e., continuous maps between closure spaces. These maps do not introduce any conceptual error [8]. Moreover, ordinal motif coverings can function as a basis for the complete conceptual structure of the data set with respect to Proposition 4.1. Therefore an accurate mapping of an explanation onto the represented information is guaranteed.
- **Scope** For the scope of the introduced explanations we differed between global and local explanations which is determined by the choice of scale-measures, i.e., local vs non-local. In addition to that we can differentiate between two kinds of **coverage**, i.e., full and non-full scale-measures. However, with our experiments and the ordinal motif covering we focus mainly on local full explanations. Altogether, we can serve different task requirements with the explanations.
- **Explanation Form** The main question addressed by ordinal motifs is dependent on the type of scale-measure. For full scale-measures we answer the question on *"What is the conceptual relation between a given set of objects."* and for non-full scale-measures we answer *"What is a conceptual relation between a given set of objects."*.
- **Simplicity** The presented explanations are written using terms familiar for readers with basic knowledge about graphs and mathematical descriptions. Formulations that require prior knowledge about conceptual structures have been avoided. In addition to that, the textual structure is kept simple and explanations are composed of at most two short sentences.
- **Knowledge Base** The generated textual explanations describe the conceptual relations between objects and can thus be considered to be factual statements.

All proposed textual explanations are designed to be applicable in every data domain that is representable by formal contexts. However, different data domains and applications come with different requirements for the design of human-centered textual explanations. Thus, a development of domain specific explanations for a large variety of settings is advisable. Given more general principles of HCI [1], user studies with the prospective users of a system are the gold standard in evaluating any kind of interaction [11]. Since the focus of this work is to introduce the theoretical foundation on how to derive human-centered explanations we deem the execution of a user study future work.

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## 6 Application Example

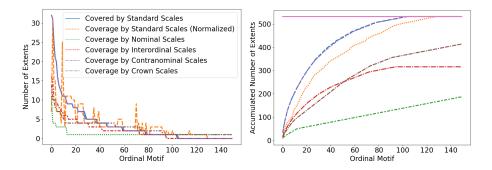
To show the applicability of our method, we compute the ordinal motif covering for the *spices planner* data set [6]. This context contains fifty-six meals as objects and thirty-seven spices and food categories as attributes. The context has 531 formal concepts for which we found over ten-thousands local full scale-measures into standard scales. In Table 1 we recall results [9] on how many local full scalemeasures there are per family of standard scales. The most frequent ordinal motif of the spices planner context is the interordinal motif. The motif having the largest scale size is the nominal scale motif, which includes up to nine objects. There are no non-trivial ordinal scale motifs in the spices planner context, i.e., the size of all local full scale-measure domains into ordinal scales within the spices planner context is one. Therefore we exclude the ordinal scales from the following analysis.

Table 1. Results for ordinal motifs [9] of the spices planner context. Every column represents ordinal motifs of a particular standard scale family. Maximal lf-sm is the number of local full scale-measures for which there is no lf-sm with a larger domain. Largest lf-sm refers to the largest domain size that occurs in the set of local full scale-measures.

	nominal	ordinal	interordinal	$\operatorname{contranominal}$	crown
local full sm	2342	37	4643	2910	2145
maximal lf-sm	527	37	2550	1498	2145
largest lf-sm	9	1	5	5	6

In our experiment we applied the introduced greedy strategy. In Figure 1 we report the extent sizes of selected ordinal motifs. In the left diagram we depict in the abscissa the steps of the greedy selection and in the ordinate the number of newly covered concepts. We report the results for the standard scales individually and combined, for the later we also experimented with the normalized heuristic. In the right diagram we depict the accumulated values, i.e., the value *c*. First we observe that the normalized heuristic does not decrease monotonously in contrast to all other results. From the right diagram we can infer that the crown, interordinal and nominal are unable to cover all extents. The contranominal and the combined scale family took the fewest selection steps to achieve complete extent coverage. This followed by the normalized heuristic on the combined scale family which about thirty percent more steps. Out of the other scale families the crown scales achieved the highest coverage followed by the interordinal and nominal scales.

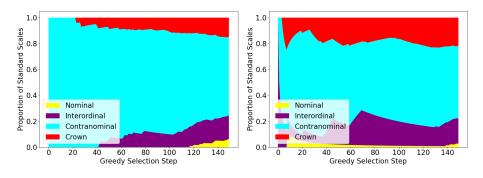
With Figure 2 we investigate the influence of the normalization on the greedy selection process. For this we depict the relative proportion of selected scale types up to a step i (abscissa). The left diagram shows the proportions for the standard heuristic and the right reports the proportions for the normalized heuristic. We count ordinal motifs that belong to multiple standard scale families relatively. For example we count the contranominal scale of size three half for the crown family. In the first diagram we see that a majority of the selected ordinal motifs are of



**Figure 1.** The extent coverage (left) for the ordinal motif covering computation for all and each standard scale family individually. The right diagram displays the accumulated coverage at each step in the ordinal motif covering computation. The legend of the left diagram does also apply to right diagram with the addition of the total number of extents (pink) in the context.

contranominal scales. This is not surprising since they have the most concepts among all standard scales. The interordinal and crown scales are almost equally represented and the nominal motifs are the least frequent. In contrast to this the normalized heuristic selects crown and interordinal motifs more frequently (right diagram).

Overall we would argue that while the normalized heuristic produces slightly worse coverage scores they provide a more diverse selection in terms of the standard scales. Therefore, the normalized heuristic may result in potentially more insightful explanations.



**Figure 2.** The ratio of each standard scale family in the ordinal motif covering computation for the standard (left) and normalized heuristic.

We conclude by providing automatically generated textual explanations for spices planner context. For this we report the top ten selections for the standard and normalized heuristic. First we depict the explanations for the standard heuris-

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tic which consist solely of contranominal motifs. Thereafter we will turn to the normalized heuristic results.

- 1. Each combination of the elements *Thyme*, *Sweet Paprika*, *Oregano*, *Caraway* and *Black Pepper* has a unique set of properties they have in common.
- 2. Each combination of the elements *Curry*, *Garlic*, *White Pepper*, *Curcuma* and *Cayenne Pepper* has a unique set of properties they have in common.
- 3. Each combination of the elements *Paprika Roses*, *Thyme*, *Sweet Paprika*, *White Pepper* and *Cayenne Pepper* has a unique set of properties they have in common.
- 4. Each combination of the elements *Paprika Roses*, *Thyme*, *Allspice*, *Curry* and *Curcuma* has a unique set of properties they have in common.
- 5. Each combination of the elements *Thyme*, *Basil*, *Garlic*, *White Pepper* and *Cayenne Pepper* has a unique set of properties they have in common.
- 6. Each combination of the elements *Tarragon*, *Thyme*, *Oregano*, *Curry*, and *Basil* has a unique set of properties they have in common.
- 7. Each combination of the elements *Vegetables*, *Caraway*, *Bay Leef* and *Juniper Berries* has a unique set of properties they have in common.
- 8. Each combination of the elements *Meat*, *Garlic*, *Mugwort* and *Cloves* has a unique set of properties they have in common.
- 9. Each combination of the elements Oregano, Caraway, Rosemary, White Pepper and Black Pepper has a unique set of properties they have in common.
- 10. Each combination of the elements *Curry*, *Ginger*, *Nutmeg* and *Garlic* has a unique set of properties they have in common.

These explanations cover a total of 195 concepts out of 531. An interesting observation is that explanation number eight has only four objects compared to the five objects of explanation number nine. Yet, explanation eight was selected first. The reason for this is that number eight has more non-redundant concepts with respect to the previous selections.

The results for the normalized heuristic are very different compared to the standard heuristic. The ten selected motifs cover a total of 125 concepts. They consist of one interordinal motif, four contranominal, one nominal and four motifs that are crown and contranominal at the same time. For the ordinal motifs that are of crown and contranominal scale we report explanations for both.

- 1. The elements *Thyme*, *Caraway* and *Poultry* are ordered in such a way that each interval of elements has a unique set of properties they have in common.
- 2. Each combination of the elements *Curry*, *Garlic*, *White Pepper*, *Curcuma* and *Cayenne Pepper* has a unique set of properties they have in common.
- 3. Each combination of the elements *Allspice*, *Ginger*, *Mugwort* and *Cloves* has a unique set of properties they have in common.
- 4. Each combination of the elements *Sweet Paprika*, *Oregano*, *Rosemary* and *Black Pepper* has a unique set of properties they have in common.
- 5. Each combination of the elements *Sauces*, *Basil* and *Mugwort* has a unique set of properties they have in common.

The elements *Basil*, *Sauces* and *Mugwort* are incomparable. Furthermore, there is a closed cycle from *Basil* over *Sauces* and *Mugwort* back to *Basil* by pairwise shared properties.

- 6. Each combination of the elements Paprika Roses, Meat and Bay Leef has a unique set of properties they have in common. The elements Paprika Roses, Meat and Bay Leef are incomparable. Furthermore, there is a closed cycle from Paprika Roses over Meat and Bay Leef back to Paprika Roses by pairwise shared properties.
- 7. Each combination of the elements Saffron, Anisey and Rice has a unique set of properties they have in common. The elements Saffron, Anisey and Rice are incomparable. Furthermore, there is a closed cycle from Saffron over Anisey and Rice back to Saffron by pairwise shared properties.
- 8. Each combination of the elements *Vegetables*, *Savory* and *Cilantro* has a unique set of properties they have in common. The elements *Savory*, *Cilantro* and *Vegetables* are incomparable. Furthermore, there is a closed cycle from *Savory* over *Cilantro* and *Vegetables* back to *Savory* by pairwise shared properties.
- 9. The elements *Tarragon*, *Potatos* and *Majoram* are incomparable, i.e., all elements have at least one property that the other elements do not have.
- 10. Each combination of the elements *Paprika Roses*, *Thyme*, *Sweet Paprika*, *White Pepper* and *Cayenne Pepper* has a unique set of properties they have in common.

## 7 Conclusion

To the best of our knowledge our presented method is the first approach for the automatic generation of textual explanations of concept lattices. It is a first step towards making Formal Concept Analysis accessible to users without prior training in mathematics. Our contribution comprises the theoretical foundations as well as the preparation of human-centered textual explanations for ordinal motifs of standard scale.

In particular, we have shown that the recognition of standard scales can be done in polynomial time in the size of the context. This is also the case when the standard scale has exponential many concepts. This is a positive result for the generation of textual explanations of large real world data sets.

Based on ordinal motif coverings we are able to limit the generated textual explanations to a low number of non-redundant conceptual relations. In detail, we proposed a greedy method for the computation of ordinal motif coverings based on two heuristics.

To asses the complexity of potential textual explanations of a concept lattice, we showed the relation between ordinal motif coverings and the scaling dimension. For the later we proved that the computational complexity of the related decision problem is NP-complete.

Accompanying our theoretical investigation, we derived criteria on how to derive textual explanations for ordinal motifs with principles from human-computer interaction. In addition to that, we demonstrated the applicability of our approach based on a real world data set.

As a next logical step, we envision a participatory user study. This will lead to improved textual explanations for ordinal motifs that are easier to comprehend 14 Johannes Hirth , Viktoria Horn, Gerd Stumme, and Tom Hanika

by humans. Moreover, the development of domain specific textual explanations may increase the number of applications for our proposed methods.

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