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Models for the computational design of microfarms

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Abstract. On microfarms, diversified crops are typically cultivated as densely as possible, and their heterogeneous growth dynamics require complex design and management of crop rotations. Models in computational agroecology may assist farmers in the maintenance of such farms. We use a phenomenological model in which the plants are represented as disks with a growing radius. We show how it can account for the variability observed in the fields and help visualize the spatiotemporal patterns involved in a microfarm cultivated according to the French intensive method. In the last part of the paper, we consider monoculture and a mixture of two vegetables with different maximum radii and growth rates. In the planning strategy, the planting positions and times are randomly chosen, taking inspiration from models of random sequential adsorption. We describe two different event-driven algorithms (1D and 2D) to simulate the dynamics of this system. The steady state of the field consists of disordered configurations of the plants. We study the evolution of the effective planting rate as a function of the nominal planting rate. When the model plants both vegetables with equal probability, simulations show that the proportion of big plants starts decreasing above a given threshold. This model and the algorithms describing the planting strategies may be extended to more species and other planting strategies to suggest original farm designs.

Keywords: agro-ecosystem · Monte-Carlo simulations · plant growth dynamics.

1 Introduction

Computational agroecology has recently emerged as a set of digital tools dedicated to the design and management of farms cultivated according to agroecological principles [1,2]. It can be organised around three themes: tools, plants, and people [3]. The first aims at helping the farmer to perform tasks on the field. An example is a robotic weeder with precise mechanical weeding [4,5,6]. The second is dedicated to the monitoring of culture crops, and their modeling

for easing the work of farmers in maintaining their field [7]. The third theme is about people, their knowledge and their know-how. It is focused on gathering knowledge about market farming on small surfaces and creating tools to share this knowledge. In all these aspects, digital technologies may be fruitful [8], and we are particularly interested in the role of computational aspects. Digital tools are deployed mainly in the precision agriculture framework, or agriculture 4.0 [9], but we are interested in some applications which are specific to microfarms cultivating according to the principles of agroecology. For example, the usual context for precision agriculture is that of a large farm in monoculture. In contrast, the methods we are interested in, like the French intensive method [10,11], recommends planting diversified crops as densely as possible. Also, precision agriculture aims at framing agricultural practices within a prespecified environment. Computational agroecology should explore the interactions between the farmer and the plants to reveal their possible dynamics.

Recently, plant phenotyping has rapidly progressed thanks to computer vision and machine learning and is now available in the field. Field phenomics data can be acquired using, for example, a motorised camera mounted on a cable (cablebot), and it is thus possible to monitor on a daily basis the growth of individual plants in a greenhouse [12]. We show below how this data can be integrated into models for the simulation of plant growth in various applications. We first show how to model spatial variability resulting from environmental conditions in a greenhouse. Then, we present the simulations for a plan of culture rotations according to the French intensive method. Finally, we suggest a random planting algorithm optimizing the density of crops for a mixture of lettuces and cabbages.

1.1 Related work

Models of plant growth are studied both in agronomy, with crop models [13], and ecology, with agent-based models [14]. Crop models include details about the physiology of the plant with distinct compartments for its various organs and corresponding fluxes of chemical components (carbon, nitrogen,...). Agent-based models have a more straightforward description of the growth of each plant but may also include the interactions between neighbouring plants, like competition or facilitation. Coarser modeling is used to design realistic ecosystems at large scale (e.g., for video games [15]). Some recent works propose a multi-scale approach to render realistic architecture at the individual plant level, and spatial patterns for the dispersion of vegetation [16]. A simple model of plants was also used to help manage the pruning and irrigation of a small robotised garden [17].

Models for the intercropping of diverse species are sometimes considered for strip intercropping configurations using patches or strips [18] but don't consider variations in the spatial positions at the individual plant level. The plant arrangements can be approximated as a set of disks with various radii [19]. When considering random plantation times or positions, the model, including growth dynamics, is an extension of the random sequential adsorption model,

also known, for the 1D case, as the parking lot model, which describes the dynamics of cars parking randomly on a street [20]. This model is relevant to the study of granular matter with jamming and to the adsorption of proteins on a solid surface, and it has been extended to reversible adsorption [21], bidisperse populations [22][23] and particles having a growth dynamics [24]. For the bidisperse case, it was shown that the selectivity towards a given particle type depends subtly on the pressures of the two populations [25].

2 A phenomenological model for plant growth

We consider simplified dynamics for the growth of plants, where each plant size is represented by its radius $R(t)$. For salads or cabbages, for example, the plant volume or biomass $M(t)$ can be estimated as that of a half-sphere of radius $R(t)$. In the simplest model, we consider the growth to be linear in time: $R(t) = \alpha(t - t_0)$, with $t_0 < t < t_0 + \tau_h$ when the plant is planted at time t_0 and harvested at $t_0 + \tau_h$. In a more refined model, we consider the growth of the radius to be a sigmoid function of time: $R_t = R_0 / (1 + e^{-\alpha(t-t_0)+\beta})$ where α corresponds to the growth rate, β to the offset that is associated with germination and/or slow growth in the first phenological stages of the plant and R_0 the maximal radius at which growth of the plant saturates when reaching maturity.

When modeling the growth of plants, it is important to take environmental parameters into account since plants develop quicker when the temperature is higher. The relevant variable, instead of time, is the accumulated temperature over a threshold T_0 (Réaumur⁴ model), growing degree day being a standard measure for estimating the phenological stage [26]. The radius then grows as follows:

$$R_t = \frac{R_0}{1 + e^{-\alpha \int_{t_0}^t T(s) \cdot H(T(s) - T_0) ds + \beta}}.$$

where H is the Heaviside function so that only the temperatures above T_0 are accumulated and contribute to the growth of the plant.

For a greenhouse, for example, the temperature is maximum at the centre and falls gradually to a minimum at the edge of the greenhouse (see Fig.1). The model shows a spatial variation in plant sizes similar to those observed in greenhouses [27]

3 Simulation of a micro-farm using the French intensive method

As a first example of simulations of the growth of plants in a field, we consider a microfarm with five culture beds. The culture plan (Fig. 2), which includes

⁴ René Antoine Ferchault de Réaumur was a French scientist in the XVIIIth century who pioneered the modeling of plant phenology.

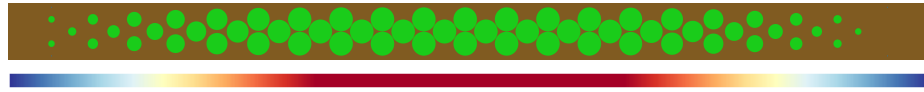


Fig. 1. Simulated culture bed of plants in a greenhouse with a temperature profile depicted on the bottom colour bar. The maximum temperature at the center is 40°C and the minimum at the borders is 25°C.

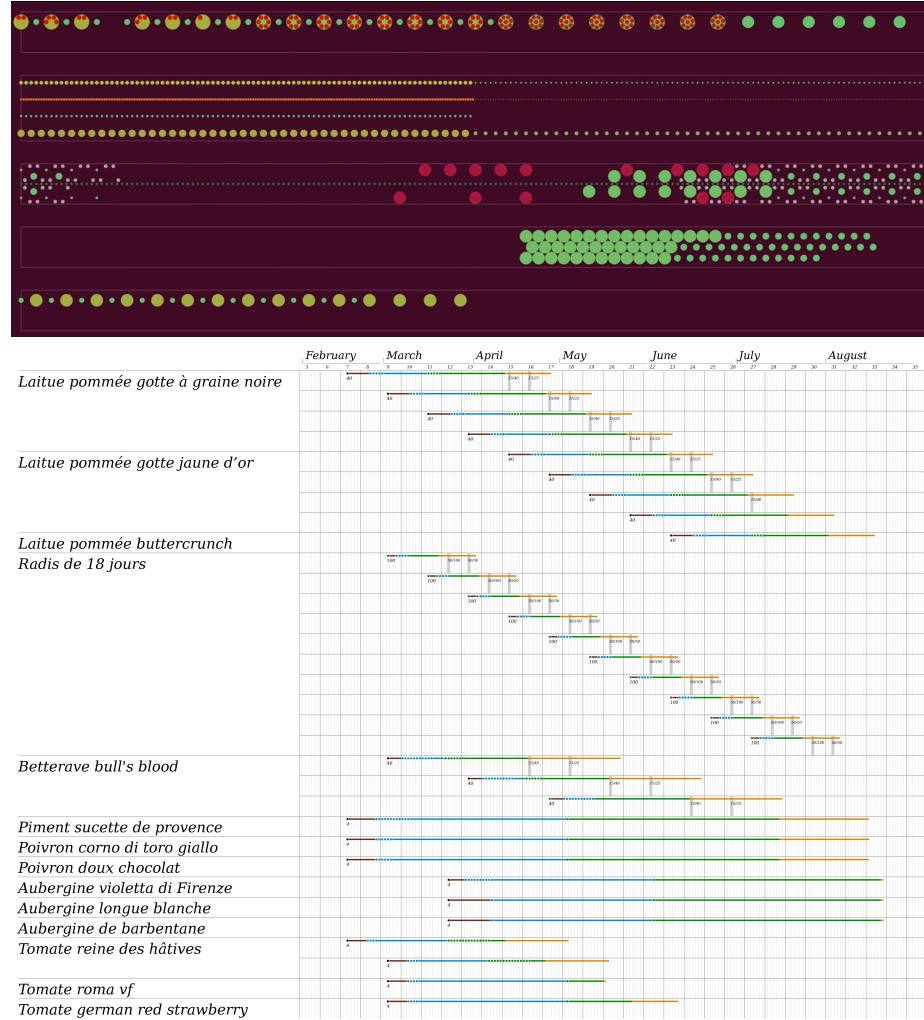


Fig. 2. (Top) Snapshot of a simulation of a microfarm with culture plan according to the French intensive method. Each disk depicts a plant with the different colors corresponding to the species listed in the culture plan. (Bottom) Culture plan for the microfarm: the brown line indicates the sowing period, the blue line indicates the time for germination before transplanting, the green line indicates the growth period in the soil, and the orange line indicates the harvesting period.

data about the average germination, transplanting and harvesting times, shows the diversity of crops and their intricate interplay. The sowing times follow the recommendation of a recent manual [11] explaining how to cultivate with the principles of the French method. This method is inspired by the practices of Parisian market gardeners of the 19th century [10]. In total, we simulate the culture of 14 species over a time period of 7 months with the linear growth model. Fig 2 shows a snapshot of the field. An animation showing the simulation for the whole season with a simple linear growth model is available online ⁵.

We observe that the dynamics may be quite complex and that the computational tools may be helpful in forecasting harvest and exploring how changes in the calendar and spatial layout of the plants affect the simulations.

4 Intercropping lettuces and cabbages

In order to explore planting strategies, we consider a field planted with lettuces and cabbages. For simplification, for both plants, we consider identical duration before harvesting ($\tau_h = 30$ days) and the same offset $\beta = 4$. On the other hand, the plants have different growth characteristics, like saturation radius ($R_0^c = 0.8$, $R_0^l = 0.3$) and growth rate ($\alpha^c = 0.4$, $\alpha^l = 0.3$).

We start with a 1-dimensional field. Plants are usually planted in a series of batches with regular spacing (Δ). Planting may be synchronised every τ_h or desynchronised so that alternated batches are planted every $\tau_h/2$ with the plants of the new batch being in between the plants of the previous batch (see Fig.3). For planting both lettuces and cabbages, we may consider two configurations. The first option is to split the field into two parts, one for lettuces and one for cabbages, each planted with the monoculture configuration. Another method is to alternate lettuces and cabbages (see Fig. 3). The latter strategy, intercropping different plants, is often used in market gardening. There are thus four patterns we may consider as reference depending on whether the batches are synchronised or desynchronised and the field being split or intercropped with lettuces and cabbages. In 2D, there are further options depending on the lattice used to position the plants. We use a triangular grid for synchronous monoculture and dual square grids for desynchronised and/or intercropped configurations.

When N plants are planted over a duration t , the effective planting rate, $\rho = N/t$, is helpful in comparing the different configurations. For example, it can be easily calculated that in 1D, the split and intercropped configurations have different effective rates. Also, in 2D, for a monoculture, we can plant more vegetables with the desynchronised configuration as long as $R(\tau_h)/R(\tau_h/2) < \sqrt{2\sqrt{3}} - 1$.

In the following, we will use the desynchronised configuration as a reference both for monoculture and intercropping, except in 2D, for the monoculture of cabbages, where the effective planting rate is higher for synchronised planting.

⁵ The video is visible on this page

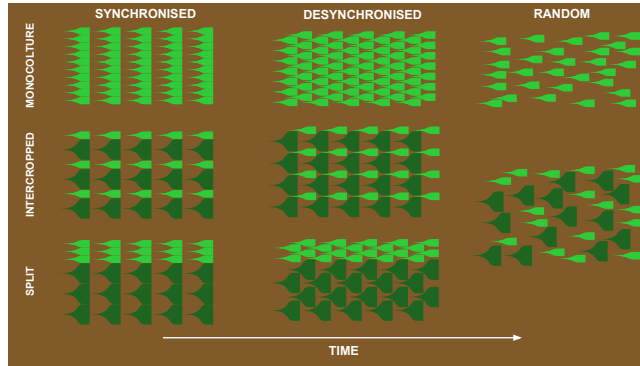


Fig. 3. Reference configurations and samples from the random plantation algorithm.

		Monoculture		Mixture	
		Lettuces	Cabbages	Intercropped	Split
1D (L=100)	Synchronised	5.59	2.08	3.04	3.04
	Desynchronised	6.88	2.21	3.32	3.35
2D (L=20)	Synchronised	43.34	6.02	11.06	10.57
	Desynchronised	56.75	5.89	13.26	10.67

Table 1. Planting rate for different reference configurations.

5 A random planting algorithm for intercropped lettuces and cabbages

Although regularly spaced planting strategies are widespread in farming practices, we are willing to explore alternative planting strategies, as they could be implemented by a robot, for example. The most straightforward strategy apart from regular grids is to choose randomly the planting times and positions, as well as the plant species, when considering mixtures, with the constraint that the newly planted individual doesn't overlap with others over the course of its growth. For simplicity, we first consider the planting of crops along a line of length $L = 100$ and then on a plane of dimensions 20×20 . We now describe the algorithms for simulating the random planting strategy (see also the detailed description in the boxes below ⁶this repository).

⁶ Both the 1D algorithm and 2D algorithms are available at [this repository](#).

Algorithm 1 1D intercropping algorithm

```

1:  $t \leftarrow 0$ 
2:  $N \leftarrow 0$ 
3: while  $N < N_p$  do
4:    $\phi \leftarrow 0$ 
5:   while  $\phi = 0$  do
6:      $dt \leftarrow$  random time interval (exponential with rate  $\lambda$ )
7:      $s \leftarrow$  random species in ['c', 'l'] with probabilities (p, 1-p)
8:      $t \leftarrow t + dt$ 
9:      $\phi \leftarrow L - \sum \sigma_i(t, s)$ 
10:   end while
11:    $\eta \leftarrow \text{uniform in } [0, \phi]$ 
12:    $k_0 \leftarrow \max k \text{ s.t. } \phi_k \leq \eta$ 
13:    $x \leftarrow \sum_{k=0}^{k_0} \sigma_k + \eta$ 
14:   plants.add(Plant(x, t, s))
15:   Remove plants reaching maturity before t
16:    $N \leftarrow N + 1$ 
17: end while

```

The algorithm attempts to plant seeds by randomly selecting the species, the planting time and the location at each trial. In fact, three properties can fully describe each plant i : its species s_i (and the associated growth curve parameter for lettuce or cabbage), its planting time t_i , and its planting position ($\mathbf{x}_i$). The species $s_i \in [c, l]$, where l stands for lettuces and c stands for cabbages. For monocultures, only one of the two will be possible. Otherwise, we can draw one of the two with uniform probability. Still, we have two options:

- *mix 1*, where the species is drawn at the beginning of each trial;
- *mix 2*, where the species is drawn at the beginning of each iteration and kept fixed for all trials in this iteration.

In the latter option, the plant population have an equal number of lettuces and cabbages ($N_c = N_l$). After a successful trial, the time for the subsequent trial is drawn from an exponential law of nominal planting rate $r = \frac{\lambda}{\tau}$. So, the time interval between 2 successive planting events is the sum of time intervals sampled overall K trials in an iteration: $t_i - t_{i-1} = \sum_{k=1}^K \tau_k$ where τ_k is drawn from the exponential distribution. When there is no rejection, planting events occur as a Poisson process.

The position x_i is sampled with uniform probability in the available space (1D) or in the entire space (2D). The available space in the 1D case is calculated by excluding all the space occupied by plants that are already present but also the place they will occupy in the future and the future size of the plant we should plant. If we consider the generic plant j already sown, the new plant i must be sown at a minimum distance $v_{ij} = R^{s_i}(t_j + \tau_h) + R^{s_j}(t_j + \tau_h)$, which calculates the future size of both plants i and j at the moment when j will be harvested, thus freeing more space. So, each existing plant j has a virtual size v_{ij} that cannot be

used to sample position for i . All the intervals $[x_j - v_{ij}, x_j + v_{ij}]$ must be excluded from the available space. In the calculation, we have to take into account that these intervals might be overlapping. After we merge the overlapping intervals, we have a certain number N' of forbidden intervals of generic size σ_f . After we exclude these intervals, the remaining available intervals $[x_j^{min}, x_j^{max}]$ can finally be summed to obtain the available space measure $\phi_N = \sum_{j=1}^N \phi_j = \sum_{j=1}^N |x_j^{max} - x_j^{min}|$. It is worth noting that the number of available intervals is $N' + 1$, but if we neglect border effects or we consider periodic conditions, the interval number can be considered equal to N' . We then draw a random number η uniformly in $[0, \phi_N]$ and find the interval n_0 so that $\sum_{j=1}^{n_0} \phi_j < \eta < \sum_{j=1}^{n_0+1} \phi_j$ and x_i is then chosen as $x_i = \sum_{j=1}^{n_0} \sigma_j + \eta$, with σ_j the length of the j^{th} forbidden interval. There will be rejections at trial k only if the space is fully occupied, in which case, time is incremented by τ_k , and another sample is drawn.

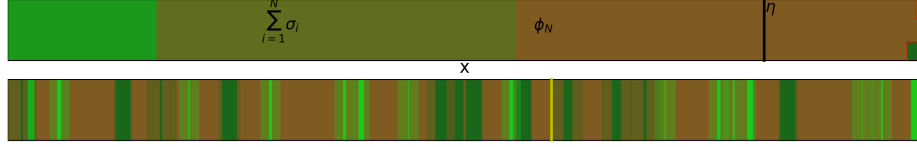


Fig. 4. Position sampling for the 1D model: (Bottom) Each plant is represented by a green interval of width $2R$, with a lighter green extension corresponding to the virtual size. The yellow bar indicates the position which is sampled at this particular iteration. The rectangles surrounded by a red borders sho the maximal size of lettuces (light green) and cabbages (dark green) for scale reference. (Top) The occupancy (shown in green) is computed as the sum of occupied intervals (merged overlapping virtual plants). The random number η is chosen uniformly in the remaining space ϕ_N .

In the 2D simulations, after drawing the position $\mathbf{x}_i$ of plant i uniformly in the space, we compute, for each of its neighbours j , the time t_{ij} at which it will intersect. If t_{ij} is smaller than the harvesting time of plant j , $t_j + \tau_h$, we reject it, increment time, and make another trial.

Let us now analyse the dynamics of the system when the planting ratio is increased. For monoculture in 1D, for a very low planting rate, there is always available space, so the effective planting rate is equal to the nominal one. Things cannot change before the configuration where the plants are regularly spaced along all L with a gap between pairs of neighbouring plants smaller than $\Delta = 4R$. This will make planting impossible, and all trials will be rejected (even with a lot of unoccupied space, roughly equal to $L/2$). The corresponding effective planting rate is $\rho_{SAT} = \frac{L}{\Delta_{SAT} \tau_h}$ (with $L = 100$, $\rho'_{SAT} = 2.77$, $\rho^c_{SAT} = 1.04$ and $\rho^{lc}_{SAT} = 1.51$). Even if this scenario is improbable, it marks the beginning of a shift in the dynamics. The average number of rejections in simulations is shown in Fig. 5, and it is 0 before those values. On the other hand, theoretical saturation configurations are shown in Fig. 3, and relative planting ratio values are shown in

Tab. 1. The results tell us that the best reference configuration (desynchronised one) is not achieved, probably because it is a very unlikely one. The effective planting rate seems, in fact, closer to the synchronised configuration.

As shown in Fig.5, above this threshold, in $1D$, the average number of trials per iteration grows linearly except for the option where species are sampled at every trial (*mix 1*), where the linear relation appears at a much higher rate. For the $2D$ simulations, Fig.5 also show how the effective planting rate was measured in simulations for the four conditions. All curves grow in the same manner (linear in $1D$) at a low rate below the threshold identified previously, where the effective rate starts saturating around the reference value computed in the previous section for monocultures and *mix 2* option. In the *mix 1* option, there are two linear regimes before saturation; in the first linear regime, there is almost no rejection, and there is an equal proportion of lettuces and cabbages ($N_c = N_l$). After the first transition point (around the cabbage monoculture saturation point), cabbages start getting rejected. We observe a decrease in the cabbage share until the field is fully covered with lettuce. A snapshot of simulations for the $1D$ model and the $2D$ model is shown in Fig.6 in the *mix 2* option ($N_c = N_l$). An animation of the $2D$ planting strategy is also available online ⁷.

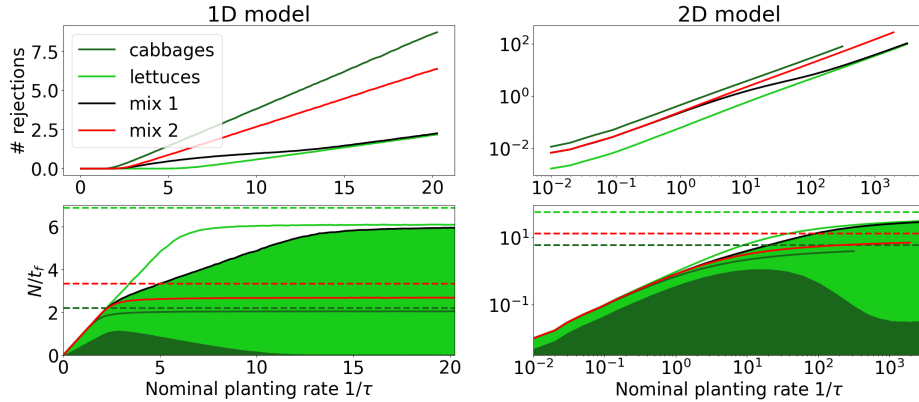


Fig. 5. Measures from the simulations in $1D$ (Left) and $2D$ (Right) with lettuce monoculture in light green, cabbage monoculture in dark green, intercropped culture with species randomly sampled at each trial (*mix 1*) in black or fixed sampled species at each iteration (*mix 2*) in red. The top graphs show the number of rejections before a plant is sown depending on the nominal planting rate ($1/\tau$). The bottom graphs show how the Effective planting rate (N/t) depends on the Nominal planting rate. The regions filled in light and dark green indicate the effective planting rates for lettuces and cabbages with the *mix 1* option. The dashed lines are a visualization of effective planting rate for the reference configurations listed in Table 1. Solid lines are the results of simulations of the random planting strategy.

⁷ The video is accessible at on this webpage

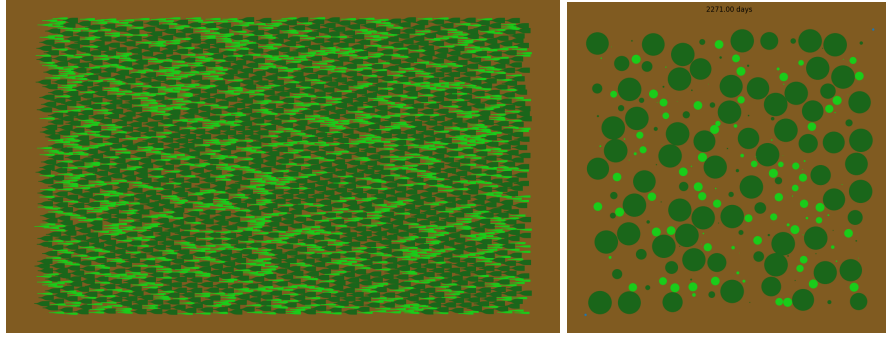


Fig. 6. Simulation of the intercropped field constrained so that $N_c = N_l$: 1D on the left and 2D on the right

6 Discussion

Optimising space and time in a farming plan is a task that could appear as an utterly theoretical one. It could be formulated as a tiling or a close-packing problem, just to be solved with a few geometrical considerations. But on the field, things are much more complex. Different areas of the same field might offer different conditions, like humidity or temperature, as shown in Fig. 1. And these conditions might change in time in unpredictable ways (e.g. weather). Furthermore, the complexity of the planning might combinatorially explode as a real farm can host tens of different species at the same time, and each of those species' features (size, maturation time, etc.) might fluctuate. Not to mention the fact that different species might mutually benefit from proximity in sowing, adding an additional constraint to the complexity. In this paper, we present the idea that a priori geometrical planning limitations can be overcome by a dynamic decision-making algorithm, aware of the current status of the field and of the plants. In principle, such an algorithm can react to the evolving conditions of the field as well as to fluctuations in crop features, elaborating dynamic plans that will result in more robustness and flexibility at the same time. To explain this concept, we introduce an algorithm operating in simplified conditions, and we analyse its performance. The algorithms we presented may be helpful in the management of the complex rotation plan, in particular in agroecological microfarms. It may also drive the development of new design strategies. It would be helpful to define an ontology adapted to agroecological principles [7] to have a comprehensive toolbox for the simulation of microfarms.

The distribution of both species in the 1D distribution shows highly correlated patterns, probably existing also in 2D, but harder to spot, which we wish to study in future work.

We conceived a conceptual architecture that will also easily allow the future introduction of more complex scenarios. For example, we plan to add variability in the field condition, seasonality effects, and variable species ratios. All these

do not require specific modifications to the algorithm but only to the simulation model. On the other hand, to handle fluctuations in plants' features, the algorithm forecasting capabilities must be improved to take that into account.

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