A Logical Interpretation of Asynchronous Multiparty Compatibility

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Abstract. Session types are types for specifying the protocols that communicating processes must follow in a concurrent system. When composing two or more well-typed processes, a session typing system must check whether such processes are *multiparty compatible*, a property that guarantees that all sent messages are eventually received and no deadlock ever occurs. Previous work has shown that duality and the more general notion of coherence are sufficient syntactic conditions for guaranteeing the multiparty compatibility property.

In this paper, following a propositions-as-types fashion which relates session types to linear logic, we generalise coherence to forwarders. Forwarders are processes that act as middleware by forwarding messages according to a given protocol. Our main result shows that forwarders not only generalise coherence, but fully capture all well-typed multiparty compatible processes.

Keywords: Linear Logic · Session Types · Process Compatibility.

1 Introduction

Session types, originally proposed by Honda et al. [18], are type annotations that ascribe protocols to processes in a concurrent system and determine how they behave when communicating together. *Binary session types* found a logical justification in *linear logic*, identified by Caires and Pfenning [4,3] and later by Wadler [31,32], which establishes the following correspondences: session types as linear logic propositions, processes as proofs, reductions in the operational semantics as cut reductions in linear logic, and duality as a notion of compatibility ensuring that two processes communication pattern match.

In binary session types, a sufficient condition for two protocols to be *compat-ible* is that their type annotations are dual: the send action of one party must match a corresponding receive action of the other party, and vice versa. Because of asynchronous interleavings, however, there are protocols that are compatible but not dual. The situation is even more complex for *multiparty session*

types [20], which generalise binary session types to types for protocols with more than two participants. A central observation is that compatibility of sessions requires a property stronger than duality, ensuring that all messages sent by any participating party will eventually be collected by another. In [12], Deniélou and Yoshida proposed the semantic notion of *multiparty compatibility*. The concept has then found many successful applications in the literature [12,24,14]. The question whether this notion "would be applicable to extend other theoretical foundations such as the correspondence with linear logic to multiparty communications" has however not been answered since Deniélou and Yoshida's original work.

As a first step in defining a logical correspondent to multiparty compatibility, Carbone et al. [9,6] extended Wadler's embedding of binary session types into classical linear logic (CLL) to the multiparty setting by generalising logical duality to the notion of *coherence* [20]. Coherence is a compatibility condition: coherent processes are multiparty compatible, which ensures that their execution never leads to a communication error. Coherence is characterised prooftheoretically, and each coherence proof corresponds precisely to a multiparty protocol specification (*global type*). An interesting observation is that coherence proofs correspond to a definable subset of the processes typable in linear logic, so-called *arbiters* [6]. In retrospect, the concept of coherence has sharpened our proof-theoretic understanding of how to characterise compatibility in multiparty session types. However, coherence, similarly to duality, cannot capture completely the notion of multiparty compatibility.

In this paper, we show that coherence can be generalised to a more expressive logic of asynchronous forwarders. Forwarders are processes that forward messages between endpoints according to a protocol specification. Forwarders are more general than arbiters (every arbiter corresponds to a forwarder but not vice versa) but still can be used to guide the communication of multiple processes and guarantee they communicate safely, as done by coherence in [6]. Our main result is that forwarders fully capture multiparty compatibility which let us answer Deniélou and Yoshida's original question positively. In this work, we show that i) any possible interleaving of a set of multiparty compatible processes can be represented as a forwarder, and, conversely, ii) if a set of processes can be composed by a forwarder (describing a possible execution), then such processes are indeed multiparty compatible.

Operationally, forwarders are a central process that decide how to dispatch messages between peers, providing a global description of a multiparty protocol (hence they can be seen as a generalisation of global types). They capture the message flow by preventing messages from being *duplicated*, as superfluous messages would not be accounted for, and by preventing messages from being *lost*, otherwise a process might get stuck, awaiting a message. However, when data-dependencies allow, forwarders can choose to receive messages from different endpoints and forward such messages at a later point, or decide to buffer a certain number of messages from a given sender. Eventually, they simply retransmit messages after receiving them, without *computing* with them. Intuitively, this captures an interleaving of the communications between the given endpoints.

Besides showing that they precisely capture multiparty compatibility, this paper shows that forwarders can also replace duality or coherence for composing well-typed processes. We achieve this logically: we replace the linear logic cut rule with a new rule called MCutF which allows to compose two or more processes (proofs) using a forwarder as a condition, instead of duality [4] or coherence [6]. Our second main result is that MCutF can be eliminated by proof reductions that correspond to asynchronous process communications.

Although most of this paper's interest is on their logical properties, forwarders have other interesting features related to explaining communication patterns as they occur in practice, including message routing, proxy services, and runtime monitors for message flows [22].

Outline and key contributions. The key contributions of this paper include

- a definition of *multiparty compatibility* for classical linear logic (§ 4);
- a logical characterisation of *forwarders* that enjoys a sound and complete correspondence with multiparty compatibility (§ 5);
- a composition mechanism (MCutF) for processes with asynchronous communication that uses forwarders instead of coherence, and guarantees lack of communication errors (§ 6).

Additionally, § 2 introduces the main concepts on an example and § 3 recaps types, processes, and CP-typing. § 7 discusses related and future work. Finally, concluding remarks are in § 8.

2 Preview

We start with a gentle introduction to asynchronous forwarders by informally describing the classic 2-buyer protocol [19,20], where two buyers intend to buy a book jointly from a seller. The first buyer sends the title of the book to the seller, who, in turn, sends a quote to both buyers. Then, the first buyer decides how much she wishes to contribute and informs the second buyer, who either pays the rest or cancels the transaction by informing the seller. If the decision was made to buy the book, the second buyer will provide the seller with an address, where to ship the book to.

The three participants are connected through endpoints b_1 , b_2 , and s respectively: an endpoint acts like a socket where processes can write/read messages to/from. Each endpoint must be used according to its respective session type which gives a precise description of how each endpoint has to act. For example, b_1 : name $\otimes \cot^{\perp} \Re \cot \otimes 1$ says that buyer b_1 first sends (expressed by \otimes) a value of type name (the book title), then receives (expressed by \Re) a value of type \cot^{\perp} (the price of the book), then sends a value of type \cot the amount of money she wishes to contribute), and finally terminates. Here, following classical linear logic, the annotation $^{\perp}$ stands

for dual. The behaviour of buyer b_2 and seller s can similarly be specified by session types, respectively $b_2 : \mathbf{cost}^{\perp} \mathfrak{V} \mathbf{cost}^{\perp} \mathfrak{V} ((\mathbf{addr} \otimes \mathbf{1}) \oplus \mathbf{1})$ and $s: \mathbf{name}^{\perp} \mathfrak{V} \mathbf{cost} \otimes \mathbf{cost} \otimes ((\mathbf{addr} \mathfrak{V} \perp) \& \perp).$

Any well-typed processes with respective endpoints b_1 , b_2 , and s are going to execute this protocol correctly because their type specifications are *compatible*, meaning that they match. In a multiparty setting, such compatibility can be expressed as *coherence* [6] and more generally with the semantic notion of *multiparty compatibility* [12]. Our theory of forwarders logically captures this notion of multiparty compatibility precisely, namely that i) anything that has been received is eventually sent, ii) anything that is sent must have been previously received, and iii) the order of messages between any two points must be preserved. A *forwarder* captures the message flows between two or more different endpoints. In the example above, a forwarder process may receive a name from b_1 , forward it to s, and then proceed to receiving the price from s, forward it to b_1 and b_2 , and so on. A forwarder acts as a medium that, according to a prescribed protocol, dispatches messages processes send. In the 2-buyers case, for P_s , P_{b_1} , P_{b_2} respectively implementing the seller and the two buyers, we can depict this with the following diagram:



In order to capture the message flows between several processes, forwarders need to support buffering, reordering and other properties, which are not necessarily coherent. Consider for example two endpoints x and y willing to communicate with the following protocol – called a *criss-cross*: they both send a message to each other, and then the messages are received, according to the following types $x : name \otimes \cot \Im 1$ and $y : \cot^{\perp} \otimes name^{\perp} \Im \perp$. Such protocol leads to no error (assuming processes implement an asynchronous semantics), still the two types above are not coherent [6] and not even dual to each other. On the other hand, we can easily write a forwarder typable in the context $x : name^{\perp} \Im \cot^{\perp} \otimes \perp, y : \cot \Im name \otimes 1$ formed by their duals, i.e., a process that first receives on both x and y and then forwards the received messages over to y and x, respectively.

3 CP and Classical Linear Logic

In this section, we give an introduction to the proposition-as-sessions approach [32]. This comprises the syntax of types and processes and the interpretation of processes as sequent proofs in classical linear logic (CLL).

Types. Following the propositions-as-types approach, types, taken to be propositions (formulas) of CLL, denote the way an endpoint (a channel end) must be

used at runtime. Their formal syntax is given by the following grammar:

Types
$$A ::= a \mid a^{\perp} \mid \mathbf{1} \mid \perp \mid A \otimes A \mid A \ \mathfrak{F} A \mid A \oplus A \mid A \ \& A \mid !A \mid ?A$$
(1)

Atoms a and negated atoms a^{\perp} are basic dual types. Types 1 and \perp denote an endpoint that must close with a last synchronisation. A type $A \otimes B$ is assigned to an endpoint that outputs a message of type A and then is used as B, and similarly, an endpoint of type $A \Im B$, receives a message of type A and continues as B. In the situation of a branching choice, $A \oplus B$ is the type of an endpoint that may select to go left or right and continues as A or B, respectively, and A & B is the type of an endpoint that offers two choices (left or right) and then, based on such choice, continues as A or B. Finally, !A types an endpoint offering an unbounded number of copies of a service of type A, while ?A types an endpoint of a client invoking some replicated/unbounded service with behaviour A.

Duality. Operators can be grouped in pairs of duals that reflect the input-output duality. Consequently, standard duality $(\cdot)^{\perp}$ on types is inductively defined as: $(a^{\perp})^{\perp} = a$ $\mathbf{1}^{\perp} = \perp$ $(A \otimes B)^{\perp} = A^{\perp} \Im B^{\perp}$ $(A \oplus B)^{\perp} = A^{\perp} \& B^{\perp}$ $(!A)^{\perp} = ?A^{\perp}$ In the remainder, for any binary operators $\oslash, \odot \in \{\bigotimes, \Im, \oplus, \&\}$, we sometimes write $A \oslash B \odot C$ to mean $A \oslash (B \odot C)$.

Processes. We use a standard language of *processes* to represent communicating entities (including forwarders) which is a variant of the π -calculus [26] with specific communication primitives as usually done for session calculi. Moreover, given that the theory of this paper is based on the proposition-as-sessions correspondence with CLL, we adopt a syntax akin to that of Wadler [32]. Each process can be found on the left-hand side of the turnstyle \vdash in the conclusion of each rule in Figure 1. We briefly comment each process term. A link $x \leftrightarrow y$ is a binary forwarder, i.e., a process that forwards any communication between endpoints x and y. This yields a sort of equality relation on names: it says that endpoints x and y are equivalent, and communicating something over x is like communicating it over y. Note that we use endpoints instead of channels [30]. The difference is subtle: the restriction (νxy) connects the two endpoints x and y, instead of referring to the channel between them. The terms x().P and x[handle synchronisation (no message passing); x().P can be seen as an empty input on x, while x[] terminates the execution of the process. The term $x[y \triangleright P].Q$ denotes a process that creates a fresh name y (hence a new session), spawns a new process P, and then continues as Q. The intuition behind this communication operation is that P uses y as an interface for dealing with the continuation of the dual primitive (denoted by term x(y), R, for some R). Note that output messages are always fresh, as for the internal π -calculus [27], hence the output term $x[y \triangleright P].Q$ is a compact version of the π -calculus term $(\nu y) \overline{x}y.(P \mid Q)$. Branching computations are handled by x.case(P,Q), x[inl], P and x[inr], P. The former denotes a process offering two options (external choice) from which some other process can make a selection with x[in].P or x[inr].P (internal choice). Finally, |x(y)| P denotes a persistently available service that can be invoked by ?x[z].Q which will spawn a new session to be handled by a copy of process P.

$$\begin{array}{c} \overline{x \leftrightarrow y \vdash x: a^{\perp}, y:a} \quad Ax \quad \overline{x[] \vdash x:1} \quad \mathbf{1} \quad \frac{P \vdash \Delta}{x().P \vdash \Delta, x: \perp} \quad \bot \\ \frac{P \vdash \Delta_1, y: A_1 \quad Q \vdash \Delta_2, x: A_2}{x[y \triangleright P].Q \vdash \Delta_1, \Delta_2, x: A_1 \otimes A_2} \otimes \quad \frac{P \vdash \Delta, y: A_1, x: A_2}{x(y).P \vdash \Delta, x: A_1 \quad \Im A_2} \quad \Im \\ \frac{P \vdash \Delta, x: A_1}{x[\mathsf{inl}].P \vdash \Delta, x: A_1 \oplus A_2} \oplus_1 \quad \frac{P \vdash \Delta, x: A_2}{x[\mathsf{inrl}].P \vdash \Delta, x: A_1 \oplus A_2} \oplus_2 \quad \frac{P \vdash \Delta, x: A_1 \quad Q \vdash \Delta, x: A_2}{x.\mathsf{case}(P,Q) \vdash \Delta, x: A_1 \& A_2} \& \\ \frac{P \vdash 2A, y: A}{!x(y).P \vdash ?\Delta, x: !A} \mid \quad \frac{P \vdash \Delta, y: A}{?x[y].P \vdash \Delta, x: ?A} ? \quad \frac{P \vdash \Delta}{P \vdash \Delta, x: ?A} \otimes \frac{P \vdash \Delta, x: ?A}{P \vdash \Delta, x: ?A} \subset \end{array}$$

Fig. 1. Sequent Calculus for CP and Classical Linear Logic

Example 1. For some P_i , Q_j , R_k , we provide possible implementations for processes P_s , P_{b_1} , and P_{b_2} from the 2-buyer example.

 $\begin{array}{lll} P_{s} &=& s(book). \; s[price \triangleright P_{1}]. \; s[price \triangleright P_{2}]. \; s. {\tt case}(s(addr).P_{3},P_{4}) \\ P_{b_{1}} &=& b_{1}[book \triangleright Q_{1}]. \; b_{1}(price). \; b_{1}[contr \triangleright Q_{2}].Q_{3} \\ P_{b_{2}} &=& b_{2}(price). \; b_{2}(contr). \; b_{2}[{\tt inl}].b_{2}[addr \triangleright R_{1}].R_{2} \end{array}$

Note that the order the two buyers receive the price is not relevant.

CP-typing. As shown by Wadler [32], CLL proofs can be associated to a subset of well-behaved processes, that satisfy deadlock freedom and session fidelity.

Judgements are defined as $P \vdash \Delta$ with Δ a set of named types, i.e., $\Delta ::= \emptyset \mid x : A, \Delta$. The system, called CP and reported in Figure 1, uses CLL to type processes.

CP can be extended with a structural rule for defining composition of processes which corresponds to the CUT rule from classical linear logic:

$$\frac{P \vdash \Sigma, x : A \quad Q \vdash \Delta, y : A^{\perp}}{(\nu x y) (P \mid Q) \vdash \Sigma, \Delta} \text{ Cut}$$

In linear logic this rule is admissible, i.e., the CLL derivations of the two premises can be combined into a derivation of the conclusion with no occurrence of the CUT rule. Moreover, this is a constructive procedure, called *cut-elimination*, meaning that the proof with cut is inductively transformed into a proof without cut. The strength of the proposition-as-type correspondence stems from the fact that it carries on to the proof level, as it was shown that the cut-elimination steps correspond to reductions in the π -calculus [4,31].

4 Multiparty Compatibility

Duality is a condition that guarantees composed processes (proofs) to be wellbehaved. This can be seen in rule CUT which composes two processes through endpoints that are dual: cut elimination guarantees that such processes can communicate with each other without getting stuck or reaching an error. Multiparty compatibility [12,24,14], a semantic notion that allows for the composition of multiple processes while guaranteeing the same properties, uses session types as an abstraction of process behaviours and simulates their execution. If no error occurs during any such simulation then the composition is considered compatible.

Extended types and queues. In order to define multiparty compatibility to our logical setting with CLL formulas, we extend the syntax of types (formulas) with annotations that make explicit where messages should be forwarded from and to. This is similar to local types !p.T and ?p.T [11] expressing an output and an input to and from role p respectively. The meaning of each operator and the definition of duality remains the same as in CP.

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Types A ::= a \mid a^{\perp} \mid \mathbf{1} \mid \perp \mid A \otimes A \mid A \otimes A \mid A \oplus A \mid A \& A \mid !A \mid ?A
Local types B ::= a \mid a^{\perp} \mid \mathbf{1}^{\tilde{u}} \mid \perp^{u} \mid (A \otimes^{\tilde{u}} B) \mid (A \otimes^{u} B)
\mid !^{\tilde{u}}B \mid ?^{u}B \mid (B \oplus^{u} B) \mid (B \&^{\tilde{u}} B)
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Annotations are either single endpoints x or a set of endpoints u_1, \ldots, u_n . The left-hand side A of \otimes and \Im are not annotated (they will become dynamically labelled when needed). We will see that units demonstrate some *gathering* behaviour which explains the need to annotate $\mathbf{1}$ with a non-empty list of an arbitrary number of distinct names. We may write \tilde{u} for u_1, \ldots, u_n when the size of the list is irrelevant.

Additionally to annotated types, in order to give a semantics to types, we introduce queues defined by as $\Psi ::= \epsilon \mid A \cdot \Psi \mid * \cdot \Psi \mid \mathcal{L} \cdot \Psi \mid \mathcal{R} \cdot \Psi \mid \mathcal{Q} \cdot \Psi$. Intuitively, a queue (FIFO) is a standard ordered list of messages. A message can be a proposition A, a session termination *, a choice \mathcal{L} or \mathcal{R} , or an exponential \mathcal{Q} . Since every ordered pair of endpoints has an associated queue, we define a queue environment σ as a mapping from ordered pairs of endpoints to queues: $\sigma : (x, y) \mapsto \Psi$. In the sequel, σ_{ϵ} denotes the queue environment with only empty queues. The notation $\sigma[(x, y) \mapsto \Psi]$ denotes a new environment where the entry for (x, y) has been updated to Ψ . Finally, we are ready to define type-context semantics for an annotated environment, i.e., an environment Δ where each formula has been annotated (with a slight abuse of notation, we overload the category Δ):

Definition 1 (Type-Context Semantics). We define $\xrightarrow{\alpha}$ as the minimum relation on annotated Δ satisfying the following rules:

$\Delta, x : \perp^y \bullet \sigma[(x, y) \mapsto \Psi]$	$\xrightarrow{x \perp y}$	$arDelta ullet \sigma[(x,y)\mapsto arPsi^{}\cdot *]$
$x: 1^{ ilde{y}} ullet \ \sigma_{\epsilon}[\{(y_i, x) \mapsto *\}_i]$	$\xrightarrow{\tilde{y}1x}$	$\varnothing \bullet \sigma_\epsilon$
$x:a^{\perp},y:a\bullet\sigma_{\epsilon}$	$\xrightarrow{x \leftrightarrow y}$	$\varnothing ullet \sigma_\epsilon$
$\Delta, x : A \mathcal{B}^{y} B \bullet \sigma[(x, y) \mapsto \Psi]$	$\xrightarrow{x \mathcal{B} y}$	$\Delta, x: B ullet \sigma[(x,y) \mapsto \Psi \cdot A]$
$\Delta, x : A \otimes^{\tilde{y}} B \bullet \sigma[\{(y_i, x) \mapsto A_i \cdot \Psi_i\}_i]$	$\xrightarrow{\tilde{\boldsymbol{y}}\otimes\boldsymbol{x}[A,\{A_i\}_i]}$	$arDelta, x: B ullet \sigma[\{(y_i, x) \mapsto \Psi_i\}_i]$
$\Delta, x : A \&^{\tilde{y}} B \bullet \sigma[\{(x, y_i) \mapsto \Psi_i\}_i]$	$\xrightarrow{x\&_{\mathcal{L}}\tilde{y}}$	$\Delta, x : A \bullet \sigma[\{(x, y_i) \mapsto \Psi_i \cdot \mathcal{L}\}_i]$
$\Delta, x : A \&^{\tilde{y}} B \bullet \sigma[\{(x, y_i) \mapsto \Psi_i\}_i]$	$\xrightarrow{x\&_{\mathcal{R}}\tilde{y}}$	$\Delta, x : B \bullet \sigma[\{(x, y_i) \mapsto \Psi_i \cdot \mathcal{R}\}_i]$
$\Delta, x : A \oplus^y B \bullet \sigma[(y, x) \mapsto \mathcal{L} \cdot \Psi]$	$\xrightarrow{y \oplus_{\mathcal{L}} x}$	$\varDelta, x: A \bullet \sigma[(y, x) \mapsto \Psi]$
$\Delta, x : A \oplus^y B \bullet \sigma[(y, x) \mapsto \mathcal{R} \cdot \Psi]$	$\xrightarrow{y \oplus_{\mathcal{R}} x}$	$\varDelta, x: B \bullet \sigma[(y, x) \mapsto \Psi]$
$\{y_i:?A_i\}_i, x: !^y A \bullet \sigma_\epsilon$	$\xrightarrow{x!y}$	$\{y_i:?A_i\}_i, x: A \bullet \sigma_{\epsilon}[\{(x, y_i) \mapsto \mathcal{Q}\}_i]$
$\Delta, x : ?^{\tilde{y}} A \bullet \sigma[(y, x) \mapsto \mathcal{Q} \cdot \Psi]$	$\xrightarrow{y?x}$	$\Delta, \mathbf{x} : A \bullet \sigma[(\mathbf{y}, \mathbf{x}) \mapsto \Psi]$

The rules above capture an asynchronous semantics for typing contexts. We clarify further with an example of how the semantics above works. Assume we wish to compose three CP proofs through endpoints $\Delta = x : A^{\perp} \ \mathfrak{P} B, y : A^{\perp} \ \mathfrak{P} A \otimes C, z : A \otimes D$. The context says that x is receiving something of type A^{\perp}, y is receiving something of type A^{\perp} and then sending something of type A, and, finally, z is sending something of type A. In order to obtain an execution of Δ , we first dualise and annotate Δ (for some annotation) to, e.g., $\Delta^{\perp} = x : A^{\perp} \ \mathfrak{P} B^{\perp}, y : A \otimes^{z} A^{\perp} \ \mathfrak{P}^{x} C^{\perp}, z : A^{\perp} \ \mathfrak{P}^{y} D^{\perp}$. Then, obtain an execution, e.g.,:

$$\begin{array}{cccc} \Delta^{\perp} \bullet \sigma_{\epsilon} & \xrightarrow{z \, \Im y} x : A \otimes^{y} B, y : A \otimes^{z} A^{\perp} \, \Im^{x} \, C^{\perp}, z : D & \bullet & \sigma_{\epsilon}[(z, y) \mapsto A^{\perp}] \\ & \xrightarrow{z \otimes y[A, A^{\perp}]} x : A \otimes^{y} B, y : A^{\perp} \, \Im^{x} \, C^{\perp}, z : D & \bullet & \sigma_{\epsilon} \\ & \xrightarrow{y \, \Im x} x : A \otimes^{y} B, y : C^{\perp}, z : D & \bullet & \sigma_{\epsilon}[(y, x) \mapsto A^{\perp}] \\ & \xrightarrow{y \otimes x[A, A^{\perp}]} x : B, y : C, z : D & \bullet & \sigma_{\epsilon} \end{array}$$

Note the general rule for the multiplicative connectors \otimes and \mathfrak{P} . In their multiparty interpretation [6], they implement a gathering communication, where many $A_i \otimes B_i$ can communicate with a single $A \mathfrak{P} B$. As a consequence, the A_i 's are enqueued to a single endpoint which will consume such messages. The effect of a gathering communication with such connectives is to spawn a new session with the environment $\{A_i\}_i$ shown in the label. Ideally, we could have enriched the semantics so that it can work on different contexts running in parallel, where $\{A_i\}_i$ would be added to. However, since the need for the semantics is to define compatibility we decided to just observe in the label. Units also have a similar gathering behaviour. On the other hand, additives and exponentials use broadcasting.

Using the relation on contexts above, we can define when a set of endpoints successfully progresses without reaching an error. This can be formalised by the concept of live path. In the sequel, let α range over all the possible labels of the relation above. Moreover, let $\alpha_1, \ldots, \alpha_n$ be a *path* for some annotated Δ whenever

there exist $\Delta_1, \sigma_1, \ldots, \Delta_n, \sigma_n$ such that $\Delta \bullet \sigma_{\epsilon} \xrightarrow{\alpha_1} \Delta_1 \bullet \sigma_1 \ldots \xrightarrow{\alpha_n} \Delta_n \bullet \sigma_n$. This path is *maximal* if there is no $\Delta_{n+1}, \sigma_{n+1}, \alpha_{n+1}$ such that $\Delta_n \bullet \sigma_n \xrightarrow{\alpha_{n+1}} \Delta_{n+1} \bullet \sigma_{n+1}$.

Definition 2 (Live Path). A path $\tilde{\alpha}$ for an environment $\Delta \bullet \sigma$ is live if $\Delta \bullet \sigma \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} \varnothing \bullet \sigma_{\epsilon}$.

Intuitively, a maximal path is live whenever we can consume all sends and receives specified in the type context and all queues are empty, i.e., an error is never reached. With this notion, we are ready to define multiparty compatibility:

Definition 3 (Multiparty Compatibility). An environment $\Delta \bullet \sigma$ is executable if all maximal paths $\alpha_1, \ldots, \alpha_n$ for Δ are live and such that $\alpha_i = \tilde{y} \otimes x[A, \{A_i\}_i]$ implies $x : A^{\perp}, \{y_i : A_i^{\perp}\}_i$ is multiparty compatible for some annotation. A context $\Delta \neq \emptyset$ is multiparty compatible if there exists an annotation such that $\Delta^{\perp} \bullet \sigma_{\epsilon}$ is executable.

Multiparty compatibility states that all maximal paths are live, i.e., an error is never reached. Note that the definition above is well-founded since propositions get smaller when reduced.

Relationship to Previous Definitions. Definition 1 is an adaptation to CLL of the typing environment reduction (Definition 4.3, [14]) with a little twist: in order not to overload notation (cf. § 5), we are defining it on the dual of formulas. For example, following an approach as that of [14], a process with an endpoint x of type $A \otimes^y B$ is meant to store something of type A in the queue from x to y. In our notation, we dualise the type of x to $A^{\perp} \mathfrak{Y}^{y} B^{\perp}$ but keep the same behaviour, i.e., storing something of type A^{\perp} in the queue from x to y. Moreover, for the sake of simplicity, we are using a single queue environment σ as a function from pairs of endpoints to a FIFO, while Ghilezan et al. [14] use labelled queues attached to each endpoint of the typing context: the two approaches are equivalent. Finally, our definition being an adaptation to CLL, uses different language constructs. In particular, we do not combine value passing and branching, and our \otimes and \Im cases spawn new sessions (hence the well-founded recursive definition). The original definition of compatibility given by Deniélou and Yoshida is for communicating automata. Therefore, our starting point is the adaptation to local types given by Ghilezan et al. [14].

Properties of Multiparty Compatibility. As a consequence of multiparty compatibility, we can formalise the lack of errors with the following

Proposition 1 (No Error). Let Δ be multiparty compatible and $\alpha_1, \ldots, \alpha_n$ be a maximal path for an annotated Δ^{\perp} such that $\Delta^{\perp} \bullet \sigma_{\epsilon} \xrightarrow{\alpha_1} \Delta_1 \bullet \sigma_1 \ldots \xrightarrow{\alpha_n} \varnothing \bullet \sigma_{\epsilon}$. Then, for i < n,

1. (a) $\sigma_i(x, y) = * \cdot \Psi$ implies that $\alpha_n = x\tilde{z}\mathbf{1}y$;

- (b) $\sigma_i(x, y) = A \cdot \Psi$ implies that there exists k > i such that $\alpha_k = x\tilde{z} \otimes y[A, \{A_i\}_i];$
- (c) $\sigma_i(x, y) = \mathcal{L} \cdot \Psi$ implies that there exists k > i such that $\alpha_k = x \oplus_{\mathcal{L}} y$;
- (d) $\sigma_i(x, y) = \mathcal{R} \cdot \Psi$ implies that there exists k > i such that $\alpha_k = x \oplus_{\mathcal{R}} y$;

(e) $\sigma_i(x, y) = \mathcal{Q} \cdot \Psi$ implies that there exists k > i such that $\alpha_k = x ? y$; 2. (a) $\Delta_i = \Delta'_i, x : \mathbf{1}^{\tilde{y}}$, then $\alpha_n = \tilde{y}\mathbf{1}x$;

- (b) $\Delta_i = \Delta'_i, x : A \otimes^{\tilde{y}} B$, then there exists k > i such that $\alpha_k = \tilde{y} \otimes x[\{A_i\}_i];$
- (c) $\Delta_i = \Delta'_i, x : A \oplus^y B$, then there exists k > i such that $\alpha_k = y \oplus_{\mathcal{L}} x$ or $\alpha_k = y \oplus_{\mathcal{R}} x;$
- (d) $\Delta_i = \Delta'_i, x : ?^y A$, then there exists k > i such that $\alpha_k = y ? x$.

Conditions in (1) state that every message that has been enqueued is eventually consumed. On the other hand, conditions in (2) state that every input instruction is eventually executed. Note also that since we have no infinite computations, we do not need to consider fairness.

Are annotations important? A careful reader may be wondering why the definitions of type-context semantics and multiparty compatibility are not given for annotation-free contexts. Unfortunately, doing so would make multiparty compatibility too strong since we would have to allow for messages to be sent to different endpoints in different paths. As an example, $x : A \otimes B^{\perp}, y : A \otimes A^{\perp} \mathcal{P}$ $C^{\perp}, z : A^{\perp} \mathcal{P} D^{\perp}$ can get stuck if z communicates with x first, violating property (2b) in Proposition 1. Note that previous definitions of multiparty compatibility for multiparty session types [24,14] do indeed use annotations.

5 Asynchronous Forwarders

Forwarders form a subclass of processes with some special features but that are also typable in classical linear logic. I.e., our goal is to identify all those CP processes that are also forwarders. In order to do so, we must add further information in the standard CP contexts.

Contexts. What we need is to be able to enforce the main features that characterise a forwarder, namely i) anything received must be forwarded, ii) anything that is going to be sent must be something that has been previously received, and iii) the order of such messages between any two points must be preserved. In order to enforce these requirements, we add more information to the standard CP judgement. For example, let us consider the input process x(y). P. In CP, the typing environment for such process must be such that endpoint x has type $A \ \mathfrak{P} B$ such that **P** has type y : A and x : B. However, the context is not telling us at all that y is actually a message that has been received and, as such, it should not be used by P for further communications but just forwarded over some other channel. In order to remember this fact when we type the subprocess P, we actually insert y: A into a queue that belongs to endpoint x where we put all the types of messages received over it. I.e., when typing P, the context will contain $\llbracket \Psi \rrbracket \llbracket u y : A] x : B$. That still means that x must have type B and y must have type A in P, but also that y: A has been received over x (it is in x's queue) and we are intending to forward it to endpoint u. Moreover, Ψ contains the types of messages that have been previously received over x. The forwarders behave asynchronously. They can input arbitrarily many messages, which are enqueued at the arrival point, without blocking the possibility of producing an

output from the same endpoint. This behaviour is captured by the notion of queues of *boxed* messages, i.e. messages that are in-transit.

$$\llbracket \Psi \rrbracket ::= \varnothing \mid [u*] \llbracket \Psi \rrbracket \mid [uy:A] \llbracket \Psi \rrbracket \mid [uQ] \llbracket \Psi \rrbracket \mid [uL] \llbracket \Psi \rrbracket \mid [uR] \llbracket \Psi \rrbracket$$

A queue element $[{}^{u}x:A]$ expresses that x of type A has been received and will need to later be forwarded to endpoint u. Similarly, $[{}^{u}*]$ indicates that a request for closing a session has been received and must be forwarded to u. $[{}^{u}\mathcal{L}]$ (or $[{}^{u}\mathcal{R}]$) and $[{}^{u}\mathcal{Q}]$ indicate that a branching request and server invocation, respectively, has been received and must be forwarded.

The order of messages needing to be forwarded to *independent* endpoints is irrelevant. Hence, we consider queue $\llbracket \Psi_1 \rrbracket \llbracket x \dots \rrbracket \llbracket \Psi_2 \rrbracket$ equivalent to queue $\llbracket \Psi_1 \rrbracket \llbracket y \dots \rrbracket \llbracket \Psi_2 \rrbracket$ whenever $x \neq y$. For a given endpoint x however the order of two messages $\llbracket x \dots \rrbracket \llbracket x \dots \rrbracket \llbracket x \dots \rrbracket \rrbracket$ is crucial and must be maintained throughout the forwarding. This follows the idea of having a queue for every ordered pair of endpoints in the type-context semantics in Definition 1. By attaching a queue to each endpoint we get a typing context

$$\Gamma ::= \emptyset \mid \Gamma, \llbracket \Psi \rrbracket x : B \mid \Gamma, \llbracket \Psi \rrbracket x : \cdot$$

The element $\llbracket \Psi \rrbracket x : B$ of a context Γ indicates that the messages in $\llbracket \Psi \rrbracket$ have been received at endpoint x. The special case $\llbracket \Psi \rrbracket x : \cdot$ is denoting the situation when endpoint x no longer needs to be used for communication, but still has a non-empty queue of messages to forward.

When forwarding to many endpoints, we use $[\tilde{u} \mathcal{X}]$ for denoting $[u_1 \mathcal{X}] \dots [u_n \mathcal{X}]$, with $\tilde{u} = u_1, \dots, u_n$. In this case, we also assume the implicit rewriting $[\mathcal{O} \mathcal{X}] \llbracket \Psi \rrbracket \equiv \llbracket \Psi \rrbracket$.

Judgements and rules. A judgement denoted by $P \Vdash \Gamma$ types the forwarder processes P that connects the endpoints in Γ . The rules enforce the asynchronous forwarding behaviour by adding elements to queues using rules for \bot and \Im , which forces them to be later removed from queues by the corresponding rules for **1** and \otimes . The rules are reported in Fig. 2.

Rule Ax is identical to the one of CP. Rules 1 and \perp forward a request to close a session. Rule \perp receives the request on endpoint x and enqueues it as $[^{u}*]$ if it needs to forward it to u. Note that in the premiss of \perp the endpoint is terminated pending the remaining messages in the corresponding queue being dispatched. Eventually all endpoints but one will be terminated in the same manner. Rule 1 will then be applicable. Note that the behaviour of x().P and x[] work as gathering, several terminated endpoints connect to the last active endpoint typed with a 1. Rules \otimes and \Im forward a message. Rule \Im receives the message y : A and enqueues it as $[^{u}y : A]$ to be forwarded to endpoint u. Dually, rule \otimes applied to a \otimes sends the messages are sent at the same time. Messages will be picked from queues belonging to distinct endpoints, as a consequence, the left premiss of \otimes rule spawns a new forwarder consisting of the gathered messages.

In the case of additives and exponentials, the behaviour is actually *broad*casting, that is, an external choice $[{}^{u}\mathcal{L}]$ or $[{}^{u}\mathcal{R}]$, or a server opening $[{}^{u}\mathcal{Q}]$, resp.,

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$$\begin{aligned} \frac{P \Vdash \Gamma, \llbracket \Psi \rrbracket \llbracket^{u} x : a^{\perp}, y : a}{x ().P \Vdash \Gamma, \llbracket \Psi \rrbracket \llbracket^{u} x : \bot^{u}} \perp & \frac{x [\llbracket + \{\llbracket^{x} \rrbracket u_{i} : \cdot\}_{i}, x : \mathbf{1}^{\tilde{u}} \ \mathbf{1} \ (\tilde{u} \neq \emptyset) \\ \frac{P \Vdash \Gamma, \llbracket \Psi \rrbracket \llbracket^{u} y : A] x : B}{x (y).P \Vdash \Gamma, \llbracket \Psi \rrbracket x : A^{\mathfrak{N}} B} & \mathfrak{N} & \frac{P \Vdash \{y_{i} : A_{i}\}_{i}, y : A \quad Q \Vdash \Gamma, \{\llbracket \Psi_{i} \rrbracket u_{i} : A_{i}\}_{i}, \llbracket \Psi \rrbracket x : B}{x [\llbracket \vee \Gamma, \llbracket \Psi \rrbracket \rrbracket x : A^{\mathfrak{N}} B} & \mathfrak{N} & \frac{P \Vdash \{y_{i} : A_{i}\}_{i}, y : A \quad Q \Vdash \Gamma, \{\llbracket \Psi_{i} \rrbracket u_{i} : A_{i}\}_{i}, \llbracket \Psi \rrbracket x : B}{x [\llbracket \vee \Gamma, \llbracket \Psi \rrbracket \rrbracket x : A^{\mathfrak{N}} B} & \mathfrak{N} & \frac{P \Vdash \{y_{i} : A_{i}\}_{i}, y : A \quad Q \Vdash \Gamma, \{\llbracket \Psi_{i} \rrbracket u_{i} : C_{i}\}_{i}, \llbracket \Psi \rrbracket x : A \otimes^{\tilde{u}} B} & \otimes (\tilde{u} \neq \emptyset) \\ & \frac{P \Vdash \Gamma, \llbracket \Psi \rrbracket \llbracket^{\tilde{u}} \mathcal{L}] x : A \quad Q \Vdash \Gamma, \llbracket \Psi \rrbracket \llbracket^{\tilde{u}} \mathcal{R}] x : B}{x (\operatorname{case}(P, Q) \Vdash \Gamma, \llbracket \Psi \rrbracket x : A \otimes^{\tilde{u}} B} & \& (\tilde{u} \neq \emptyset) \\ & \frac{P \Vdash \Gamma, \llbracket \Psi_{z} \rrbracket z : C, \llbracket \Psi_{z} \rrbracket x : A \quad \oplus^{z} B}{x (\operatorname{case}(P, Q) \Vdash \Gamma, \llbracket \Psi \rrbracket x : A \otimes^{\tilde{u}} B} & \& (\tilde{u} \neq \emptyset) \\ & \frac{P \Vdash \{u_{i} : ?B_{i}\}_{i}, \llbracket^{\tilde{u}} \mathcal{Q}] y : A}{[\operatorname{tabular}(y, P \Vdash \{u_{i} : ?B_{i}\}_{i}, x : !^{\tilde{u}} A} ! (\tilde{u} \neq \emptyset) & \frac{P \Vdash \Gamma, \llbracket \Psi_{z} \rrbracket z : C, \llbracket \Psi_{z} \rrbracket x : A \oplus^{z} B}{?x [\Psi \sqcup [\Psi \sqcup [\Psi] [\Psi \sqcup \chi] x : A \oplus^{z} B} & \frac{P \Vdash \Gamma, \llbracket \Psi_{z} \rrbracket z : C, \llbracket \Psi_{z} \rrbracket x : A \oplus^{z} B}{?x [\Psi \amalg [\Psi \amalg \chi] x : A \oplus^{z} B} & \mathbb{C}_{T} & \frac{P \Vdash \{u_{i} : ?B_{i}\}_{i}, \llbracket^{\tilde{u}} \mathcal{Q}\}_{i} x : A \oplus^{z} B}{?x [\Psi \amalg [\Psi \amalg \chi] z : C, \llbracket \Psi_{z} \rrbracket x : A \oplus^{z} B} & \mathbb{C}_{T} & \frac{P \Vdash \Gamma, \llbracket \Psi_{z} \rrbracket z : C, \llbracket \Psi_{z} \rrbracket x : A \oplus^{z} B}{?x [\Psi \amalg \llbracket \Psi_{z} \rrbracket Z : C, \llbracket \Psi_{z} \rrbracket x : A \oplus^{z} B} & \mathbb{C}_{T} & \frac{P \Vdash \Gamma, \llbracket \Psi_{z} \rrbracket Z : C, \llbracket \Psi_{z} \rrbracket x : A \oplus^{z} B}{?x [\Psi \amalg \Vert [\Psi \amalg \Vert \chi] [\Psi \amalg \Vert \chi] x : A \oplus^{z} A} ? (\tilde{u} \neq \emptyset) & \frac{P \Vdash \Gamma, \llbracket \Psi_{z} \rrbracket Z : C, \llbracket \Psi_{z} \rrbracket x : ?^{\mathfrak{u}} A \cong^{\mathfrak{u}} ? \mathbb{C}_{T} & \frac{P \amalg \Psi}{?x \amalg \Psi} x : ?^{\mathfrak{u}} A} & \frac{P \amalg \Psi}{?x \amalg \Psi} X = \frac{P \amalg \Psi}{?x \amalg \Psi} x : ?^{\mathfrak{u}} X = \mathbb{C}_{T} & \frac{P \amalg \Psi}{?x \amalg \Psi} x : A \oplus^{z} A} ? \mathbb{C}_{T} & \frac{P \amalg \Psi}{?\pi} x : ?^{\mathfrak{u}} X = A \oplus^{z} A}{?x [\Psi \amalg \Vert \Vert \Vert \Vert \Vert \Vert \Vert \Vert \Vert} X = \frac{P \amalg \Psi}{?\pi} X = \mathbb{C}_{T} & \frac{P \amalg \Psi}{?$$

Fig. 2. Proof System for Forwarders

is received and can be used several (at least one) times to guide internal choices or server requests, resp., later on.

Note how annotations put constraints on how the proof is constructed, e.g., annotating $x : A \ \mathfrak{P} B$ with u ensures us that the proof will contain a \otimes -rule application on endpoint u.

Example 2 (Multiplicative Criss-cross). $P := x(u).y(v).y[u' \triangleright u \leftrightarrow u'].x[v' \triangleright v' \leftrightarrow v].x().y[]$ is one of the forwarders that can prove the compatibility of the types involved in the criss-cross protocol (in § 2), as illustrated by the derivation below.

$$\frac{F_{1} := u \leftrightarrow u'}{\Vdash u : \operatorname{name}^{\perp}, u' : \operatorname{name}} \operatorname{Ax} \begin{array}{c} \frac{F_{2} := v' \leftrightarrow v}{\Vdash v' : \operatorname{cost}^{\perp}, v : \operatorname{cost}} \operatorname{Ax} \begin{array}{c} \frac{F_{3} := x().y[]}{y[] \Vdash [^{y}*]x : ., y : \mathbf{1}} \mathbf{1} \\ \frac{y[] \Vdash [^{y}*]x : ., y : \mathbf{1}}{F_{3} \Vdash x : \bot^{y}, y : \mathbf{1}^{x}} \bot \\ \frac{y[u' \triangleright F_{1}].x[v' \triangleright F_{2}].F_{3} \Vdash [^{y}u : \operatorname{name}^{\perp}]x : \operatorname{cost}^{\perp} \otimes^{y} \bot^{y}, [^{x}v : \operatorname{cost}]y : \mathbf{1}^{x}}{y[v \cdot y[u' \triangleright F_{1}].x[v' \triangleright F_{2}].F_{3} \Vdash [^{y}u : \operatorname{name}^{\perp}]x : \operatorname{cost}^{\perp} \otimes^{y} \bot^{y}, y : \operatorname{cost}^{\gamma} x \operatorname{name}^{\otimes x} \mathbf{1}^{x}} \\ \frac{y[v \cdot y[u' \triangleright F_{1}].x[v' \triangleright F_{2}].F_{3} \Vdash [^{y}u : \operatorname{name}^{\perp}]x : \operatorname{cost}^{\perp} \otimes^{y} \bot^{y}, y : \operatorname{cost}^{\gamma} x \operatorname{name}^{\otimes x} \mathbf{1}^{x}}{y : v : \operatorname{cost}^{\gamma} x \operatorname{name}^{\otimes x} \mathbf{1}^{x}} \end{array}$$

Example 3. The following is a forwarder for the 2-buyer protocol, for some T_i 's.

 $\begin{array}{l} b_1'(book). \ s'[book \triangleright T_1]. \ s'(price). \ s'(price). \ b_1'[price \triangleright T_2]. \ b_2'[price \triangleright T_3]. \\ b_1'(contr). \ b_2'[contr \triangleright T_4]. \ b_2'. \\ \text{case}(s'[\mathsf{inl}]. \ b_2'(addr). \ s[addr \triangleright T_5]. \ T_6 \ , \ T_7) \end{array}$

Properties of Forwarders. We write $\lfloor B \rfloor$ for the formula obtained from any *B* by removing all the annotations. We state that every forwarder is also a CP process, the embedding $\lfloor \cdot \rfloor$ being extended to contexts and queue as:

$\mathbf{T}[\![\Psi]\!]\mathbf{x}:B,\Gamma \lrcorner = \mathbf{T}[\![\Psi]\!] \lrcorner,\mathbf{x}:\mathbf{T} \lrcorner = \mathbf{T}[\![\Psi]\!] \lrcorner,\mathbf{x}:\mathbf{T} \lrcorner$	$\operatorname{L}[\![\varPsi]\!]x:\cdot,\varGamma\lrcorner=\operatorname{L}[\![\varPsi]\!]\lrcorner,\operatorname{L}\varGamma\lrcorner$
$\mathbf{k} [[{}^{u}y:A] [\! [\varPsi] \!] \mathbf{k} = y:A, \mathbf{k} [\! [\varPsi] \!] \mathbf{k}$	$ \lfloor [{}^{u}\mathcal{X}]\llbracket \Psi \rrbracket \lrcorner = \lfloor [{}^{u}*]\llbracket \Psi \rrbracket \lrcorner = \lfloor \llbracket \Psi \rrbracket \lrcorner $ where $\mathcal{X} \in \{\mathcal{L}, \mathcal{R}, \mathcal{Q}\}$

Proposition 2. Any forwarder is typable in CP, i.e., if $P \Vdash \Gamma$, then $P \vdash \llcorner \Gamma \lrcorner$.

Moreover, forwarders enjoy an invertibility property, i.e., all its rules are invertible. In CLL, the rules \otimes or \oplus are not invertible because of the choice involved either in splitting the context in the conclusion of \otimes into the two premisses or the choice of either disjuncts for \oplus . In our case on the other hand, the annotations put extra syntactic constraints on what can be derived and hence are restricting these choices to a unique one and as a result the rules are invertible. This is formalised by the following.

Proposition 3. All the forwarder rules are invertible, that is, for any rule if there exists a forwarder F such that $F \Vdash \Gamma$, the conclusion of the rule, there is a forwarder $F_i \Vdash \Gamma_i$, for each of its premiss, i = 1 or 2.

Proof. For each rule, the proof is standard and follows by induction on F. For example, for \oplus_l , let us suppose that $F \Vdash \Gamma$, $[{}^x \mathcal{L}][\![\Psi_z]\!]z : C$, $[\![\Psi_x]\!]x : A \oplus^z B$. If $F = x[\mathsf{inl}].F_1$, then we directly get that $F_1 \Vdash \Gamma$, $[\![\Psi_z]\!]z : C$, $[\![\Psi_x]\!]x : A$.

Otherwise, if F starts with any other process operator, we apply an inductive step. E.g., let F = u(v).F', meaning that $\Gamma = \Gamma', \llbracket \Psi_u \rrbracket u : A \mathfrak{N}^w B$ and that $F' \Vdash \Gamma', \llbracket \Psi_u \rrbracket \llbracket^w v : A \rrbracket u : B, \llbracket^x \mathcal{L} \rrbracket \llbracket \Psi_z \rrbracket z : C, \llbracket \Psi_x \rrbracket x : A \oplus^z B$. By induction hypothesis (given that F' is a subprocess of F), we know there exists F'_1 such that $F'_1 \Vdash \Gamma', \llbracket \Psi_u \rrbracket \llbracket^w v : A \rrbracket u : B, \llbracket \Psi_z \rrbracket z : C, \llbracket \Psi_x \rrbracket x : A$. From there we can derive $F_1 = u(v).F'_1$. The rest of the cases follow in a similar way.

A Note on Compositionality and Cut Elimination. The sequent calculus we presented enjoys cut elimination. That means that forwarders can be composed, and their composition is still a forwarder. The way we compose forwarders follows a standard cut rule augmented with extra machinery to deal with annotations. Additionally, the cut elimination proof provides a semantics for forwarders, in the proposition-as-types style. For space reasons, we have not included its technical development, but it can be found in our extended note [7].

Relation to Multiparty Compatibility. Forwarders relate to transitions in the type-context semantics introduced in the previous section. In order to formalise this, we first give a translation from type-contexts into forwarder contexts:

- $-\operatorname{tr}(\varnothing \bullet \sigma_{\epsilon}) = \varnothing.$
- $\operatorname{tr}(\Delta, x : A \bullet \sigma \cup \{(y_i, x) \mapsto \Psi_i\}_i) := \llbracket^{y_1} \Psi_1 \rrbracket \dots \llbracket^{y_n} \Psi_n \rrbracket x : A, \operatorname{tr}(\Delta \bullet \sigma) \text{ for a type environment } \Delta = \{y_i : \Psi_i\}_i \text{ and a queue environment } \sigma \text{ mapping the endpoints } y_i;$

We use the extended notation $\llbracket^{u}\Psi\rrbracket$ to signify that all the brackets in $\llbracket\Psi\rrbracket$ are labelled by u.

Lemma 1. Let $\Delta \bullet \sigma$ be a type-context and $\Gamma = tr(\Delta \bullet \sigma)$.

- 1. if there exists α and $\Delta' \bullet \sigma'$ such that $\Delta \bullet \sigma \xrightarrow{\alpha} \Delta' \bullet \sigma'$ then there exists a rule from forwarders such that Γ is an instance of its conclusion and $\Gamma' = \operatorname{tr}(\Delta' \bullet \sigma')$ is an instance of (one of) the premiss(es);
- 2. otherwise, either $\Delta = \emptyset$ and $\sigma = \sigma_{\epsilon}$ or there is no forwarder F such that $F \Vdash \Gamma$.

Proof. Follows from a simple inspection of the transitions in Def. 1 and the rules in Fig. 2.

Thus, we can conclude this section by proving that forwarders are actually characterising multiparty compatibility for CP processes (processes that are welltyped in CLL).

Theorem 1. Δ is multiparty compatible iff there exists a forwarder F such that $F \Vdash \Delta^{\perp}$ where each connective in Δ^{\perp} is annotated.

Proof (Proof Sketch). See Appendix ?? for the full proof. From left to right, we need to prove more generally that if $\Delta \bullet \sigma$ is executable, then there exists a forwarder F such that $F \Vdash \operatorname{tr}(\Delta \bullet \sigma)$, by induction on the size of Δ , defined as the sum of the formula sizes in Δ . From right to left can be proven by contrapositive, using Lemma 1 and Proposition 3.

6 Composing Processes with Asynchronous Forwarders

In this section, we show how to use forwarders for logically composing CP processes.

Multiparty Process Composition. We start by focusing on the *structural* rule that can be added to CP, namely the CUT, as seen in Section 3. Rule CUT corresponds to parallel composition of processes. The implicit side condition that this rule uses is *duality*, i.e., we can compose two processes if endpoints x and y have a dual type. Carbone et al. [6] generalise the concept of duality to that of *coherence*. Coherence, denoted by \vDash , generalises duality to many endpoints, allowing for a cut rule that composes many processes in parallel

$$\frac{\{R_i \vdash \Sigma_i, x_i : A_i\}_{i \le n} \quad G \vDash \{x_i : A_i\}_{i \le n}}{(\boldsymbol{\nu}\tilde{x} : G) (R_1 \mid \dots \mid R_n) \vdash \{\Sigma_i\}_{i \le n}} \text{ MCut}$$

The judgement $G \vDash \{x_i : A_i\}_{i \le n}$ intuitively says that the $x_i : A_i$'s are compatible and the execution of the R_i will proceed without any error. Such a result is formalised by an MCUT elimination theorem analogous to the one of CP. We leave G abstract here: it is a proof term and it corresponds to a global type (see [6]). Our goal here is to replace the notion of coherence with an asynchronous forwarder Q, yielding the rule

$$\frac{\{R_i \vdash \Sigma_i, x_i : A_i\}_{i \le n} \quad Q \Vdash \{x_i : A_i^{\perp}\}_{i \le n}}{(\boldsymbol{\nu}\tilde{x} : Q) (R_1 \mid \dots \mid R_n) \vdash \{\Sigma_i\}_{i \le n}} \text{ MCutF}$$

Asynchronous forwarders are more general than coherence: every coherence proof can be transformed into an *arbiter* process [6], which is indeed a forwarder, while there are judgements that are not coherent but are provable in our forwarders (see Example 2). In the rule MCUTF, the role of a forwarder (replacing coherence) is to be a middleware that decides whom to forward messages to. This means that when a process R_i sends a message to the middleware, the message must be stored by the forwarder, who will later forward it to the right receiver. Since our goal is to show that MCUTF is admissible (and hence we can eliminate it from any correct proof), we extend such rule to account for messages in transit that are temporarily held by the forwarder. In order to do so, we use the forwarders queues and some extra premises and define MCUTQ as:

$$\frac{\{P_j \vdash \Delta_j, y_j : A_j\}_{j \le m} \quad \{R_i \vdash \Sigma_i, x_i : B_i\}_{i \le n} \quad Q \Vdash \{\llbracket \Psi_i \rrbracket x_i : B_i^{\perp}\}_{i \le n}, \{\llbracket \Psi_i \rrbracket x_i : \cdot\}_{n < i \le p}}{(\boldsymbol{\nu} \tilde{x} : Q[\tilde{y} \lhd P_1, \dots, P_m]) (R_1 \mid \dots \mid R_n) \vdash \{\Delta_j\}_{j \le m}, \{\Sigma_i\}_{i \le n}}$$

We have three types of process terms: P_j 's, R_i 's and Q. Processes R_i 's are the processes that we are composing, implementing a multiparty session. Q is the forwarder whose role is to certify compatibility and to determine, at run time, who talks to whom. Finally, processes P_i 's must be linked to messages in the forwarder queue. Such processes are there because of the way \otimes and \Im work in linear logic. This will become clearer when we look at the reduction steps that lead to cut admissibility. This imposes a side condition on the rule, namely that

$$igcup_{i\leq p} arPsi_i\setminus\{*\}=\left\{y_j:A_j^ot
ight\}_{j\leq n}$$

Note that we need to introduce a new syntax for this new structural rule: in the process $(\nu \tilde{x} : Q[\tilde{y} \triangleleft P_1, \ldots, P_m])(R_1 \mid \ldots \mid R_n)$, the list P_1, \ldots, P_m denotes those messages (processes) in transit that are going to form a new session after the communication has taken place. In the remainder we (slightly abusively) abbreviate both $\{P_1, \ldots, P_m\}$ and $(R_1 \mid \ldots \mid R_n)$ as \tilde{P} and \tilde{R} respectively.

Semantics and MCUTF-admissibility. We now formally show that MCUTF is admissible, yielding a semantics for our extended CP (with MCUTF) in a proposition-as-types fashion. In order to do so, we consider some cases from the multiplicative fragment (see appendix for all cases). In the sequel, $\Gamma = \{ \llbracket \Psi_i \rrbracket x_i : B_i^{\perp} \}_{i \leq n}, \{ \llbracket \Psi_i \rrbracket x_i : \cdot \}_{n < i \leq p}$ and $\Gamma - k = \Gamma \setminus \{ \llbracket \Psi_k \rrbracket x_k : B_k^{\perp} \}$. Also, we omit (indicated as "...") the premises of the MCUTQ that do not play a role in the reduction at hand, and assume that they are always the same as above, that is, $\{ P_j \vdash \Delta_j, y_j : A_j \}_{j \leq m}$ and $\{ R_i \vdash \Sigma_i, x_i : B_i \}_{i \leq n}$.

Send Message (\otimes). This is the case when a process intends to send a message, which corresponds to a \otimes rule. As a consequence, the forwarder has to be ready to receive the message (to then forward it later):

$$\frac{P \vdash \Delta, y : A \quad R \vdash \Sigma, x : B}{x[y \triangleright P].R \vdash \Delta, \Sigma, x : A \otimes B} \otimes \dots \qquad \frac{Q \Vdash \llbracket \Psi \rrbracket [x_k y : A^{\perp}] x : B^{\perp}, \Gamma}{x(y).Q \Vdash \llbracket \Psi \rrbracket x : A^{\perp} \mathfrak{P}^{x_k} B^{\perp}, \Gamma} \qquad \mathfrak{P}$$
$$\frac{Q \vdash \llbracket \Psi \rrbracket x : A^{\perp} \mathfrak{P}^{x_k} B^{\perp}, \Gamma}{(\nu x \tilde{x} : x(y).Q[\tilde{y} \lhd \tilde{P}]) (x[y \triangleright P].R \mid \tilde{R}) \vdash \Delta, \Sigma, \{\Delta_j\}_{j \le m}, \{\Sigma_i\}_{i \le n}} \qquad \mathfrak{P}$$

The process on the left is ready to send the message to the forwarder. By the annotation on the forwarder, it follows that the message will have to be forwarded to endpoint x_k , at a later stage. Observe that the nature of \otimes forces us to deal with the process P: the idea is that when the forwarder will finalise the communication (by sending to a process R' owning endpoint x_k) process P will be composed with R'. For now, we obtain the reductum:

$$\frac{P \vdash \Delta, y : A \quad R \vdash \Sigma, x : B \quad \dots \quad Q \Vdash \llbracket \Psi \rrbracket \llbracket^{x_k} y : A^{\perp}] x : B^{\perp}, \Gamma}{(\nu x \tilde{x} : Q[y, \tilde{y} \triangleleft P, \tilde{P}]) (R \mid \tilde{R}) \vdash \Delta, \Sigma, \{\Delta_j\}_{j \le m}, \{\Sigma_i\}_{i \le n}} \text{ MCutQ}$$

Receive Message (\mathfrak{P}). At a later point, the forwarder will be able to complete the forwarding operation by connecting with a process ready to receive (\mathfrak{P} rule):

$$\begin{array}{c} P\vdash \Delta, z: A^{\perp} & \dots \\ \frac{R\vdash \Sigma, y: A, x: B}{x(y).R\vdash \Sigma, x: A \,\mathfrak{P} \, B} \,\mathfrak{P} & \frac{S\Vdash z: A, y: A^{\perp} \quad Q \Vdash \llbracket \Psi_x \rrbracket x: B^{\perp}, \Gamma}{x[y \triangleright S].Q \Vdash \llbracket \Psi_x \rrbracket x: A^{\perp} \otimes^{x_k} B^{\perp}, [^xz: A] \llbracket \Psi_k \rrbracket x_k: B_k^{\perp}, \Gamma - k} \otimes \\ \frac{(\nu x \tilde{x}: x[y \triangleright S].Q[z, \tilde{y} \lhd P, \tilde{P}]) \left(x(y).R \mid \tilde{R}\right) \vdash \Delta, \Sigma, \{\Delta_j\}_{j \le m}, \{\Sigma_i\}_{i \le n}} \end{array}$$

Key ingredients are process P with endpoint z of type A^{\perp} , endpoint x_k in the forwarder with a boxed endpoint z with type A, and process x(y). R ready to receive.

After reduction, we obtain the following:

$$\frac{(\boldsymbol{\nu} yz:S)\left(R \mid P\right) \vdash \boldsymbol{\Sigma}, \boldsymbol{\Delta}, x:B \quad \dots \quad Q \Vdash \llbracket \boldsymbol{\Psi}_x \rrbracket x:B^{\perp}, \boldsymbol{\Gamma}}{(\boldsymbol{\nu} x \tilde{x}:Q[\tilde{y} \triangleleft \tilde{P}])\left((\boldsymbol{\nu} yz:S)\left(R \mid P\right) \mid \tilde{R}\right) \vdash \boldsymbol{\Delta}, \boldsymbol{\Sigma}, \{\boldsymbol{\Delta}_j\}_{j \leq m}, \{\boldsymbol{\Sigma}_i\}_{i \leq n}} \text{ MCutQ}$$

Where the left premiss is obtained as follows:

$$\frac{R \vdash \Sigma, y : A, x : B \quad P \vdash \Delta, z : A^{\perp} \quad S \Vdash z : A, y : A^{\perp}}{(\boldsymbol{\nu} y z : S) (R \mid P) \vdash \Sigma, \Delta, x : B} \quad \text{MCutQ}$$

meaning that now the message (namely process P) has finally been delivered and it can be directly linked to R with a new (but smaller) MCUTQ.

These reductions (the full set can be found in Appendix ??) allow us to prove the key lemma of this section.

Lemma 2 (MCUTQ Admissibility). If $\{P_j \vdash \Delta_j, y_j : A_j\}_{j \le m}$ and $\{R_i \vdash \Sigma_i, x_i : B_i\}_{i \le n}$ and $Q \Vdash \{\llbracket \Psi_i \rrbracket x_i : B_i^\perp\}_{i \le n}, \{\llbracket \Psi_i \rrbracket x_i : \cdot\}_{n < i \le p}$ then there exists a process S such that $(\boldsymbol{\nu} \tilde{x} : Q[\tilde{y} \lhd \tilde{P}]) \tilde{R} \Rightarrow^* S$ and $S \vdash \{\Delta_j\}_{j \le m}, \{\Sigma_i\}_{i \le n}$.

Proof (Proof Sketch). By lexicographic induction on (i) the sum of sizes of the B_i 's and (ii) the sum of sizes of the R_i 's. Some of the key and base cases have been detailed above; the others can be found in Appendix ??. The commutative

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cases are straightforward and only need to consider the possible last rule applied to a premise of the form $R_i \vdash \Sigma_i, x_i : B_i$.

We can finally conclude with the following theorem as a special case.

Theorem 2 (MCUTF Admissibility). If $\{R_i \vdash \Sigma_i, x_i : A_i\}_{i \le n}$ and $Q \Vdash \{x_i : A_i^{\perp}\}_{i \le n}$ then there exists a process S such that $(\nu \tilde{x} : Q) (R_1 \mid \ldots \mid R_n) \Rightarrow^* S$ and $S \vdash \{\Delta_j\}_{j \le n}, \{\Sigma_i\}_{i \le n}$.

7 Related Work

Our work takes [6] as a starting point. Guided by CLL, we set out to explore if coherence can be broken down into more elementary logical rules which led us to introduce forwarders. As a result, forwarders provide a more general notion of compatibility. An earlier unpublished version of this work [8] proposes synchronous forwarders, i.e., the restriction of forwarders with only buffers of size one. In that case, we show that we can always construct a coherence proof from a synchronous forwarder. However, synchronous forwarders fail to capture all the possible interleaving of an arbiter (encoding of coherence to processes).

Caires and Perez [3] also study multiparty session types in the context of intuitionistic linear logic by translating global types to processes, called *mediums*. Their work does not start from a logical account of global types (their global types are just syntactic terms). But, as previous work [6], they do generate arbitres as linear logic proofs, which are special instances of forwarders. In this work, we generalise this approach to characterise exactly which processes can justify the compatibility of several processes. In a more recent work, van den Heuvel and Pérez [29] use *routers* in order to provide a decentralised analysis of multiparty protocols. Routers act as point-to-point forwarders but their types, called *relative types*, carry extra information on causality of events that are not local. As in [3], this approach is confined to global types and is therefore not complete wrt multiparty compatibility.

Another logical interpretation of multiparty compatibility is proposed by Horne [21]. It uses the additonal expressivity of BV, a generalisation of CLL with a non-commutative sequential operator, but only considers a fragment that is expressible in the sequent calculus, unlike their previous work which relied on the full strength of deep inference [10]. It also allows one to consider compatible processes beyond duality but only for simply typed processes, which cannot spawn other processes. The main advantage of this approach is the fact that annotations are not needed (but can be recovered in the typing).

Sangiorgi [27], probably the first to treat forwarders for the π -calculus, uses binary forwarders, i.e., processes that only forward between two channels, which are equivalent to our $x \leftrightarrow y$. We attribute our result to the line of work that originated in 2010 by Caires and Pfenning [4], where forwarders à la Sangiorgi were introduced as processes to be typed by the axiom rule in linear logic. Van den Heuvel and Perez [17] have recently developed a version of linear logic that

encompasses both classical and intuitionistic logic, presenting a unified view on binary forwarders in both logics.

Gardner et al. [13] study the expressivity of the linear forwarder calculus, by encoding the asynchronous π -calculus (since it can encode distributed choice). The linear forwarder calculus is a variant of the (asynchronous) π -calculus that has binary forwarders and a restriction on the input x(y). P such that y cannot be used for communicating (but only for forwarding). Such a restriction is similar to the intuition behind our forwarders, with the key difference that their methodology would not apply to some of our session-based primitives.

Barbanera and Dezani [2] study multiparty session types as *gateways* that work as a medium among many interacting parties, forwarding communications between two multiparty sessions. Such mechanism reminds us of our forwarder composition: indeed, in their related work discussion they do mention that their gateways could be modelled by a "connection-cut".

Recent work [22,15] proposes a variant of linear logic that models *identity providers*, monitors that are like our forwarders but restricted to between two channels. Identity providers are asynchronous, i.e., they allow unbounded buffering of messages before forwarding. Our forwarders can be seen as a generalisation to multiparty monitors. Multiparty monitors are also addressed by Hamers and Jongmans [16], but not in a linear logic context.

Our forwarding mechanism may be confused with that of locality [25], which is addressed logically by Caires et al. [5]. Locality requires that received channels cannot be used for inputs (which must occur at the location where the channel was created). In our case, we do not allow received channels to be used at all until a new forwarder is spawned.

8 Conclusions and Future Work

We showed that forwarders are a logical characterisation of multiparty compatibility and they can safely replace coherence for composing all well-typed compatible communicating processes. Below, we discuss some aspects of forwarders and identify possible future extensions.

Improving Multiparty Compatibility? Multiparty compatibility concerns the error-free composition of processes that communicate by enqueueing/dequeueing messages into/from pair-wise distinct FIFO queues. This work is not about improving multiparty compatibility, unlike, e.g., [14]. In this paper, we take the definition of multiparty compatibility and give it a logical characterisation, in the spirit of the research line started in [4]. Forwarders are derived from our logical characterisation, this is our novel contribution.

Are Forwarders Centralised? Following the approach taken for arbiters [6] and mediums [3], forwarders provide an orchestration of the message flows between the composed processes. In order to step to a fully decentralised setting, it is necessary to redefine rule MCut such that i) queues are no longer embedded in forwarders and ii) annotations in the forwarders are transferred to the composed processes. This two steps are immediate and the correctness of this follows, also immediately, by Theorem 1, since the type-context semantics in Definition 1 is indeed fully decentralised. Note that a similar decentralisation approach is also done for coherence in [6].

Cut Elimination for Forwarders. An unpublished longer version of this work [7] addresses forwarder composition through a CUT rule for forwarders. Due to annotations, such rule requires some subtle technical details for which there was no space in this paper. This CUT rule for forwarders is proven to be admissible through an algorithm of CUT elimination which mimics how messages are transferred from one forwarder to another.

Process Language. Our process language is based Wadler's CP [32], without polymorphic communications. We conjecture that our forwarders can be smoothly extended to polymorphic types $\exists X.A$ and $\forall X.A$. As future work, we plan to consider a further extension to support recursion in the style of Toninho et al. [28]. That will require an extended notion of multiparty compatibility dealing with infinite paths as done by Ghilezan et al. [14].

Variants of Linear Logic. In this paper, we have chosen to base our theory on CLL for two main reasons. Coherence is indeed defined by Carbone et al. [6] in terms of CLL and therefore our results can immediately be related to theirs without further investigations. An earlier version of forwarders was based on intuitionistic linear logic, but moving to CLL required fewer rules and greatly improved the presentation. Nevertheless, our results should be easily reproducible in intuitionistic linear logic. A different approach could be to include non-commutative operators which could encode our FIFO queues, e.g., using the work on non-commutative subexponentials by Kanovich et al. [23]. We leave this as future work.

Beyond Linear Logic. Another interesting avenue would be to understand how the queueing mechanism of forwarders can be treated within the graphical proof system of [1]. Indeed, they observed that queues of length greater than 3 could not be expressed as linear logic formulas so they designed a proof system that works not only on LL formulas (or even its generalisation with a sequential operator called BV) but on more general graphs.

Variants of Coherence. Our results show that forwarders are a generalisation of coherence proofs. Indeed, coherence would correspond to the notion of *synchronous forwarders* [8], the restriction of forwarders with only buffers of size one. As a follow-up, we would like to investigate, whether other syntactic restrictions of forwarders also induce interesting generalised notions of coherence, and, as a consequence, generalisations of global types.

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