Reinforcement Learning under Partial Observability Guided by Learned Environment Models

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Abstract. In practical applications, we can rarely assume full observability of a system's environment, despite such knowledge being important for determining a reactive control system's precise interaction with its environment. Therefore, we propose an approach for reinforcement learning (RL) in partially observable environments. While assuming that the environment behaves like a partially observable Markov decision process with known discrete actions, we assume no knowledge about its structure or transition probabilities.

Our approach combines Q-learning with IoAlergia, a method for learning Markov decision processes (MDP). By learning MDP models of the environment from episodes of the RL agent, we enable RL in partially observable domains without explicit, additional memory to track previous interactions for dealing with ambiguities stemming from partial observability. We instead provide RL with additional observations in the form of abstract environment states by simulating new experiences on learned environment models to track the explored states. In our evaluation we report on the validity of our approach and its promising performance in comparison to six state-of-the-art deep RL techniques with recurrent neural networks and fixed memory.

Keywords: Reinforcement Learning · Automata Learning · Partially Observable Markov Decision Processes · Markov Decision Processes.

1 Introduction

Reinforcement learning (RL) enables the automatic creation of controllers in stochastic environments through exploration guided by rewards. Partial observability presents a challenge to RL, which naturally arises in various control problems. Unreliable or inaccurate sensor readings may provide incomplete state information, e.g., static images provided by visual sensors do not capture the agent's movement trajectory and speed. Formally, partial observability occurs when observations of the environment do not allow to deduce the environment state directly. In such settings, optimal control based on observations only is generally not possible.

For this reason, RL methods often include some form of memory to cope with partial observability, such as the hidden state in recurrent neural networks [10] or fixed-size memory obtained by concatenating previous observations [18]. In this paper, we propose a method for RL in partially observable environments that combines Q-learning [23] with IoAlergia [15], a technique for learning deterministic labeled Markov decision processes (MDPs). With IoAlergia, we regularly learn and update MDPs based on the experiences of the RL agent. The learned MDPs approximate the dynamics of the partially observable Markov decision process (POMDP) underlying the environment and their states extend the observation space of Q-learning. To enable this extension, we trace every step of the RL agent on the most recently learned MDP and add the explored MDP state as an observation. Hence, we provide memory by tracking learned environmental states. With this approach, we follow the tradition of state estimation in RL under partial observability [6,16]. In comparison to earlier work, we overcome strict assumptions on the underlying POMDP, such as knowledge about the number of states, e.g., criticized by Singh et al. [21].

Our contributions comprise: (1) an approach for RL under partial observability aided by automata learning, which we term Q^A-learning, (2) an implementation of the approach in an environment conforming to the OpenAI gym interface [2], and (3) its evaluation against three baseline deep RL methods with fixed memory and three RL methods with LSTMs providing memory.

Structure. In Sect. 2, we introduce preliminaries like passive learning of stochastic automata. We present our method for reinforcement learning in partially observable environments in Sect. 3, followed by a corresponding evaluation in Sect. 4. After discussing related work in Sect. 5, we conclude by summarizing our findings and providing an outlook on future work in Sect. 6.

2 Preliminaries

2.1 Models

In RL, we commonly assume that the environment behaves like an MDP (see Def. 1). An agent observes the environment's state and based on that reacts by choosing from a given set of actions—causing a probabilistic state transition.

Definition 1 (Markov decision processes (MDPs)). A Markov decision process (MDP) is a tuple $\mathcal{M} = (S, s_0, A, \delta)$, where S is a finite set of states, $s_0 \in S$ is the initial state, A is a finite set of actions, $\delta : S \times A \to Dist(S)$ is a probabilistic transition function.

Please note that for a simplified presentation, we assume MDPs to support all actions in all states, s.t. δ is total. In our work, we consider settings where the agent cannot observe the environment directly, but where it has only limited information—like a room's number, but not its position in the room (see Fig. 4)—and where we assume discrete states as well as finite action and state spaces. Such scenarios are commonly modeled as partially observable Markov decision

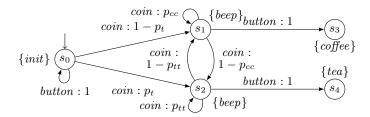


Fig. 1. A POMDP producing hot beverages.

processes (POMDPs) (Def. 2), see, e.g., Bork et al. [1]. Alternative POMDP definitions including probabilistic observation functions can also be handled [5].

Definition 2 (Partially observable Markov decision processes). A partially observable Markov decision process (POMDP) is a triple (\mathcal{M}, Z, O) , where $\mathcal{M} = (S, s_0, A, \delta)$ is the underlying MDP, Z is a finite set of observations and $O: S \to Z$ is the observation function.

Example 1 (Hot Beverage POMDP). Fig. 1 shows a POMDP of a vending machine that, depending on parameterized probabilities, produces either tea or coffee. For the individual states s_i , we show the respective observation in curly braces. For each probabilistic transition reported (for brevity we ommit the transitions from s_3 and s_4 , but they would loop back to s_0 for any action) we show a corresponding edge, labeled by the action and the transition's probability. While the parameterized probabilities will become more important later on, let us for now assume $p_t = 0.5$, $p_{cc} = 0.9$, and $p_{tt} = 0.1$. In the initial state s_0 , for the action coin, we would now progress to $either\ s_1$ or s_2 , but where the resulting observation would be beep for both. Pressing a button, we would then move to s_3 or s_4 receiving either a coffee or tea. Alternatively, we can add another coin to move to s_1 with a probability of 0.9 for increasing the chances to get a coffee.

Paths, Traces & Policies. The interaction of an agent with its environment can be described by a path that is defined by an alternating sequence of states and actions $s_0 \cdot a_1 \cdot s_1, \dots, s_n$ starting in the initial state. We denote the set of paths in an MDP \mathcal{M} by $Paths_{\mathcal{M}}$. For partially observable scenarios, traces basically replace the states in a path with the corresponding observations. We correspondingly lift observation functions O to paths, applying O on every state to derive trace $O(p) = L(s_0) \cdot a_1 \cdot L(s_1)$ from path $p = s_0 \cdot a_1 \cdot s_1$. An agent selects actions based on a policy that is a mapping from $Paths_{\mathcal{M}}$ to distributions over actions Dist(A). If a policy σ depends only on the current state, we say that σ is memoryless. With policies relating to action choices, an MDP controlled by a policy defines a probability distribution over paths.

Rewards define the crucial feedback an agent needs during learning for judging whether the actions it chose were "good or bad". That is, the goal is to learn a policy that maximizes the reward. To this end, we consider a reward function R:

 $S \to \mathbb{R}$ that returns a real value³. For a path $p = s_0 \cdot a_1 \cdot s_1, \cdots, s_n$, we can define a discounted cumulative reward at time step t as $Ret(p,t) = \sum_{i=0}^{n-t-1} \gamma^i R(s_{t+i+1})$, taking a (time) discount factor γ into account. For a memoryless policy σ , we can define a value function for a state s as $v_{\sigma}(s) = \mathbb{E}_{\sigma} \left[Ret(p,t) \mid s_t = s \right]$. To accommodate partial observability, we define reward-observation traces that extend traces with rewards, e.g., $rt = RO(p) = O(s_0) \cdot R(s_0) \cdot a_1 \cdot O(s_1) \cdot R(s_1), \cdots, O(s_n) \cdot R(s_n)$ for path $p = s_0 \cdot a_1 \cdot s_1, \cdots, s_n$.

Note that in RL, we usually consider memoryless policies—enabled by the assumption of a Markovian environment (modeled as MDP) which guarantees that there is an optimal, memoryless policy for maximizing the reward. With partial observability it is impossible to precisely identify the current state (consider the Hot Beverage example and s1 vs. s2), meaning that creating optimal policies for POMDPs entails taking the history of previous actions into account—rendering the problem non-Markovian. Alternatively, deriving policies under partial observability can be approached by creating belief-MDPs from POMDPs [4].

Belief-MDPs & Deterministic Labeled MDPs. Deterministic Labeled MDPs (DLMDPs) feature an observation function and adhere to a specific determinism property that guarantees that any possible (observation) trace reaches exactly one state. Belief MDPs (BMDPs) are special DLMDPs that represent the dynamics of a POMDP and are defined over so-called belief states. These belief states (beliefs for short) describe probability distributions over states in a POMDP, i.e., over those states that the paths relating to a trace would reach in the POMDP. That is, for any given trace, a BMDP progresses to a unique state that in turn defines a distribution over possible POMDP states.

Definition 3 (Deterministic Labeled MDPs). A deterministic labeled MDP is a triple (\mathcal{M}, Z, O) , where $\mathcal{M} = (S, s_0, A, \delta)$ is the underlying MDP, Z is a set of observations, and O is an observation function, satisfying

$$\forall s, s', s'' \in S, \forall a \in A \colon \delta(s, a, s') > 0 \land \delta(s, a, s'') > 0 \land O(s') = O(s'')$$

$$\implies s' = s''.$$

We introduce BMDPs with some auxiliary definitions: Let $P = (\mathcal{M}, Z, O)$ be a POMDP over an MDP $\mathcal{M} = (S, s_0, A, \delta)$. This defines the beliefs as the set $B = \{\mathbf{b} \in Dist(S) | \forall s, s' \in supp(\mathbf{b}) : O(s) = O(s')\}$, where supp() returns the support of a probability distribution. Then the probability of observing $z \in Z$ after executing $a \in A$ in $s \in S$ is defined as $\mathbf{P}(s, a, z) = \sum_{s' \in S, O(s') = z} \delta(s, a, s')$ and in a belief (state) \mathbf{b} it is $\mathbf{P}(\mathbf{b}, a, z) = \sum_{s \in S} \mathbf{b}(s) \cdot \mathbf{P}(s, a, z)$. If O(s') = z, the subsequent belief update is defined as $[\![\mathbf{b}|a,z]\!](s') = \frac{\sum_{s \in S} \mathbf{b}(s) \cdot \delta(s,a,s')}{\mathbf{P}(\mathbf{b},a,z)}$.

Definition 4 (Belief MDPs). The BMDP for a POMDP P as of Def. 2 is a DLMDP (\mathcal{M}_B, Z, O_B) over an MDP $\mathcal{M}_B = (B, s_{0B}, A, \delta_B)$, with B as defined above, $s_{0B} = \{s_0 \mapsto 1\}$, $O_B(\mathbf{b}) = O(s)$ for an $s \in supp(\mathbf{b})$, and

$$\delta_B(\mathbf{b}, a, \mathbf{b}') = \begin{cases} \mathbf{P}(\mathbf{b}, a, O(\mathbf{b}')) & \text{if } \mathbf{b}' = [\![\mathbf{b}|a, O(\mathbf{b}')]\!], \\ 0 & \text{otherwise.} \end{cases}$$

³ Alternative definitions including actions are possible as well.

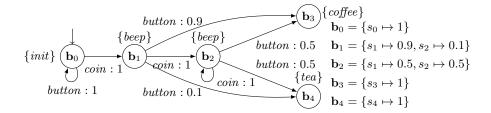


Fig. 2. A finite BMDP for the POMDP from Fig. 1 and parameters $p_t = 0.1$ and $p_{tt} = p_{cc} = 0.5$. For brevity reasons, we do not show transitions from \mathbf{b}_3 and \mathbf{b}_4 .

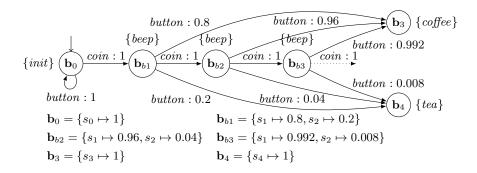


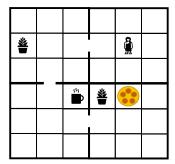
Fig. 3. An infinite BMDP for the POMDP from Fig. 1 for parameters $p_t = 0.2$, $p_{tt} = 0.2$, and $p_{cc} = 1$. For brevity reasons, we do not show transitions from \mathbf{b}_3 and \mathbf{b}_4 .

BMDPs allow to synthesize policies under partial observability, i.e., it was shown that an optimal policy for a BMDP is optimal also for the corresponding POMDP [4]. Since they are Markovian, there are furthermore methods for synthesizing memoryless policies. Please note that while in principle there are finite BMDPs (e.g., Fig. 2), in general they are of infinite size [1] (see, e.g., Fig. 3).

Example 2 (Hot Beverage BMDPs). Let us consider again the POMDP from Fig. 1 and parameters $p_t = 0.1$ and $p_{tt} = p_{cc} = 0.5$. The corresponding finite BMDP is shown in Fig. 2. Now suppose that we get a reward for observing tea, i.e., when reaching b_4 . The BMDP then supports a memoryless policy where we choose the actions coin in \mathbf{b}_0 and \mathbf{b}_1 , and button in \mathbf{b}_2 . That is, unless γ is very small (like 0) s.t. choosing button in \mathbf{b}_1 would be optimal for maximizing the immediate reward. A second, infinite BMDP for parameters $p_t = 0.2$, $p_{tt} = 0.2$ and $p_{cc} = 1$ is shown in Figure 3.

2.2 Learning MDPs

We learn MDPs using the IoAlergia algorithm [15]. IoAlergia takes samples \mathcal{T} , which is a multiset of traces, and an ϵ_{AL} controlling the significance level of a statistical check as inputs and returns a deterministic labelled MDP.



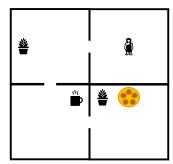


Fig. 4. Fully and partially observable *Office World*. With full observations (left), the (x,y) coordinates can be observed, otherwise (right) it is only the room's number.

The algorithm first creates an input/output frequency prefix tree acceptor (IOFPTA) from \mathcal{T} , a tree where common prefixes of traces are merged. Every node of the tree is labeled with an observation and every edge is labeled with input and a frequency. The frequency denotes the multiplicity of the trace prefix in \mathcal{T} that corresponds to the path from the root node to the edge. After creating a tree, IOALERGIA creates an MDP by merging nodes that are compatible and promoting nodes to MDP states that are not compatible with other states. Initially, the root node is promoted to be the initial MDP state and labeled red. Then, the algorithm performs a loop comprising the following steps. All immediate successors of red states are labeled blue. A blue node b is selected and checked for compatibility with all red states. If there is a compatible red state r, b and r are merged and the subtree originating in b is folded into the currently created MDP. Otherwise, b is labeled red, thus being promoted to an MDP state. The loop terminates when there are only red states.

Nodes b and r are compatible if their observation labels are the same and the probability distributions of future observations conditioned on actions are not statistically different. The latter check is also performed recursively on all successors of b and r. The statistical difference is based on Hoeffding bounds [12], where a parameter ϵ_{AL} controls significance. A data-dependent ϵ_{AL} guarantees convergence in the limit to an MDP isomorphic to the canonical MDP underlying the distribution of traces. For finite sample sizes, we can use ϵ_{AL} to influence the MDP size. For more information, we refer to Mao et al [15].

3 Q^A-learning: RL Assisted by Automata Learning

In this section, we present Q^A-learning, an approach to reinforcement learning under partial observability. First, we describe the setting and the general intuition behind the approach. Then, we present the state space perceived by the learning agent, followed by a presentation of the complete approach.

3.1 Overview

Setting. We consider reinforcement learning in partially observable environments. That is, we assume that the environment behaves like a POMDP, where we cannot observe the state directly. Moreover, we do not assume to have a POMDP model of the environment. Initially we only know the available actions and as we learn, we learn more about the available observations and the environment dynamics and refine our policy.

Interface. We formalize the setting via an interface comprising two operations through which the RL agent interacts with the environment: (1) **reset** and (2) **step**. Following the conventions of OpenAI gym [2], the **reset** operation resets the environment into its initial state. The **step** operation takes an action as input, performs the actions, which changes the environment state, and returns the immediate reward, a Boolean flag *done*, and the observation in the new state. The flag *done* indicates whether a goal state was reached.

Execution & Traces. The agent learns in episodes, where it traverses a finite path in each episode. We want to note again that the agent cannot see the state that it visited. Each executed path yields a finite reward-observation trace rt consisting of observations, immediate rewards, and the performed actions. We store these reward-observation traces in a multiset \mathcal{RT} .

3.2 Extended State Space

The Q-table in Q-learning is a function $Q: S \times A \to \mathbb{R}$, where S are the observable states of the environment. Since we only observe observations from a set Z, we cannot use this function definition directly. As individual observations are insufficient to facilitate learning, we extend the observation space with states of a learned MDP leading to an extended state space. Suppose we are in episode i, we combine Z with the states of the last labeled MDP (\mathcal{M}_i, Z, O_i) , with $\mathcal{M}_i = (S_i, s_{0i}, A, \delta_i)$, learned via IoAlergia. During training, we continuously simulate the observation traces perceived by the RL agent on the learned automaton and use the visited states from S_i as additional observations. Since a learned MDP may not define transitions for all action-observation pairs, we represent the current state of \mathcal{M}_i as a pair $(s,d) \in S_i \times \{\top, \bot\}$, where the first element encodes the last visited state of \mathcal{M}_i and the second element denotes whether the simulation encountered an undefined transition. To work with these state pairs, we define two functions, where for $a \in A$ and $o \in O$:

$$resetToInitial() = (s_{0i}, \top)$$

$$stepTo((s, d), a, o) = \begin{cases} (s', \top) & \text{if } d = \top \land \delta_i(s, a)(s') > 0 \land O_i(s') = o \\ (s, \bot) & \text{otherwise} \end{cases}$$

The extended state space uses these state pairs, i.e., the Q-function is defined as $Q: S_i^e \times A \to \mathbb{R}$ with $S_i^e = O \times S_i \times \{\top, \bot\}$. Due to \mathcal{M}_i being deterministic, s' in the above definition is either uniquely defined or not defined at all, denoted by \bot . Once we reach undefined behavior, we remember the last visited state

Algorithm 1 Algorithm implementing Q^A-learning

Input: reinforcement learning environment env, fully configured partially observable agent agent, update interval updateInterval, number of training episodes maxEp
 Output: trained agent implementing the policy for the env

```
1: agent.S_E \leftarrow env.O \times agent.model.states \times \{\top, \bot\}
                                                                       ▷ Init. extended state space
 2: agent.A \leftarrow env.A
                                                               \triangleright Get action state space from env
 3: agent.Q(s, a) = 0, \forall s \in S_E, a \in A
                                                                             ▶ Initialize the Q-table
 4: \mathcal{RT} \leftarrow \{\}
                                                                                  ▶ Multiset of traces
 5: for trainingEpisode \leftarrow 0 to maxExp do
         initialObs, initialRew \leftarrow env.reset()
 6:
 7:
         rt \leftarrow \langle initialObs, initialRew \rangle
                                                                          ▶ Trace of a single episode
         agentState \leftarrow agent.model.resetToInitial()
 8:
         epDone \leftarrow False
 9:
         while not epDone do
10:
                         \triangleright Select an action using \epsilon-greedy policy and extended state space
11:
             act \leftarrow agent.getAction(agent.state)
12:
                                ▷ Record all observed (state, action, reward, newState) pairs
13:
             obs, reward, done, newObs \leftarrow env.step(act)
14.
             agentState \leftarrow agent.model.stepTo(agentState, act, newObs)
15:
             updateQValues(agent, obs, act, reward, newObs)
16:
17:
             rt \leftarrow rt \cdot \langle act, reward, newObs \rangle
         \mathcal{RT} \leftarrow \mathcal{RT} \uplus \mathit{rt}
18:
        if trainingEpisode \geq agent.freezeAutomaton then
                                                                                 ▶ Freeze automaton
19:
             continue
20:
        if trainingEpisode \ \mathbf{mod}\ updateInterval = 0 \ \mathbf{then}
21:
             agent.model \leftarrow runIOAlergia(\mathcal{RT})
                                                           ▶ Learn the new environment model
22:
             agent.S_E \leftarrow agent.S_{Init} \times agent.model.states \times \{\top, \bot\}
23:
             agent. Q(s, a) = 0, \forall s \in S_E, a \in A
                                                             ▶ Reinitialize the extended Q-table
24:
             for episode \in \mathcal{RT} do
25:
                 agentState \leftarrow agent.model.resetToInitial()
26:
                 for obs, action, reward, newObs \in episode do
27:
                      agentState \leftarrow agent.model.stepTo(agentState, act, newObs)
28:
                      updateQValues(agent, obs, action, reward, newObs)
29:
30: return agent
```

and leave it unchanged. The intuition is that when behavior is encountered after reaching some (s, \perp) is important for RL performance, due to achieving high reward, this will be reflected in updates of the Q-function. As a result, learning will be directed towards s. This leads to more sampling in the vicinity of s s.t. subsequently learned MDPs are more accurate in this region. Consequently, previously undefined behavior will eventually become defined in the learned MDP.

3.3 Partially Observable Q-Learning

We apply tabular, ϵ -greedy Q-learning [23] combined with MDP learning. Deterministic labeled MDPs learned by IoAlergia provide the Q-learning agent

with additional information in order to make the learning problem Markovian despite partial observability.

We regularly learn new MDPs via IoALERGIA from the growing sample of reward-observation traces, where we discard the rewards, so that at each episode i there is an approximate MDP \mathcal{M}_i with states S_i . To take information from \mathcal{M}_i into account during RL, we extend the Q-table with observations corresponding to the states S_i . At every step performing action a and observing o during RL, we simulate the step in \mathcal{M}_i . This yields a unique state in S_i due to \mathcal{M}_i being deterministic, which we feed to the RL agent as an additional observation.

We actually perform two stages of learning. First, we perform Q-learning while regularly updating \mathcal{M}_i . In the second stage, we fix the final MDP \mathcal{M}_i , referred to *freezing* below, and perform Q-learning without learning new MDPs with IoAlergia. We term the resulting learning approach Q^A -learning.

Algorithm 1 implements this learning approach, i.e., training of a Q^A-learning agent. For a more detailed view of the training algorithm and agent parameterization, we point an interested reader to the implementation ⁴.

Algorithm 1 assumes that the Q^A -learning agent interacts with the environment *env* as described in Section. 3.1. The parameter maxEp defines the maximum number of training episodes. The other parameter updateInterval defines how often the agent recomputes the model, thus extending the state space perceived by the learning agent and the Q-table.

Lines 1-2 initialize the extended state space and set the actions of the agent to those of the environment. For the state-space initialization, we assume an initial approximate MDP to be given. In our implementation, we learn such an MDP from a small number of randomly generated traces. Alternatively, the extended state space S_E can be initialized to the observation space of the environment. It will in any case be extended as the algorithm progresses. Line 3 initializes the Q-table with the initial observation space and action space.

Training progresses until the maximum number of episodes is reached. In the implementation, we have added an early stopping criterion to end the training as soon as the agent achieves satisfactory performance on a predefined number of test episodes. Lines 5-17 show the steps taken in a single training episode. At the beginning of each episode, the environment and the agent's internal state are reset to their initial states (Lines 6 and 8). Until an episode terminates, either by reaching a goal or exceeding the maximum number of allowed steps, the Q^A -learning agent selects an action using an ϵ -greedy policy and executes it in the environment (Lines 12-14). Based on the selected action act and received observation newObs, the agent updates its current model state by tracing the pair (act, newObs) in the learned MDP (Line 15). After performing a step, the agent updates the values in the Q-tables based on observations and received reward. Alg. 2 describes the process of updating Q-values. It follows the same procedure as in standard Q-learning with the notable difference of using a state space extended with learned MDP states instead of the observation space of the environment. The extended state space is discussed in more detail in Sect. 3.2

⁴ https://anonymous.4open.science/r/Q-learning-under-Partial-Observability-4BEC

In Line 19, we check whether the automaton should be *frozen*. Freezing of the automaton prevents further updates of the model and extensions of the state space. This way once the automaton is frozen, the Q-table will continue to be optimized with respect to the current extended state space. Automaton freezing operates under the assumption that once a model is computed that is "good enough", computing a new model in the next update interval is unnecessary and might even be detrimental to the performance of the agent (in the short term).

If the automaton freezing is not enabled or its episode threshold has not yet been reached, we proceed with the update of the model and the Q-table (Lines 22-24). This update happens every updateInterval episodes. IoAlergia computes a new model that conforms to the sample \mathcal{RT} with rewards discarded. In Line 23 we extend the state space with state identifiers of the learned model. After that, we recompute the Q-table by initializing it with the extended state space and action space (Line 24). To recompute the values in the extended Q-table we perform an experience replay [8] with all traces in \mathcal{RT} (Lines 25-29).

Demonstrating Example. We use a simple *Office World* example to demonstrate state extension. In this example, an agent selects one of four actions: {up, down, left, right} to move into the given direction. The agent may also slip into a different direction with a location-specific probability.

Whereas [20,13] use Office World in a non-Markovian reward setting under full observability, we modify the OfficeWorld layout as shown in Fig. 4 to introduce partial observability. On the right-hand side of Fig. 4, abstraction is applied over the state space. The reinforcement learning agent can only observe in which room he is located, but not the x and y coordinates, which would truly identify a Markov state. Note that each observation, e.g. Room1, is shared by nine different POMDP states identified by their x, y coordinates, each with different future and stochastic behavior.

Table 1. Non-extended Q-table.

State\Action	Up	Down	Left	Right
Room1		-0.35		
Room2	-0.67	-0.67	-0.32	-0.66
Room3		4.65	-	
Room4	24.72	24.45	23.26	24.79

Table 2. Extended O-table

Table 2. Extended Q-table.							
State\Action	Up	Down	Left	Right			
(Room1, s0)	-0.35	-0.35	0.75	-0.39			
(Room1, s1)	-0.33	-0.35	0.90	-0.36			
(Room1, s2)	-0.31	-0.31	1.06	-0.30			
(Room1, s3)	0.4	-0.41	0.06	-0.35			
(Room2, s0)	0	0	0	0			
(Room2, s5)	-0.67	1.23	1.07	-0.66			
(Room2, s6)	-0.67	1.42	0.4	-0.66			
(Room4, s10)	97.3	74.1	78.2	93.8			
(Room4, s11)	97.9	74.7	78.8	92.1			
(Room5, s12)	98.3	75.2	79.2	95.3			

Table 1 shows a Q-table obtained from observations only. The Q-values indicate that the Q-learning agent is unable to find optimal actions due to partial observability. Table 2 shows an extended Q-table, but we do not include the definedness flag from $\{\top, \bot\}$ for brevity. Each observation is extended with a learned MDP state as discussed in Sect. 3.2. We observe that each (observation,

Algorithm 2 Algorithm implementing update of Q-values of the agent.

Input: Q^A-learning agent, environment state, reward, reached environment newState

- $1: \ extendedNewState \leftarrow agent.model.stepTo(action, newState)$
- 2: $oldValue \leftarrow agent.Q(extendedState, action)$
- $3: maxNextStateValue \leftarrow max(agent.Q(extendedNewState))$
- 4: $agent.Q(extendedState, action) \leftarrow (1 \alpha) * oldValue + \alpha * (reward + \gamma * maxNextStateValue)$

state) pair approximates the underlying BMDP to such an extend that every such pair has a clear optimal action defined by the Q-values. For example, in states (Room1, s0) and (Room1, s1) the agent needs to perform a left action, whereas in state (Room1, s3) the agent needs to move up. We can also observe that the Q-values of the state (Room2, s0) are set to zero. This results from s0 being unreachable while observing Room2.

3.4 Correctness

Q^A-learning learns an optimal policy in the limit, when the number of episodes tends to infinity, if the BMDP of the POMDP environment is finite.

This property follows from the convergence of IoAlergia and Q-learning. We sample traces from a POMDP that, by assumption, is equivalent to a finite BMDP. The BMDP itself is a deterministic labeled MDP. IoAlergia in the limit learns an MDP isomorphic to the canonical deterministic labeled MDP producing the traces when every action always has a non-zero probability to be executed, as has been shown by Mao et al. [15]. That is, every pair of belief-state and action would be explored infinitely often in the limit.

This also ensures convergence of Q-learning in a Markovian environment [23]. The environment is Markovian once we learned the BMDP and add its current state to the observations of the Q-learning agent. Hence, we will learn an optimal policy for the BMDP and thus the POMDP in the limit.

When the BMDP is not finite and for finite sample sizes, we learn an approximation of the BMDP. For instance, in the example shown in Fig. 3, we might learn the three belief states labeled with *beep*, but beyond that the probability of observing *tea* is likely too small to detect additional states. As we will demonstrate in the evaluation, such approximate MDPs encode sufficient information to aid reinforcement learning. The learned automaton and its states can be thought of as providing memory to the reinforcement learning agent.

4 Evaluation

To evaluate the proposed method we have implemented Q^A-learning in Python. The implementation uses AALPY's [19] IOALERGIA implementation and it interfaces OpenAI gym [2]. The implementation can be used on all gym environments with discrete action and observation space. We have evaluated Q^A-

learning by comparing its performance on four partially observable environments with multiple state-of-the-art RL algorithms implemented in OpenAI's stable-baselines [11], considering: non-recurrent policies with a stacked history of observations and LSTM-based policies.

Stacked history of observations. In a first set of experiments, we have compared Q^A -learning with DQN [18], A2C [17], and ACKTR [24]. To aid those algorithms to cope with partial observability, we encoded the history of observations as a stacked frame. Stacked observation frames encode an observation history by using the last n observations observed during training and evaluation. By using stacked observations, we extend the observation space from initially i observations to $(i+1)^n$ observations 5 . For all experiments we have set the size of stacked frames to 5. A similar approach was, e.g., used in [18] as a method to encode movement in ATARI games.

LSTM-based policies. Deep-recurrent Q-learning [10] has been used to solve ATARI games without stacking the history of observations. Hausknecht and Stone [10] show that recurrence is a viable alternative to frame stacking, and while no significant advantages were noticed during training of the agent, LSTM-based policies were more adaptable in the evaluation phase in the presence of previously unseen observations.

Setup. All experiments were conducted on a laptop with an Intel[®] Core[™]i7-11800H at 2.3 GHz, 32 GB RAM, and an NVIDIA RTX[™]3050 Ti graphics card using Python3.6. For all experiments we have set the maximum number of training episodes to 30,000. A training episode ends if an agent reaches a goal or the maximum number of steps is exceeded. The training performance was periodically evaluated and training was halted when reaching satisfactory performance.

Table 3 summarizes the results of the experiments. There are columns for each partially observable environment, where the first shows the average number of actions required to reach a dedicated goal with the best policy found by RL, and the second column shows the number of training episodes needed to learn a policy. The symbol \times denotes that no policy was found that reaches the goal within the allotted maximum number of steps. The rows correspond to: the optimal policy, the policy found by Q^A -learning, the three RL approaches with stacked observations, and the three approaches with LSTM-based policies. All experiments were repeated multiple times and we chose the best training run as a representative for each approach. As the agent performance was evaluated on 100 episodes, the average number of steps to reach a goal was rounded to the closest integer. In the remainder of this section, we will explain the partially observable environments on which the agents were trained and discuss the obtained results.

The Office World domain is depicted on the right of Fig. 4. Q^A -learning was able to find an optimal policy in this environment, but with a higher total number of training episodes compared to LSTM-based approaches. Stacked-frame based approaches also performed well, but were not able to find an optimal policy. This environment was solvable by all approaches despite its partial observability as each room has two actions which when executed repeatedly will lead the agent

 $^{^{5}}$ +1 due to the padding observation present in the first n steps of each training episode

into the next room (e.g., in Room2 the agent needs to repeatedly perform down and right actions).

ConfusingOfficeWorld found on the left-hand side of Fig. 6 is a variation of the OfficeWorld. ConfusingOfficeWorld is harder to solve as the agent receives the same observations in the upper right and the lower left rooms, likewise in the upper left and the lower right rooms. The rooms labeled with Room1 and have two sets of opposite actions that need to be taken, depending on the actual agent location. The same holds for Room2. Q^A-learning was able to find a solution for this world in 16 thousand episodes, while other approaches failed due to insufficient state differentiation.

GravityDomain was inspired by the environment discussed in [13]. In GravityDomain, gravity will pull the agent down in each state with 50% probability. By reaching a toggle indicated by a blue switch in Fig. 5, gravity is turned off and the environment becomes deterministic. We observed that both stacked-frame and LSTM-based approaches learned a policy in which they repeatedly performed the up action, thus reaching the goal in only 50% of the test episodes within the maximum number of 100 steps. Q^A -learning was able to learn an optimal strategy in which it first reached the blue toggle and then proceeded to the goal, depicted by a cookie. Note that the approach presented in [13] is generally not able to solve the GravityDomain.

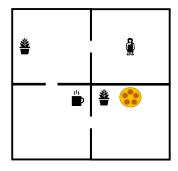
ThinMaze is depicted on the left-hand side of Fig. 6. In ThinMaze, the only observations are "cookie" and "wall", which signals that the agent performs an action that is blocked by a wall. Due to the lack of observations both stacked-frame and LSTM-based approaches failed to find a solution to ThinMaze. Q^A-learning was able to find a non-optimal solution in 27 thousand training episodes. This is due to the fact that IOALERGIA requires a high number of traces to approximate the underlying belief-MDP with sufficient accuracy.

Runtime. We only briefly comment on runtime, considering OfficeWorld for a fair comparison, as all approaches found a decent policy. Other results might be skewed due to different training lengths. Stacked-frame DQN, A2C, and ACKTR, require 312s, 373s, and 48s, respectively. Stopping after only 1k episodes, the LSTM-backed ACER, A2C, and ACKTR take 51s, 74s, and 80s, respectively. Our approach is considerably faster, finishing after about 3s. The reason is that we apply tabular Q-learning and IoAlergia adds very little runtime overhead. The automata-learning technique has cubic worst-case runtime in the sample size, but has been reported to have linear runtime in practice [3].

5 Related Work

In recent years, different forms of automata learning have been applied in combination with RL. Automata learning can aid RL by providing a stateful memory. This memory can be exploited either to further differentiate environment states or to capture steps required for non-Markovian rewards.

Early work closely related to our approach is [6,16]. Both techniques combine model learning and Q-learning for RL under partial observability, but they



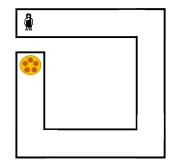


Fig. 6. Confusing Office World (left) and partially observable ThinMaze (right).

		Offic	eWorld	Confusing OfficeWorld		GravityDomain		ThinMaze	
Algorithm			# Training						
		Goal	Episodes	Goal	Episodes	Goal	Episodes	Goal	Episodes
Optimal So	olution	12	-	12	-	18	-	20	-
Q ^A -lear	ning	12	3k	18	16k	18	2k	32	27k
Stacked observation	DQN	17	2k	Х	30k	75	30k	Х	30k
	A2C	24	12k	Х	30k	75	30k	Х	30k
	ACKTR	14	2k	Х	30k	75	30k	Х	30k
LSTM-based policy	ACER	12	1k	X	30k	75	30k	X	30k
	A2C	12	1k	Х	30k	75	30k	Х	30k
	ACKTR.	12	1k	Х	30k	75	30k	Х	30k

Table 3. Representative evaluation results.

place stricter assumptions on the environment, like knowledge about the number of environmental states. More recently, Toro Icarte et al. [14] described optimization-based learning of finite-state models, called reward machines, to aid RL. However, they require a labeling function on observations that meets certain criteria and generally cannot handle changes in the transition probabilities, when observations stay the same. DeepSynth [9] follows a similar approach, but focuses on sparse rewards rather than partial observability. They learn automata via satisfiability checking to provide structure to complex tasks, where they also impose requirements on a labeling function.

Learning of reward machines has also been proposed to enable RL with non-Markovian rewards [25,26,20], where the gained rewards depend on the history of experiences rather than the current state and actions. In this context, different approaches to automata learning are applied to learn Mealy machines that keep track of previous experiences in an episode. Velasquez et al. [22] extend reward-machine learning to a setting with stochastic non-Markovian rewards. Our approach could be extended to non-Markovian rewards by adding rewards to the observations. Subgoal automata inferred by Furelos-Blanco et al. [7] through answer set programming serve a similar purpose as reward machines, by capturing interaction sequences that need to occur for the successful completion of a task. Brafman et al. [8] learn deterministic finite automata that also encode which in-

teractions lead to a reward in RL with non-Markovian rewards. Similarly to our approach, the states of learned automata are used as additional observations.

6 Conclusion

We propose an approach for reinforcement learning under partial observability. For this purpose, we combine Q-learning with the automata learning technique IoAlergia. With automata learning, we learn hidden information about the environment's state structure that provides additional observations to Q-learning, thus enabling this form of learning in POMDPs. We evaluate our approach in partially observable environments and show that it can outperform the baseline deep RL approach with LSTMs and fixed memory.

For future work, we plan to generalize our approach to other (deep) RL approaches by integrating explored learned MDP states as observations. Approaches already including experience replay naturally lend themselves to such extensions, since we merely need to change the replay

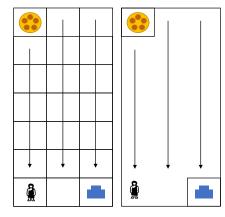


Fig. 5. Fully and partially observable gravity domain. Once the button in the lower right corner is reached, gravity is turned off. Please note that for conciseness, we show a width of three, while we used six states in our evaluation.

mechanism and execute it after updating the learned MDP. To scale the proposed approach to larger environments, we intend to explore and develop automatalearning techniques that can model high-dimensional data.

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References

- Bork, A., Junges, S., Katoen, J., Quatmann, T.: Verification of indefinite-horizon POMDPs. In: ATVA (2020)
- Brockman, G., Cheung, V., Pettersson, L., Schneider, J., Schulman, J., Tang, J., Zaremba, W.: OpenAI gym. CoRR abs/1606.01540 (2016)
- 3. Carrasco, R.C., Oncina, J.: Learning stochastic regular grammars by means of a state merging method. In: ICGI. LNCS, vol. 862, pp. 139–152. Springer (1994)
- 4. Cassandra, A.R., Kaelbling, L.P., Littman, M.L.: Acting optimally in partially observable stochastic domains. In: AAAI (1994)
- 5. Chatterjee, K., Chmelik, M., Gupta, R., Kanodia, A.: Qualitative analysis of POMDPs with temporal logic specifications for robotics applications. In: ICRA (2015)

- Chrisman, L.: Reinforcement learning with perceptual aliasing: The perceptual distinctions approach. In: AAAI. pp. 183–188 (1992)
- Furelos-Blanco, D., Law, M., Russo, A., Broda, K., Jonsson, A.: Induction of subgoal automata for reinforcement learning. In: AAAI (2020)
- 8. Gaon, M., Brafman, R.I.: Reinforcement learning with non-Markovian rewards. In: AAAI (2020)
- 9. Hasanbeig, M., Jeppu, N.Y., Abate, A., Melham, T., Kroening, D.: DeepSynth: Automata synthesis for automatic task segmentation in deep reinforcement learning. In: AAAI (2021)
- Hausknecht, M.J., Stone, P.: Deep recurrent Q-learning for partially observable MDPs. In: AAAI (2015)
- 11. Hill, A., Raffin, A., Ernestus, M., Gleave, A., Kanervisto, A., Traore, R., Dhariwal, P., Hesse, C., Klimov, O., Nichol, A., Plappert, M., Radford, A., Schulman, J., Sidor, S., Wu, Y.: Stable baselines. https://github.com/hill-a/stable-baselines (2018)
- 12. Hoeffding, W.: Probability inequalities for sums of bounded random variables. Journal of the American Statistical Association **58**(301), 13–30 (1963)
- Icarte, R.T., Klassen, T.Q., Valenzano, R.A., McIlraith, S.A.: Using reward machines for high-level task specification and decomposition in reinforcement learning. In: ICML (2018)
- Icarte, R.T., Waldie, E., Klassen, T.Q., Valenzano, R.A., Castro, M.P., McIlraith, S.A.: Learning reward machines for partially observable reinforcement learning. In: NeurIPS (2019)
- 15. Mao, H., Chen, Y., Jaeger, M., Nielsen, T.D., Larsen, K.G., Nielsen, B.: Learning deterministic probabilistic automata from a model checking perspective. Machine Learning 105(2) (2016)
- McCallum, A.: Overcoming incomplete perception with utile distinction memory. In: ICML. pp. 190–196 (1993)
- Mnih, V., Badia, A.P., Mirza, M., Graves, A., Lillicrap, T.P., Harley, T., Silver, D., Kavukcuoglu, K.: Asynchronous methods for deep reinforcement learning. In: ICML (2016)
- Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Wierstra, D., Riedmiller, M.A.: Playing Atari with deep reinforcement learning. CoRR abs/1312.5602 (2013)
- 19. Muškardin, E., Aichernig, B.K., Pill, I., Pferscher, A., Tappler, M.: AALpy: An active automata learning library. In: ATVA (2021)
- 20. Neider, D., Gaglione, J., Gavran, I., Topcu, U., Wu, B., Xu, Z.: Advice-guided reinforcement learning in a non-Markovian environment. In: AAAI (2021)
- 21. Singh, S.P., Jaakkola, T.S., Jordan, M.I.: Learning without state-estimation in partially observable Markovian decision processes. In: ICML. pp. 284–292. Morgan Kaufmann (1994)
- Velasquez, A., Beckus, A., Dohmen, T., Trivedi, A., Topper, N., Atia, G.K.: Learning probabilistic reward machines from non-Markovian stochastic reward processes. CoRR (2021)
- 23. Watkins, C.J.C.H., Dayan, P.: Q-learning. Machine Learning 8(3) (1992)
- 24. Wu, Y., Mansimov, E., Grosse, R.B., Liao, S., Ba, J.: Scalable trust-region method for deep reinforcement learning using Kronecker-factored approximation. In: NIPS (2017)
- Xu, Z., Gavran, I., Ahmad, Y., Majumdar, R., Neider, D., Topcu, U., Wu, B.: Joint inference of reward machines and policies for reinforcement learning. In: ICAPS (2020)

26. Xu, Z., Wu, B., Ojha, A., Neider, D., Topcu, U.: Active finite reward automaton inference and reinforcement learning using queries and counterexamples. In: CD-MAKE (2021)