A Navigation Logic for Recursive Programs with Dynamic Thread Creation

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Abstract. Dynamic Pushdown Networks (DPNs) are a model for multithreaded programs with recursion and dynamic creation of threads. In this paper, we propose a temporal logic called NTL for reasoning about the call- and return- as well as thread creation behaviour of DPNs. Using tree automata techniques, we investigate the model checking problem for the novel logic and show that its complexity is not higher than that of LTL model checking against pushdown systems despite a more expressive logic and a more powerful system model. The same holds true for the satisfiability problem when compared to the satisfiability problem for a related logic for reasoning about the call- and return-behaviour of pushdown systems. Overall, this novel logic offers a promising approach for the verification of recursive programs with dynamic thread creation.

Keywords: Concurrency, Dynamic Pushdown Networks, Navigation Logic, Model Checking, Satisfiability, Tree Automata

1 Introduction

Model Checking is an established technique for the verification of hardware and software systems. Conceptually, it consists of checking whether a property given in a specification logic holds for a model of a system. While logics such as LTL or CTL and finite Kripke models were considered early on [12,22], later more expressive logics as well as infinite state systems have been studied. The use of pushdown systems, for instance, allows for a more precise analysis of recursive software systems due to the presence of a call stack of a program while still retaining a decidable model checking problem against LTL specifications [8]. In the context of pushdown systems, an example of a logic more expressive than LTL is the logic CaRet [4] which extends LTL by operators for non-regular properties of the call and return behaviour of pushdown systems. This extension does not lead to increased complexity for the model checking problem against pushdown systems compared to LTL.

However, there are even more powerful system models than pushdown systems for which model checking of variants of LTL is decidable. In this paper, we consider Dynamic Pushdown Networks (DPNs) [9], a model for software systems that cannot model only recursion, but also multithreading with dynamic thread creation. So far, the model checking problem for DPNs has only been considered for single indexed LTL, a variant of LTL for multithreaded systems in which an LTL formula is assigned to each thread [26]. Here, we consider the model checking problem for a more expressive logic. More specifically, we propose a fixpoint calculus with CaRet-like operators for the verification of DPNs via model checking. The logic allows to specify non-regular properties concerning the call and return behaviour of the different execution threads of a DPN. Unlike CaRet, it can additionally specify properties concerning the thread-spawn behaviour of programs. For example, consider a scenario where a program has a method for bookkeeping information about spawned threads and it is required that new threads be only spawned from this method in order to keep the bookkeeping consistent. The property $\mathcal{G}^r(\bigcirc^c \psi \to \mathcal{F}^- pr)$ specified in our new logic expresses that in all positions of all threads (expressed through the modality \mathcal{G}^r), new threads fulfilling the property ψ are only spawned (expressed through $\bigcirc^c \psi$) when the procedure pr is in the call stack (expressed through $\mathcal{F}^- pr$). This formalises the requirement. Properties regarding such relationships between parent and child threads cannot be expressed in the variant of LTL from [26] or other specification logics for DPNs we are aware of. Our logic thus constitutes the first specification logic able to reason about the thread spawning behaviour of DPNs.

Contributions and structure of the paper. After introducing some notation and results (Section 2), we present a semantics for DPNs based on graphs (Section 3). As our first main contribution, we then introduce a novel specification logic called Navigation Temporal Logic (NTL) with the ability to reason about the call/return and thread creation behaviour of DPNs (Section 4). We discuss some example properties and applications of our logic in Section 5. Towards algorithmic verification, we then switch from a semantics based on graphs to a semantic based on trees (Section 6). As our second main contribution, we then investigate the model checking and satisfiability problems for the new logic (Section 7). In particular, we show that the model checking problem is decidable in time exponential in the size of the specification and polynomial in the size of the system model, i.e. the same as for LTL model checking against pushdown systems, and that the satisfiability problem is solvable in time exponential in the size of the specification, i.e. the same as for the satisfiability problem for $VP-\mu-TL$ [10], a temporal logic subsuming CaRet and subsumed by our logic. For both problems, we establish matching lower bounds. Section 8 concludes the paper. Due to lack of space, some technical proofs can be found in an appendix.

Related work. There are several specification logics related to the logic we present in this paper. The temporal logic LTL was considered for model checking finite state systems [22] as well as pushdown systems [8]. For pushdown models, CaRet was developed with different successor types that allow the inspection of the call and return behaviour of the system [4]. Also, variants of CaRet have been studied in the literature [3,2,11,17]. As mentioned, CaRet is one inspiration for the logic presented in this paper and we adopt and complement its successor types in our logic. Other inspirations are the linear time μ -calculus from [28] and the logic $VP-\mu-TL$ from [10]. In these logics, fixpoint operators can be used to express arbitrary ω -regular (resp. ω -visibly pushdown) properties on paths, which

makes them more expressive than LTL and CaRet, respectively. From these logics, we take fixpoint operators. There is also a plethora of work on dynamic pushdown networks. The model was first introduced in [9]. Different methods for reachability analysis of DPNs have been proposed [9,20]. Additionally, different variants of the model were investigated. [24] and [19] consider variants of DPNs that communicate via locks. Another variant is the model of Dynamic Networks of concurrent pushdown systems from [7] in which threads can communicate via global variables. However, none of the above works on DPNs is concerned with model checking. The only approach to model checking DPNs we are aware of consists of checking different variants of DPNs against a variant of LTL called single indexed LTL [26,13]. Compared to NTL, this variant cannot specify properties concerning the call and return behaviour of a thread or the relationship between different threads. In Section 5, we show that single indexed LTL can be embedded into NTL.

2 Preliminaries

Without further ado, we introduce tools and notation used throughout the paper. This section can be skipped on first reading and be consulted for reference later.

Trees. An \mathbb{N}_0 -tree T is a prefix-closed subset of \mathbb{N}_0^* , i.e. for all nodes $t \in \mathbb{N}_0^*$ and directions $d \in \mathbb{N}_0$, $t \cdot d \in T$ implies $t \in T$. Moreover, we require that $t \cdot d \in T$ for some $d \in \mathbb{N}_0$ implies $t \cdot d' \in T$ for all $d' \leq d$. We call an element $t \in T$ a node of T with special node ε , which we call the root. A node of the form $t \cdot d$ is called a *child* of t and t is called the *parent* of $t \cdot d$. Additionally, for sequences $w \in \mathbb{N}_0^*$, we call $t \cdot w$ a descendant of t and t an ancestor of $t \cdot w$. Nodes $t \in T$ that have no children are called *leaves*. A path in a tree T is a finite or infinite sequence $t_0t_1 \dots$ of nodes such that $t_0 = \varepsilon$ and for all $i \in \mathbb{N}_0$, t_{i+1} is a child of t_i . An \mathbb{N}_0 -tree that is a subset of $\{0, 1\}^*$ is also called a binary tree. In this case, we call a node of the form $t \cdot 0$ the *left child* of t and a node of the form $t \cdot 1$ the right child of t. Let Σ be a finite set of labels and $ar: \Sigma \to \{0, 1, 2\}$ be a function assigning an arity to each of these labels. A (Σ, ar) -labelled binary tree is a pair (T, l) such that T is a binary tree and $l: T \to \Sigma$ is a labelling function such that each node $t \in T$ has exactly ar(l(t)) children.

2-way alternating tree automata. For a finite set X, let $\mathcal{B}^+(X)$ be the set of positive boolean combinations over X, i.e. boolean formulae built with elements of X, conjunction and disjunction. For $Y \subseteq X$ and $\vartheta \in \mathcal{B}^+(X)$, we say that Y satisfies ϑ iff assigning the value true to the elements of Y and false to the elements of $X \setminus Y$ makes the formula ϑ true. Let $\text{Dir} = \{0, 1, \varepsilon, \uparrow\}$ be the set of moves in the tree with directions 0 for the left child, 1 for the right child, ε for standing still and \uparrow for moving upwards. We define $u \cdot \varepsilon = u$ and $u \cdot d \cdot \uparrow = u$ for all $u \in \{0, 1\}^*$ and $d \in \{0, 1\}$. A 2-way alternating tree automaton (2ATA) [29] over (Σ, ar) -labelled binary trees is a tuple $\mathcal{A} = (Q, q_0, \rho, \Omega)$ where Q is a finite set of states, $q_0 \in Q$ is an initial state, $\rho: Q \times \Sigma \to \mathcal{B}^+(\text{Dir} \times Q)$ is a transition function and $\Omega: Q \to \{0, \ldots, k\}$ is a priority mapping. The size $|\mathcal{A}|$ of a 2-way alternating tree automaton is defined as the sum of the sizes of its

constituents. We sometimes also refer to the size of individual constituents of an automaton. In particular, we refer to the number of states, i.e. |Q|, and the size of the acceptance condition, i.e. k. If the transition function of a 2-way alternating tree automaton uses only symbols from $\{0, 1\}$ instead of Dir and additionally maps all nodes t either to true or to disjunctions over conjunctions that consist of exactly one pair (d, q) for each d < ar(l(t)), it is called a *nondeterministic parity tree automaton* (NPTA).

For a (Σ, ar) -labelled binary tree $\mathcal{T} = (T, l)$, a node $t \in T$ and a state $q \in Q$, a (t,q)-run of \mathcal{A} over \mathcal{T} is a pair (T_r,r) such that T_r is an \mathbb{N}_0 -tree and $r: T_r \to T \times Q$ assigns a pair of a node of T and a state of A to all nodes in T_r . Additionally, (T_r, r) has to satisfy the following conditions: (i) $r(\varepsilon) = (t, q)$ and (ii) for all nodes $y \in T_r$ with r(y) = (x, s) and $\rho(s, l(x)) = \vartheta$, there is a set $Y \subseteq \text{Dir} \times Q$ satisfying ϑ and for all $(d, s') \in Y$, there is $n \in \mathbb{N}_0$ such that $y \cdot n \in T_r$ and $r(y \cdot n) = (x \cdot d, s')$. In particular, for all leaves $y \in T_r$ with r(y) = (x, s), we thus require $\rho(s, l(x)) = true$. A (t, q)-run (T_r, r) is accepting iff on each infinite path in T_r the lowest priority occurring infinitely often is even. If \mathcal{A} is a nondeterministic parity tree automaton, a minimal set Y satisfying $\rho(s, l(x))$ in the above definition moves to each child of the current node x in the tree T. For an (ε, q) -run, we can thus simply identify T with T_r and consider a map $r_A: T \to Q$ as an (ε, q) -run over \mathcal{A} . The set of nodes $t \in T$ such that there is an accepting (t, q)-run of \mathcal{A} over \mathcal{T} is denoted by $\mathcal{L}_q^{\mathcal{T}}(\mathcal{A})$. We say that \mathcal{A} accepts a tree \mathcal{T} iff there is an accepting (ε, q_0) -run of \mathcal{A} over \mathcal{T} . The set of trees accepted by \mathcal{A} is denoted by $\mathcal{L}(\mathcal{A})$. We use the following theorems:

Proposition 1 ([29]). For every 2ATA \mathcal{A} , there is an equivalent NPTA \mathcal{A}' . The number of states in \mathcal{A}' is at most exponential in the number of states of \mathcal{A} and the size of the acceptance condition of \mathcal{A}' is linear in the size of the acceptance condition of \mathcal{A} .

Proposition 2 ([14,18,25]). The emptiness problem for NPTA can be solved in time polynomial in the number of states and exponential in the size of the acceptance condition.

Proposition 3. (i) For any two NPTA A_1 and A_2 , there is a NPTA A with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$.

(ii) If either acceptance condition is trivial, the size of \mathcal{A} is in $\mathcal{O}(|\mathcal{A}_1| \cdot |\mathcal{A}_2|)$.

Proposition 3 (i) can be found e.g. in [23]. For (ii), a straightforward product construction can be used and yields an automaton of the size claimed.

Dynamic Pushdown Networks. Let AP be a set of atomic propositions, Γ be a finite set of stack symbols and $\perp \notin \Gamma$ be a special bottom of stack symbol. A Dynamic Pushdown Network (DPN) [9] is a tuple $\mathcal{M} = (S, s_0, \gamma_0, \Delta, L)$ where S is a finite set of control locations, $s_0 \in S$ is an initial control location, $\gamma_0 \in \Gamma$ is an initial stack symbol and $L: S \times \Gamma \to 2^{AP}$ is a labelling function. The transition relation $\Delta = \Delta_I \cup \Delta_C \cup \Delta_R \cup \Delta_S$ is a finite set of internal rules (Δ_I) , calling rules (Δ_C) , returning rules (Δ_R) and spawning rules (Δ_S) . Internal rules $s\gamma \to s'\gamma' \in \Delta_I \subseteq S\Gamma \times S\Gamma$, call rules $s\gamma \to s'\gamma'\gamma'' \in \Delta_C \subseteq S\Gamma \times S\Gamma^2$ or returning rules $s\gamma \to s' \in \Delta_R \subseteq S\Gamma \times S$ enable transitions of a single pushdown process in control location s with top of stack γ to the new control location s' with new top of stack γ' , $\gamma'\gamma''$ and ε , respectively. A spawning rule $s\gamma \to s'\gamma' \triangleright s_n\gamma_n \in \Delta_S \subseteq S\Gamma \times S\Gamma \times S\Gamma$ is an internal rule with the additional side effect of spawning a new process in control location s_n and stack content γ_n . We formally develop a semantics for DPNs in Section 3.

3 Graph Semantics of Dynamic Pushdown Networks

The semantics of DPNs is often defined as an *interleaving semantics*. In such semantics, a configuration of a DPN is a collection of local configurations of the underlying pushdown systems representing the currently active threads. A step in this semantics consists of a step of one of the active threads, possibly adding a configuration of a new thread to the collection. This way, the semantics accurately reflects different interleavings of the steps of the threads issued by arbitrary schedulers, hence the name. For our intents, interleaving semantics has some drawbacks, however. First, an encoding of the intersection problem for contextfree languages is often possible in interleaving semantics, which leads to undecidability of the investigated verification problem. Second, we are mostly interested in temporal properties of individual threads, not necessarily temporal properties of interleaving. This is because the behaviour of a thread is in most cases independent of what types of steps other threads currently make in a specific interleaving. Third, we want to reason about the parent-child relationship of threads which is lost in most formalisations of interleaving semantics.

We thus instead adopt a semantics based on graphs. Intuitively, in an *execu*tion graph, each thread is modelled by a linear sequence of positions connected by *int-*, *call-* and *ret-*edges based on the types of transitions taken in the thread. In order to model the parent-child relationship between threads, a position where a spawn-transition is taken is connected to the first position of the spawned thread via a *spawn-*edge. This is analogous to the notion of *action trees* from [20,15]. Additionally, similar to *nested words* [5], calls and their matching returns are connected via *nesting edges*. We formalise these graphs in the next paragraph.

Execution graphs. Let Moves = {*int*, *call*, *ret*, *spawn*} be the set of moves for dynamic pushdown networks, V be a set of nodes, $l: V \to 2^{AP}$ be a labelling function, $\to^d \subseteq V^2$ be a transition relation for all $d \in Moves$ and $\cap \subseteq V^2$ be a nesting relation. For $x, y \in V$, we call $y \in (d)$ -successor of x and $x \in (d)$ predecessor of y if $x \to^d y$ for some $d \in Moves$. A tuple $G = (V, l, (\to^d)_{d \in Moves}, \cap)$ is called an *execution graph*, iff the following conditions hold:

- 1. Every node has exactly one predecessor with respect to $\bigcup \{ \rightarrow^d | d \in \mathsf{Moves} \}$ except for a special node v_0 without predecessor.
- 2. For all $x \in V$ we have $(v_0, x) \in (\bigcup \{ \rightarrow^d | d \in \mathsf{Moves} \})^*$.
- 3. Every node either has (a) exactly one *int*-successor and at most one *spawn*-successor, (b) exactly one *call*-successor, (c) exactly one *ret*-successor or (d) no successors.

- 6 R. Lakenbrink et al.
- 4. On every finite path starting in v_0 or a node with a *spawn*-predecessor and following only Moves $\{spawn\}$ -successors, the number of *call*-moves on that path is greater than or equal to the number of *ret*-moves on that path.
- 5. For all $x \in V$ having a *call*-successor, let A_x be the set of nodes $y \neq x$ such that there is a path π from x to y following only Moves $\setminus \{spawn\}$ -successors where the number of *call*-moves on π is equal to the number of *ret*-moves on π . Then we have $x \curvearrowright y$ for a node $y \in V$ iff y is a node in A_x such that the witnessing path has minimal length.

The set of execution graphs is denoted by ExGraphs.

An example of an execution graph can be found in Fig. 1. In this example, a main thread spawns two additional threads. Additionally, there are two nested procedure calls in the main thread and one procedure call in a spawned thread.



Fig. 1: Example of an execution graph. Labelled edges represent edges \rightarrow^d for $d \in Moves$ and dashed edges represent nesting edges \sim .

Graph semantics. In most cases, we care only about graphs generated by a given DPN instead of arbitrary execution graphs. This is formalised in the graph semantics of DPN. In the definition of this semantics, we make use of configurations of the processes of a given DPN $\mathcal{M} = (S, s_0, \gamma_0, \Delta, L)$. Formally, a configuration of a pushdown process of \mathcal{M} is a pair c = (s, u) where $s \in S$ is a control location and $u \in \Gamma^* \perp$ is a stack content ending in \perp . We define successor relations on configurations corresponding to the different types of transition rules of DPNs. For this purpose, let c = (s, u), c' = (s', u') and c'' = (s'', u'') be configurations. We call c' an internal successor of c, denoted by $c \rightarrow_{int} c'$, if there is a transition $s\gamma \to s'\gamma' \in \Delta_I$ and $u = \gamma w, u' = \gamma' w$ for some stack content $w \in \Gamma^* \perp$. We call c' a call successor of c, denoted by $c \to_{call} c'$, if there is a transition $s\gamma \to s'\gamma'\gamma'' \in \Delta_C$ and $u = \gamma w, u' = \gamma'\gamma'' w$ for some stack content $w \in \Gamma^* \perp$. We call c' a return successor of c, denoted by $c \to_{ret} c'$, if there is a transition $s\gamma \to s' \in \Delta_R$ and $u = \gamma w, u' = w$ for some stack content $w \in \Gamma^* \perp$. Finally, we call c' a successor of c with spawned process c'', denoted $c \to c' \triangleright c''$, if there is a transition $s\gamma \to s'\gamma' \triangleright s''\gamma'' \in \Delta_S$ and $u = \gamma w$, $u' = \gamma' w$ for some stack content $w \in \Gamma^* \perp$ as well as $u'' = \gamma'' \perp$. Using the notion of configurations and the successor relations just introduced, we now define the graph semantics. We say that an execution graph $(V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \sim)$ is generated by \mathcal{M} if there is

an assignment $as: V \to S \times \Gamma^* \bot$ satisfying (i) $as(v_0) = (s_0, \gamma_0 \bot)$ and (ii) for all $x \in V$, we have $l(x) = L(s, \gamma)$ where $as(x) = (s, \gamma w)$ for some control location $s \in S$, stack symbol $\gamma \in \Gamma$ and stack content $w \in \Gamma^* \bot$ and

- if x has only one d-successor y with $d \in \{int, call, ret\}$, then $as(x) \rightarrow_d as(y)$,
- if x has an *int*-successor y and a *spawn*-successor z, then $as(x) \rightarrow as(y) \triangleright as(z)$ and
- if x has no successor, then as(x) has no successor.

The set of execution graphs generated by \mathcal{M} is denoted by $\llbracket \mathcal{M} \rrbracket$.

Successor functions. On execution graphs $G = (V, l, (\rightarrow^d)_{d \in \text{Moves}}, \sim)$, we define multiple successor functions $succ_g^G$, $succ_a^G$, $succ_a^G$, $succ_p^G$ and $succ_c^G$ with signature $V \rightsquigarrow V$, i.e. partial functions from V to V. The first four of these successor functions come from logics like CaRet [4] and allow us to progress single threads and their call-return behaviour in different ways. The latter two functions are new and give means to reason about the thread spawning behaviour of DPNs. For $x \in V$, the functions are defined as follows:

- The global successor $succ_g^G(x)$ of x is the *int-*, *call-* or *ret-*successor of x, if it exists, and undefined otherwise.
- The global predecessor $succ^{G}_{\uparrow}(x)$ of x is the *int*-, *call* or *ret*-predecessor of x, if it exists, and undefined otherwise.
- The abstract successor $succ_a^G(x)$ of x is the node y with $x \frown y$ or $x \rightarrow^{int} y$, if it exists, and undefined otherwise.
- The caller $succ_{-}^{G}(x)$ of x is the node y with a call-successor y' such that there is a path from y' to x following abstract successors, if it exists, and undefined otherwise.
- The parent $succ_p^G(x)$ of x is the node y with a spawn-successor z such that there is a path from z to x with only Moves $\{spawn\}$ -transitions, if it exists, and undefined otherwise.
- The child $succ_c^G(x)$ of x is the spawn-successor of x, if it exists, and undefined otherwise.

We illustrate these successor functions for parts of the execution graph from Fig. 1 in Fig. 2a and Fig. 2b. Abstract successors (seen in dashdotted red in Fig. 2a) follow the execution of a procedure on the same stack level and skip over executions of additional procedures via nesting edges. If a procedure is left in the next step, i.e. if the next step is a return, the abstract successor is undefined. Callers (seen in dotted blue in Fig. 2a) are defined if the stack level is at least one in a configuration and move to the latest previous call on a lower stack level. Parents (seen in dotted green in Fig. 2b) are defined in every branch of an execution graph representing a thread except for the thread starting in v_0 and move to the position in the graph where the current thread was spawned. Children (seen in dashdotted yellow in Fig. 2b) are defined only if the current thread currently executes a spawn transition and move to the initial position of the spawned thread.



(a) Abstract successors (red, dashdotted) and callers (blue, dotted). Irrelevant edges are gray and some internal edges coinciding with abstract successors are omitted to improve readability.



(b) Parents (green, dotted) and children (yellow, dashdotted). Irrelevant edges are gray to improve readability.

Fig. 2: Successor types in parts of the execution graph from Fig. 1.

4 A Navigation Logic for Dynamic Pushdown Networks

Syntax. We now define the new logic Navigation Temporal Logic (NTL) for expressing properties of execution graphs. As mentioned in the introduction, we have three main inspirations. From the logics CaRet [4] and VP- μ -TL [10], we take different next operators inspecting the call and return behaviour of a thread. We complement these by additional next operators expressing parent and child relationships between different processes. From the linear time μ -calculus [28] and logics like VP- μ -TL [10], we take fixpoint operators for additional expressivity beyond LTL modalities. First, we define the syntax of NTL.

Definition 4 (Syntax of NTL). The syntax of NTL formulae is defined by

$$\varphi ::= ap \mid \neg \varphi \mid \varphi_1 \lor \varphi_2 \mid X \mid \bigcirc^f \varphi \mid \mu X.\varphi$$

where $ap \in AP$ is an atomic proposition, X is a fixpoint variable and $f \in \{g, \uparrow, a, -, p, c\}$ is a successor type.

An NTL formula φ is called *closed*, if every fixpoint variable X is bound in φ , i.e. it only appears in a subformula of the form $\mu X.\psi$. A formula φ is called *well-formed*, if every fixpoint variable X occurring in φ (i) is bound by only one fixpoint formula which we then denote by fp(X), (ii) appears only in the scope of an even number of negations inside fp(X) and (iii) is in scope of at least one next operator inside fp(X). We use $Sub(\varphi)$ for the set of subformulae of a formula φ . The size $|\varphi|$ of a formula φ is defined as the number of its distinct subformulae. We also need a notion of substitution: $\varphi[\varphi'/X]$ is the formula that is obtained from φ by replacing every occurrence of the fixpoint variable X with φ' .

Let us explain the intuition behind each construct. Atomic formulae ap express that $ap \in AP$ holds in the current node of the graph. Next operators $\bigcirc^f \varphi$ can be used to navigate and express that the corresponding successor exists in the current node and additionally satisfies φ . Negation and disjunction are interpreted as usual. Finally, we have fixpoint variables X and least fixpoint operators $\mu X.\varphi$ for more involved properties. Intuitively, $\mu X.\varphi$ is the least fixpoint of a function that unrolls the formula by replacing $\mu X.\varphi$ with $\varphi[\mu X.\varphi/X]$.

We use some common syntactic sugar such as $true \equiv ap \lor \neg ap$, $false \equiv \neg true$, $\varphi_1 \land \varphi_2 \equiv \neg (\neg \varphi_1 \lor \neg \varphi_2), \varphi_1 \to \varphi_2 \equiv \neg \varphi_1 \lor \varphi_2, \varphi_1 \leftrightarrow \varphi_2 \equiv (\varphi_1 \to \varphi_2) \land (\varphi_2 \to \varphi_1)$ and $\nu X.\varphi \equiv \neg \mu X.\neg \varphi[\neg X/X]$. We also introduce a dual operator $\bigcirc^{\overline{f}}\varphi \equiv \neg \bigcirc^{f}$ $\neg \varphi$ of $\bigcirc^f \varphi$ for each successor type $f \in \{g, \uparrow, a, -, p, c\}$ which is needed for a special form in the next paragraph. It is necessary to explicitly define these dual operators since $\neg \bigcirc^f \varphi$ is not equivalent to $\bigcirc^f \neg \varphi$ as the corresponding successors can be undefined for certain nodes in an execution graph. Unlike $\bigcirc^{f}\varphi, \bigcirc^{\overline{f}}\varphi$ is equivalent to *true* for nodes that do not have an *f*-successor. We also introduce some variants of LTL modalities as derived operators. In particular we use $\varphi_1 \mathcal{U}^f \varphi_2 \equiv \mu X.(\varphi_2 \lor (\varphi_1 \land \bigcirc^f X)), \mathcal{F}^f \varphi \equiv true \mathcal{U}^f \varphi \text{ and } \mathcal{G}^f \varphi \equiv$ $\neg \mathcal{F}^f \neg \varphi$ for $f \in \{g, \uparrow, a, -, p, c\}$. For f = g, we sometimes omit the superscript and write $\mathcal{F}\varphi$ etc. Intuitively, these modalities correspond to the usual LTL modalities evaluated on the path starting in the current position and taking fsuccessors. Additionally, we introduce modalities $\mathcal{F}^r \varphi \equiv \mu X.(\varphi \vee \bigcirc^g X \vee \bigcirc^c X)$ and $\mathcal{G}^r \varphi \equiv \neg \mathcal{F}^r \neg \varphi$ to express that φ holds in some position or all positions, respectively, reachable from the current position. Using these abbreviations, dual operators and the equivalence $\neg \neg \varphi \equiv \varphi$ we can transform every well-formed formula into an equivalent formula in which negation only appears in front of atomic propositions. We call this form *positive normal form* and assume formulae to be given in this form in the algorithms presented in this paper.

Semantics. We now formally define the semantics of NTL. It is defined with respect to an execution graph $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \curvearrowright)$ and a fixpoint variable assignment \mathcal{V} assigning sets of nodes of G to fixpoint variables. Intuitively, $\llbracket \varphi \rrbracket_{\mathcal{V}}^G$ is the set of nodes of G satisfying φ when each free fixpoint variable X is interpreted to hold at nodes $\mathcal{V}(X)$. In the following, for a fixpoint variable assignment \mathcal{V} , a fixpoint variable X and a set of nodes $M \subseteq V$, we write $\mathcal{V}[X \mapsto M]$ for the fixpoint variable assignment with $\mathcal{V}[X \mapsto M](X) = M$ and $\mathcal{V}[X \mapsto M](Y) = \mathcal{V}(Y)$ for all variables $Y \neq X$.

Definition 5 (Semantics of NTL). Let $G = (V, l, (\rightarrow^d)_{d \in Moves}, \curvearrowright)$ be an execution graph and \mathcal{V} be a fixpoint variable assignment. The semantics of an NTL formula with respect to G and \mathcal{V} is defined by

$$\begin{split} \|ap\|_{\mathcal{V}}^{G} &:= \{x \in V \mid ap \in l(x)\} \\ \|\neg\varphi\|_{\mathcal{V}}^{G} &:= V \setminus \|\varphi\|_{\mathcal{V}}^{G} \\ \|\varphi_{1} \vee \varphi_{2}\|_{\mathcal{V}}^{G} &:= \|\varphi_{1}\|_{\mathcal{V}}^{G} \cup \|\varphi_{2}\|_{\mathcal{V}}^{G} \\ \|X\|_{\mathcal{V}}^{G} &:= \mathcal{V}(X) \\ \|\bigcirc^{f}\varphi\|_{\mathcal{V}}^{G} &:= \{x \in V \mid succ_{f}^{G}(x) \text{ is defined and } succ_{f}^{G}(x) \in \|\varphi\|_{\mathcal{V}}^{G} \\ \|\mu X.\varphi\|_{\mathcal{V}}^{G} &:= \bigcap \{M \subseteq V \mid \|\varphi\|_{\mathcal{V}[X \mapsto M]}^{G} \subseteq M \} \end{split}$$

where $ap \in AP$ is an atomic proposition, X is a fixpoint variable and $f \in \{g, \uparrow, a, -, p, c\}$ is a successor type.

In this semantics definition, two remarks are in order. First, it is easy to see using Knaster-Tarski's fixpoint theorem [27] that for formulae φ in positive normal form, $\llbracket \mu X. \varphi \rrbracket_{\mathcal{V}}^{G}$ characterises the least fixpoint of the monotone function

 $\alpha_S \colon 2^V \to 2^V$ with $\alpha_S(M) = \llbracket \varphi \rrbracket_{\mathcal{V}[X \mapsto M]}^G$ for $S = (G, \mathcal{V}, X, \varphi)$. Second, for closed NTL formulae φ , the semantics does not depend on the fixpoint variable assignment. For such formulae, we introduce additional semantic notations. We write $\llbracket \varphi \rrbracket^G$ for $\llbracket \varphi \rrbracket_{\mathcal{V}}^G$ where \mathcal{V} is an arbitrary fixpoint variable assignment and set $\llbracket \varphi \rrbracket := \{G \in \mathsf{ExGraphs} \mid v_0 \in \llbracket \varphi \rrbracket^G\}$. For an execution graph G, we write $G \models \varphi$ for $G \in \llbracket \varphi \rrbracket$. Finally, for a DPN \mathcal{M} , we write $\mathcal{M} \models \varphi$, iff $G \models \varphi$ for all $G \in \llbracket \mathcal{M} \rrbracket$. In this paper, we consider the following decision problems for NTL:

- *Model Checking:* Given a DPN \mathcal{M} and a closed well-formed NTL formula φ , does $\mathcal{M} \models \varphi$ hold?

- DPN Satisfiability: Given a closed well-formed NTL formula φ , is there a DPN \mathcal{M} such that $\mathcal{M} \models \varphi$?
- Graph Satisfiability: Given a closed well-formed NTL formula φ , is there an execution graph G such that $G \models \varphi$?

5 Example properties

We motivate the introduction of our new logic with some examples.

Locking policies. In programming languages like Java, mutual exclusion between different threads on certain procedures or code blocks is realised via synchronized procedures or blocks. Internally, this feature works by acquiring a lock upon entering a synchronized procedure or block that is released when leaving the synchronized part of the code [1]. Locks are thus acquired and released in a nested manner. In DPNs, this synchronization mechanism can be modelled by including symbols for locks in the stack alphabet that are pushed onto the stack when acquiring a lock and removed from the stack when releasing it. A call or return of a synchronized procedure is then modelled by taking two *call*- or *ret*transitions of the DPN, respectively, one for pushing or popping the lock symbol and another one as usual. We also include the lock symbols as atomic propositions that are assigned to corresponding configuration heads. In this setup, the formula $\varphi_l := \mathcal{F}^{-l}$ expresses that the lock l is currently held using the caller modality \mathcal{F}^- . This form of modelling also works for reentrant locks, i.e. locks that can be acquired multiple times. When threads acquire multiple locks, problems with deadlocks can occur when different threads acquire locks in a different order. Assume, for example, that we have two locks where thread one acquires lock one first and then lock two and thread two acquires lock two first and then lock one. In this case, a deadlock can occur when the threads are scheduled such that thread one acquires lock one and thread two acquires lock two. A common policy to avoid deadlocks is to ensure that all threads acquire locks in the same order. The formula $\varphi_{ij} := \mathcal{F}^{-}(l_i \wedge \mathcal{G}^{-} \neg l_j)$ expresses that lock l_i is currently held and when it was acquired, lock l_i was not held. It can be used in the formula $\mathcal{G}^r(\varphi_{l_i} \land \varphi_{l_j}) \to \varphi_{ij}$ to express that l_i is always acquired before l_j , if both locks are held. The disjunction $\mathcal{G}^r(\varphi_{l_i} \land \varphi_{l_j}) \to \varphi_{ij} \lor \mathcal{G}^r(\varphi_{l_i} \land \varphi_{l_j}) \to \varphi_{ji}$ then expresses the existence of a global order for locks l_i and l_j and the existence of a global order for all locks can be expressed by a boolean combination of a quadratic number of variants of this formula. Another problem with locking arises when certain

threads wait for a lock that is held by another thread for an infinite amount of time, e.g. if a synchronized method is never left. A policy addressing this problem is to ensure that all locks that are acquired are released at some point in the future. We can express this using the formula $\mathcal{G}^r \bigwedge_{l \in Locks} \varphi_l \to \mathcal{F} \neg \varphi_l$. Under these two policies, a necessary and sufficient condition for mutual exclusion of two program points labelled s_1 and s_2 is that a common lock is held at the two program points. This can also be expressed in a formula from our logic: $\bigvee_{l \in Locks} \mathcal{G}^r((s_1 \to \varphi_l) \land (s_2 \to \varphi_l)).$

Behaviour of main and worker threads. We elaborate on a motivating example for single indexed LTL from [26] expressible in NTL. In this example, a main thread of a server processes requests from clients by starting a worker thread responding to the specific request. Then, the main thread should repeatedly accept new requests, expressed by the formula $\mathcal{G}^r(main \to \mathcal{GF}accept)$. Also, each worker thread should respond with a correct acknowledgement to each type of request, i.e. it should respond exactly with ack to req and exactly with ack' to req'. This is expressed by the formula $\mathcal{G}^r(worker \to (req \to req))$ $(\mathcal{F}ack \wedge \mathcal{G}\neg ack') \wedge req' \rightarrow (\mathcal{F}ack' \wedge \mathcal{G}\neg ack))$). Such requirements were already expressible in single indexed LTL. However, using the different types of successor operators in NTL, we can further expand on this scenario and express properties not expressible in single indexed LTL. For example, it is a reasonable requirement that worker threads are only spawned by the main thread and only if the main thread has accepted a request. This requirement can be expressed in the formula $\mathcal{G}^r(worker \to \bigcirc^p(main \land accept))$. Another desirable property in this scenario is a variant of the property from the introduction. In particular, we may want worker threads to only be spawned from a procedure pr which performs bookkeeping about the currently active worker threads. This is expressed by the formula $\mathcal{G}^r(\bigcirc^c worker \to \mathcal{F}^- pr).$

Single indexed LTL model checking. It is no surprise that the previous motivating example for single indexed LTL is expressible in NTL. Indeed, we show that the full approach of single indexed LTL DPN model checking from [26] can also be handled using our logic. We first sketch their setup. In [26], a DPN $\mathcal{M} = \{\mathcal{P}_1, \ldots, \mathcal{P}_n\}$ is defined as a set of pushdown systems \mathcal{P}_i with the ability to spawn threads executing one of the pushdown systems of $\mathcal{M}.$ A single indexed LTL formula is a conjunction $\varphi = \bigwedge_{i=1}^{n} \varphi_i$ of LTL formulae φ_i that are each assigned to a specific pushdown system \mathcal{P}_i . Then, $\mathcal{M} \models \varphi$ holds iff \mathcal{M} has a global run such that for all i, every local run of \mathcal{P}_i in the global run satisfies φ_i . In our setup, their global runs correspond to execution graphs and their local runs correspond to the paths in the execution graph starting in positions where new threads are spawned and following the global successors. Since in single indexed LTL model checking, the existence of a global run is checked, whereas in NTL model checking, it is checked that all execution graphs satisfy a property, we can check that $\mathcal{M} \not\models \varphi$ for a single indexed LTL formula $\varphi = \bigwedge_{i=1}^{n} \varphi_i$ using NTL model checking. This is done as follows. We model the partition of a DPN \mathcal{M} from their setup into its pushdown systems \mathcal{P}_i by labelling every control location of \mathcal{P}_i with a fresh atomic proposition p_i in its translation $\overline{\mathcal{M}}$ in our setup. LTL

formulae φ_i can trivially be translated to NTL by encoding until operators using least fixpoints. Then, the NTL formula $\bar{\varphi} = (p_1 \wedge \neg \varphi_1) \vee \mathcal{F}^r(\bigvee_{i=1}^n \bigcirc^c (p_i \wedge \neg \varphi_i))$ expresses that there is a local run of \mathcal{P}_i for some *i* that does not satisfy φ_i . In this formula, the disjunct $(p_1 \wedge \neg \varphi_1)$ identifies a violation by the root process \mathcal{P}_1 and the disjunct $\mathcal{F}^r(\bigvee_{i=1}^n \bigcirc^c (p_i \wedge \neg \varphi_i))$ identifies violations by spawned processes. Accordingly, $\mathcal{M} \models \varphi$ (in the single indexed LTL setup) iff $\overline{\mathcal{M}} \not\models \overline{\varphi}$ (in our setup).

6 From Graph Semantics to Tree Semantics

In order to enable algorithmic verification with tree automata, we introduce an additional structure called *execution tree*. In a nutshell, these trees are obtained from execution graphs by keeping the same set of nodes and adjusting the edge relation a little. In particular, we discard *ret*-edges. In order to properly interpret left and right children in this adjusted structure, we add labels (l, d, p) where l represents the label of the current node, d represents the transition types from this node to its children and p represents the transition type from the parent to this node. This yields us a structure simpler than execution graphs that still contains the same information and can be analysed using tree automata.

Execution trees. Let $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \frown)$ be an execution graph. We inductively define a map $\delta_G \colon V \to \{0, 1\}^*$ assigning a tree node to each graph node $x \in V$ as follows.

- If $x = v_0$, we set $\delta_G(x) := \varepsilon$.
- If x has a call- or int-predecessor y, we set $\delta_G(x) := \delta_G(y) \cdot 0$. In this case, we also call $\delta_G(x)$ a call- or int-child of $\delta_G(y)$, respectively.
- If there is $y \in V$ such that y is a spawn-predecessor of x or $y \curvearrowright x$, we set $\delta_G(x) := \delta_G(y) \cdot 1$. If y is a spawn-predecessor of x, we also call $\delta_G(x)$ a spawn-child of $\delta_G(y)$ and if $y \curvearrowright x$, we also call $\delta_G(x)$ a ret-child of $\delta_G(y)$.

Additionally, for a subset $M \subseteq$ Moves and nodes $x, y \in V$, we call $\delta_G(y)$ an M-descendant of $\delta_G(x)$ and $\delta_G(x)$ an M-ancestor of $\delta_G(y)$, if there is a path from $\delta_G(x)$ to $\delta_G(y)$ in the tree following only M-children. Let $TL = 2^{AP} \times \{int, call, callRet, spawn, ret, end\} \times (Moves \cup \{\bot\})$ be the

Let $TL = 2^{AP} \times \{int, call, callRet, spawn, ret, end\} \times (\mathsf{Moves} \cup \{\bot\})$ be the set of labels for tree nodes and the arity function $ar: TL \to \{0, 1, 2\}$ be defined by ar(l, ret, p) = ar(l, end, p) = 0, ar(l, int, p) = ar(l, call, p) = 1 and ar(l, callRet, p) = ar(l, spawn, p) = 2. The tree representation $\mathcal{T}(G)$ of G is the (TL, ar)-labelled binary tree $(\operatorname{im}(\delta_G), r)$ where $\operatorname{im}(\delta_G) = \{\delta_G(x) \mid x \in V\}$ denotes the image of δ_G and for all $x \in V$ we have $r(\delta_G(x)) = (l(x), d(x), p(x))$ where (i) either $p(x) \neq \bot$ and x has a p(x)-predecessor or $p(x) = \bot$ and $x = v_0$ and (ii) one of the following conditions hold:

- -x has only one d(x)-successor and $d(x) \in \{int, ret\}.$
- -x has only one *int* and one *spawn*-successor and d(x) = spawn.
- x has only one call-successor, there is no $y \in V$ with $x \frown y$, and d(x) = call.
- x has only one call-successor, there is $y \in V$ with $x \curvearrowright y$, and d(x) = callRet.
- -x has no successors and d(x) = end.



Fig. 3: Execution tree for the execution graph in Fig. 1. An edge from node t to node t' labelled d means that $t' = t \cdot d$. Labels are depicted for nodes 0, 00 and 0000. Gray edges exist in the execution graph but not in the execution tree.

A tree representation of an execution graph is also called an *execution tree*. An example of an execution tree can be found in Fig. 3.

Adapted successor functions. We adapt the successor functions previously defined on execution graphs to execution trees in order to allow us to check the satisfaction of formulae directly on execution trees. Specifically, we define multiple successor functions $succ_g^{\mathcal{T}}$, $succ_{\uparrow}^{\mathcal{T}}$, $succ_{-}^{\mathcal{T}}$, $succ_p^{\mathcal{T}}$ and $succ_c^{\mathcal{T}}$ with signature $T \rightsquigarrow T$ for execution trees $\mathcal{T} = (T, r)$. For $t \in T$ with r(t) = (l, d, p), the successor functions are given as follows:

- The abstract successor $succ_a^{\mathcal{T}}(t)$ of t is defined as the left child of t, if $d \in \{int, spawn\}$, the right child of t, if d = callRet, and undefined else.
- The caller predecessor $succ_{-}^{\mathcal{T}}(t)$ of t is defined as the parent node of t, if p = call, the caller predecessor of its parent node, if $p \in \{int, ret\}$ and this is defined, and undefined else.
- The global successor $succ_g^{\mathcal{T}}(t)$ of t is defined as the left child of t, if $d \in \{int, call, callRet, spawn\}, succ_a^{\mathcal{T}}(succ_{-}^{\mathcal{T}}(t)), \text{ if } d = ret, \text{ and undefined else.}$
- The global predecessor $succ^{\mathcal{T}}_{\uparrow}(t)$ of t is defined as the parent node of t, if $p \in \{int, call\}$, the $\{int, ret\}$ -descendant leaf of the left child of its parent node, if p = ret, and undefined else.
- The parent predecessor $succ_p^{\mathcal{T}}(t)$ of t is defined as the parent node of t, if p = spawn, the parent predecessor of its parent node, if $p \in \{int, call, ret\}$ and this is defined, and undefined else.
- The child successor $succ_c^{\mathcal{T}}(t)$ of t is defined as the right child of t, if d = spawn, and undefined else.

We show in the following lemma that these adapted successor functions behave exactly like their counterparts on execution graphs.

Lemma 6. Let $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \sim)$ be an execution graph with $\mathcal{T}(G) = \mathcal{T}$. For all $f \in \{g, \uparrow, a, -, p, c\}$ we have $\delta_G \circ \operatorname{succ}_f^G = \operatorname{succ}_f^{\mathcal{T}} \circ \delta_G$, i.e. for all nodes $x \in V$, $\delta_G(\operatorname{succ}_f^G(x))$ is defined iff $\operatorname{succ}_f^{\mathcal{T}}(\delta_G(x))$ is defined and in this case $\delta_G(\operatorname{succ}_f^G(x)) = \operatorname{succ}_f^{\mathcal{T}}(\delta_G(x))$.

A detailed proof of this lemma can be found in Appendix D.

7 Model Checking and Satisfiability

We now use execution trees to decide the model checking and satisfiability problems for NTL. For this, we construct three tree automata: one automaton for checking whether a tree is an execution tree, a second automaton for checking whether an execution graph (given by its tree representation) satisfies a given formula, and another automaton for checking whether a tree represents an execution graph generated by a given DPN.

An automaton for execution trees. We first construct a nondeterministic parity tree automaton that checks whether a (TL, ar)-labelled binary tree is an execution tree. At each node labelled by (l, d, p), the automaton needs to ensure that the node is a *p*-child, if $p \neq \bot$, and that it is the root, if $p = \bot$. Moreover, if d = callRet, it has to check that its *call*-child does have an $\{int, ret\}$ descendant leaf. Finally, it has to ensure that for leaves *t* labelled by (l, d, p) we have d = ret iff *t* is the $\{int, ret\}$ -descendant leaf of a *call*-child of a node labelled by (l', callRet, p') for some $l' \in 2^{AP}$ and $p' \in Moves \cup \{\bot\}$. Thus, we can define the automaton as $\mathcal{A}_{\mathsf{ET}} = (Q, q_0, \rho, \Omega)$ with state set $Q = (\mathsf{Moves} \cup \{\bot\}) \times \{0, 1\}$ and initial state $q_0 = (\bot, 0)$. Intuitively, in a state (p, c), *p* denotes the parent edge type and the bit *c* indicates whether the current node is an $\{int, ret\}$ descendant of a *call*-child of a node labelled by (l', callRet, p') for some $l' \in 2^{AP}$ and $p' \in \mathsf{Moves} \cup \{\bot\}$. The transition function ρ is defined by

$$\rho((p,c),(l,d,p')) := \begin{cases} (0,(int,c)) & \text{if } d = int \\ (0,(call,0)) & \text{if } d = call \text{ and } c = 0 \\ (0,(call,1)) \wedge (1,(ret,c)) & \text{if } d = callRet \\ (0,(int,c)) \wedge (1,(spawn,0)) & \text{if } d = spawn \\ true & \text{if } (d,c) \in \{(ret,1),(end,0)\} \end{cases}$$

for p = p' and $\rho((p, c), (l, d, p')) := false$ in all other cases. The priority assignment is given by $\Omega(p, c) = c$ for all $(p, c) \in Q$.

We establish the following theorem. A proof can be found in Appendix E.

Theorem 7. One can construct a NPTA $\mathcal{A}_{\mathsf{ET}}$ over (TL, ar)-labelled binary trees with a constant size such that $\mathcal{L}(\mathcal{A}_{\mathsf{ET}}) = \{\mathcal{T}(G) \mid G \text{ is an execution graph}\}.$

An automaton for formulae. For the next automaton, we define a 2-way alternating tree automaton evaluating φ on execution trees, intersect it with the automaton recognising execution trees and then transform this automaton into a nondeterministic parity tree automaton. In the following, let φ be a closed, well-formed NTL formula in positive normal form. We define the automaton for φ as $\tilde{\mathcal{A}}_{\varphi} = (Q, q_0, \rho, \Omega)$ where Q, q_0, ρ and Ω are described in more detail in the following paragraphs.

The states of the automaton are given by

$$Q = Sub(\varphi) \cup Q_1 \cup Q_2 \text{ where}$$
$$Q_1 = \{\bigcirc^- \bigcirc^a \psi, \bigcirc^a \psi \mid \bigcirc^g \psi \in Sub(\varphi) \text{ or } \bigcirc^{\overline{g}} \psi \in Sub(\varphi)\} \text{ and}$$

A Navigation Logic for Recursive Programs with Dynamic Thread Creation

$$Q_2 = \{ call, leaf \} \times \{ \psi \mid \bigcirc^{\uparrow} \psi \in Sub(\varphi) \text{ or } \bigcirc^{\uparrow} \psi \in Sub(\varphi) \}$$

with initial state $q_0 = \varphi$. Since we use another automaton to check that the given tree indeed represents an execution graph, we care only about execution trees as inputs in this construction. Intuitively, being in a state $\psi \in Sub(\varphi) \cup Q_1$ at the position $\delta_G(x)$ in the input execution tree $\mathcal{T}(G)$, the automaton checks whether the node x satisfies ψ , i.e. whether $x \in [\![\psi]\!]^G$. The states in Q_2 are used to handle the global predecessor next modality and its dual version. We use states of the form $(call, \psi)$ to denote that we should move to the *call*-child of the current node and switch to state $(leaf, \psi)$; states of the form $(leaf, \psi)$ denote that we should check ψ for the $\{int, ret\}$ -descendant leaf of the current node.

The transition function ρ is defined as described next. Recall that \mathcal{A}_{φ} operates on execution trees which are labelled by triples (l, d, p) where $l \in 2^{AP}$ are the atomic propositions, $d \in \{int, call, callRet, spawn, ret, end\}$ specifies the successor types of the current node and $p \in \mathsf{Moves} \cup \{\bot\}$ denotes the type of its predecessor. If the current state is an atomic proposition or a negation of an atomic proposition, we can check directly whether the tree node is labelled by this proposition and thus determine whether the formula holds:

$$\rho(ap, (l, d, p)) := \begin{cases} true \text{ if } ap \in l \\ false \text{ if } ap \notin l \end{cases} \quad \rho(\neg ap, (l, d, p)) := \begin{cases} false \text{ if } ap \in l \\ true \text{ if } ap \notin l. \end{cases}$$

For a disjunction or conjunction of two formulae, we can use the power of alternation and set

$$\rho(\psi_1 \vee \psi_2, \sigma) := (\varepsilon, \psi_1) \vee (\varepsilon, \psi_2) \text{ and } \rho(\psi_1 \wedge \psi_2, \sigma) := (\varepsilon, \psi_1) \wedge (\varepsilon, \psi_2).$$

For a formula of the form $\bigcirc^f \psi$, we move to the corresponding successor of the given node and then switch to state ψ . In most cases, the according transitions can be defined straightforwardly using the characterisation from the successor functions on execution trees:

$$\rho(\bigcirc^{g}\psi, (l, d, p)) := \begin{cases} (0, \psi) \text{ if } d \in \{int, call, callRet, spawn\} \\ (\varepsilon, \bigcirc^{-} \bigcirc^{a} \psi) \text{ if } d = ret \\ false \text{ if } d = end \end{cases}$$

$$\begin{split} \rho(\bigcirc^{a}\psi,(l,d,p)) & \rho(\bigcirc^{-}\psi,(l,d,p)) \\ &:= \begin{cases} (0,\psi) \text{ if } d \in \{int,spawn\} \\ (1,\psi) \text{ if } d = callRet \\ false \text{ if } d \in \{call,ret,end\} \end{cases} & contended \\ &:= \begin{cases} (\uparrow,\psi) \text{ if } p = call \\ (\uparrow,\bigcirc^{-}\psi) \text{ if } p \in \{int,ret\} \\ false \text{ if } p \in \{spawn,\bot\} \end{cases} \\ &\rho(\bigcirc^{p}\psi,(l,d,p)) & \rho(\bigcirc^{c}\psi,(l,d,p)) \\ &:= \begin{cases} (\uparrow,\psi) \text{ if } p = spawn \\ (\uparrow,\bigcirc^{p}\psi) \text{ if } p \in \{int,call,ret\} \\ false \text{ if } p = \bot \end{cases} & := \begin{cases} (1,\psi) \text{ if } d = spawn \\ false \text{ if } d \neq spawn \end{cases} \end{split}$$

15

In the above definition, we move to *false* when we see that the desired successor does not exist and the formula is not satisfied. The transition function for dual next operators is defined analogously but moves to *true* instead of *false* in case the successor does not exist.

For the global predecessor, we additionally use states of the form $(call, \psi)$ and $(leaf, \psi)$ for moving to the $\{int, ret\}$ -descendant leaf of the *call*-child of the parent of a node in cases where the global predecessor is defined this way:

$$\begin{split} \rho(\bigcirc^{\uparrow}\psi,(l,d,p)) & \rho((leaf,\psi),(l,d,p)) \\ &:= \begin{cases} (\uparrow,\psi) \text{ if } p \in \{int, call\} \\ (\uparrow,(call,\psi)) \text{ if } p = ret \\ false \text{ if } p \in \{spawn,\bot\}, \end{cases} & := \begin{cases} (0,(leaf,\psi)) \text{ if } d \in \{int, spawn\} \\ (1,(leaf,\psi)) \text{ if } d = callRet \\ (\varepsilon,\psi) \text{ if } d \in \{ret, call, end\} \end{cases} \\ \text{and } \rho((call,\psi),\sigma) := (0,(leaf,\psi)). \end{split}$$

Note that if we are in a state $(leaf, \psi)$ at position $\delta_G(x)$ in the tree, $d(x) \in \{call, end\}$ cannot hold if the tree represents an execution graph since in this case x lies on the path between nodes y and z following Moves $\setminus \{spawn\}$ -successors with $y \cap z$ and $x \neq z$.

Finally, fixpoint formulae lead to loops:

$$\rho(\lambda X.\psi,\sigma) := (\varepsilon,\psi) \text{ for } \lambda \in \{\mu,\nu\} \text{ and } \rho(X,\sigma) := (\varepsilon,fp(X)).$$

The acceptance condition specifies whether a fixpoint formula may be visited at most a finite number of times or an infinite number of visits is allowed. In this definition, higher priorities are assigned to fixpoint formulae binding variables which *depend* on other fixpoint variables. Formally, we say that a fixpoint variable X' depends on the variable X in φ , written $X \prec_{\varphi} X'$, if X is a free variable in fp(X'). We consider all maximal chains $X_1 \prec_{\varphi} \ldots \prec_{\varphi} X_n$ of fixpoint variables appearing in φ . If $fp(X_1)$ is a formula of the form $\mu X.\psi$, we set $\Omega(fp(X_1)) = 1$, otherwise we set $\Omega(fp(X_1)) = 0$. Then, we move through the chains and assign this priority to $fp(X_i)$ as long as the fixpoint type does not change. In that case, we increase the currently assigned priority by one and keep going. Then, we set $\Omega(q)$ to the highest priority assigned so far for all other states q.

We establish the following theorem.

Theorem 8. Let φ be a closed, well-formed NTL formula, G be an execution graph and $\tilde{\mathcal{A}}_{\varphi}$ be the 2ATA defined above. Then $\tilde{\mathcal{A}}_{\varphi}$ accepts $\mathcal{T}(G)$ iff $G \in [\![\varphi]\!]$.

Proof Sketch. The proof is by induction on the structure of φ . Therefore, we also have to deal with non-closed subformulae and consider valuations to decide whether a subformula is satisfied. In order to do this in a formal way, we consider automata with special states X_1, \ldots, X_n , called *holes* [21], that can be filled with sets of nodes L_1, \ldots, L_n of a given tree. Intuitively, such an automaton can operate on a tree as before, but when a hole X_i is encountered during a run and we are at the tree node t, then we do not continue on the current path and say that it is accepting iff $t \in L_i$. By $\mathcal{L}_q^{\mathcal{T}}(\mathcal{A}[X_1:L_1,\ldots,X_n:L_n])$ we denote the set

of nodes $t \in T$ such that there is an accepting (t, q)-run over \mathcal{A} where the states X_1, \ldots, X_n are holes filled by L_1, \ldots, L_n .

For the inductive proof, we assume that the free variables of the current formula $\psi \in Sub(\varphi)$ are holes in the automaton and show that the language of this automaton corresponds to the semantics of ψ . Intuitively, we fill the holes in the automaton, i.e. the free variables of ψ , with the same sets of nodes as specified by a given valuation that we consider for the semantics of ψ . More formally, the holes are filled by sets of tree nodes that correspond to given sets of graph nodes in the valuation.

We consider the case for subformulae of the form $\psi \equiv \mu X.\psi'$ with free variables X_1, \ldots, X_n . Let \mathcal{V} be a fixpoint variable assignment, $\mathcal{T}(G) = \mathcal{T} = (T, r)$ and R be a (t, ψ) -run over $\tilde{\mathcal{A}}_{\varphi}$ for a $t \in T$, where the states X_1, \ldots, X_n are holes filled by $\delta_G(L_1), \ldots, \delta_G(L_n)$ with $L_i = \mathcal{V}(X_i)$. We observe that R can only visit states φ' of the form $\mu X.\psi''$ or $\nu X.\psi''$ if φ' is a subformula of ψ . Therefore, $\Omega(\psi)$ is the lowest priority occurring in the run so that the state ψ can only be visited finitely often if the run is accepting. This means we can characterize $\mathcal{L}_{\psi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_1: \delta_G(L_1), \ldots, X_n: \delta_G(L_n)])$ as the least fixpoint of the function $f: 2^T \to 2^T$ with $f(\delta_G(L)) := \mathcal{L}_{\psi'}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_1: \delta_G(L_1), \ldots, X_n: \delta_G(L_n), X: \delta_G(L)])$. Thus, we can use the induction hypothesis and the fixpoint characterization of the semantics of ψ obtained by Knaster-Tarski's fixpoint theorem to get the desired result in this inductive step.

Since φ is closed, the induction establishes in particular that $\tilde{\mathcal{A}}_{\varphi}$ accepts $\mathcal{T}(G)$ iff $G \in \llbracket \varphi \rrbracket$. \Box

As mentioned, we do not use this automaton directly but instead intersect it with $\mathcal{A}_{\mathsf{ET}}$ from Theorem 7 and then transform it into a nondeterministic parity tree automaton using Proposition 1. We obtain:

Corollary 9. Let φ be a closed, well-formed NTL formula. Then we can construct an NPTA \mathcal{A}_{φ} over (TL, ar)-labelled binary trees with a number of states exponential and an acceptance condition linear in $|\varphi|$ such that $\mathcal{L}(\mathcal{A}_{\varphi}) = \{\mathcal{T}(G) \mid G \text{ is an execution graph with } G \in [\![\varphi]\!] \}.$

An automaton for DPNs. We proceed with an automaton for a DPN $\mathcal{M} = (S, s_0, \gamma_0, \Delta, L)$. We define $\mathcal{A}_{\mathcal{M}}$ as an NPTA that checks whether an execution tree represents an execution graph generated by \mathcal{M} . We set $\mathcal{A}_{\mathcal{M}} := (Q, q_0, \rho, \Omega)$ where Q, q_0, ρ and Ω are described in more detail next.

The state set is given by $Q = S \times \Gamma \times ((S \times \Gamma) \cup \{\bot\})$ with initial state $q_0 = (s_0, \gamma_0, \bot)$. Being in a state $(s, \gamma, c) \in Q$ at the position $\delta_G(x)$ in the tree labelled by (l, d, p) means that there is a suitable assignment *as* assigning configurations to the graph nodes whose corresponding tree nodes have been visited so far where $as(x) = (s, \gamma w)$ for some stack content $w \in \Gamma^* \bot$. If d = callRet, we also have to know the configuration assigned to the global predecessor of the *ret*-child of the current node to check that we can extend *as* suitably for the children of the current node. We thus guess this configuration in this case and use $c \in S \times \Gamma$ to indicate that we must assign *c* to the $\{int, ret\}$ -descendant leaf of the call successor of the current node in order to fulfill the requirements for

the assignment as. Note that the $\{int, ret\}$ -descendant leaf exists in this case, if the input tree is an execution tree. The transition function ρ then checks that (i) l = L(as(x)), (ii) if $c \in S \times \Gamma$, then the configuration c is assigned to the $\{int, ret\}$ -descendant leaf of $\delta_G(x)$ and (iii) the assignment as can be properly extended to the children of $\delta_G(x)$. We set

$$\begin{split} \rho((s,\gamma,c),(l,int,p)) &:= \bigvee \{(0,(s',\gamma',c)) \mid s\gamma \to s'\gamma' \in \Delta_I\},\\ \rho((s,\gamma,\bot),(l,call,p)) &:= \bigvee \{(0,(s',\gamma',\bot)) \mid \exists \gamma'' \in \Gamma \text{ s.t. } s\gamma \to s'\gamma'\gamma'' \in \Delta_C\},\\ \rho((s,\gamma,c),(l,callRet,p)) &:= \bigvee \{(0,(s',\gamma',(s_r,\gamma_r))) \land (1,(s'',\gamma'',c)) \mid s\gamma \to s'\gamma'\gamma'' \in \Delta_C \text{ and } s_r\gamma_r \to s'' \in \Delta_R\},\\ \rho((s,\gamma,c),(l,spawn,p)) &:= \bigvee \{(0,(s',\gamma',c)) \land (1,(s_n,\gamma_n,\bot)) \mid s\gamma \to s'\gamma' \rhd s_n\gamma_n \in \Delta_S\},\\ \rho((s,\gamma,(s,\gamma)),(l,ret,p)) &:= true \text{ and} \end{split}$$

$$\rho((s,\gamma,\perp),(l,end,p)) := \begin{cases} true \text{ if there is no transition for } s\gamma \text{ in } \Delta \\ false \text{ else} \end{cases}$$

for $l = L(s, \gamma)$ and $\rho((s, \gamma, c), (l, d, p)) := false$ in all other cases. Since we are only concerned with execution trees as inputs, all conditions necessary to determine if the input tree is generated by \mathcal{M} are already checked by the transition function of $\mathcal{A}_{\mathcal{M}}$. We thus set $\Omega(q) := 0$ for all $q \in Q$. We establish the following theorem. A detailed proof can be found in Appendix E.

Theorem 10. Let \mathcal{M} be a DPN. We can construct an NPTA $\mathcal{A}_{\mathcal{M}}$ over (TL, ar)labelled binary trees with a number of states quadratic in $|\mathcal{M}|$ and a trivial acceptance condition such that for all execution graphs $G, \mathcal{T}(G) \in \mathcal{L}(\mathcal{A}_{\mathcal{M}})$ iff $G \in [\mathcal{M}]$.

Complexity of Model Checking and Satisfiability. These automata constructions can be used to obtain a decision procedure for the model checking and satisfiability problems. For the former, we obtain the following theorem:

Theorem 11. The model checking problem for NTL is EXPTIME-complete. For fixed formulae, the problem is in PTIME.

Proof. For the upper bound, we construct an automaton for the negation of the formula using Corollary 9 and intersect it with an automaton for the DPN from Theorem 10. Since the acceptance condition of the latter is trivial, the resulting automaton is quadratic in the size of the DPN and exponential in the size of the formula by Proposition 3 (ii). It is tested for emptiness using Proposition 2 in time exponential in $|\varphi|$ and polynomial in $|\mathcal{M}|$ to answer the model checking problem.

The lower bound follows by a reduction from the LTL pushdown model checking problem which was shown to be EXPTIME-hard in [8]. The reduction is trivial since LTL is a sublogic of NTL for single threads and pushdown systems can be trivially embedded into DPNs with a single thread. For satisfiability, we can show that the two problems defined in Section 4 are equivalent and thus only need to solve one of the problems by a direct procedure.

Theorem 12. The graph and DPN satisfiability problems are equivalent.

Proof. For the first direction, assume that a formula φ is satisfiable by a DPN \mathcal{M} . Then $G \models \varphi$ for all $G \in \llbracket \mathcal{M} \rrbracket$. Since $\llbracket \mathcal{M} \rrbracket \neq \emptyset$ (this indeed holds for all DPNs), we can thus choose an arbitrary graph $G \in \llbracket \mathcal{M} \rrbracket$ to show that φ is satisfiable by a graph.

For the other direction, assume that a formula φ is satisfiable by a graph G. By Corollary 9, we know that $\mathcal{T}(G) \in \mathcal{L}(\mathcal{A}_{\varphi})$. Since $\mathcal{L}(\mathcal{A}_{\varphi})$ is a nonempty ω regular tree language, we know that $\mathcal{T} \in \mathcal{L}(\mathcal{A}_{\varphi})$ for a regular tree $\mathcal{T} = (T, r)$, i.e. a tree with only finitely many non-isomorphic subtrees (see e.g. Cor 8.20. in [16]). Let x_1, \ldots, x_n be the finitely many classes of nodes associated with the roots of the distinct subtrees of T such that x_1 is the class of ε and let (l_i, d_i, p_i) be the label of the nodes from class x_i . We construct a DPN $\mathcal{M} = (\{s\}, s, x_1, \Delta, L)$ with stack alphabet $\Gamma = \{x_1, \ldots, x_n\}$. The labeling L is defined such that $L(s, x_i) =$ l_i . Transition rules are defined from the parent-child relationships between the different classes of nodes: (i) if $d_i = int$, then nodes of class x_i have exactly one child of class x_j and we include $sx_i \to sx_j \in \Delta$, (ii) if $d_i = spawn$, then nodes of class x_i have exactly one left child of class x_j and one right child of class x_k and we include $sx_i \to sx_j \triangleright sx_k \in \Delta$, (iii) if $d_i = callRet$, then nodes of class x_i have exactly one left child of class x_j and one right child of class x_k and we include $sx_i \to sx_jx_k \in \Delta$, (iv) if $d_i = call$, then nodes of class x_i have exactly one child of class x_i and we include $sx_i \to sx_jx_i \in \Delta$, (v) if $d_i = ret$, then nodes of class x_i have no children and we include $sx_i \to s \in \Delta$ and (vi) if $d_i = end$, then nodes of class x_i have no children and we do not include a transition. It is easy to see that $\llbracket \mathcal{M} \rrbracket$ is a singleton set since \mathcal{M} is deterministic. We show that $\llbracket \mathcal{M} \rrbracket = \{H\}$ where $\mathcal{T} = \mathcal{T}(H)$ and thus $\mathcal{M} \models \varphi$. For this, let $\mathcal{T}(H) = (T_H, r_H)$.

We show by induction on the length of x that for all $x \in \{0,1\}^*$, $x \in T$ iff $x \in T_H$ and in that case (a) $r(x) = r_H(x)$ and (b) if x belongs to class x_i , then the configuration in $\delta_G^{-1}(x)$ is $(s, x_i w)$ for some stack content w. In the base case, we know that $\varepsilon \in T$ and $\varepsilon \in T_H$. We know that $r(\varepsilon) = (l_1, d_1, p_1)$ since Tis rooted in x_1 and $p_1 = \bot$ since \mathcal{T} is an execution tree. Let $r_H(\varepsilon) = (l, d, p)$. Since $(s, x_1 \bot)$ is the starting configuration of \mathcal{M} , we know that it is also the configuration in $\delta_G^{-1}(\varepsilon)$ and that $l = l_1$. Additionally, we can show that $d = d_1$ by a case distinction on d_1 . We only sketch the case $d_1 = int$, the other cases are similar. In this case, the only enabled transition in $(s, x_1 \bot)$ is $sx_1 \to sx_j$, an internal transition. Thus, $\delta_G^{-1}(\varepsilon)$ has exactly one *int*-successor in H which means that d = int. Finally, since $\mathcal{T}(H)$ is an execution tree, we have $p = \bot$.

In the inductive step, we consider $x \cdot d$ for $d \in \{0, 1\}$. From the induction hypothesis, we know that the claim holds for x. If $x \notin T$ and $x \notin T_H$, then also $x \cdot d \notin T$ and $x \cdot d \notin T_H$ since trees are prefix-closed. In the other case, let x_i be the class of x. We have $x \in T$ and $x \in T_H$ with $r(x) = r_H(x) = (l_i, d_i, p_i)$ and the configuration in $\delta_G^{-1}(x)$ is $(s, x_i w)$ for some stack content w. We distinguish cases based on d_i . We consider the most involved case where $d_i = callRet$. Since \mathcal{T} is

an execution tree, we know that $x \cdot d \in T$ for $d \in \{0, 1\}$. Let x_j be the class of $x \cdot 0$ and x_k be the class of $x \cdot 1$. We know that the only enabled transition in $(s, x_i w)$ is $sx_i \to sx_j x_k$. Since $d_i = callRet$ and since $\mathcal{T}(G)$ is an execution tree, we know that $\delta_G^{-1}(x \cdot 0)$ continues with the configuration after this call transition and $\delta_G^{-1}(x \cdot 1)$ continues with the configuration after the matching return transition (which exists in this case). Thus, the configuration in $\delta_G^{-1}(x \cdot 0)$ is $(s, x_j x_k w)$ and the configuration in $\delta_G^{-1}(x \cdot 1)$ is $(s, x_k w)$, establishing this part of the claim. We now establish that $r(x \cdot d) = r_H(x \cdot d)$. For the first and second component, this is established by the fact that the configuration in $\delta_G^{-1}(x \cdot d)$ determines both the label and the unique enabled transition. For the third component, this follows from the fact that both \mathcal{T} and $\mathcal{T}(H)$ are execution trees and the fact that r(x)and $r_H(x)$ match in the second component.

We obtain the following theorem for the two satisfiability problems:

Theorem 13. The graph and DPN satisfiability problems for NTL are EXPTIMEcomplete.

Proof. Since the two problems are equivalent by Theorem 12, we need to only give an upper and lower bound for the graph satisfiability problem.

For the upper bound, we can construct an automaton for the formula using Corollary 9 and test it for emptiness using Proposition 2 in time exponential in $|\varphi|$ for an answer to the graph satisfiability problem.

The lower bound follows by a reduction from the $VP-\mu-TL$ satisfiability problem which was shown to be EXPTIME-hard in [10]. The reduction is straightforward since $VP-\mu-TL$ is a sublogic of NTL and we can easily extract a nested word satisfying a formula interpreted in $VP-\mu-TL$ from the execution graph satisfying the same formula interpreted in NTL.

8 Conclusion

We introduced a novel specification logic called NTL for reasoning about the call-return and thread creation behaviour of dynamic pushdown networks. We showed that a variety of interesting properties regarding the behaviour of multi-threaded software is expressible in NTL. Further, the model checking and satisfiability problems were investigated. The complexity of these problems is not higher than that of the corresponding problems for related logics for pushdown systems despite a more powerful logic and system model. For future work, it would be interesting to consider more powerful variants of DPNs that allow communication and synchronization of different threads via locking or messages.

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23

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A Fixpoint Theory

In some of the proofs in this paper, we need results from fixpoint theory. We provide the necessary definitions and results in this section. A partial order is a pair (L, \sqsubseteq) such that \sqsubseteq is a reflexive, transitive and antisymmetric binary relation on L. For $X \subseteq L$ and $x \in L$, we call x a *lower bound* on X iff $x \sqsubseteq x'$ for all $x' \in X$. Similarly, x is called an upper bound on X iff $x' \sqsubseteq x$ for all $x' \in X$. A lower bound x of X is called the greatest lower bound of X, denoted $x = \prod X$, iff $x' \sqsubseteq x$ for all lower bounds x' of X. Analogously, an upper bound x of X is called the least upper bound of X, denoted $x = \bigsqcup X$, iff $x \sqsubseteq x'$ for all upper bounds x' of X. A partial order (L, \sqsubseteq) is called a *complete lattice* iff the least upper bound ||X| exists for every set $X \subseteq L$. For a function $f: L \to L'$ on partial orders (L, \sqsubseteq) and (L', \sqsubseteq') , we call f monotone iff $x \sqsubseteq x'$ implies $f(x) \sqsubseteq' f(x')$ for all $x, x' \in L$. For $(L, \sqsubseteq) = (L', \sqsubseteq')$, a fixpoint of f is an element $x \in L$ with f(x) = x. We call a fixpoint x of f the *least fixpoint* of f, denoted μf , iff $x \sqsubseteq x'$ for all fixpoints x' of f. Analogously, a fixpoint x is called the greatest fixpoint of f, denoted νf , iff $x' \sqsubseteq x$ for all fixpoints x' of f. We use the classical Knaster-Tarski fixpoint theorem:

Proposition 14 ([27]). Let (L, \sqsubseteq) be a complete lattice and $f: L \to L$ be a monotone function. Then f has a least fixpoint that is characterised by $\mu f = \prod \{x \in L \mid f(x) \sqsubseteq x\}.$

Additionally, we need a lemma about the relationship of least fixpoints in different partial orders. This lemma is a variant of a similar transfer lemma found e.g. in [6].

Lemma 15. Let (L, \sqsubseteq) and (L', \sqsubseteq') be partial orders with functions $f: L \to L$, $f': L' \to L'$ and μf be the least fixpoint of f. Let further $h: L \to L'$ be a bijective, monotone function with $h \circ f = f' \circ h$. Then $\mu f' = h(\mu f)$.

Proof. We first show that $h(\mu f)$ is a fixpoint of f'. Since μf is a fixpoint of f, we have $f'(h(\mu f)) = f' \circ h(\mu f) = h \circ f(\mu f) = h(f(\mu f)) = h(\mu f)$, i.e. $h(\mu f)$ is a fixpoint of f'.

It remains to show that $h(\mu f)$ is the *least* fixpoint of f'. Therefore, let y be an arbitrary fixpoint of f'. We show that $h(\mu f) \sqsubseteq y$. Since y is a fixpoint of f', we have $f(h^{-1}(y)) = h^{-1} \circ h \circ f \circ h^{-1}(y) = h^{-1} \circ f' \circ h \circ h^{-1}(y) = h^{-1}(f'(y)) =$ $h^{-1}(y)$, i.e. $h^{-1}(y)$ is a fixpoint of f. Since μf is the least fixpoint of f, we have $\mu f \sqsubseteq h^{-1}(y)$. Since h is monotone, we infer that $h(\mu f) \sqsubseteq' h(h^{-1}(y)) = y$. Since y was an arbitrary fixpoint of f', the fixpoint $h(\mu f)$ must be the least fixpoint of f'.

In this paper, we consider complete lattices of the form $(2^A, \subseteq)$ for a set A. In these lattices, greatest lower and least upper bounds are given by intersections and unions over sets, respectively.

25

B Proofs from Section 4

Lemma 16. Let $S = (G, \mathcal{V}, X, \varphi)$ where $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \frown)$ is an execution graph, \mathcal{V} is a fixpoint variable assignment, X is a fixpoint variable and φ is a well-formed NTL formula in positive normal form. Then, the function $\alpha_S : 2^V \rightarrow 2^V$ with $\alpha_S(M) = [\![\varphi]\!]_{\mathcal{V}[X \mapsto M]}^G$ is monotone.

Proof. The proof is by induction on the structure of φ .

- $\frac{\varphi = ap \text{ for } ap \in AP}{\alpha_S(M') \text{ for } M \subseteq M'}$. Since α_S is constant in this case, we have $\alpha_S(M) =$
- $\varphi = \neg ap$ for $ap \in AP$: Analogous to the previous case.
- $-\frac{\overline{\varphi = Y \text{ for a fixpoint variable } Y \text{: For } Y \neq X, \, \alpha_S \text{ is constant in this case, and}}{\text{we have } \alpha_S(M) = \alpha_S(M') \text{ for } M \subseteq M'. \text{ For } Y = X, \text{ we have } \alpha_S(M) = M \subseteq M' = \alpha_S(M') \text{ for } M \subseteq M'.}$
- $\frac{\varphi = \varphi_1 \vee \varphi_2}{\alpha_{S_1}(M) \cup \alpha_{S_2}(M)} = (G, \mathcal{V}, X, \varphi_1) \text{ and } S_2 = (G, \mathcal{V}, X, \varphi_2). \text{ We have } \alpha_S(M) = \alpha_{S_1}(M) \cup \alpha_{S_2}(M) \subseteq \alpha_{S_1}(M') \cup \alpha_{S_2}(M') = \alpha_S(M') \text{ for } M \subseteq M' \text{ by the induction hypothesis.}$
- $-\varphi = \varphi_1 \wedge \varphi_2$: Analogous to the previous case.
- $\overline{\varphi = \bigcirc^f \varphi_1 \text{ for } f \in \{g, \uparrow, a, -, p, c\}}: \text{Let } S_1 = (G, \mathcal{V}, X, \varphi_1). \text{ We have}$

$$\alpha_S(M) = \{ x \in V \mid succ_f^G(x) \text{ is defined and } succ_f^G(x) \in \alpha_{S_1}(M) \}$$
$$\subseteq \{ x \in V \mid succ_f^G(x) \text{ is defined and } succ_f^G(x) \in \alpha_{S_1}(M') \}$$
$$= \alpha_S(M')$$

for $M \subseteq M'$ by the induction hypothesis.

 $- \frac{\varphi = \bigcirc^{\bar{f}} \varphi_1 \text{ for } f \in \{g, \uparrow, a, -, p, c\}:}{\varphi = \mu Y \cdot \varphi_1: \text{ For } Y = X, \ \alpha_S \text{ is constant and we have } \alpha_S(M) = \alpha_S(M') \text{ for }$

 $- \underline{\varphi = \mu Y. \varphi_1}$: For Y = X, α_S is constant and we have $\alpha_S(M) = \alpha_S(M')$ for $M \subseteq M'$. For $Y \neq X$, let $S_1^{M''} = (G, \mathcal{V}[Y \mapsto M''], X, \varphi_1)$. We have

$$\alpha_{S}(M) = \bigcap \{ M'' \subseteq V \mid \llbracket \varphi_{1} \rrbracket_{\mathcal{V}[X \mapsto M][Y \mapsto M'']}^{G} \subseteq M'' \}$$
$$= \bigcap \{ M'' \subseteq V \mid \alpha_{S_{1}^{M''}}(M) \subseteq M'' \}$$
$$\stackrel{(*)}{\subseteq} \bigcap \{ M'' \subseteq V \mid \alpha_{S_{1}^{M''}}(M') \subseteq M'' \}$$
$$= \bigcap \{ M'' \subseteq V \mid \llbracket \varphi_{1} \rrbracket_{\mathcal{V}[X \mapsto M'][Y \mapsto M'']}^{G} \subseteq M'' \}$$
$$= \alpha_{S}(M')$$

for $M \subseteq M'$. In step (*), the induction hypothesis implies $\alpha_{S_1^{M''}}(M) \subseteq \alpha_{S_1^{M''}}(M')$ for all $M'' \subseteq V$, which then means that $\{M'' \subseteq V \mid \alpha_{S_1^{M''}}(M) \subseteq M''\} \supseteq \{M'' \subseteq V \mid \alpha_{S_1^{M''}}(M') \subseteq M''\}$ which in turn implies the inclusion (*).

 $- \underline{\varphi} = \nu Y \cdot \varphi_1$: Analogous to the previous case.

Using Proposition 14, a corollary from this lemma is:

Corollary 17. Let $S = (G, \mathcal{V}, X, \varphi)$ where $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \frown)$ is an execution graph, \mathcal{V} is a fixpoint variable assignment, X is a fixpoint variable and φ is a well-formed NTL formula in positive normal form. Then, $[\![\mu X.\varphi]\!]_{\mathcal{V}}^G$ is the least fixpoint of α_S .

C Properties of Successor Functions

We establish some properties of the successor functions defined on execution graphs that are used in some of the proofs in this paper.

Lemma 18. Let $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \frown)$ be an execution graph.

- (i) For all $y \in V$, there is $z \in V$ with $z \frown y$ iff y has a ret-predecessor x. In this case we have $z = succ_{-}^{G}(x)$.
- (ii) For all $x, y \in V$ with $y = succ_a^G(x)$, the caller of x is defined iff the caller of y is defined and in this case $succ_-^G(x) = succ_-^G(y)$.
- (iii) For all $x, y \in V$ with $x \to^{int} y, x \to^{call} y$ or $x \frown y$, the parent of x is defined iff the parent of y is defined and in this case $succ_p^G(x) = succ_p^G(y)$.

Proof. (i) Let $y \in V$ be a node.

For the first direction, assume that there is a node $z \in V$ with $z \curvearrowright y$. Then there is a path from z to y following only Moves $\setminus \{spawn\}$ -successors such that the number n of *call*-moves on that path is equal to the number of *ret*moves on that path. Since $y \neq z$ and z has a *call*-successor, we have n > 0. Since y is defined as the node such that this path has minimal length, the predecessor of y must be a *ret*-predecessor.

For the other direction, assume that y has a *ret*-predecessor x. Since $(v_0, y) \in (\bigcup_{d \in \mathsf{Moves}} \to^d)^*$, there is a node $u \in V$ that is either v_0 or has a *spawn*-

predecessor such that there is a path π from u to y following only Moves $\{spawn\}$ -successors. Since y is a *ret*-successor and the number of *call*-moves on π has to be greater or equal to the number of *ret*-moves on π , we can consider the last node z on π with a *call*-successor z' such that the number of *call*-moves on the path π between z and y is equal to the number of *ret*-moves on π between z and y.

It remains to show that $z = succ_{-}^{G}(x)$. Let *n* be the number of *call*-transitions on the path π from the *call*-successor z' of z to x following only Moves $\langle spawn \rangle$ -transitions. Since $z \curvearrowright y$, z' is the *call*-successor of z and y is the *ret*-successor of x, this is also the number of *ret*-transitions on π . We now show by induction on n that we can transform the path π to a path π' from z'to x following abstract successors. Since $z \rightarrow call z'$, this implies in particular that $z = succ_{-}^{G}(x)$.

If n = 0, the path π follows only *int*-successors, i.e. it is also a path following abstract successors.

If n > 0, let c and c' be the first nodes on the path π with $c \to call c'$. Since the number of *call*-moves on π is equal to the number of *ret*-moves on π , there is a node $r \in V$ on the path π between c and x with $c \curvearrowright r$. Thus, π can be written as $\pi = \pi_1 \pi_2 \pi_3$ for paths π_1 from z' to c, π_2 from c to rand π_3 from r to x following Moves $\setminus \{spawn\}$ -transitions. By construction, there are no *call*-moves on π_1 . Moreover, there are no *ret*-moves on π_1 since this would imply $z \curvearrowright v$ for a node $v \neq y$. Thus, π_1 is also a path following abstract successors. Moreover, the number m of *call*-moves on π_3 is also equal to the number of *ret*-moves on π_3 , since the same holds true for π , π_1 and π_2 . Since we clearly have m < n, we can transform π_3 to a path π_a from r to x following abstract successors by the induction hypothesis. Since r is the abstract successor of c, the concatenation of the paths π_1 and π_a is thus a path from z' to x following abstract successors.

- (ii) $succ_{a}^{G}(x)$ is defined iff there is a node $z \in V$ with a *call*-successor z' such that there is a path from z' to x following abstract successors. Since $y = succ_{a}^{G}(x)$ and abstract successors are uniquely determined, we can demand equivalently that there is a node $z \in V$ with a *call*-successor z' such that there is a path from z' to y following abstract successors, i.e. $succ_{-}^{G}(y)$ is defined and in this case $succ_{-}^{G}(y) = z = succ_{-}^{G}(x)$.
- (iii) $succ_p^G(x)$ is defined iff there is a node $z \in V$ with a spawn-successor z' such that there is a path π from z' to x following only Moves $\setminus \{spawn\}$ -transitions. If $x \sim y$, there is also a path π' from x to y following only Moves $\setminus \{spawn\}$ -transitions. This also holds trivially if $x \to int y$ or $x \to call y$. Thus, since Moves $\setminus \{spawn\}$ -successors are uniquely determined, we can demand equivalently that there is a node $z \in V$ with a spawn-successor z' such that there is a path π from z' to y following only Moves $\setminus \{spawn\}$ -transitions, i.e. $succ_p^G(y)$ is defined and in this case $succ_p^G(y) = z = succ_p^G(x)$.

D Proofs from Section 6

Proof of Lemma 6. Let $x \in V$ be an arbitrary node. Since $\delta_G(y)$ is defined for all nodes $y \in V$, we only have to show that $succ_f^G(x)$ is defined iff $succ_f^T(\delta_G(x))$ is defined and that $\delta_G(succ_f^G(x)) = succ_f^T(\delta_G(x))$ holds in this case. In order to improve readability, we also write $succ_f^G(x) = succ_f^G(y)$ for $f \in \{g, \uparrow, a, -, p, c\}$ and $y \in V$, if both $succ_f^G(x)$ and $succ_f^G(y)$ are undefined. We show the claim for each successor type separately.

- -f = a: The claim is shown by a case distinction on d(x).
 - If $d(x) \in \{int, spawn\}$, then x has an *int*-successor y, i.e. $succ_a^G(x) = y$. Moreover, we have $\delta_G(y) = \delta_G(x) \cdot 0$ and thus $succ_a^T(\delta_G(x)) = \delta_G(x) \cdot 0 = \delta_G(y) = \delta_G(succ_a^G(x))$.

If d(x) = callRet, there is $y \in V$ with $x \curvearrowright y$, i.e. $succ_a^G(x) = y$. Moreover, we have $\delta_G(y) = \delta_G(x) \cdot 1$ and thus $succ_a^T(\delta_G(x)) = \delta_G(x) \cdot 1 = \delta_G(y) = \delta_G(succ_a^G(x))$.

If $d(x) \in \{call, ret, end\}$, there is no $y \in V$ with $x \curvearrowright y$ and x has no *int*-successor, i.e. $succ_a^G(x)$ is undefined. Moreover, $succ_a^T(\delta_G(x))$ is also undefined in this case.

Thus, we have established $\delta_G \circ succ_a^G(x) = succ_a^T \circ \delta_G(x)$ in each of the cases.

 $- \underline{f} = -:$ Since G is an execution graph, we have $(v_0, x) \in (\bigcup_{d \in \mathsf{Moves}} \to^d)^*$.

Thus, let π be a path from v_0 to x following Moves-successors. We show the claim by induction on the length n of π .

If n = 0, we have $x = v_0$. In this case x has no predecessor and $succ_{-}^{G}(x)$ is undefined. Moreover, we have $p(x) = \bot$ and thus $succ_{-}^{\mathcal{T}}(\delta_{G}(x))$ is also undefined.

If n > 0, let y be the predecessor node of x in π . We show the claim by a case distinction based on what type of predecessor y is.

If $y \to^{call} x$, we have $succ_{-}^{G}(x) = y$ and $\delta_{G}(x) = \delta_{G}(y) \cdot 0$, i.e. $\delta_{G}(y)$ is the parent node of $\delta_{G}(x)$. Moreover, we have p(x) = call and hence $succ_{-}^{T}(\delta_{G}(x)) = \delta_{G}(y) = \delta_{G}(succ_{-}^{G}(x))$.

If $y \to^{int} x$, then $x = succ_a^G(y)$. Thus, by Lemma 18(ii), $succ_-^G(x) = succ_-^G(y)$. Moreover, we have $\delta_G(x) = \delta_G(y) \cdot 0$, i.e. $\delta_G(y)$ is the parent node of $\delta_G(x)$ and we have p(x) = int, i.e. $succ_-^T(\delta_G(x)) = succ_-^T(\delta_G(y))$. By the induction hypothesis, we obtain $\delta_G(succ_-^G(x)) = \delta_G(succ_-^G(y)) = succ_-^T(\delta_G(y))$.

If $y \to^{ret} x$, then by Lemma 18(i), there is $z \in V$ with $z \curvearrowright x$. We then have $x = succ_a^G(z)$ and $\delta_G(x) = \delta_G(z) \cdot 1$, i.e. $\delta_G(z)$ is the parent node of $\delta_G(x)$. The claim follows as in the previous case.

If $y \to spawn x$, then $succ_{-}^{G}(x)$ is undefined since x has no call-, int- or ret-predecessor, i.e. there is no $y \in V$ such that $x = succ_{a}^{G}(y)$ or x is a call-successor of y. Moreover, we have p(x) = spawn and thus $succ_{-}^{T}(\delta_{G}(x))$ is also undefined.

Thus, we have established $\delta_G \circ succ_-^G(x) = succ_-^T \circ \delta_G(x)$ in each of the cases. f = g: The claim is shown by a case distinction on d(x).

If $d(x) \in \{int, call, callRet, spawn\}$, then x has an *int*- or *call*-successor y, i.e. $succ_g^G(x) = y$. Moreover, we have $\delta_G(y) = \delta_G(x) \cdot 0$ and thus $succ_g^{\mathcal{T}}(\delta_G(x)) = \delta_G(x) \cdot 0 = \delta_G(y) = \delta_G(succ_g^G(x))$.

If d(x) = ret, then x has a ret-successor y, i.e. $succ_g^G(x) = y$. By Lemma 18(i), there is a node $z \in V$ with $z \curvearrowright y$ and $z = succ_{-}^G(x)$ and thus $y = succ_a^G(z) = succ_a^G(succ_{-}^G(x))$. Since we have already shown the claim for the abstract successor and the caller, we conclude that $succ_g^T(\delta_G(x)) = succ_a^T(succ_{-}^T(\delta_G(x))) = succ_a^T(\delta_G(succ_{-}^G(x))) = \delta_G(succ_a^G(succ_{-}^G(x))) = \delta_G(succ_g^G(x))$.

If d(x) = end, then x has no successor, i.e. $succ_g^G(x)$ is undefined. Moreover, $succ_g^T(\delta_G(x))$ is also undefined in this case.

Thus, we have established $\delta_G \circ succ_g^G(x) = succ_g^T \circ \delta_G(x)$ in each of the cases. $f = \uparrow$: We show the claim by a case distinction on p(x).

If $p(x) \in \{int, call\}$, then x has a p(x)-predecessor y, i.e. $succ_{\uparrow}^{G}(x) = y$. Moreover, $\delta_{G}(y)$ is the parent node of $\delta_{G}(x) = \delta_{G}(y) \cdot 0$. Thus, we have $\delta_{G}(succ_{\uparrow}^{G}(x)) = \delta_{G}(y) = succ_{\uparrow}^{T}(\delta_{G}(x))$.

If p(x) = ret, then x has a ret-predecessor y, i.e. $succ^{G}_{\uparrow}(x) = y$, and by Lemma 18(i) there is a node $z \in V$ with $z \curvearrowright x$ and $succ^{G}_{-}(y) = z$. Using the claim for f = -, which we have already seen, we thus have $\delta_{G}(z) =$ A Navigation Logic for Recursive Programs with Dynamic Thread Creation

29

 $\delta_G(succ^G_{-}(y)) = succ^T_{-}(\delta_G(y))$. By definition, the caller predecessor of $\delta_G(y)$ has to be a node with a *call*-child $\delta_G(z')$ such that $\delta_G(z')$ is an $\{int, ret\}$ ancestor of $\delta_G(y)$. Moreover, since x is a ret-successor of y, we have d(y) = retand hence $\delta_G(y)$ is a leaf. Thus, $\delta_G(y)$ is the $\{int, ret\}$ -descendant leaf of $\delta_G(z')$, which is the left child of the parent node $\delta_G(z)$ of $\delta_G(x) = \delta_G(z) \cdot 1$, i.e. $succ^{\mathcal{T}}_{\uparrow}(\delta_G(x)) = \delta_G(y)$. Hence, $\delta_G(succ^G_{\uparrow}(x)) = \delta_G(y) = succ^{\mathcal{T}}_{\uparrow}(\delta_G(x))$. If $p(x) \in \{spawn, \bot\}$, then x has no *int-*, call- or ret-predecessor, i.e. $succ^G_{\uparrow}(x)$ is undefined. Moreover, $succ^{\mathcal{T}}(\delta_G(x))$ is also undefined in this case.

Thus, we have established $\delta_G \circ succ^G_{\uparrow}(x) = succ^{\mathcal{T}}_{\uparrow} \circ \delta_G(x)$ in each of the cases. $-\underline{f} = p$: Since G is an execution graph, we have $(v_0, x) \in (\bigcup \to^d)^*$. Thus, $d \in Moves$ let π be a path from v_0 to x following Moves-successors. We show the claim by induction via the length n of π .

If n = 0, we have $x = v_0$. In this case x has no predecessor and $succ_n^G(x)$ is undefined. Moreover, we have $p(x) = \bot$ and thus $succ_p^{\mathcal{T}}(\delta_G(x))$ is also undefined.

If n > 0, let y be the predecessor node of x in π . We show the claim by a case distinction based on what type of predecessor y is.

If $y \to^{ret} x$, by Lemma 18(i), there is $z \in V$ with $z \curvearrowright x$. Hence, if $y \to^{int} x$, $y \to^{call} x$ or $y \to^{ret} x$, there is a node $z \in V$ with $z \to^{int} x$, $z \to^{call} x$ or $z \curvearrowright x$. Thus, by Lemma 18(ii), $succ_p^G(x) = succ_p^G(z)$. Moreover, $\delta_G(z)$ is the parent node of $\delta_G(x)$ and we have $p(x) \in \{int, call, ret\}$, i.e. $succ_p^{\mathcal{T}}(\delta_G(x)) =$ $succ_n^{\mathcal{T}}(\delta_G(z))$. By the induction hypothesis, we obtain

$$\begin{split} &\delta_G(succ_p^G(x)) = \delta_G(succ_p^G(z)) = succ_p^{\mathcal{T}}(\delta_G(z)) = succ_p^{\mathcal{T}}(\delta_G(x)). \\ &\text{If } y \to^{spawn} x, \text{ we have } succ_p^G(x) = y \text{ and } \delta_G(x) = \delta_G(y) \cdot 1, \text{ i.e. } \delta_G(y) \end{split}$$
is the parent node of $\delta_G(x)$. Moreover, we have p(x) = spawn and hence $succ_p^{\mathcal{T}}(\delta_G(x)) = \delta_G(y) = \delta_G(succ_p^G(x)).$

Thus, we have established $\delta_G \circ succ_p^G(x) = succ_p^T \circ \delta_G(x)$ in each of the cases. $\underline{f = c}$: We distinguish two cases for d(x).

If d(x) = spawn, then x has a spawn-successor y, i.e. we have $succ_c^G(x) = y$. Moreover, we have $\delta_G(y) = \delta_G(x) \cdot 1$ and thus $succ_c^T(\delta_G(x)) = \delta_G(y) =$ $\delta_G(succ_c^G(x)).$

If $d(x) \neq spawn$, then x has no spawn-successor, i.e. $succ_c^G(x)$ is undefined. Moreover, $\delta_G(x)$ is also undefined in this case.

Thus, we have established $\delta_G \circ succ_c^G(x) = succ_c^T \circ \delta_G(x)$ in both cases.

Proofs from Section 7 \mathbf{E}

Proof of Theorem 7. We first show that $\mathcal{A}_{\mathsf{ET}}$ accepts all execution trees.

For this, let $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \sim)$ be an execution graph and $\mathcal{T}(G) =$ (T,r) be the tree representation of G. We inductively define a map $r_R: V \to$

- $\{0,1\}$ as follows. First, we set $r_R(v_0) := 0$. Then, for each node $x \in V$,
- if there is a node $y \in V$ such that y is an *int*-predecessor of x or $y \curvearrowright x$ (the latter holds by Lemma 18(i) iff x has a ret-predecessor), we set $r_R(x) :=$ $r_R(y),$

- 30 R. Lakenbrink et al.
- if x (i) has a spawn-predecessor or (ii) it has a call-predecessor y and there is no node $z \in V$ with $y \curvearrowright z$, we set $r_B(x) := 0$ and
- if x has a *call*-predecessor y and there is a node $z \in V$ with $y \curvearrowright z$, we set $r_R(x) := 1$.

Next, we define a map $r_A : T \to Q$ by $r_A(\delta_G(x)) := (p(x), r_R(x))$ and show the following claim:

<u>Claim</u>: r_A is an accepting (ε, q_0) -run of $\mathcal{A}_{\mathsf{ET}}$ over $\mathcal{T}(G)$.

We first show that r_A is an (ε, q_0) -run of $\mathcal{A}_{\mathsf{ET}}$ over $\mathcal{T}(G)$. For the initial node, we have $p(v_0) = \bot$ and $r_R(v_0) = 0$ and thus $r_A(\varepsilon) = r_A(\delta_G(v_0)) = (\bot, 0) = q_0$.

Let $t = \delta_G(x) \in T$ be an arbitrary node with r(t) = (l(x), d(x), p(x)) and $r_A(t) = (p(x), c)$. By a case distinction on d(x), we show that the children of t satisfy the transition function in this node.

- If d(x) = int, then x has exactly one *int*-successor y with p(y) = int, $r_R(y) = r_R(x) = c$ and $\delta_G(y) = t \cdot 0$. Thus, $\{(0, r_A(t \cdot 0))\} = \{(0, (int, c))\}$ satisfies $\rho((p(x), c), r(t))$.
- If d(x) = call, then x has exactly one call-successor y with p(y) = call, there is no node $z \in V$ with $x \curvearrowright z$, i.e. $r_R(x) = 0$, and $\delta_G(y) = t \cdot 0$. Thus $\{(0, r_A(t \cdot 0))\} = \{(0, (call, 0))\}$ satisfies $\rho((p(x), c), r(t))$.
- If d(x) = callRet, then x has a call-successor y with p(y) = call, there is a node $z \in V$ with $x \curvearrowright z$ and by Lemma 18(i), there is a node $z' \in V$ with $z' \rightarrow^{ret} z$, i.e. p(z) = ret. Thus, we have $r_R(y) = 1$, $\delta_G(y) = t \cdot 0$, $r_R(z) = r_R(x) = c$, and $\delta_G(z) = t \cdot 1$. Thus, $\{(0, r_A(t \cdot 0)), (1, r_A(t \cdot 1))\} =$ $\{(0, (call, 1)), (1, (ret, c))\}$ satisfies $\rho((p(x), c), r(t))$.
- If d(x) = spawn, then x has an *int*-successor y and a spawn-successor z with p(y) = int, $r_R(y) = r_R(x) = c$, $\delta_G(y) = t \cdot 0$, p(z) = spawn, $r_R(z) = 0$, and $\delta_G(z) = t \cdot 1$. Thus, $\{(0, r_A(t \cdot 0)), (1, r_A(t \cdot 1))\} = \{(0, (int, c)), (1, (spawn, 0))\}$ satisfies $\rho((p(x), c), r(t))$.
- If d(x) = ret, then x has a ret-successor y. By Lemma 18(i), there is a node $z \in V$ with $z \curvearrowright y$ and $succ_{-}^{G}(x) = z$, i.e. there is a path from the call-successor z' of z to x following abstract successors. Then we clearly have $c = r_{R}(x) = r_{R}(z') = 1$ by construction. Thus, \emptyset satisfies $true = \rho((p(x), c), r(t))$.
- If d(x) = end, then x has no successors. Assume towards contradiction that $r_R(x) = 1$. Clearly, by construction, there is a node $z \in V$ with $z \curvearrowright y$ and a path from the *call*-successor z' of z to x following abstract successors. Thus, $\delta_G(x)$ is the $\{int, ret\}$ -descendant leaf of the left child $\delta_G(z') = \delta_G(z) \cdot 0$ of the parent node $\delta_G(z)$ of $\delta_G(y) = \delta_G(z) \cdot 1$. Then we have $succ^{\mathcal{T}(G)}(\delta_G(y)) = \delta_G(x)$. Using Lemma 6, we obtain

 $\delta_G(succ_{\uparrow}^G(y)) = succ_{\uparrow}^{\mathcal{T}(G)}(\delta_G(y)) = \delta_G(x)$ and thus $succ_{\uparrow}^G(y) = x$ since δ_G is injective. This means that y is a successor of x, which contradicts our assumption that x has no successor. Thus, $c = r_R(x) = 0$, and \emptyset satisfies $true = \rho((p(x), c), r(t))$.

Thus, r_A is an (ε, q_0) -run of $\mathcal{A}_{\mathsf{ET}}$ over $\mathcal{T}(G)$.

It remains to show that the run is accepting. Assume towards contradiction that there is an infinite path $\delta_G(x_0)\delta_G(x_1)\ldots$ in T where the priority 0 occurs

only finitely often, i.e. there is a minimal i > 0 such that for all $j \ge i$ we have $r_R(x_i) = 1$. By construction, there must be a node $y \in V$ with $x_{i-1} \curvearrowright y$ such that x_{i-1} is the *call*-predecessor of x_i and we have $r_R(y) = r_R(x_{i-1}) = 0$. Thus, there is a finite path in G from x_{i-1} to $y \neq x_{i-1}$ following only Moves \{spawn\}successors such that the number of *call*-moves on the path is equal to the number of *ret*-moves on the path. On the other hand, the infinite path in the tree T up from $\delta_G(x_i)$ cannot contain a spawn-child by construction and thus provides an infinite path in G starting in x_i and following only call- or abstract successors and thus also an infinite path in G starting in x_{i-1} following only Moves $\{spawn\}$ successors. Since $Moves \setminus \{spawn\}$ -successors are uniquely determined, y must be contained in this path. But since the paths between nodes z and z' with $z \sim z'$ are the minimal paths of length greater than zero from z such that the number of *call*-moves is equal to the number of *ret*-moves, the number of *call*-moves on the infinite path up from x_i so far is always greater or equal to the number of ret-moves on this path so far. Since x_i is the call-successor of x_{i-1} , the number of *call*-moves on the infinite path in G from x_{i-1} so far is always greater than the number of *ret*-moves on this path so far after the first move. This contradicts the fact that $y \neq x_{i-1}$ is contained in this path. Thus, for all infinite paths in T, the priority 0 occurs infinitely often, and the run is accepting.

We now show that all trees accepted by $\mathcal{A}_{\mathsf{ET}}$ are execution trees. For this, let $\mathcal{T}_A = (T, r)$ be a tree accepted by $\mathcal{A}_{\mathsf{ET}}$, witnessed by the accepting (ε, q_0) -run r_A of $\mathcal{A}_{\mathsf{ET}}$ over \mathcal{T}_A . We define an execution graph $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \curvearrowright)$ and show that \mathcal{T}_A is the tree representation of G.

The components of G are defined as follows. First, we set V := T and $l(t) := l_t$ where $r(t) = (l_t, d_t, p_t)$. For the definition of the transitions of G, let $t \in V$ be a node with $r_A(t) = q$ and r(t) = (l', d, p). Outgoing transitions in t are defined based on d.

- If $d \in \{int, call\}$, then ar(r(t)) = 1, i.e. t has a child $t \cdot 0$ and we include $t \rightarrow^d t \cdot 0$.
- If $d \in \{ callRet, spawn \}$, then ar(r(t)) = 2, i.e. t has two children $t \cdot 0$ and $t \cdot 1$. For d = callRet, we include $t \rightarrow^{call} t \cdot 0$ and $t \frown t \cdot 1$. For d = spawn, we include $t \rightarrow^{int} t \cdot 0$ and $t \rightarrow^{spawn} t \cdot 1$.

In order to define the transition relation \rightarrow^{ret} , we show by induction on the length of t that for all $t \in V$ with $r_A(t) = (p, c)$:

(*) c = 1 iff there are $t_1, t_2, t_3 \in V$ with $t_1 \rightarrow^{call} t_2, t_1 \frown t_3$ and there is a path from t_2 to t following only *int*- and *ret*-children.

In the base case, where $t = \varepsilon$, we have c = 0 and t has no parent node and no *call*-predecessor.

In the inductive step, let t be of the form $t' \cdot i$ for $i \in \{0, 1\}$ with $r_A(t') = (p', c')$ and r(t') = (l', d, p''). The claim is shown by a case distinction on d.

- If d = int, we have i = 0 and $\rho(r_A(t'), r(t')) = (0, (int, c'))$, i.e. c = c'. Since $t = t' \cdot 0$ is the *int*-child of t' and *int*- or *ret*-children are uniquely determined, the required nodes and the path exist for t iff they exist for t', and the latter holds by induction hypothesis iff c = c' = 1.

- 32 R. Lakenbrink et al.
- If d = call, we have i = 0 and $\rho(r_A(t'), r(t')) = (0, (call, 0))$, i.e. c = 0. Since t is no *int* or *ret*-child and there is no node $\tilde{t} \in V$ with $t' \curvearrowright \tilde{t}$, the required nodes and the path do not exist for $t' \cdot 0 = t$.
- If d = callRet, we have $\rho(r_A(t'), r(t')) = (0, (call, 1)) \land (1, (ret, c'))$. If i = 0, then c = 1. Since $t' \rightarrow^{call} t' \cdot 0$ and $t' \frown t' \cdot 1$, the nodes $t_1 = t', t_2 = t' \cdot 0$ and $t_3 = t' \cdot 1$ and the empty path from $t_2 = t' \cdot 0 = t$ to t witness that (*) holds. If i = 1, then c = c' and $t' \frown t$. Since *int*- or *ret*-children are uniquely determined, the required nodes and the path exist for t iff they exist for t', and the latter holds by induction hypothesis iff c = 1.
- If d = spawn, we have $\rho(r_A(t'), r(t')) = (0, (int, c')) \land (1, (spawn, 0))$. If i = 0, then c = c'. Since $t' \to int t' \cdot 0 = t$ and *int*- or *ret*-children are uniquely determined, the required nodes and the path exist for t iff they exist for t', and the latter holds by induction hypothesis iff c = c' = 1. If i = 1, then c = 0. Since t is no *int*- or *ret*-child and it has no *call*-predecessor, the required nodes and the path do not exist for $t' \cdot 1 = t$.

Given (*), for each node $t \in V$ with $r_A(t) = (p, 1)$ and r(t) = (l', ret, p'), there are nodes $t_1, t_2, t_3 \in V$ with $t_1 \rightarrow^{call} t_2, t_1 \frown t_3$ and there is a path from t_2 to t following only *int*- and *ret*-children. We then include the transition $t \rightarrow^{ret} t_3$.

We now show that G is indeed an execution graph. For this, we separately check each of the conditions from the definition of execution graphs.

- 1. Clearly, the node ε has no predecessor with respect to $(\rightarrow^d)_{d \in \mathsf{Moves}}$. Moreover, every node $t \neq \varepsilon$ has exactly one predecessor with respect to $(\rightarrow^d$ $d \in Moves \setminus \{ret\}$ and \frown . Additionally, t can only have a ret-predecessor, if t has a predecessor with respect to \uparrow . Now let $t' \in V$ be a node with $t' \uparrow t$. We show that t has a unique ret-predecessor in this case. We know that t' has a call-successor $t' \cdot 0 \in V$. Now consider the unique maximal path in T from $t' \cdot 0$ following only *int*- and *ret*-chlidren. By (*), we have $r_A(x) = (p, 1)$ for some $p \in \mathsf{Moves} \cup \{\bot\}$ for all nodes x on the path. Since $\Omega(q) = 1$ for all states q visited on the path and the run r_A is accepting, the path cannot be infinite. Thus, there is a node x in the path that has no *int*- or *ret*-child. Assume towards contradiction that x has a call-child y. For $r_A(x) = (p, c)$ and r(x) = (l', d, p'), we must have d = call in this case. Since $y = x \cdot 0$ has to satisfy $\rho((p,c), (l', call, p'))$, we thus have c = 0 which contradicts our assumption that x is on the given path. Therefore, x also has no *call*-child, i.e. we have $d \in \{ret, end\}$. Since $\rho(r_A(x), r(x)) = \rho((p, 1), (l', d, p))$ must be satisfied by the children of x, we then have d = ret, since otherwise we had $\rho(r_A(x), r(x)) = false$. Thus, $x \to^{ret} t$ holds by construction. Clearly, since *int-* or *ret-*children are uniquely determined, we can only have $y \to^{ret} t$ for a node $y \in T$, if y is on the given path and it is a leaf, i.e. if y = x.
- 2. By construction of V, there is a path π from ε to x following only Moves $\{ret\}$ -transitions or nesting edges for all nodes $x \in V$. Consider the first nodes y, z on the path π with $y \curvearrowright z$, if they exist. As shown in (i), there is a path π' from the *call*-successor y' of y to the *ret*-predecessor z' of z following only *int* and *ret*-children, i.e. *int*-transitions and nesting edges. Now include the concatenation of the paths $y \rightarrow^{call} y', \pi'$ and $z' \rightarrow^{ret} z$ in

the initial path π between the nodes y and z and repeat this construction. If this provided an infinite procedure, we would obtain an infinite path from y'following only *call*- or *int*-successors. However, since $y \curvearrowright z$, all states visited on the path are of the form (p, 1), which contradicts the fact that the run r_A is accepting. Thus, the given procedure terminates and we finally obtain a path from ε to x following only Moves-transitions.

- 3. By construction, each node clearly either has (a) exactly one *int*-successor and at most one *spawn*-successor, (b) one *call*-successor, (c) one *ret*-successor or (d) no successors.
- 4. Let $x \in V$ be a node. In the construction of the path from ε to x following only Moves-transitions in (ii), we never add a *spawn*-transition on the path between nodes y and z with $y \frown z$. Moreover, for each new nesting edge we add one *call*-transition and one *ret*-transition afterwards on the path. Thus, on the path from ε or a node with a *spawn*-predecessor to x, the number of *call*-moves is greater or equal to the number of *ret*-moves, since we start without any *ret*-moves in this construction.
- 5. Let $x \in V$ be a node with a *call*-successor. For the first direction, let $y \in V$ be a node with $x \frown y$. By the construction given in (ii), we obtain a path from x to y following only Moves $\{spawn\}$ -transitions from x to y such that the number of *call*-moves on the path is equal to the number of *ret*-moves on the path. Moreover, for all nodes z between x and y, the number of *call*-moves on the path between x and z is greater than the number of *ret*-moves on this path, i.e. y is given as the node with the stated property such that the witnessing path has minimal length.

For the other direction, let $y \in V$ with $y \neq x$ be a node such that there is a path from x to y following only Moves $\setminus \{spawn\}$ -transitions where the number of *call*-moves on the path is equal to the number of *ret*-moves on the path and the path has minimal length. Consider a subpath of this path from a node z to a node z' starting with a *call*-transition, then following only *int*-transitions and finally ending with a *ret*-transition. By the construction in (ii), we only obtain such a path if $z \cap z'$. Now remove the nodes between z and z' from the path and repeat this procedure. Since the initial path has minimal length, it must end with a *ret*-transition, it starts with a *call*transition and we finally see that $x \cap y$.

Thus, we have shown that G is indeed an execution graph. It remains to show that \mathcal{T}_A is the tree representation of G.

For this, we show by induction over the length of t that for all $t \in T$ with $r_A(t) = (p, c)$ we have $\delta_G(t) = t$ and either $p \neq \bot$ and t has a p-predecessor or $p = \bot$ and $t = \varepsilon$.

In the base case, where $t = \varepsilon$, we have $\delta_G(t) = \varepsilon = t$ since $\varepsilon \in V$ has no predecessor. Moreover, we have $r_A(t) = q_0 = (\bot, 0)$, since r_A is an (ε, q_0) -run, i.e. $p = \bot$.

In the inductive step, let t be of the form $t' \cdot i$ for $i \in \{0, 1\}$ with r(t') = (l', d, p'). Since T is prefix-closed, we have $t' \in T$ and thus, by the induction hypothesis, $\delta_G(t') = t'$. Here, we make a case distinction on d.

- If $d \in \{int, call\}$, then ar(r(t')) = 1, i.e. i = 0 since $t = t' \cdot i \in T$. Moreover, we have $t' \to^{int} t$ and thus $\delta_G(t) = \delta_G(t') \cdot 0 = t' \cdot 0 = t$ and $r_A(t) = (d, c)$ for a $c \in \{0, 1\}$ and t has a d-predecessor t'.
- If d = callRet, then t' has a call-successor $t' \cdot 0$ and we have $t' \frown t' \cdot 1$. If additionally i = 0, we have $\delta_G(t) = \delta_G(t') \cdot 0 = t' \cdot 0 = t$ and $r_A(t) = (call, 1)$ and t has a call-predecessor t'. If instead i = 1, we have $\delta_G(t) = \delta_G(t') \cdot 1 =$ $t' \cdot 1 = t$ and $r_A(t) = (ret, c)$ for a $c \in \{0, 1\}$ and since $t' \frown t' \cdot 1$ and G is an execution graph, $t = t' \cdot 1$ has a ret-predecessor by Lemma 18(i).
- If d = spawn, then t' has an *int*-successor $t' \cdot 0$ and a spawn-successor $t' \cdot 1$. If additionally i = 0, we have $\delta_G(t) = \delta_G(t') \cdot 0 = t' \cdot 0 = t$ and $r_A(t) = (int, c)$ for a $c \in \{0, 1\}$ and t has an *int*-predecessor t'. If instead i = 1, we have $\delta_G(t) = \delta_G(t') \cdot 1 = t' \cdot 1 = t$ and $r_A(t) = (spawn, 0)$ and t has a spawnpredecessor t'.
- We cannot have $d \in \{ret, end\}$, since that would mean ar(r(t')) = 0, i.e. t' has no children.

Moreover, for all nodes $t \in V$ with $r_A(t) = (p, c)$ and r(t) = (l', d, p'), the boolean formula $\rho((p, c), (l', d, p'))$ must be satisfied by the children of t, i.e we have p' = p since otherwise we had $\rho((p, c), (l', d, p')) = false$. Thus, p' characterizes the predecessor type of t as required. Finally, it is straightforward to see that for all $t \in T$ with r(t) = (l, d, p), d specifies the successor types of $t = \delta_G^{-1}(t)$ as required for tree representations.

Overall, we conclude $\mathcal{T}_A = (T, r) = (im(\delta_G), r) = \mathcal{T}(G).$

Proof of Theorem 8. We prove the theorem by induction on the structure of φ . Therefore, we also have do deal with non-closed subformulae and consider valuations to decide whether a subformula is satisfied. In order to do this in a formal way, we consider automata with special states X_1, \ldots, X_n , called *holes*, that can be filled with sets of nodes L_1, \ldots, L_n of a given tree. Intuitively, such an automaton can operate on a tree as before, but when a hole X_i is encountered during a run and we are at the tree node t, then we do not continue on the current path and say that it is accepting iff $t \in L_i$.

Formally, let $\mathcal{A} = (Q, q_0, \rho, \Omega)$ be a 2-way alternating tree automaton over (Σ, ar) -labelled binary trees with states $q, X_1, ..., X_n \in Q, \mathcal{T} = (T, l)$ be a (Σ, ar) -labelled binary tree, $t \in T$ be a tree node and $L_1, ..., L_n \subseteq T$ be sets of tree nodes. A (t, q)-run over $\mathcal{A}[X_1 : L_1, ..., X_n : L_n]$ is defined as a (t, q)-run (T_r, r) over \mathcal{A} except that for nodes $x \in T_r$ with $r(x) = (t', X_i)$ for a $t' \in T$, the positive boolean formula $\rho(X_i, l(t'))$ that has to be satisfied by the children of x is replaced by true, if $t' \in L_i$, and by false, if $t' \notin L_i$. The acceptance of such a path is then defined as before. By $\mathcal{L}_q^{\mathcal{T}}(\mathcal{A}[X_1 : L_1, ..., X_n : L_n])$ we denote the set of nodes $t \in T$ such that there is an accepting (t, q)-run over $\mathcal{A}[X_1 : L_1, ..., X_n : L_n]$.

For the inductive proof, we now assume that the free variables of the current formula $\psi \in Sub(\varphi)$ are holes in the automaton and show that the language of this automaton corresponds to the semantics of ψ . Intuitively, we fill the holes in the automaton, i.e. the free variables of ψ , with the same sets of nodes as specified by a given valuation that we consider for the semantics of ψ . More formally, the holes are filled by sets of tree nodes that correspond to given sets of graph nodes in the valuation. Therefore, we lift the function $\delta_G \colon V \to T$ for the execution graph $G = (V, l, (\to^d)_{d \in \text{Moves}}, \frown)$ with $\mathcal{T}(G) = \mathcal{T} = (T, r)$ to a function $\tilde{\delta}_G \colon 2^V \to 2^T$ by $\tilde{\delta}_G(A) \coloneqq \{\delta_G(a) \mid a \in A\}$ and show the following claim:

<u>Claim</u>: For all fixpoint variable assignments \mathcal{V} , subformulae $\psi \in Sub(\varphi)$ with free variables X_1, \ldots, X_n and $L_1, \ldots, L_n \subseteq V$ we have

 $\mathcal{L}^{\mathcal{T}}_{\psi}(\tilde{\mathcal{A}}_{\varphi}[X_1:\tilde{\delta}_G(L_1),\ldots,X_n:\tilde{\delta}_G(L_n)]) = \tilde{\delta}_G(\llbracket\psi\rrbracket^G_{\mathcal{V}[X_1\mapsto L_1,\ldots,X_n\mapsto L_n]}).$

Since φ is closed, this implies in particular that

$$\mathcal{T} \in \mathcal{L}(\tilde{\mathcal{A}}_{\varphi}) \Leftrightarrow \varepsilon \in \mathcal{L}_{\varphi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}) \Leftrightarrow \varepsilon \in \tilde{\delta}_{G}(\llbracket \varphi \rrbracket^{G}) \Leftrightarrow v_{0} \in \llbracket \varphi \rrbracket^{G} \Leftrightarrow G \in \llbracket \varphi \rrbracket.$$

We now proceed with the structural induction.

– For $\psi \equiv ap \in AP$ we have

$$\mathcal{L}_{\psi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}) = \{\delta_G(x) \in T \mid ap \in l(x)\}\$$

= $\tilde{\delta}_G(\{x \in V \mid ap \in l(x)\})$
= $\tilde{\delta}_G(\llbracket \psi \rrbracket_{\mathcal{V}}^G).$

- For $\psi \equiv \neg ap$ with $ap \in AP$, the claim follows analogously.
- For $\psi \equiv X$ and $L \subseteq V$ we have

$$\mathcal{L}^{\mathcal{T}}_{\psi}(\tilde{\mathcal{A}}_{\varphi}[X:\tilde{\delta}_{G}(L)])$$

= $\tilde{\delta}_{G}(L)$
= $\tilde{\delta}_{G}(\mathcal{V}[X\mapsto L](X))$
= $\tilde{\delta}_{G}(\llbracket\psi\rrbracket^{G}_{\mathcal{V}[X\mapsto L]}).$

- For $\psi \equiv \psi_1 \lor \psi_2$, let $X_{k_1^i}, \ldots, X_{k_{n_i}^i}$ be the free variables of ψ_i for $i \in \{1, 2\}$. We clearly have

$$\mathcal{L}_{\psi_i}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_1:\tilde{\delta}_G(L_1),\ldots,X_n:\tilde{\delta}_G(L_n)]) = \mathcal{L}_{\psi_i}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{k_1^i}:\tilde{\delta}_G(L_{k_1^i}),\ldots,X_{k_{n_i}^i}:\tilde{\delta}_G(L_{k_1^i})])$$

since a (t, ψ_i) -run over $\tilde{\mathcal{A}}_{\varphi}[X_{k_1^i} : \tilde{\delta}_G(L_{k_1^i}), \ldots, X_{k_{n_i}^i} : \tilde{\delta}_G(L_{k_{n_i}^i})]$ can only reach nodes labelled by a fixpoint variable X if X occurs in ψ_i . Moreover, $\llbracket \psi_i \rrbracket_{\mathcal{V}[X_{k_1^i} \mapsto L_{k_{n_i}^i} \mapsto L_{k_{n_i}^i}]}^G = \llbracket \psi \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \ldots, X_n \mapsto L_n]}^G$, since all the free variables $X_{k_1^i}, \ldots, X_{k_{n_i}^i}$ of ψ_i are also among the free variables X_1, \ldots, X_n of ψ . Thus, we have

$$\mathcal{L}_{\psi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{1}:\tilde{\delta}_{G}(L_{1}),\ldots,X_{n}:\tilde{\delta}_{G}(L_{n})])$$

=
$$\bigcup_{i\in\{0,1\}}\mathcal{L}_{\psi_{i}}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{1}:\tilde{\delta}_{G}(L_{1}),\ldots,X_{n}:\tilde{\delta}_{G}(L_{n})])$$

$$= \bigcup_{i \in \{0,1\}} \mathcal{L}_{\psi_i}^{\mathcal{T}} (\tilde{\mathcal{A}}_{\varphi}[X_{k_1^i} : \tilde{\delta}_G(L_{k_1^i}), \dots, X_{k_{n_i}^i} : \tilde{\delta}_G(L_{k_{n_i}^i})])$$

$$\stackrel{(\mathrm{IH})}{=} \bigcup_{i \in \{0,1\}} \tilde{\delta}_G(\llbracket \psi_i \rrbracket_{\mathcal{V}[X_{k_1^i} \mapsto L_{k_1^i}, \dots, X_{k_{n_i}^i} \mapsto L_{k_{n_i}^i}])$$

$$= \bigcup_{i \in \{0,1\}} \tilde{\delta}_G(\llbracket \psi_i \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \dots, X_n \mapsto L_n]}^G)$$

$$= \tilde{\delta}_G(\bigcup_{i \in \{0,1\}} \llbracket \psi_i \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \dots, X_n \mapsto L_n]}^G)$$

$$= \tilde{\delta}_G(\llbracket \psi \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \dots, X_n \mapsto L_n]}^G)$$

where equation (IH) uses the induction hypothesis.

- For $\psi \equiv \psi_1 \wedge \psi_2$, the claim follows analogously since $\tilde{\delta}_G$ is injective. For $\psi \equiv \bigcirc^f \psi'$ with $f \in \{g, \uparrow, a, -, p, c\}$, the free variables X_1, \ldots, X_n of ψ are also the free variables of ψ' . Following the definition of the tree successor functions, we easily see that

$$\begin{aligned} \mathcal{L}_{\psi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{1}:\tilde{\delta}_{G}(L_{1}),\ldots,X_{n}:\tilde{\delta}_{G}(L_{n})]) \\ &= \{t \in T \mid succ_{f}^{\mathcal{T}}(t) \text{ is defined and} \\ succ_{f}^{\mathcal{T}}(t) \in \mathcal{L}_{\psi'}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{1}:\tilde{\delta}_{G}(L_{1}),\ldots,X_{n}:\tilde{\delta}_{G}(L_{n})]) \} \\ \stackrel{(\mathrm{IH})}{=} \{t \in T \mid succ_{f}^{\mathcal{T}}(t) \text{ is defined and } succ_{f}^{\mathcal{T}}(t) \in \tilde{\delta}_{G}(\llbracket \psi' \rrbracket_{\mathcal{V}[X_{1} \mapsto L_{1},\ldots,X_{n} \mapsto L_{n}]}^{G}) \} \\ \stackrel{(*)}{=} \{\delta_{G}(x) \in T \mid succ_{f}^{G}(x) \text{ is defined and} \\ \delta_{G}(succ_{f}^{G}(x)) \in \tilde{\delta}_{G}(\llbracket \psi' \rrbracket_{\mathcal{V}[X_{1} \mapsto L_{1},\ldots,X_{n} \mapsto L_{n}]}^{G}) \} \\ &= \tilde{\delta}_{G}(\{x \in V \mid succ_{f}^{G}(x) \text{ is defined and } succ_{f}^{G}(x) \in \llbracket \psi' \rrbracket_{\mathcal{V}[X_{1} \mapsto L_{1},\ldots,X_{n} \mapsto L_{n}]}^{G}\} \\ &= \tilde{\delta}_{G}(\llbracket \psi \rrbracket_{\mathcal{V}[X_{1} \mapsto L_{1},\ldots,X_{n} \mapsto L_{n}]}^{G}), \end{aligned}$$

where equation (IH) uses the induction hypothesis and equation (*) uses Lemma 6.

- For $\psi \equiv \bigcirc^{\bar{f}} \psi'$ with $f \in \{g, \uparrow, a, -, p, c\}$ we observe that a (t, ψ) -run behaves as a $(t, \bigcirc^f \psi')$ -run except that we move to *true* instead of *false*, if a desired successor or predecessor does not exist. Thus, analogously to the previous case, it is easy to see that

$$\begin{aligned} \mathcal{L}_{\psi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{1}:\tilde{\delta}_{G}(L_{1}),\ldots,X_{n}:\tilde{\delta}_{G}(L_{n})]) \\ &= \{t \in T \mid succ_{f}^{\mathcal{T}}(t) \text{ is undefined or} \\ succ_{f}^{\mathcal{T}}(t) \in \mathcal{L}_{\psi'}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_{1}:\tilde{\delta}_{G}(L_{1}),\ldots,X_{n}:\tilde{\delta}_{G}(L_{n})])\} \\ &= \tilde{\delta}_{G}(V \setminus \{x \in V \mid succ_{f}^{G}(x) \text{ is defined and} \\ succ_{f}^{G}(x) \notin \llbracket \psi' \rrbracket_{\mathcal{V}[X_{1} \mapsto L_{1},\ldots,X_{n} \mapsto L_{n}]}^{G}\}) \\ &= \tilde{\delta}_{G}(V \setminus \{x \in V \mid succ_{f}^{G}(x) \text{ is defined and} \\ succ_{f}^{G}(x) \in \llbracket \neg \psi' \rrbracket_{\mathcal{V}[X_{1} \mapsto L_{1},\ldots,X_{n} \mapsto L_{n}]}^{G}\}) \end{aligned}$$

A Navigation Logic for Recursive Programs with Dynamic Thread Creation

$$= \tilde{\delta}_G(V \setminus \llbracket \bigcirc^f \neg \psi' \rrbracket^G_{\mathcal{V}[X_1 \mapsto L_1, \dots, X_n \mapsto L_n]})$$

= $\tilde{\delta}_G(\llbracket \neg \bigcirc^f \neg \psi' \rrbracket^G_{\mathcal{V}[X_1 \mapsto L_1, \dots, X_n \mapsto L_n]})$
= $\tilde{\delta}_G(\llbracket \psi \rrbracket^G_{\mathcal{V}[X_1 \mapsto L_1, \dots, X_n \mapsto L_n]}).$

- For $\psi \equiv \mu X.\psi'$, the free variables of ψ' are given by X_1, \ldots, X_n, X . We observe that a (t, ψ) -run over $\tilde{\mathcal{A}}_{\varphi}[X_1 : \tilde{\delta}_G(L_1), \ldots, X_n : \tilde{\delta}_G(L_n)]$ can only visit states φ' of the form $\mu X.\psi''$ or $\nu X.\psi''$ if φ' is a subformula of ψ . Therefore, $\Omega(\psi)$ is the lowest priority occurring in the run so that the state ψ can only be visited finitely often if the run is accepting. This means we can characterize $\mathcal{L}_{\psi}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_1 : \tilde{\delta}_G(L_1), \ldots, X_n : \tilde{\delta}_G(L_n)])$ as the least fixpoint of the function $f: 2^T \to 2^T$ with $f(\tilde{\delta}_G(L)) := \mathcal{L}_{\psi'}^{\mathcal{T}}(\tilde{\mathcal{A}}_{\varphi}[X_1 : \tilde{\delta}_G(L_1), \ldots, X_n : \tilde{\delta}_G(L_n), X : \tilde{\delta}_G(L)])$. By the induction hypothesis, we obtain $f(\tilde{\delta}_G(L)) = \tilde{\delta}_G(\llbracket \psi' \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \ldots, X_n \mapsto L_n], X, \psi')$. By Corollary 17, the least fixpoint of α_S is given by $\llbracket \psi \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \ldots, X_n \mapsto L_n]}^{\mathcal{T}}$. Since the function $\tilde{\delta}_G$ is trivially monotone and bijective because δ_G is bijective, we conclude by Lemma 15 that $\tilde{\delta}_G(\llbracket \psi \rrbracket_{\mathcal{V}[X_1 \mapsto L_1, \ldots, X_n \mapsto L_n]}^{\mathcal{T}})$ is the least fixpoint of f, i.e.

$$\mathcal{L}^{\mathcal{T}}_{\psi}(\tilde{\mathcal{A}}_{\varphi}[X_1:\tilde{\delta}_G(L_1),\ldots,X_n:\tilde{\delta}_G(L_n)]) = \tilde{\delta}_G(\llbracket\psi\rrbracket^G_{\mathcal{V}[X_1\mapsto L_1,\ldots,X_n\mapsto L_n]}).$$

- For $\psi \equiv \nu X.\psi'$, the claim follows analogously.

In order to prove Theorem 10, we establish the following lemma:

Lemma 19. Let $\mathcal{M} = (S, s_0, \gamma_0, \Delta, L)$ be a DPN and $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \curvearrowright)$ be an execution graph of \mathcal{M} witnessed by the assignment $as: V \to S \times \Gamma^* \bot$. Further, let $x, y \in V$ be nodes with $y = \operatorname{succ}_a^G(x)$. Then there are control locations $s, s' \in S$, stack symbols $\gamma, \gamma' \in \Gamma$ and a stack content $w \in \Gamma^* \bot$ such that $as(x) = (s, \gamma w)$ and $as(y) = (s', \gamma' w)$.

Proof. Since $y = succ_a^G(x)$, we either have $x \to^{int} y$ or $x \curvearrowright y$. In both cases, there is a path π from x to y following only Moves $\setminus \{spawn\}$ -transitions such that the number n of *call*-moves on π is equal to the number of *ret*-moves on π . We show the claim by induction over n.

If n = 0, we have $x \to^{int} y$. Since G is generated by \mathcal{M} , we thus have $as(x) \to_{int} as(y)$, i.e. there are control locations $s, s' \in S$, stack symbols $\gamma, \gamma' \in \Gamma$ and a stack content $w \in \Gamma^* \bot$ such that $as(x) = (s, \gamma w)$ and $as(y) = (s', \gamma' w)$.

If n > 0, we have $x \frown y$. Thus, x has a call-successor x' and by Lemma 18(i), y has a ret-predecessor y' with $succ_{-}^{G}(z') = x$, i.e. there is a path π' from x' to z' following abstract successors. In particular, in each step in π' from a node u to its abstract successor v, there is a path π'' from u to v following only Moves $\{spawn\}$ -transitions such that the number $m_{\pi''}$ of call-moves on π'' is equal to the number of ret-moves on π'' . Since all of these paths π'' are proper subpaths of π , we clearly have $m_{\pi''} < n$ in each step. Thus, by the induction hypothesis, there are control locations $s_1, s_2 \in S$, stack symbols $\gamma_1, \gamma_2 \in \Gamma$ and a stack content $w' \in \Gamma^* \bot$ such that $as(x') = (s_1, \gamma_1 w')$ and as(y') =

37

 $(s_2, \gamma_2 w')$. Since further $x \to^{call} x'$ and hence $as(x) \to_{call} as(x')$, we also have $as(x) = (s, \gamma w)$ for some $s \in S$ and $\gamma \in \Gamma$ as well as $w' = \gamma' w$ for some $\gamma' \in \Gamma$. Finally, since $y' \to^{ret} y$ and hence $as(y') \to_{ret} as(y)$, we also have $as(y) = (s', w') = (s', \gamma' w)$ for some $s' \in S$.

Proof of Theorem 10. We first show that $\mathcal{A}_{\mathcal{M}}$ accepts all tree representations of executions graphs generated by \mathcal{M} .

For this, let $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \frown)$ be an execution graph of \mathcal{M} witnessed by the assignment $as: V \to S \times \Gamma^* \bot$. Further, let $\mathcal{T}(G) = (T, r)$ be the tree representation of G. We first define maps $r_S: V \to S$ and $r_{\Gamma}: V \to \Gamma$ as follows. For nodes $x \in V$ with $as(x) = (s, \gamma w)$ for some stack symbol $\gamma \in \Gamma$ and stack content $w \in \Gamma^* \bot$ we set $r_S(x) := s$ and $r_{\Gamma}(x) = \gamma$. Moreover, we inductively define a map $r_R: V \to (S \times \Gamma) \cup \{\bot\}$ as follows. First, we set $r_R(v_0) := \bot$. Then, for all nodes $x \in V$,

- if there is a node $y \in V$ such that y is an *int*-predecessor of x or $y \curvearrowright x$ (the latter holds by Lemma 18(i) iff x has a *ret*-predecessor), we set $r_R(x) := r_R(y)$,
- if x (i) has a spawn-predecessor or (ii) it has a call-predecessor y and there is no $z \in V$ with $y \curvearrowright z$, we set $r_R(x) := \bot$ and
- if x has a call-predecessor y and there is $z \in V$ with $y \curvearrowright z$, we set $r_R(x) := (r_S(z'), r_{\Gamma}(z'))$, where z' is the ret-predecessor of z (this node exists in this case by Lemma 18(i)).

Finally, we define a map $r_A : T \to Q$ by $r_A(\delta_G(x)) := (r_S(x), r_\Gamma(x), r_R(x))$ and show the following claim:

<u>Claim:</u> r_A is an accepting (ε, q_0) -run of $\mathcal{A}_{\mathcal{M}}$ over $\mathcal{T}(G)$.

First, we show that r_A is an (ε, q_0) -run of $\mathcal{A}_{\mathcal{M}}$ over $\mathcal{T}(G)$.

For the initial node, we have $as(v_0) = (s_0, \gamma_0 \perp)$ as well as $r_R(v_0) = \perp$ and thus $r_A(\varepsilon) = r_A(\delta_G(v_0)) = (s_0, \gamma_0, \perp) = q_0$.

Now let $t = \delta_G(x) \in T$ be a node with r(t) = (l(x), d(x), p(x)) and $r_A(t) = (s, \gamma, c)$. Then we have $as(x) = (s, \gamma w)$ for a stack content $w \in \Gamma^* \bot$ and the node is labelled by $l(x) = L(s, \gamma)$ since G is generated by \mathcal{M} . By a case distinction on d(x), we show that the successors of t satisfy the transition function of $\mathcal{A}_{\mathcal{M}}$.

- If d(x) = int, then x has an *int*-successor y with $as(x) \to_{int} as(y)$. Thus, there are $s' \in S$ and $\gamma' \in \Gamma$ with $as(y) = (s', \gamma'w)$ and $s\gamma \to s'\gamma' \in \Delta_I$. Moreover, we have $r_R(y) = r_R(x) = c$ and $\delta_G(y) = t \cdot 0$. Thus, $\{(0, r_A(t \cdot 0))\} = \{(0, (s', \gamma', c))\}$ satisfies $\rho((s, \gamma, c), r(t))$.
- If d(x) = call, then x has a call-successor y with $as(x) \to_{call} as(y)$. Thus, there are $s' \in S$ and $\gamma', \gamma'' \in \Gamma$ with $as(y) = (s', \gamma'\gamma''w)$ and $s\gamma \to s'\gamma'\gamma'' \in \Delta_C$. Moreover, there is no node $z \in V$ with $x \curvearrowright z$, i.e. $r_R(x) = \bot$, and $\delta_G(y) = t \cdot 0$. Thus $\{(0, r_A(t \cdot 0))\} = \{(0, (s', \gamma', \bot))\}$ satisfies $\rho((s, \gamma, c), r(t))$.
- If d(x) = callRet, then x has a call-successor y with $as(x) \rightarrow_{call} as(y)$. Thus, there are $s' \in S$ and $\gamma', \gamma'' \in \Gamma$ with $as(y) = (s', \gamma'\gamma''w)$ and $s\gamma \rightarrow s'\gamma'\gamma'' \in \Delta_C$. Moreover, there is a node $z \in V$ with $x \curvearrowright z$. By Lemma 18(i), z has a ret-predecessor z' with $succ_{-}^{G}(z') = x$, i.e. there is a path from the callsuccessor y of x to z' following abstract successors. Hence, by Lemma 19,

A Navigation Logic for Recursive Programs with Dynamic Thread Creation

there are $s_r \in S$ and $\gamma_r \in \Gamma$ with $as(z') = (s_r, \gamma_r \gamma''w)$. Moreover, since $z' \to^{ret} z$, we have $as(z') \to_{ret} as(z)$, i.e. there is $s'' \in S$ with $as(z) = (s'', \gamma''w)$ and $s_r \gamma_r \to s'' \in \Delta_R$. Thus, we have $r_R(y) = (r_S(z'), r_\Gamma(z')) = (s_r, \gamma_r), \ \delta_G(y) = t \cdot 0, \ r_R(z) = r_R(x) = c \text{ and } \delta_G(z) = t \cdot 1$. Hence, $\{(0, r_A(t \cdot 0)), (1, r_A(t \cdot 1))\} = \{(0, (s', \gamma', (s_r, \gamma_r))), (1, (s'', \gamma'', c))\}$ satisfies $\rho((s, \gamma, c), r(t))$.

- If d(x) = spawn, then x has an *int*-successor y and a spawn-successor z with $as(x) \to as(y) \rhd as(z)$. Thus, there are $s', s_n \in S$ and $\gamma', \gamma_n \in \Gamma$ with $as(y) = (s', \gamma'w), as(z) = (s_n, \gamma_n \perp)$ and $s\gamma \to s'\gamma' \rhd s_n\gamma_n \in \Delta_S$. Moreover, we have $r_R(y) = r_R(x) = c, \ \delta_G(y) = t \cdot 0, \ r_R(z) = \perp$, and $\delta_G(z) = t \cdot 1$. Thus, $\{(0, r_A(t \cdot 0)), (1, r_A(t \cdot 1))\} = \{(0, (s', \gamma', c)), (1, (s_n, \gamma_n, \perp))\}$ satisfies $\rho((s, \gamma, c), r(t)).$
- If d(x) = ret, then x has a ret-successor y with $as(x) \rightarrow_{ret} as(y)$. Thus, there is $s' \in S$ with as(y) = (s', w) and $s\gamma \rightarrow s' \in \Delta_R$. Moreover, by Lemma 18(i), there is a node $z \in V$ with $z \frown y$ and $z = succ_-^G(x)$, i.e. there is a path from the call-successor z' of z to x following abstract successors. Thus, we have $c = r_R(x) = r_R(z') = (r_S(x), r_\Gamma(x)) = (s, \gamma)$, i.e. \emptyset satisfies $true = \rho((s, \gamma, (s, \gamma)), r(t)) = \rho((s, \gamma, c), r(t))$.
- If d(x) = end, then x has no successors and hence as(x) has no successor, i.e. there is no rule for $s\gamma$ in Δ . Assume towards contradiction that $c \neq \bot$. By construction, there must be nodes $y, z \in V$ with $y \curvearrowright z$ and a path from the call-successor y' of y to x following abstract successors. Thus, $\delta_G(x)$ is the $\{int, ret\}$ -descendant leaf of the left child $\delta_G(y') = \delta_G(y) \cdot 0$ of the parent node $\delta_G(y)$ of $\delta_G(z) = \delta_G(y) \cdot 1$. Then we have $succ_{\uparrow}^{\mathcal{T}(G)}(\delta_G(z)) = \delta_G(x)$. Using Lemma 6, we obtain $\delta_G(succ_{\uparrow}^G(z)) = succ_{\uparrow}^{\mathcal{T}(G)}(\delta_G(z)) = \delta_G(x)$ and thus $succ_{\uparrow}^G(z) = x$ since δ_G is injective. This means that z is a successor of x, which contradicts our assumption that x has no successor. Thus, we have $c = \bot$ and hence \emptyset satisfies $true = \rho((s, \gamma, \bot), r(t)) = \rho((s, \gamma, c), r(t))$.

Thus, we have established that r_A is an (ε, q_0) -run of $\mathcal{A}_{\mathcal{M}}$ over $\mathcal{T}(G)$. Since we have $\Omega(q) = 0$ for all $q \in Q$, the run is clearly accepting, and thus $\mathcal{T}(G) \in \mathcal{L}(\mathcal{A}_{\mathcal{M}})$.

For the other direction of the theorem, we show that all execution trees accepted by $\mathcal{A}_{\mathcal{M}}$ are tree representations of execution graphs of \mathcal{M} . For this, let $G = (V, l, (\rightarrow^d)_{d \in \mathsf{Moves}}, \sim)$ be an execution graph with $\mathcal{T}(G) = (T, r) \in \mathcal{L}(\mathcal{A}_{\mathcal{M}})$. Then there is an accepting (ε, q_0) -run r_A of $\mathcal{A}_{\mathcal{M}}$ over $\mathcal{T}(G)$. We now inductively define an assignment $as \colon V \to S \times \Gamma^* \bot$ such that for all nodes $v \in V$ with $as(v) = (s, \gamma w)$ for a stack symbol $\gamma \in \Gamma$ and a stack content $w \in \Gamma^* \bot$, we have $r_A(\delta_G(v)) = (s, \gamma, c)$ for a $c \in (S \times \Gamma) \cup \{\bot\}$.

In the base case, we set $as(v_0) := (s_0, \gamma_0 \perp)$. Since r_A is an (ε, q_0) -run of $\mathcal{A}_{\mathcal{M}}$, we have $r_A(\delta_G(v_0)) = r_A(\varepsilon) = q_0 = (s_0, \gamma_0, \perp)$.

In the inductive step, let as be defined for some node $v \in V$ with $r(\delta_G(v)) = (l', d, p)$ such that there are $\gamma \in \Gamma$ and $w \in \Gamma^* \perp$ with $as(v) = (s, \gamma w)$ and $r_A(\delta_G(v)) = (s, \gamma, c)$ for a $c \in (S \times \Gamma) \cup \{\perp\}$. We define the mapping for the successors of v by a case distinction on d.

- 40 R. Lakenbrink et al.
- If d = int, v has exactly one *int*-successor v' and we have $r_A(\delta_G(v')) = r_A(\delta_G(v) \cdot 0) = (s', \gamma', c)$ for some $s' \in S$ and $\gamma' \in \Gamma$ such that $s\gamma \to s'\gamma' \in \Delta_I$. Then we set $as(v') := (s', \gamma'w)$.
- If d = call, v has exactly one call-successor v' and we have $r_A(\delta_G(v')) = r_A(\delta_G(v) \cdot 0) = (s', \gamma', \bot)$ for some $s' \in S$ and $\gamma' \in \Gamma$ such that $s\gamma \to s'\gamma'\gamma'' \in \Delta_C$ for a $\gamma'' \in \Gamma$. Then we set $as(v') := (s', \gamma'\gamma''w)$.
- If d = callRet, v has exactly one call-successor v' and there is $v'' \in V$ with $v \curvearrowright v''$. Then we have $r_A(\delta_G(v')) = r_A(\delta_G(v) \cdot 0) = (s', \gamma', (s_r, \gamma_r))$ and $r_A(\delta_G(v'')) = r_A(\delta_G(v) \cdot 1) = (s'', \gamma'', c)$ for some $s', s'', s_r \in S$ and $\gamma', \gamma'', \gamma_r \in \Gamma$ such that $s\gamma \to s'\gamma'\gamma'' \in \Delta_C$ and $s_r\gamma_r \to s'' \in \Delta_R$. Then we set $as(v') := (s', \gamma'\gamma''w)$ and $as(v'') := (s'', \gamma''w)$.
- If d = spawn, v has exactly one *int*-successor v' and one *spawn*-successor v''. Then we have $r_A(\delta_G(v')) = r_A(\delta_G(v) \cdot 0) = (s', \gamma', c)$ and $r_A(\delta_G(v'')) = r_A(\delta_G(v) \cdot 1) = (s_n, \gamma_n, \bot)$ for some $s', s_n \in S$ and $\gamma', \gamma_n \in \Gamma$ such that $s\gamma \to s'\gamma' \triangleright s_n\gamma_n \in \Delta_S$. Then we set $as(v') := (s', \gamma'w)$ and $as(v'') := (s_n, \gamma_n \bot)$.

Most conditions for as to witness that G is generated by \mathcal{M} are directly satisfied by construction. We only show the more involved conditions regarding nodes with *ret*-successors and nodes without successors. In order to do this, we inductively show that the following holds for all nodes $v \in V$ with $r_A(\delta_G(v)) = (s, \gamma, c)$:

(*) We have $c = (s_r, \gamma_r)$ for a $s_r \in S$ and $\gamma_r \in \Gamma$ iff there are nodes $v_1, v_2, v_3 \in V$ with $v_1 \rightarrow^{call} v_2, v_1 \frown v_3$ and $s' \in S$ and $\gamma' \in \Gamma$ with $r_A(\delta_G(v_2)) = (s', \gamma', c)$ and there is a path from v_2 to v following abstract successors.

In the base case, where $v = v_0$, we have $c = \bot$ and there is no incoming *int*-transition or nesting edge to v.

In the inductive step, let $v \in V$ be a node with $r_A(\delta_G(v)) = (s, \gamma, c)$ such that $\delta_G(v) = \delta_G(v') \cdot i$ for an $i \in \{0, 1\}$ and a node $v' \in V$ with $r(\delta_G(v')) = (l', d, p)$ and $r_A(\delta_G(v')) = (s', \gamma', c')$. We show (*) for v by a case distinction on d.

- If d = int, we have i = 0 and $\rho(r_A(\delta_G(v')), r(\delta_G(v')))$ is a disjunction of formulae of the form $(0, (s'', \gamma'', c'))$ for $s'' \in S$ and $\gamma'' \in \Gamma$, i.e. c = c'. Since $v' \to int v$ and abstract successors are uniquely determined, the right side of the equivalence in (*) is satisfied for v iff it is satisfied for v'. Thus, by the induction hypothesis, (*) also holds for v.
- If d = call, we have i = 0 and $\rho(r_A(\delta_G(v')), r(\delta_G(v')))$ is a disjunction of formulae of the form $= (0, (s'', \gamma'', \bot))$ for $s'' \in S$ and $\gamma'' \in \Gamma$, i.e. $c = \bot$. Since there is no node $\tilde{v} \in V$ with $\tilde{v} \to^{int} v$, $\tilde{v} \curvearrowright v$ or $v' \curvearrowright \tilde{v}$, the required nodes and the path for any $c \neq \bot$ do not exist for v.
- If d = callRet, then $\rho(r_A(\delta_G(v')), r(\delta_G(v')))$ is a disjunction of formulae of the form $(0, (s_1, \gamma_1, (s_r, \gamma_r))) \land (1, (s_2, \gamma_2, c'))$ for $s_1, s_2, s_r \in S$ and $\gamma_1, \gamma_2, \gamma_r \in \Gamma$. If additionally i = 0, then $c = (s_r, \gamma_r)$ for some $s_r \in S$ and $\gamma_r \in \Gamma$. Since $v' \rightarrow^{call} v$ and $v' \curvearrowright v''$ for some $v'' \in V$, the nodes $v_1 = v', v_2 = v$ and $v_3 = v''$ and the empty path from $v_2 = v$ to v witness that (*) holds. If instead i = 1, then c = c' and $v' \curvearrowright v$. Since abstract successors are uniquely determined, the right side of the equivalence in (*) is satisfied for v iff it is satisfied for v'. Thus, by the induction hypothesis, (*) also holds for v.

- A Navigation Logic for Recursive Programs with Dynamic Thread Creation
- If d = spawn, then $\rho(r_A(\delta_G(v')), r(\delta_G(v')))$ is a disjunction of formulae of the form $(0, (s_1, \gamma_1, c')) \land (1, (s_2, \gamma_2, \bot))$ for $s_1, s_2 \in S$ and $\gamma_1, \gamma_2 \in \Gamma$. If additionally i = 0, then c = c'. Since $v' \to^{int} v$ and abstract successors are uniquely determined, the right side of the equivalence in (*) is satisfied for v iff it is satisfied for v'. Thus, by induction hypothesis, (*) also holds for v. If instead i = 1, then $c = \bot$. Since there is no node $\tilde{v} \in V$ with $\tilde{v} \to^{int} v$, $\tilde{v} \curvearrowright v$ or $v' \curvearrowright \tilde{v}$, the required nodes and the path for any $c \neq \bot$ do not exist for v.

Given (*), we now show the more involved conditions regarding nodes with ret-successors. For this, let $x, y \in V$ be nodes with $x \to^{ret} y$. By Lemma 18(i), there is a node $z \in V$ with $z \curvearrowright y$ and $succ_{-}^{G}(x) = z$, i.e. there is a path from the call-successor z' of z to x following abstract successors. Let $r_A(\delta_G(x)) = (s_x, \gamma_x, c_x)$ and $r_A(\delta_G(z)) = (s, \gamma, c)$. Then we have $as(z) = (s, \gamma w)$ for a $w \in \Gamma^* \bot$. Since $r(\delta_G(z)) = (l(z), callRet, p(z)), \rho(r_A(\delta_G(z)), r(\delta_G(z)))$ is a disjunction of formulae of the form $(0, (s', \gamma', (s_r, \gamma_r))) \land (1, (s'', \gamma'', c))$ with $s\gamma \to s'\gamma'\gamma'' \in \Delta_C$ and $s_r\gamma_r \to s'' \in \Delta_R$. Thus, $r_A(\delta_G(z')) = (s', \gamma', (s_r, \gamma_r))$ as well as $r_A(\delta_G(y)) = (s'', \gamma'', c)$ for some $s', s'', s_r \in S$ and $\gamma', \gamma'', \gamma_r \in \Gamma$ with $s\gamma \to s'\gamma'\gamma'' \in \Delta_C$ and $s_r\gamma_r \to s'' \in \Delta_R$. By (*) we infer $c_x = (s_r, \gamma_r)$. Moreover, since x has a ret-successor, we have d(x) = ret and thus $\rho(r_A(\delta_G(x)), r(\delta_G(x)))$ is true, if $s_x = s_r$ and $\gamma_x = \gamma_r$, and false otherwise. Since the transition function must be satisfied by the children of $\delta_G(x)$, we thus have $s_x = s_r$ and $\gamma_x = \gamma_r$, i.e. $as(x) = (s_r, \gamma_r w')$ for a $w' \in \Gamma^* \bot$. By construction of as, we clearly have $w' = \gamma''w$. Thus, since $as(x) = (s_r, \gamma_r \gamma''w)$, $as(y) = (s'', \gamma''w)$ and $s_r\gamma_r \to s'' \in \Delta_R$, we have $as(x) \to_{ret} as(y)$.

Finally, let $x \in V$ be a node without successors. Then we have $r(\delta_G(x)) = (l(x), end, p(x))$. Let $r_A(\delta_G(x)) = (s, \gamma, c)$, i.e. $as(x) = (s, \gamma w)$ for some $w \in \Gamma^* \bot$. Assume towards contradiction that we have $c = (s_r, \gamma_r)$ for some $s_r \in S$ and $\gamma_r \in \Gamma$. By (*), there are $v_1, v_2, v_3 \in V$ with $v_1 \to^{call} v_2$ and $v_1 \curvearrowright v_3$ and there is a path from v_2 to x following abstract successors. Thus, $\delta_G(x)$ is the $\{int, ret\}$ -descendant leaf of the left child $\delta_G(v_2) = \delta_G(v_1) \cdot 0$ of the parent node $\delta_G(v_1)$ of $\delta_G(v_3) = \delta_G(v_1) \cdot 1$. Then we have $succ_{\uparrow}^{T(G)}(\delta_G(v_3)) = \delta_G(x)$. Using Lemma 6, we obtain $\delta_G(succ_{\uparrow}^G(v_3)) = succ_{\uparrow}^{T(G)}(\delta_G(v_3)) = \delta_G(x)$ and thus $succ_{\uparrow}^G(v_3) = x$ since δ_G is injective. This means that v_3 is a successor of x, which contradicts our assumption that x has no successor. Thus, we must have $c = \bot$, i.e. $\rho(r_A(\delta_G(x)), r(\delta_G(x)))$ is true, if there is no rule for $s\gamma$ in Δ , and false otherwise. Since the transition function must be satisfied by the children of $\delta_G(x)$, there is thus no successor of as(x).