Correctness Witness Validation by Abstract Interpretation

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Abstract. Witnesses record automated program analysis results and make them exchangeable. To validate correctness witnesses through abstract interpretation, we introduce a novel abstract operation unassume. This operator incorporates witness invariants into the abstract program state. Given suitable invariants, the unassume operation can accelerate fixpoint convergence and yield more precise results. We demonstrate the feasibility of this approach by augmenting an abstract interpreter with unassume operators and evaluating the impact of incorporating witnesses on performance and precision. Using manually crafted witnesses, we can confirm verification results for multi-threaded programs with a reduction in effort ranging from 7% to 47% in CPU time. More intriguingly, we discover that using witnesses from model checkers can guide our analyzer to verify program properties that it could not verify on its own.

Keywords: Correctness Witness · Witness Validation · Software Verification · Program Analysis · Abstract Interpretation

1 Introduction

Automated software verifiers can be faulty and may produce incorrect results. To increase trust in their verdicts, verifiers may produce *witnesses* that expose their reasoning. Such proof objects allow independent validators to confirm analysis results. The use of witnesses as a standardized way to communicate between different automated software verifiers was pioneered by Beyer et al. [17]. They introduced an analyzer-agnostic automaton-based format for explaining property violations. The witness automaton guides the validator towards a feasible counterexample. This witness format was later extended to explain program correctness using invariants [15]. Witnesses form a cornerstone of the annual software verification competition SV-COMP [14] and have played a key role in the emergence of Cooperative Verification [19, 38], where independent verifiers collaborate by exchanging witnesses [24].

This paper aims to show how *correctness witnesses* can be validated using abstract interpretation. Existing validators are based on model checking [16], test

execution [18], interpretation [7, 58], or SMT-based verification [43]; whereas, validators for correctness witnesses at SV-COMP 2023 were all based on model checking [14, 15]. In that year's competition report, the long-time organizer Dirk Beyer highlighted the scarcity of validators — which leaves many verification outcomes without independent confirmation — as a "remarkable gap in software-verification research" [14]. Abstract interpretation, originally proposed by Cousot and Cousot [29], has proven successful in the efficient verification of large real-world software [9, 31] and multi-threaded programs [33, 46, 48, 52, 53]. To complement existing validators, we propose enhancing the framework of abstract interpretation to incorporate invariants from witnesses.

For communication across technological boundaries, correctness witnesses must be restricted to invariants that do not expose internal abstractions of tools. For example, as each tool may abstract dynamically allocated memory differently, invariants about the content of such memory may only be expressed indirectly, e.g., via C invariants such as $*p \ge 0$. The challenge is how to incorporate such tool-independent invariants into an abstract interpreter. A key technical contribution of this paper are techniques to incorporate witness invariants, given as expressions, into abstract domains without relying on those invariants to actually hold. This differs from existing work on witnesses for abstract interpretation (detailed in Section 7), which does not allow for or aim at the exchange of witnesses across tool boundaries.

Our solution is to introduce a new abstract operation *unassume*. This operator can be used to selectively inject imprecision (hence the name) to speed up fixpoint computation. Suitably increasing abstract values during fixpoint computation can also improve the precision of an existing analysis, most notably due to the non-monotonicity of widening operators. The following example illustrates both the speedup and increase in precision.

Example 1. Consider the example (shown right) from $_{1}$ int x = 40; Miné [47]. An abstract interpreter using interval abstrac- $_{2}$ while (x != 0) { tion first reaches the loop head on line 2 with the in- $_{3}$ x--; terval [40,40] for x. After one iteration, the loop head $_{4}$ } is reached with [39,39], so the abstract value at that point is [39,40]. To accelerate fixpoint iteration for termination, standard interval widening is applied, which abandons the unstable lower bound, resulting in $[-\infty, 40]$. Another iteration with this interval reaches the loop head with $[-\infty, 39]$, which is subsumed by the previous abstract value. Subsequent

the loop head; therefore, the analysis fails to establish a lower bound for x. Now, suppose that a witness provides the invariant $0 \le x \le 40$ for the loop head at line 2. When guiding the fixpoint iteration with this witness, the loop head is again first reached with the singleton interval [40, 40]. Using the provided invariant, the unassume operator relaxes the lower bound of the interval:

standard interval narrowing cannot improve the inferred invariant $[-\infty, 40]$ at

$$[[unassume(0 \le x \le 40)]]^{\sharp} \{ x \mapsto [40, 40] \} = \{ x \mapsto [0, 40] \}.$$

After one iteration, the loop head is now reached with [0, 39], which makes the abstract value at that point [0, 40]. Thus, a fixpoint is reached without the need for widening or narrowing, and the stronger invariant [0, 40] is confirmed. This demonstrates how the same analysis, when guided, can validate an invariant that it could not infer on its own. A well-chosen witness invariant can prevent precision loss during widening that cannot be recovered by narrowing, serving as a proxy for providing known widening thresholds. Additionally, the witness-guided analysis required fewer steps (transfer function evaluations and fixpoint iterations). Using the same invariant as a widening threshold does not yield such speedup.

After introducing relevant terms (Section 2) and discussing the shortcomings of intuitive approaches to validate witnesses with abstract interpretation (Section 3), the paper presents the following main contributions:

- a specification of the unassume operator, and a general realization for relational abstract domains using dual-narrowing (Section 4);
- an efficient algorithm for unassuming in non-relational abstract domains (Section 5), with generalization to pointer variables (Appendix A);
- an implementation of an abstract-interpretation-based witness validator, which is evaluated using hand-crafted invariants for multi-threaded programs and invariants produced by state-of-the-art model checkers for intricate literature examples (Section 6).

Our evaluation results provide practical evidence of the unassumed witness invariants making the analysis faster and more precise.

2 Preliminaries

In the following, we formally introduce the notion of location-based correctness witnesses, subsequently referred to simply as *witnesses*, and recall the basics of abstract interpretation.

2.1 Witnesses

Following the refined definitions of Beyer and Strejček [23], a correctness witness should contain hints for the proof of program correctness. Witness automata [15] are a powerful way to provide such hints, but Strejček [56] has observed that their control-flow semantics are ambiguous, impairing interoperability. In practice, however, invariants per program location are often sufficient [15, 22], which has led the SV-COMP community to adopt them [57]. Therefore, we consider here correctness witnesses consisting of location-based invariants.

Let \mathcal{N} denote the set of program locations. For clarity of exposition, we consider a fixed set \mathcal{V} of program variables. Invariants from some language \mathcal{E} , which we do not fix, are used to specify properties of the program executions reaching a particular program location. We assume that there is a trivial invariant $e_{\text{true}} \in \mathcal{E}$ that always holds. Since the goal is to exchange invariants between tools, the choice of an invariant language involves a trade-off:

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- 1. Beyer et al. [15] use boolean-valued side-effect-free C expressions for their invariants. The chief advantage is its conceptual simplicity: the semantics of such assertions is well-known, and analyzers already come with the necessary facilities to manipulate these expressions, as they appear in the analyzed program. In C expressions, pointers allow exchanging information also about the heap between verifiers. Nevertheless, the expressivity of such invariants is limited, especially regarding more complex data structures.
- 2. ACSL [10] has more expressive power than plain C expressions by offering quantification, memory predicates, etc. On the downside, considerably fewer analyzers support it, limiting exchange possibilities.
- 3. Custom invariant languages can be arbitrarily expressive, at the cost of restricting communication to few similar tools whose re-verification to boost.

With these notions, we introduce the definitions of a witness, its validation, and witness-guided verification.

Definition 1. A witness for a safety property Φ of a program P is a tuple (W, P, Φ) , where W is a total mapping $W : \mathcal{N} \to \mathcal{E}$ from the program locations of P to invariants from \mathcal{E} .

The textual format in which witnesses are exchanged is not required to provide invariants for all program locations — we implicitly assume that if the witness contains no information for some program location, then this location is mapped to e_{true} . If the invariant language contains a contradictory expression $e_{false} \in \mathcal{E}$ that never holds, then it can be used to convey unreachability of a program location. This notion of a witness is generic and can be instantiated to different programming and invariant languages. For our examples, we use an invariant language of arithmetic and boolean expressions, enriched with basic pointers, address-taking (&) and dereferencing (*). The pointer constructs pose practical challenges, as will become apparent in subsequent sections.

Definition 2. A witness (W, P, Φ) is valid if

- 1. P satisfies the property Φ ;
- 2. whenever the execution of P reaches the location $n \in \mathcal{N}$, the invariant W n holds.

A witness validator attempts to prove that a witness is valid; specifically, it tries to recreate the proof that the program satisfies the property Φ , and checks that the witness makes only true claims about the program. However, the validation track at SV-COMP 2023 scored participants according to a limited form of validation which only confirms the first condition [14].

A witness-guided verifier uses the witness as guidance towards the verification of Φ . A sound verifier can perform this task without assuming the witness invariants to be true; therefore, it qualifies as a sound validator of the first condition. It may additionally verify the invariants in W to perform full witness validation.

2.2 Abstract Interpretation

We rely on the framework of abstract interpretation as introduced by Cousot and Cousot [29, 30], and briefly recall relevant notions here. Let S denote the set of all concrete program states. Its subsets are abstracted by an *abstract domain* \mathbb{D} satisfying the following properties [47]:

- a partial order \sqsubseteq , modeling the relative precision of abstract states;
- a monotonic *concretization* function $\gamma : \mathbb{D} \to 2^S$, mapping an abstract element to the set of concrete states it represents;
- a least element \perp , representing unreachability, i.e. $\gamma \perp = \emptyset$;
- a greatest element \top , representing triviality, i.e. $\gamma \top = S$;
- sound abstractions join (\sqcup) and meet (\sqcap) of \cup and \cap on S, respectively, i.e. $\gamma x \cup \gamma y \subseteq \gamma (x \sqcup y)$ and $\gamma x \cap \gamma y \subseteq \gamma (x \sqcap y)$ for all $x, y \in \mathbb{D}$;
- a widening (∇) operator, computing upper bounds that ensure termination in abstract domains with infinite ascending chains, i.e. $x \sqsubseteq x \nabla y$ and $y \sqsubseteq x \nabla y$ for all $x, y \in \mathbb{D}$, and for every sequence $(y_i)_{i \in \mathbb{N}}$ from \mathbb{D} , the sequence $(x_i)_{i \in \mathbb{N}}$ defined by $x_0 = y_0$, $x_{i+1} = x_i \nabla y_{i+1}$ is ultimately stable;
- a narrowing (Δ) operator, recovering some precision given up by widening, i.e. $x \sqcap y \sqsubseteq x \Delta y \sqsubseteq x$ for all $x, y \in \mathbb{D}$, and for every sequence $(y_i)_{i \in \mathbb{N}}$ from \mathbb{D} , the sequence $(x_i)_{i \in \mathbb{N}}$ defined by $x_0 = y_0, x_{i+1} = x_i \Delta y_{i+1}$ is ultimately stable.

An abstract interpreter uses an abstract domain \mathbb{D} and sound abstractions $[\![s]\!]^{\sharp} : \mathbb{D} \to \mathbb{D}$ of primitive statements s to model the abstract semantics of a program. Fixpoint iteration (potentially with widening and narrowing) is used to compute for each program location an abstract state, which represents a superset of all reaching concrete program states. The resulting abstract states may be used to check whether the program satisfies a given safety property.

For validating a witness by abstract interpretation, we assume that the analyzer provides us with a mapping $\sigma : \mathcal{N} \to \mathbb{D}$ from locations to abstract values. A witness (W, P, Φ) is validated by the abstract interpreter, if

- 1. σ is sufficient to verify that Φ holds for program P;
- 2. for each $n \in \mathcal{N}$, the invariant W n is true in every state of $\gamma(\sigma n)$.

In practice, the second condition may not be easy to check since computing γ is not always feasible. Thus, abstract expression evaluation is used instead to perform the validity check, although this is possibly less precise, as the following example shows.

Example 2. Assume the non-relational abstract domain $\mathbb{D} = \mathcal{V} \to \mathbb{V}$ of environments, where \mathbb{V} is the abstract domain of individual values. Using intervals for the latter, let $d = \{\mathbf{x} \mapsto [1, 2]\}$ be the computed abstract state at some program location where $\gamma d = \{\{\mathbf{x} \mapsto 1\}, \{\mathbf{x} \mapsto 2\}\}$ is *exact.* Consider the validation of the following two logically equivalent invariants at this location:

1. The invariant $1 \leq x \land x \leq 2$ holds for each concrete state in γd . It also evaluates abstractly to true on d using standard syntax-driven evaluation (see Section 5.1), because both conjuncts are true for the interval [1, 2].

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- 2. The invariant $\mathbf{x} = 1 \lor \mathbf{x} = 2$ also holds for each concrete state in γd . However, when evaluated abstractly and syntax-driven on d, it evaluates to an unknown boolean, because both disjuncts evaluate to an unknown boolean for the interval [1,2].

Hence, the abstract interpreter is not complete, i.e., it may fail to validate witnesses which are indeed valid, due to imprecision arising from abstraction or fixpoint acceleration. Nevertheless, validating witnesses using abstract expression evaluation is sound, i.e., all witnesses claimed to be validated by the abstract interpreter, are indeed valid.

A witness which maps all locations to the invariant e_{true} is called *trivial*. The validation of such a witness trivially passes the second validity condition, and checking of the first condition falls entirely to the analyzer itself. To be useful, a witness has to be non-trivial and aid the analyzer in proving that the program P satisfies the property Φ , either by improving the precision or the performance of the verification process. For *witness-guided verification*, it suffices if the analyzer can show that Φ holds — even if the invariants of the witness cannot be validated. Given a witness (W, P, Φ) , the challenge of witness-guided abstract interpretation is to simultaneously achieve the following:

- 1. to use the invariants in W to reach a fixpoint $\sigma_W : \mathcal{N} \to \mathbb{D}$ in fewer iterations;
- 2. to avoid overshooting the required property, i.e., σ_W suffices to prove Φ ;
- 3. to not trust the witness, i.e., σ_W should remain sound even in presence of wrong invariants.

Subsequently, we first consider some intuitive approaches to motivate our unassume operator for soundly speeding up abstract interpretation with the help of untrusted witnesses.

3 Initialization-Based Approaches

Given a witness (W, P, Φ) , one natural idea is to extract from the mapping $W : \mathcal{N} \to \mathcal{E}$ a mapping $w : \mathcal{N} \to \mathbb{D}$ of initial abstract values as non- \perp start points for constructing inductive invariants for the program. We discuss two flavors for realizing this idea, along with their shortcomings.

Total Initial Values. In the first approach, the initial value wn for program location n is chosen such that $\gamma(wn)$ includes every concrete state where Wn holds. For example, by choosing $w \ell_2 = \{x \mapsto [0, 40]\}$ in Example 1. Such a value, however, is only suitable if *all* relevant information for program location n is formalized in the invariant Wn and expressible by the abstract domain. Apart from trivial cases, both requirements are seldom fulfilled in practice.

For example, consider the invariant $*p \ge 0$ involving a pointer p dereference for some program location n. It provides no information about which variables p may point to, thus nothing can be concluded about any integer variables it intends to describe. Therefore, $w n = \top$ which leads to a complete loss of precision at location n during the analysis.

This approach also makes silent assumptions about the way in which the analyzer computes values, namely how such initial abstract values are incorporated into analysis, if at all. For example, TD fixpoint solvers [54] only use initial values at *dynamically* identified widening points for starting fixpoint iteration. Additionally, context-sensitive interprocedural analysis is known to give rise to infinite constraint systems [5], requiring dedicated changes to the analyzer to ensure that all accessed constraint variables associated with a given location are appropriately initialized.

Partial Initial Values. In order to remedy the problem that all relevant information must be provided in the invariants for program locations, one may instead rely on *partial* initialization. For that to work, we assume here that a non-relational abstract domain $\mathbb{D} = \mathcal{V} \to \mathbb{V}$ is used. Assuming that the invariant Wn only speaks of variables from $V \subseteq \mathcal{V}$, the partial initial value is the same as the total initial value w n except all unmentioned variables $x \in \mathcal{V} \setminus V$ are assigned \perp .

Example 3. Consider for a particular program location n, two integer variables i and j and a pointer variable p. Let the abstract domain \mathbb{V} of values consist of intervals for abstracting integers and points-to sets for abstracting pointers. Consider two invariants:

- 1. The witness invariant $i \ge 0 \land j \ge 0$ can be represented by the partial state $\{\mathsf{p}\mapsto\perp,\mathsf{i}\mapsto[0,\infty],\mathsf{j}\mapsto[0,\infty]\}.$
- 2. The witness invariant $*p \ge 0$, on the other hand, results in the partial state $\{p \mapsto \top, i \mapsto \bot, j \mapsto \bot\}.$

Now assume that during analysis of the program, the complete abstract state $\{\mathbf{p} \mapsto \{\&i,\&j\}, i \mapsto [0,0], j \mapsto [0,0]\}, \text{ where } \mathbf{p} \text{ may point to either } i \text{ or } j, \text{ reaches } i \in [0,0], j \mapsto [0,0]\}$ the program location n. In order to exploit the witness, this value is *joined* with the partial state constructed from the witness in the corresponding transfer function. For the two invariants above, we respectively obtain:

- 1. {p \mapsto {&i,&j}, i \mapsto [0, ∞], j \mapsto [0, ∞]}, 2. {p \mapsto \top , i \mapsto [0,0], j \mapsto [0,0]}.

The first may be useful to guide the analysis since the information for i and j is maximally relaxed such that the witness invariant can still be validated, while the information for the pointer variable p is retained. On the other hand, the second state loses all information about p, which is problematic if memory is accessed through p later in the program. At the same time, here, the values for the variables i and j remain overly precise. Instead, one would have liked to obtain the former abstract state also when using the invariant $*p \ge 0$.

By joining initial values within transfer functions, this approach is more general: it works for all program locations regardless of the analysis engine and can be seamlessly applied to infinite constraint systems. Nevertheless, the incorporation of witnesses via partial initial values is only applicable to non-relational domains and cannot depend on analysis state. Therefore, in the next section, we propose a more general solution that overcomes these issues.

4 Unassuming

We introduce new statements unassume(e) to the programming language for all invariants $e \in \mathcal{E}$. Given a witness (W, P, Φ) , we insert at every location n, the statement unassume(Wn) if it is different from e_{true} . In case the invariant is not a legal program expression, we may instead insert the statement at the location into the internal representation used by the analyzer (e.g., the controlflow graph). In the concrete semantics, unassume statements have no effect, i.e., their arguments are not evaluated and thus does not cause runtime errors or undefined behavior.

During the abstract interpretation of the program, the abstract state transformer for the statement unassume(e) for location n is meant to inject the desired imprecision into the abstract state for n. Intuitively, the abstract semantics of unassume is dual to the *assume* operation, i.e., it relaxes a state instead of refining it. Thus, e.g.,

$$\{ \mathbf{x} \mapsto [0,\infty] \} \xleftarrow[]{\text{assume}(\mathbf{x}=0)}_{\text{unassume}(\mathbf{x}\geq 0)} \{ \mathbf{x} \mapsto [0,0] \}.$$

Note that unassume is not the inverse of assume because the used expressions are different. By integrating unassume operations as statements, they can be treated path- and context-sensitively – just like all other statements – if the abstract interpreter supports such sensitivity, yielding a general approach.

4.1 Specification

Subsequently, we provide abstract operators $\llbracket unassume_V(e) \rrbracket^{\sharp} : \mathbb{D} \to \mathbb{D}$ which we use to abstractly interpret the corresponding unassume statement. The abstract operators are parameterized by the set of variables $V \subseteq \mathcal{V}$ whose values are relaxed up to the constraining invariant e. The abstract unassume operator $\llbracket unassume_V(e) \rrbracket^{\sharp}$ is *sound* if it abstracts the concrete no-op operator, i.e.,

$$\gamma d \subseteq \gamma (\llbracket \text{unassume}_V(e) \rrbracket^{\sharp} d)$$

for all $d \in \mathbb{D}$. In particular, this is the case if the operator is *extensive*, i.e.,

$$d \sqsubseteq \llbracket \text{unassume}_V(e) \rrbracket^{\sharp} d.$$

Given that the abstract interpreter is sound w.r.t. the original program, and sound unassume operations are inserted, we conclude that the resulting abstract interpreter is sound w.r.t. the modified program. Since the newly inserted statements have no effects in the concrete, the resulting abstract interpreter remains sound also w.r.t. the original program. This implies the soundness of our validation approach.

Theorem 1 (Sound witness validation). Assume a witness (W, P, Φ) is used to insert unassume statements and $\sigma_W : \mathcal{N} \to \mathbb{D}$ is the result of analyzing the instrumented program. If the sound analyzer confirms Φ and all invariants of $W : \mathcal{N} \to \mathcal{E}$ abstractly evaluate to true in σ_W , then the witness must be valid. *Example 4.* The desired behavior of unassume operators is illustrated by the following examples.

1. Unmentioned parts of the abstract state should be retained:

 $[\![unassume_{\{x\}}(x \ge 0)]\!]^{\sharp} \{x \mapsto [0,0], y \mapsto [0,0]\} = \{x \mapsto [0,\infty], y \mapsto [0,0]\}.$

2. Information on variables used in the invariant, but not contained in V should be retained:

$$\llbracket unassume_{\{i\}} (i \le n) \rrbracket^{\sharp} \{ i \mapsto [0, 0], n \mapsto [10, 10] \} = \\ = \{ i \mapsto [-\infty, 10], n \mapsto [10, 10] \}.$$

3. Relational invariants between relaxed and not relaxed variables should be preserved whenever possible without restricting the unassumed invariant; e.g., relaxing the state $0 = x \le y$ with $0 \le x$ should result in $0 \le x \le y$:

$$\llbracket \text{unassume}_{\{x\}}(x \ge 0) \rrbracket^{\sharp} \{x \le 0, -x \le 0, -y \le 0, -x - y \le 0, x - y \le 0\} = \\ = \{-x \le 0, -y \le 0, -x - y \le 0, x - y \le 0\}$$

when using the octagon domain [45].³ More specifically, this is the most precise result which, when projected to V, contains the abstract state $\{-x \le 0\}$ defined only by the unassumed invariant.

4. Information provided by the input abstract state should be leveraged to propagate imprecision to further variables and heap locations not mentioned in the invariant (cf. Example 3):

$$\begin{split} \llbracket \mathrm{unassume}_{\{\mathbf{i},\mathbf{j}\}}(\star \mathbf{p} \ge 0) \rrbracket^{\sharp} \{ \mathbf{p} \mapsto \{ \&\mathbf{i},\&\mathbf{j} \}, \mathbf{i} \mapsto [0,0], \mathbf{j} \mapsto [0,0] \} = \\ &= \{ \mathbf{p} \mapsto \{ \&\mathbf{i},\&\mathbf{j} \}, \mathbf{i} \mapsto [0,\infty], \mathbf{j} \mapsto [0,\infty] \}. \end{split}$$

We remark that Items 2 and 4 illustrate cases where V differs from the set of variables syntactically occurring in e.

4.2 Naïve Definition

We present the first unassume operator in terms of the abstract operators for non-deterministic assignments and guards. In this section, we assume the invariant language \mathcal{E} is a subset of the side-effect-free expressions used for conditional branching in the programming language.

For an expression e, let $\operatorname{assume}(e)$ denote the concrete operation which only continues execution if the condition e is true, and aborts otherwise. Let $[\operatorname{assume}(e)]^{\sharp}: \mathbb{D} \to \mathbb{D}$ be a sound abstraction.

For a set of variables $V \subseteq \mathcal{V}$, let havoc(V) denote the concrete operation which non-deterministically assigns arbitrary values to all $x \in V$, and $[havoc(V)]^{\sharp} : \mathbb{D} \to \mathbb{D}$ be a sound abstraction.

³ Redundant constraints are grayed out. They can be derived from non-redundant (non-grayed out) constraints using the octagon closure algorithm.

Definition 3 (Naïve unassume). Let $V \subseteq \mathcal{V}$, $e \in \mathcal{E}$ and $d \in \mathbb{D}$. Then the naïve unassume is defined as

$$\llbracket \text{unassume}_V(e) \rrbracket_1^{\sharp} d = d \sqcup \left(\llbracket \text{assume}(e) \rrbracket^{\sharp} \circ \llbracket \text{havoc}(V) \rrbracket^{\sharp} \right) d.$$

Intuitively, the argument state is relaxed by joining with an additional value. This value is obtained by first forgetting all information about the variables from V and then assuming the information provided by e. Due to the join, this unassume operator is *sound by construction*. The naïve unassume operator is already sufficient to gain the improvements illustrated by Example 1 when choosing $V = \{x\}$:

$$\begin{bmatrix} \text{unassume}_{\{x\}} (0 \le x \le 40) \end{bmatrix}_{1}^{\sharp} \{ \mathbf{x} \mapsto [40, 40] \} = \\ = \{ \mathbf{x} \mapsto [40, 40] \} \sqcup (\begin{bmatrix} \text{assume}(0 \le x \le 40) \end{bmatrix}^{\sharp} \circ \begin{bmatrix} \text{havoc}(\{\mathbf{x}\}) \end{bmatrix}^{\sharp}) \{ \mathbf{x} \mapsto [40, 40] \} = \\ = \{ \mathbf{x} \mapsto [40, 40] \} \sqcup \begin{bmatrix} \text{assume}(0 \le x \le 40) \end{bmatrix}^{\sharp} \{ \mathbf{x} \mapsto \top \} = \\ = \{ \mathbf{x} \mapsto [40, 40] \} \sqcup \{ \mathbf{x} \mapsto [0, 40] \} = \{ \mathbf{x} \mapsto [0, 40] \}. \end{aligned}$$

This operator also succeeds for Items 1 and 2 in Example 4:

$$\begin{split} & [\![unassume_{\{x\}}(x \ge 0)]\!]_1^{\sharp} \{ x \mapsto [0,0], y \mapsto [0,0] \} = \\ &= \{ x \mapsto [0,0], y \mapsto [0,0] \} \sqcup \\ & \sqcup ([\![assume(x \ge 0)]\!]^{\sharp} \circ [\![havoc(\{x\})]\!]^{\sharp}) \{ x \mapsto [0,0], y \mapsto [0,0] \} = \\ &= \{ x \mapsto [0,0], y \mapsto [0,0] \} \sqcup [\![assume(x \ge 0)]\!]^{\sharp} \{ x \mapsto \top, y \mapsto [0,0] \} = \\ &= \{ x \mapsto [0,0], y \mapsto [0,0] \} \sqcup \{ x \mapsto [0,\infty], y \mapsto [0,0] \} = \{ x \mapsto [0,\infty], y \mapsto [0,0] \} \end{split}$$

and

$$\begin{split} & [\![unassume_{\{i\}}(i \leq n)]\!]_{1}^{\sharp} \left\{ i \mapsto [0,0], n \mapsto [10,10] \right\} = \\ &= \left\{ i \mapsto [0,0], n \mapsto [10,10] \right\} \sqcup \\ & \sqcup ([\![assume(i \leq n)]\!]^{\sharp} \circ [\![havoc(\{i\})]\!]^{\sharp}) \left\{ i \mapsto [0,0], n \mapsto [10,10] \right\} = \\ &= \left\{ i \mapsto [0,0], n \mapsto [10,10] \right\} \sqcup [\![assume(i \leq n)]\!]^{\sharp} \left\{ i \mapsto \top, n \mapsto [10,10] \right\} = \\ &= \left\{ i \mapsto [0,0], n \mapsto [10,10] \right\} \sqcup \left\{ i \mapsto [-\infty,10], n \mapsto [10,10] \right\} = \\ &= \left\{ i \mapsto [-\infty,10], n \mapsto [10,10] \right\}. \end{split}$$

But it fails when there are relations between elements of V and $\mathcal{V} \setminus V$, e.g., for Item 3 with $d = \{x \leq 0, -x \leq 0, -y \leq 0, -x - y \leq 0, x - y \leq 0\}$:

$$\begin{bmatrix} \text{unassume}_{\{\mathsf{x}\}}(\mathsf{x} \ge 0) \end{bmatrix}_1^{\sharp} d = d \sqcup (\begin{bmatrix} \text{assume}(\mathsf{x} \ge 0) \end{bmatrix}^{\sharp} \circ \begin{bmatrix} \text{havoc}(\{\mathsf{x}\}) \end{bmatrix}^{\sharp}) d = d \sqcup \begin{bmatrix} \text{assume}(\mathsf{x} \ge 0) \end{bmatrix}^{\sharp} \{-\mathsf{y} \le 0\} = d \sqcup \{-\mathsf{x} \le 0, -\mathsf{y} \le 0, -\mathsf{x} - \mathsf{y} \le 0\} = \{-\mathsf{x} \le 0, -\mathsf{y} \le 0, -\mathsf{x} - \mathsf{y} \le 0\}.$$

In this case the octagon constraint $x-y \leq 0$ is lost by havocing and cannot be recovered by assuming.

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4.3 Dual-Narrowing

We will address the above challenge by relying on additional insights from abstract interpretation. Let us recall the term dual-narrowing, which is the lattice analogue of Craig interpolation [28]. A dual-narrowing operator $\widetilde{\Delta} : \mathbb{D} \to \mathbb{D} \to \mathbb{D}$ returns for every $d_1, d_2 \in \mathbb{D}$ with $d_1 \sqsubseteq d_2$, a value between both of them, i.e., $d_1 \sqsubseteq d_1 \widetilde{\Delta} d_2 \sqsubseteq d_2$.

Using such an operator, we can define an abstract unassume that, given d, may return an abstract value in the range from d to $[[unassume_V(e)]]^{\sharp}_{1} d$:

Definition 4 (Dual-narrowing unassume). Let $\widetilde{\Delta} : \mathbb{D} \to \mathbb{D} \to \mathbb{D}$ be a dualnarrowing. Let $V \subseteq \mathcal{V}$, $e \in \mathcal{E}$ and $d \in \mathbb{D}$. Then the dual-narrowing unassume is defined as a wrapper around the naïve unassume:

 $\llbracket \text{unassume}_V(e) \rrbracket_2^{\sharp} d = d \widetilde{\Delta} \llbracket \text{unassume}_V(e) \rrbracket_1^{\sharp} d.$

Example 5. A dual-narrowing for relational domains can be defined using *het*erogeneous environments and strengthening [42]. Let $\operatorname{dom}(d) \subseteq \mathcal{V}$ denote the environment of the abstract value $d \in \mathbb{D}$. Let $d|_V$ denote the restriction of the abstract value $d \in \mathbb{D}$ to the program variables $V \subseteq \mathcal{V}$.

An environment-aware order $\underline{\Bbbk}$ is defined for $d_1, d_2 \in \mathbb{D}$ by

$$d_1 \cong d_2 \iff \operatorname{dom}(d_1) \subseteq \operatorname{dom}(d_2) \land d_1 \sqsubseteq d_2|_{\operatorname{dom}(d_1)}.$$

Let $\exists : \mathbb{D} \to \mathbb{D} \to \mathbb{D}$ be an upper bound operator w.r.t. $\underline{\mathbb{E}}$, such that the resulting environment is minimal, i.e., $\operatorname{dom}(d_1 \exists d_2) = \operatorname{dom}(d_1) \cup \operatorname{dom}(d_2)$. Specifically, Journault et al. [42] define \exists as follows. The result of joining d_1 and d_2 in their common environment $\operatorname{dom}(d_1) \cap \operatorname{dom}(d_2)$ is extended to $\operatorname{dom}(d_1) \cup \operatorname{dom}(d_2)$ by adding unconstrained dimensions. A *strengthening* operator refines this result by iteratively adding back constraints from both arguments which would not cause the upper-boundedness w.r.t. $\underline{\mathbb{E}}$ to be violated. Note that this definition is not semantic, i.e., the result depends on the constraints representing the arguments and their processing order.

By defining $d_1 \Delta d_2 = d_1 \boxtimes d_2|_V$, which is *parametrized* by V, dual-narrowing unassume yields the following desired result for Item 3 from Example 4 with $d = \{\mathbf{x} \leq 0, -\mathbf{x} \leq 0, -\mathbf{y} \leq 0, -\mathbf{x} - \mathbf{y} \leq 0, \mathbf{x} - \mathbf{y} \leq 0\}$:

$$\begin{aligned} & [[\text{unassume}_{\{\mathsf{x}\}}(\mathsf{x} \ge 0)]]_2^{\sharp} d = d \widetilde{\Delta} \ [[\text{unassume}_{\{\mathsf{x}\}}(\mathsf{x} \ge 0)]]_1^{\sharp} d = \\ &= d \widetilde{\Delta} \{-\mathsf{x} \le 0, -\mathsf{y} \le 0, -\mathsf{x} - \mathsf{y} \le 0\} = d \bowtie \{-\mathsf{x} \le 0, -\mathsf{y} \le 0, -\mathsf{x} - \mathsf{y} \le 0\}|_{\{\mathsf{x}\}} = \\ &= d \bowtie \{-\mathsf{x} \le 0\} = \{-\mathsf{x} \le 0, -\mathsf{y} \le 0, -\mathsf{x} - \mathsf{y} \le 0, \mathsf{x} - \mathsf{y} \le 0\}. \end{aligned}$$

Although the restriction to V first destroys relations between V and $\mathcal{V} \setminus V$, the subsequent strengthening join can restore original relations which are compatible with e on V.

5 Unassuming Indirectly

We now turn to the unassuming of more complex invariants, which include indirection via pointers and dependent subexpressions. Naïve unassume is unable to achieve the desired precision for Item 4 from Example 4 with $d = \{p \mapsto \{\&i, \&j\}, i \mapsto [0, 0], j \mapsto [0, 0]\}$:

$$\begin{bmatrix} \text{unassume}_{\{\mathbf{i},\mathbf{j}\}}(\mathbf{*}\mathbf{p} \ge 0) \end{bmatrix}_{1}^{\sharp} \{\mathbf{p} \mapsto \{\mathbf{\&}\mathbf{i},\mathbf{\&}\mathbf{j}\}, d = \\ = d \sqcup (\begin{bmatrix} \text{assume}(\mathbf{*}\mathbf{p} \ge 0) \end{bmatrix}^{\sharp} \circ [\begin{bmatrix} \text{havoc}(\{\mathbf{i},\mathbf{j}\}) \end{bmatrix}^{\sharp}) d = \\ = d \sqcup \begin{bmatrix} \text{assume}(\mathbf{*}\mathbf{p} \ge 0) \end{bmatrix}^{\sharp} \{\mathbf{p} \mapsto \{\mathbf{\&}\mathbf{i},\mathbf{\&}\mathbf{j}\}, \mathbf{i} \mapsto \top, \mathbf{j} \mapsto \top\} = \\ = d \sqcup \{\mathbf{p} \mapsto \{\mathbf{\&}\mathbf{i},\mathbf{\&}\mathbf{j}\}, \mathbf{i} \mapsto \top, \mathbf{j} \mapsto \top\} = \{\mathbf{p} \mapsto \{\mathbf{\&}\mathbf{i},\mathbf{\&}\mathbf{j}\}, \mathbf{i} \mapsto \top, \mathbf{j} \mapsto \top\}.$$

This is due to both integer variables being havoced and the assume operator not being able to soundly refine via ambiguous may-point-to sets (see Appendix A). Technically, there exists a dual-narrowing that yields the desired result, but it would be ad-hoc.

To address the disjunctive nature of the may-point-to set, we propose an improved unassume operator. Suppose we are provided a family of mappings $\operatorname{explode}_V(e) : \mathbb{D} \to 2^{\mathbb{D}}$ which explode any given abstract state d into a non-empty finite subset $\operatorname{explode}_V(e) d \subseteq \mathbb{D}$ of abstract states where for each resulting element d' we have $d' \sqsubseteq d$. The *explode* operator can be used to make disjunctive information in abstract states explicit, e.g., resolve non-singleton may-point-to sets for pointer variables not contained in V.

Definition 5 (Exploding unassume). Let $V \subseteq \mathcal{V}$, $e \in \mathcal{E}$ and $d \in \mathbb{D}$. Let $explode_V(e)$ be an explode operator. Then the exploding unassume is defined as

 $\llbracket \text{unassume}_V(e) \rrbracket_3^{\sharp} d = \prod d \sqcup (\llbracket \text{assume}(e) \rrbracket^{\sharp} \circ \llbracket \text{havoc}(V) \rrbracket^{\sharp}) d'.$ $d' \in \text{explode}_V(e) d$

This improved unassume operator is extensive and therefore sound for any choice of $\operatorname{explode}_V(e)$. One might want to establish that $\bigsqcup \operatorname{explode}_V(e) d = d$ holds, but this is not necessary for soundness. Whereas $\operatorname{explode}_V(e) d = \{\bot\}$ would make the unassume a no-op.

Example 6. Consider the following explode operator, which splits ambiguous may-point-to sets:

$$\begin{aligned} & \text{explode}_{\{i,j\}}(*p \ge 0) \{ p \mapsto \{\&i,\&j\}, i \mapsto [0,0], j \mapsto [0,0] \} = \\ & = \{ \{ p \mapsto \{\&i\}, i \mapsto [0,0], j \mapsto [0,0] \}, \{ p \mapsto \{\&j\}, i \mapsto [0,0], j \mapsto [0,0] \} \}. \end{aligned}$$

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Using this explode operator, Item 4 from Example 4 is handled as desired with $d = \{p \mapsto \{\&i, \&j\}, i \mapsto [0, 0], j \mapsto [0, 0]\}$:

$$\begin{split} & [[unassume_{\{i,j\}}(*p \ge 0)]]_{3}^{\sharp} d = \\ &= (d \sqcup ([[assume(*p \ge 0)]]^{\sharp} \circ [[havoc(\{i,j\})]]^{\sharp}) \{p \mapsto \{\&i\}, i \mapsto [0,0], j \mapsto [0,0]\}) \sqcap \\ & \sqcap (d \sqcup ([[assume(*p \ge 0)]]^{\sharp} \circ [[havoc(\{i,j\})]]^{\sharp}) \{p \mapsto \{\&j\}, i \mapsto [0,0], j \mapsto [0,0]\}) = \\ &= (d \sqcup [[assume(*p \ge 0)]]^{\sharp} \{p \mapsto \{\&i\}, i \mapsto \top, j \mapsto \top\}) \sqcap \\ & \sqcap (d \sqcup [[assume(*p \ge 0)]]^{\sharp} \{p \mapsto \{\&j\}, i \mapsto \top, j \mapsto \top\}) = \\ &= (d \sqcup \{p \mapsto \{\&i\}, i \mapsto [0,\infty], j \mapsto \top\}) \sqcap \\ & \sqcap (d \sqcup \{p \mapsto \{\&i\}, i \mapsto [0,\infty], j \mapsto \top\}) \sqcap \\ & \sqcap (d \sqcup \{p \mapsto \{\&j\}, i \mapsto \top, j \mapsto [0,\infty]\}) = \\ &= \{p \mapsto \{\&i,\&j\}, i \mapsto [0,\infty], j \mapsto \top\} \sqcap \{p \mapsto \{\&i,\&j\}, i \mapsto \top, j \mapsto [0,\infty]\} = \\ &= \{p \mapsto \{\&i,\&j\}, i \mapsto [0,\infty], j \mapsto [0,\infty]\}. \end{split}$$

Example 7. However, consider the following, where different subexpressions depend on each other (here through p):

 $[[unassume_{\{\mathsf{p},\mathsf{i},\mathsf{j}\}}((\mathsf{p}=\mathtt{\&i}\lor\mathsf{p}=\mathtt{\&j})\land \star\mathsf{p}\geq 0)]]^{\sharp}\{\mathsf{p}\mapsto\{\mathtt{\&i}\},\mathsf{i}\mapsto[0,0],\mathsf{j}\mapsto[0,0]\}.$

In contrast to Example 6, there is no ambiguous may-point-to set in the abstract state supplied as the argument. All possible explosions lead to the same issue as when using the naïve unassume on this example. After havocing, the environment contains $\mathbf{p} \mapsto \top$, thus, in the assume a top pointer needs to be dereferenced and its targets refined. The semantics of this is unclear and imprecise at best, when one has to consider assignments to *all* possible (unrelated) memory locations.

5.1 Propagating Unassume

The HC4-revise algorithm by Benhamou et al. [11] can be used to implement the assume operation for complex expressions on non-relational domains in a syntax-directed manner [47, 60]. It is also known as *backwards evaluation* [27]. We describe the algorithm and then apply it to construct an unassume operator.

We loosely follow the presentation by Cousot [27]. Let the languages of expressions e and logical conditions c be defined by the grammars in Fig. 1. For each $n \in \mathbb{N}$, let \mathcal{O}_n be the set of *n*-ary operators. For simplicity of presentation, assume that the condition is in negation normal form (NNF), i.e., negations in conditions have been "pushed down" into binary comparisons according to boolean logic. The logical conditions form an invariant language (see Section 2). The following algorithms generalize from just variables to lvalues, allowing for languages with pointers like our example invariant language from before. This generalization is formalized in Appendix A.

Evaluation. Let \mathbb{V} be the abstract domain for individual values and $\mathbb{D} = \mathcal{V} \to \mathbb{V}$ the abstract domain for non-relational environments. Let \mathbb{B} be the flat boolean

e ::= k	(constant)	$c ::= e \bowtie e$	(binary comparison,
$\mid x$	(variable, $x \in \mathcal{V}$)		$\bowtie \in \{=,\neq,<,\leq,>,\geq\})$
$ \Box(e)_{i=1}^{n}$	(n-ary operator,	$ c \wedge c$	(conjunction)
	$n \in \mathbb{N}, \Box \in \mathcal{O}_n)$	$ c \lor c$	(disjunction)

Fig. 1: Syntax of expressions and conditions.

$$\begin{split} \mathbb{E}\llbracket e \rrbracket^{\sharp} : \mathbb{D} \to \mathbb{V} & \mathbb{C}\llbracket c \rrbracket^{\sharp} : \mathbb{D} \to \mathbb{B} \\ \mathbb{E}\llbracket k \rrbracket^{\sharp} d = k^{\sharp} & \mathbb{C}\llbracket e_1 \bowtie e_2 \rrbracket^{\sharp} d = \mathbb{E}\llbracket e_1 \rrbracket^{\sharp} d \bowtie^{\sharp} \mathbb{E}\llbracket e_2 \rrbracket^{\sharp} d \\ \mathbb{E}\llbracket x \rrbracket^{\sharp} d = d x & \mathbb{C}\llbracket c_1 \land c_2 \rrbracket^{\sharp} d = \mathbb{C}\llbracket c_1 \rrbracket^{\sharp} d \land^{\sharp} \mathbb{C}\llbracket c_2 \rrbracket^{\sharp} d \\ \mathbb{E}\llbracket \Box (e_i)_{i=1}^n \rrbracket^{\sharp} d = \Box^{\sharp} (\mathbb{E}\llbracket e_i \rrbracket^{\sharp} d)_{i=1}^n & \mathbb{C}\llbracket c_1 \lor c_2 \rrbracket^{\sharp} d = \mathbb{C}\llbracket c_1 \rrbracket^{\sharp} d \lor^{\sharp} \mathbb{C}\llbracket c_2 \rrbracket^{\sharp} d \end{split}$$

Fig. 2: Forward evaluation of expressions and conditions.

domain, where $\perp \sqsubseteq \{ \mathsf{true}^{\sharp}, \mathsf{false}^{\sharp} \} \sqsubseteq \top$. The standard abstract forward evaluation of expressions $\mathbb{E}\llbracket e \rrbracket^{\sharp}$ and conditions $\mathbb{C}\llbracket c \rrbracket^{\sharp}$ in the non-relational environment $d \in \mathbb{D}$ is shown in Fig. 2. For a constant k, let k^{\sharp} be its corresponding abstraction, and $\Box^{\sharp}, \bowtie^{\sharp}, \wedge^{\sharp}, \lor^{\sharp}$ be abstract versions of the corresponding operators.

Assume. The HC4-revise algorithm for the assume operation has two phases:

- 1. Bottom-up forward propagation on the expression tree abstractly evaluates the expression, as usual.
- 2. Top-down backward propagation refines each abstract value with the expected result of the sub-expression. This relies on backward abstract operators, which refine each argument based on the other arguments and the expected result, while variables are refined at the leaves.

The algorithm $\llbracket assume(e) \rrbracket^{\sharp}$ with its abstract backward evaluation of expressions $\overleftarrow{\mathbb{E}} \llbracket e \rrbracket^{\sharp}$ and conditions $\overleftarrow{\mathbb{C}} \llbracket c \rrbracket^{\sharp}$ is shown in Fig. 3. Instead of evaluating to an abstract value, they refine values of variables in the abstract environment. For each $n \in \mathbb{N}$, $\Box \in \mathcal{O}_n$, let $\overleftarrow{\Box}^{\sharp} : \mathbb{V} \to \mathbb{V}^n \to \mathbb{V}^n$ be the abstract backward version of the *n*-ary operator \Box . It returns abstract values for its arguments under the assumption that the operator evaluates to the given abstract value v' and the other arguments have the given abstract values. For example, if n = 2, then

$$\overline{\Box}^{\sharp} v'(v_1, v_2) = (v'_1, v'_2) \implies \{x_1 \in \gamma \mathbb{V} \mid \exists x_2 \in \gamma v_2 : \Box (x_1, x_2) \in \gamma v'_1 \leq \gamma v'_1 \land \langle x_2 \in \gamma \mathbb{V} \mid \exists x_1 \in \gamma v_1 : \Box (x_1, x_2) \in \gamma v'_1 \} \subseteq \gamma v'_2.$$

Unlike Cousot [27] and Miné [47], we require that the backward operators *do not* intersect an argument's backward-computed value with its current value.

Instead, we make this explicit in the algorithm like Benhamou et al. [11]. Similarly, let $\overleftarrow{\bowtie}^{\sharp} : \mathbb{V} \to \mathbb{V} \to \mathbb{V} \times \mathbb{V}$ be the abstract backward version of the comparison \bowtie . Since conditions are in NNF, the expected result is always true^{\sharp} and no v' argument is needed for it. The evaluations $\mathbb{E}[\![e]\!]^{\sharp} d$ should all be cached and reused from a single forward evaluation as the argument environment is passed around without changes [11, 47].

$$\llbracket \text{assume}(e) \rrbracket^{\sharp} d = \overleftarrow{\mathbb{C}} \llbracket e \rrbracket^{\sharp} d$$

$$\begin{split} & \overleftarrow{\mathbb{E}} \, \llbracket e \rrbracket^{\sharp} : \mathbb{V} \to \mathbb{D} \to \mathbb{D} & \overleftarrow{\mathbb{C}} \, \llbracket c \rrbracket^{\sharp} : \mathbb{D} \to \mathbb{D} \\ & \overleftarrow{\mathbb{E}} \, \llbracket k \rrbracket^{\sharp} \, v' \, d = \mathbf{if} \, k^{\sharp} \sqsubseteq v' \, \mathbf{then} \, d \, \mathbf{else} \perp & \overleftarrow{\mathbb{C}} \, \llbracket e_1 \bowtie e_2 \rrbracket^{\sharp} \, d = \\ & \overleftarrow{\mathbb{E}} \, \llbracket x \rrbracket^{\sharp} \, v' \, d = d \llbracket x \mapsto d \, x \sqcap v' \rrbracket & \mathbf{let} \, (v_1, v_2) = (\mathbb{E} \llbracket e_1 \rrbracket^{\sharp} \, d, \mathbb{E} \llbracket e_2 \rrbracket^{\sharp} \, d) \, \mathbf{in} \\ & \overleftarrow{\mathbb{E}} \, \llbracket \Box \, (e_i)_{i=1}^{n} \rrbracket^{\sharp} \, v' \, d = & \mathbf{let} \, (v_1, v_2) = (\mathbb{E} \llbracket e_1 \rrbracket^{\sharp} \, d, \mathbb{E} \llbracket e_2 \rrbracket^{\sharp} \, d) \, \mathbf{in} \\ & \mathbf{let} \, (v_i)_{i=1}^{n} = (\mathbb{E} \llbracket e_i \rrbracket^{\sharp} \, d)_{i=1}^{n} \, \mathbf{in} & \overleftarrow{\mathbb{E}} \llbracket e_1 \rrbracket^{\sharp} \, (v_1 \sqcap v_1') \, d \sqcap \, \overleftarrow{\mathbb{E}} \llbracket e_2 \rrbracket^{\sharp} \, (v_2 \sqcap v_2') \, d \\ & \mathbf{let} \, (v_i')_{i=1}^{n} = \overleftarrow{\square}^{\sharp} \, v' \, (v_i)_{i=1}^{n} \, \mathbf{in} & \overleftarrow{\mathbb{C}} \, \llbracket c_1 \varPi^{\sharp} \, d \sqcap \, \overleftarrow{\mathbb{C}} \llbracket c_2 \rrbracket^{\sharp} \, d \\ & \prod_{i=1}^{n} \overleftarrow{\mathbb{E}} \llbracket e_i \rrbracket^{\sharp} \, (v_i \sqcap v_i') \, d & \overleftarrow{\mathbb{C}} \llbracket c_1 \varPi^{\sharp} \, d \sqcup \, \overleftarrow{\mathbb{C}} \llbracket c_2 \rrbracket^{\sharp} \, d \\ & \overleftarrow{\mathbb{C}} \llbracket c_1 \lor^{\sharp} \, d \sqcup \, \overleftarrow{\mathbb{C}} \llbracket c_2 \rrbracket^{\sharp} \, d \end{bmatrix} \end{split}$$

Fig. 3: Assume via backward evaluation of expressions and conditions by the propagation algorithm.

Unassume. This algorithm can be adapted into a *propagating unassume* operator $[[unassume(e)]]^{\sharp}$ as shown in Fig. 4. Changes are required to achieve the following properties:

Variable set variance. In Example 7 the first conjunct should relax $\{p\}$, while the second should relax $\{i, j\}$ via the relaxed pointer. In order to allow different sub-expressions to relax different variable sets, the abstract environments returned by $\mathbb{E}[\![e]\!]^{\sharp}$ and $\mathbb{C}[\![c]\!]^{\sharp}$ are partial: they only contain variables which have been relaxed at leaves in the corresponding sub-expression.

Thus heterogeneous lattice join \bowtie from Example 5 is used. However, here in the non-relational case its definition is simpler: values are joined pointwise while using \perp for missing variables. The heterogeneous lattice meet \blacksquare is defined analogously, using \top for missing variables. Note that \blacksquare is *not* the meet w.r.t. $\underline{\sqsubseteq}$, because \blacksquare must preserve all relaxed variables from both operands, not just the common ones.

- **Soundness.** The result is joined with the pre-unassume environment to ensure soundness.
- **Relaxation.** Backward propagation only propagates backward values v'_i and does not refine them using abstract values v_i computed by forward propagation. Otherwise, sub-expressions cannot be relaxed at all from their current

where

values. Note that the forward values are still necessary for evaluating the backward operators. This modification on its own yields the HC4-revise^{*} algorithm described by Benhamou et al. [11].

 $\llbracket \text{unassume}(e) \rrbracket^{\sharp} d = \underline{d} \Join (\tilde{\mathbb{C}} \llbracket e \rrbracket^{\sharp} d)$

where

Fig. 4: Unassume via backward evaluation of expressions and conditions by the propagation algorithm (changes from the assume algorithm are highlighted).

Unlike our previous unassume operators, this algorithm implicitly chooses V to be those variables which are relaxed in the process. Note that it is different from the set of variables syntactically occurring in e in a more complex invariant language, such as our example language with pointers, which is described in Appendix A. This is illustrated by Example 8 below.

Local Iteration. Repeated application of the propagation algorithm for assuming can improve precision in the presence of dependent subexpressions, i.e., when the same variable occurs multiple times in the condition [27, 47]. Analogously, repeated application of the propagation algorithm for unassuming can perform more relaxation in the presence of dependent subexpressions. Both repetitions can be iterated to a local fixpoint.

Example 8. Consider using the above algorithm for the case from Example 7:

 $[\operatorname{unassume}((\mathsf{p} = \& i \lor \mathsf{p} = \& j) \land *\mathsf{p} \ge 0)]^{\sharp} \{\mathsf{p} \mapsto \{\& i\}, i \mapsto [0, 0], j \mapsto [0, 0]\}.$

As formalized in Appendix A, the first backward propagation returns for $\mathbf{p} = \&i \lor \mathbf{p} = \&j$ the partial map $\{\mathbf{p} \mapsto \{\&i,\&j\}\}$ and uses $*\mathbf{p} \ge 0$ to relax $*\mathbf{p}$. To do so, backward operator \succeq^{\sharp} uses the expected true result and its forward-evaluated right argument [0, 0] to propagate the expected value $[0, \infty]$ into its left argument. Using forward evaluated $\mathbf{p} \mapsto \{\&i\}$, backward propagation of the lvalue $*\mathbf{p}$, acting

as a leaf, returns the partial map $\{i \mapsto [0, \infty]\}$. The new constructed environment is $\{p \mapsto \{\&i, \&j\}, i \mapsto [0, \infty], j \mapsto [0, 0]\}$.

The second backward propagation does all of the above and also includes $\mathbf{j} \mapsto [0, \infty]$, due to the new points-to set. The final fixpoint environment is $\{\mathbf{p} \mapsto \{\mathbf{\&}\mathbf{i}, \mathbf{\&}\mathbf{j}\}, \mathbf{i} \mapsto [0, \infty], \mathbf{j} \mapsto [0, \infty]\}$.

In Example 8 the two iterations induced $V = \{p, i, j\}$, which used with naïve unassume yields the issue described in Example 7. Therefore propagating unassume is not equivalent to simply using its induced variable set with a naïve unassume. Propagating unassume fuses the multiple steps involved together into one algorithm which avoids intermediate imprecision and undefined behavior. We do not give an exact characterization of the result computed by the modified algorithm as it has remained an open problem for HC4-revise itself [11, 36].

In case the value lattice has infinite chains, the local iteration of propagating assume must use narrowing to ensure termination [27, 47]. Similarly, the local iteration of propagating unassume must instead use widening.

Example 9. Consider using the above algorithm to compute the following:

 $\llbracket unassume(i \le i+1) \rrbracket^{\sharp} \{i \mapsto [0,0]\}.$

The algorithm makes following iterations:

- 1. The first forward propagation computes $[0,0] \leq^{\sharp} [1,1]$. Backward propagation then returns $\{i \mapsto [-\infty,1] \sqcap [-1,\infty] = [-1,1]\}$.
- 2. The second forward propagation computes $[-1, 1] \leq [0, 2]$. Backward propagation then returns $\{i \mapsto [-\infty, 2] \sqcap [-2, \infty] = [-2, 2]\}$. If no widening is applied, then these bounds keep growing by one per iteration. If widening is applied, then we get $[-1, 1] \nabla [-2, 2] = \top$.
- 3. The third forward propagation computes $\top \leq^{\sharp} \top$. Backward propagation cannot relax anything further, so the result is $\{i \mapsto \top\}$. This is consistent with expectation: all values of i, where the tautology holds.

6 Evaluation

We implement the unassume operator in a state-of-the-art abstract interpreter. Since the analyzer is sound, this yields a sound witness validator. However, a sound validator can trivially be obtained by replacing the unassume operator with the identity operator and ignoring the witnesses entirely. Therefore, our experimental evaluation aims to demonstrate that our witness-guided verifier effectively uses witnesses. More specifically, we seek to confirm that "the effort and feasibility of validation depends on witness content" [15]. To assess the analyzer's dependency on witness content, we pose the following questions:

- **Precision.** Can the witness-guided verifier leverage witnesses to validate verification results that it could not confirm without a witness?
- **Performance.** Do witnesses influence the verification effort in the application domain of the analyzer?

It is worth noting that the performance improvement from technology-agnostic correctness witnesses is expected to be modest. In fact, Beyer et al. [15] observed no consistent trend in performance gains.

Experimental Setup and Data. Our benchmarks are executed on a laptop running Ubuntu 22.04.3 on an AMD RYZEN 7 PRO 4750U processor. For reliable measurements, all the experiments are carried out using the BENCHEXEC framework [21], where each tool execution is limited to 900 s of CPU time on one core and 4 GB of RAM. The benchmarks, tools and scripts used, as well as the raw results of the evaluation, are openly archived on Zenodo [51].

Implementation. GOBLINT is an abstract interpretation framework for C programs [59]. We have extended the framework with unassume operators and YAML witness support. The correctness witnesses proposed by Beyer et al. [15] and subsequently used in SV-COMP [14] provide invariants using an automaton in the GraphML format. The witnesses we consider (defined in Section 2.1) are much simpler and, thus, we use the newly-proposed YAML format [55, 57], which directly matches our notion. To this end, our implementation includes parsing of YAML witnesses and matching provided invariants to program locations such that the unassume operator can be applied. Our implementation contains two unassume operators:

- 1. Propagating unassume (Section 5.1) for non-relational domains. The existing propagating assume in GOBLINT could be generalized and directly reused, yielding an unassume operator capable of handling, e.g., C lvalues, not just variables, with no extra effort (see Appendix A).
- 2. Strengthening-based dual-narrowing unassume (Section 4.3) for relational domains. Although APRON [41], which GOBLINT uses for its relational domains, does not provide dual-narrowing, the generic approach described in Example 5 works for, e.g., octagons and convex polyhedra. Since the relational analysis is just numeric, V is collected syntactically.

To prevent unintended precision loss when widening from initially reached abstract values to the unassumed ones, we must take care to delay the application of widening. We tag abstract values with the identifiers of incorporated witness invariants (UUIDs from the YAML witness) and delay the widening if this set increases [44]. Such widening tokens ensure that each witness invariant can be incorporated without immediate overshooting.

6.1 Precision Evaluation

We collected and provide a set of 11 example programs (excluding duplicates) from literature [3, 26, 37, 47] where more advanced abstract interpretation techniques are developed to infer certain invariants, where standard accelerated solving strategies fail. We configure GOBLINT the same as in SV-COMP [49, 50],

except autotuning is disabled and relational analysis using polyhedra is unconditionally enabled. We manually created YAML witnesses containing suitable loop invariants for these programs. We also used two state-of-the-art verifiers from SV-COMP 2023 to generate real witnesses: CPACHECKER [20, 32] and UAUTOMIZER [39, 40]. Both verifiers are able to verify these programs and produce GraphML witnesses. Following Beyer et al. [22], we use CPACHECKER in its witness2invariant configuration to convert them into YAML witnesses that GOBLINT can consume.

Table 1: Evaluation results on literature examples (excluding duplicates). The GOBLINT column indicates whether it can verify the program without any witness. Remaining columns indicate results with corresponding witnesses: witness validated (\checkmark), program verified with witness-guidance but witness not validated (\checkmark) or program not verified with witness-guidance (\bigstar).

		Goblint	Goblint w/ witness from			
Author(s)	Example	$\rm w/o~witness$	Manual CPACHECKER UAUTOMIZER			
Miné [47]	4.6 4.7 4.8 4.10	× × ✓	\$ \$ \$	√ X √ X √ X	\$ \$ \$	
Halbwachs and Henry [37]	1.b 2.b 3	✓ × ×	\$ \$ \$	√ X √ X √ X	√ X ✓ ✓ X	
Boutonnet and Halbwachs [26]	1 (polyhedra) 3 "additional"	× × ×		√ X X X	5 X 5 5	
Amato and Scozzari [3]	hybrid	X	1	√×	1	
Total	11	√ : 3	√ : 11	√X : 8, √ : 1	√X : 3, √ : 8	

The results are summarized in Table 1. GOBLINT manages to verify the desired property for 3 of these programs without any witness, but can validate all handwritten witnesses, despite not implementing any of the advanced techniques needed for their inference. With CPACHECKER witnesses our validator can verify 9 out of 11 programs and validate 1 out of 11 witnesses. With UAU-TOMIZER witnesses our validator can verify all 11 programs and validate 8 out of 11 witnesses. Furthermore, our abstract interpreter can validate the witnesses from model checkers orders of magnitude faster than it took to generate them.

The evaluation, however, shows many instances where the program was only verified thanks to witness-guidance, but not all witness invariants could be validated, especially for CPACHECKER. This is precisely due to the phenomenon described in Example 2: in these small programs bounded model checking is suc-

cessful and yields disjunctive invariants over all the finitely-many cases. Surprisingly, an invariant is useful for witness-guided verification, even when it cannot be proven to hold abstractly.

Example 10. The invariant from Example 2 relaxes an abstract state:

$$[[unassume_{\{x\}}(x = 1 \lor x = 2)]]^{\sharp} \{x \mapsto [1, 1]\} = \{x \mapsto [1, 2]\}.$$

6.2 Performance Evaluation

To explore whether a suitable witness can reduce verification effort, we consider larger programs, as runtimes for the literature examples are negligible. Since GOBLINT specializes in the analysis of multi-threaded programs, we examine a set of multi-threaded POSIX programs previously used to evaluate GOB-LINT [52, 53]. We manually construct witnesses that contain core invariants for these programs, based on how widenings were applied during fixpoint solving. We configure GOBLINT as described earlier, but with relational analysis disabled. In addition to CPU time, we measure analysis effort without a witness and with witness-guidance via transfer function evaluation counts. This metric of evaluations is proportional to CPU time, but excludes irrelevant pre- and post-processing, and is independent of hardware.

Table 2: Evaluation results on GOBLINT benchmarks. The LLoC column counts logical lines of code, i.e., only lines with executable code, excluding declarations.

		w/o witness		w/ witness		Reduction	
Program	LLoC	Evals	CPU time (s)	Evals	CPU time (s)	Evals C	PU time
pfscan aget knot smtprc	559 587 981 3,037	$\begin{array}{r} 4,194 \\ 7,932 \\ 29,588 \\ 48,559 \end{array}$	$0.86 \\ 2.23 \\ 4.92 \\ 15.00$	2,9194,68321,43224,091	0.73 1.68 4.54 7.95	30.4% 41.0% 27.6% 50.4%	$15.4\% \\ 24.7\% \\ 7.7\% \\ 47.0\%$
Average						37.3%	23.7%

The results, aggregated in Table 2, show a noticeable performance improvement in the abstract interpreter when guided by a witness. However, the fixpointsolving process still requires numerous widening iterations. This is due to various abstractions used by GOBLINT that cannot be expressed as C expressions, including but not limited to array index ranges in abstract addresses and various concurrency aspects. Nevertheless, the average $1.23 \times \text{CPU}$ time speedup is relatively close to the average $1.63 \times$ improvement achieved by Albert et al. [2] when using analyzer-specific certificates (see Section 7).

Admittedly, we have used a limited set of benchmarks and hand-crafted witnesses because our automatically generated witnesses produce excessive information. Large witnesses that express full proofs with numerous invariants can be problematic for a validator [1, 15], which must manipulate, use, and/or verify them. In our case, the validator performs an unassume operation for each invariant each time the corresponding transfer function is evaluated. The speedup gained from using the witness must outweigh the overhead to truly benefit from witnesses. This amounts to witnesses containing partial proofs, like loop invariants [15]. Moreover, our approach does not take advantage of exact invariants, such as equalities outside of disjunctions, since these do not relax a reached state already representing such exact values. Even if such invariants are useful for some validators, they do not benefit our witness-guided verifier. Therefore, the challenge remains for us, in collaboration with other tool developers, to develop methods for generating suitable witnesses.

7 Related work

Fixpoint iterations involving widening and narrowing are well-studied [3, 4, 28, 29, 54], but focus mostly on improving precision and ensuring termination. Halbwachs and Henry [37] extend fixpoint iteration with partial restarting, which derives from the narrowing result a new initial value for the following widening iteration, hoping it improves the result. Their restarted value is analogous to our partial initialization and could be used as such. They focus on finding such values automatically, while we focus on using them to avoid all the computation leading up to it. Boutonnet and Halbwachs [26] improve the technique for finding good restarting candidates. Cousot [28] extends fixpoint iteration with dual-narrowing, hoping it improves the result further. Both approaches focus purely on improving precision with more iterations, while we aim to skip that iteration and arrive at the same result quicker, knowing the invariant. Hence, the techniques can be combined: use theirs to find a precise invariant and use ours to directly reuse it.

Arceri et al. [6] swap the abstract domain for a more precise one when switching from widening to narrowing. This can be considered a precision improvement technique, which makes the narrowing phase more expensive. However, it can also be viewed as an optimization, which makes the widening phase cheaper. Either way, it can be combined with our approach: immediately using the more precise domain with the final invariant, ideally skipping iteration in both ways.

Widening operators themselves are also well-studied [8, 25, 35, 44, 47]. Widening *up to* or *with threshold* use candidate invariants as intermediates to avoid irrecoverably losing precision. Such automated techniques can identify which candidates are true invariants. Using these as input to our approach is an effective way of supplying known good thresholds at specific program points, removing the need for retrying all the candidates on re-verification. This is one instance of dealing with the inherent non-monotonicity of widening operators [28]. Furthermore, widening thresholds require domain-specific implementation, whereas our approach is more generic.

Our naïve unassume with its havoc and assume bears some similarity to the generation of *verification conditions* from user-supplied loop invariants [34].

However, there one additionally refines the state with the loop condition and then checks that the loop invariant is preserved. We avoid the former to remain sound by construction, but effectively do the latter when validating witness invariants.

Albert et al. [2] introduce Abstraction-Carrying Code (ACC) as an abstractinterpretation-based instance of Proof-Carrying Code (PCC). For validation they use a simplified analyzer which only performs a single pass of abstract interpretation and no fixpoint iteration. Thus, it requires certificates to supply invariants for all loops. This is more restrictive than our validation approach, which runs in a single pass if all necessary invariants are provided, but also allows some fixpoint iteration if this is not the case. Nevertheless, we could handicap our analyzer with this stronger restriction.

Albert et al. [1] develop a notion of reduced certificates which can be smaller and are used by the validator to reconstruct full certificates. Besson et al. [12] propose a fixpoint compression algorithm to further compact the certificates. In follow-up work, Besson et al. [13] develop a theory for studying the issue of certificate size. Rather than using the strongest information from the least fixpoint of an analysis, they seek the weakest information still sufficient for implying correctness. This omits irrelevant information, leading to smaller witnesses. Such techniques could also be used when generating witnesses for our validator.

8 Conclusion

We have demonstrated how to turn abstract-interpretation-based tools into witness-guided verifiers and witness validators, by equipping them with unassume operations. These can be constructed from abstract transformers for assumes, non-deterministic assignments, joins and (optionally) dual-narrowings, which allow retaining more precision for relational abstract interpretation. A powerful syntax-directed unassume operation for non-relational domains can be derived from a classical algorithm with minimal changes. Our implementation and evaluation demonstrate that unassuming invariants from witnesses can both speed up the analysis and make it more precise. The experiments further show that the abstract interpreter can benefit from witnesses produced by model checkers, and thus indicate that the approach is suited even for cross-technology collaboration.

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Bibliography

- Albert, E., Arenas, P., Puebla, G., Hermenegildo, M.: Reduced certificates for abstraction-carrying code. In: Logic Programming, pp. 163–178, Springer Berlin Heidelberg (2006), https://doi.org/10.1007/11799573_14
- [2] Albert, E., Puebla, G., Hermenegildo, M.: Abstraction-carrying code. In: Logic for Programming, Artificial Intelligence, and Reasoning, pp. 380–397, Springer Berlin Heidelberg (2005), https://doi.org/10.1007/ 978-3-540-32275-7 25
- [3] Amato, G., Scozzari, F.: Localizing widening and narrowing. In: Static Analysis, pp. 25–42, Springer Berlin Heidelberg (2013), https://doi.org/10.1007/ 978-3-642-38856-9
- [4] Amato, G., Scozzari, F., Seidl, H., Apinis, K., Vojdani, V.: Efficiently intertwining widening and narrowing. Science of Computer Programming 120, 1 – 24 (may 2016), https://doi.org/10.1016/j.scico.2015.12.005
- [5] Apinis, K., Seidl, H., Vojdani, V.: Side-effecting constraint systems: A swiss army knife for program analysis. In: Programming Languages and Systems, pp. 157–172, Springer Berlin Heidelberg (2012), https://doi.org/10.1007/ 978-3-642-35182-2 12
- [6] Arceri, V., Mastroeni, I., Zaffanella, E.: Decoupling the ascending and descending phases in abstract interpretation. In: Programming Languages and Systems, pp. 25–44, Springer Nature Switzerland (2022), https://doi.org/ 10.1007/978-3-031-21037-2 2
- [7] Ayaziová, P., Chalupa, M., Strejček, J.: Symbiotic-Witch: A KLEE-based violation witness checker. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 468–473, Springer International Publishing (2022), https://doi.org/10.1007/978-3-030-99527-0 33
- [8] Bagnara, R., Hill, P.M., Ricci, E., Zaffanella, E.: Precise widening operators for convex polyhedra. Science of Computer Programming 58(1-2), 28–56 (oct 2005), https://doi.org/10.1016/j.scico.2005.02.003
- [9] Baudin, P., Bobot, F., Bühler, D., Correnson, L., Kirchner, F., Kosmatov, N., Maroneze, A., Perrelle, V., Prevosto, V., Signoles, J., Williams, N.: The dogged pursuit of bug-free C programs: The Frama-C software analysis platform. Communications of the ACM 64(8), 56–68 (jul 2021), https:// doi.org/10.1145/3470569
- [10] Baudin, P., Cuoq, P., Filliâtre, J.C., Marché, C., Monate, B., Moy, Y., Prevosto, V.: ANSI/ISO C specification language version 1.19 (2023), URL http://frama-c.com/download/acsl.pdf
- [11] Benhamou, F., Goualard, F., Granvilliers, L., Puget, J.F.: Revising hull and box consistency. In: Logic Programming, p. 230–244, The MIT Press (1999), https://doi.org/10.7551/mitpress/4304.003.0024
- [12] Besson, F., Jensen, T., Pichardie, D.: Proof-carrying code from certified abstract interpretation and fixpoint compression. Theoretical Computer Science 364(3), 273–291 (nov 2006), https://doi.org/10.1016/j.tcs.2006.08.012

- 24 S. Saan et al.
- [13] Besson, F., Jensen, T., Turpin, T.: Small witnesses for abstract interpretation-based proofs. In: Programming Languages and Systems, pp. 268–283, Springer Berlin Heidelberg (2007), https://doi.org/10.1007/ 978-3-540-71316-6 19
- [14] Beyer, D.: Competition on software verification and witness validation: SV-COMP 2023. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 495–522, Springer Nature Switzerland (2023), https://doi.org/10.1007/978-3-031-30820-8 29
- [15] Beyer, D., Dangl, M., Dietsch, D., Heizmann, M.: Correctness witnesses: Exchanging verification results between verifiers. In: Proceedings of the 2016 24th ACM SIGSOFT International Symposium on Foundations of Software Engineering, pp. 326–337, ACM (nov 2016), https://doi.org/10.1145/ 2950290.2950351
- [16] Beyer, D., Dangl, M., Dietsch, D., Heizmann, M., Lemberger, T., Tautschnig, M.: Verification witnesses. ACM Transactions on Software Engineering and Methodology **31**(4), 1–69 (sep 2022), https://doi.org/10. 1145/3477579
- [17] Beyer, D., Dangl, M., Dietsch, D., Heizmann, M., Stahlbauer, A.: Witness validation and stepwise testification across software verifiers. In: Proceedings of the 2015 10th Joint Meeting on Foundations of Software Engineering, pp. 721–733, ACM (aug 2015), https://doi.org/10.1145/2786805.2786867
- Beyer, D., Dangl, M., Lemberger, T., Tautschnig, M.: Tests from witnesses: Execution-based validation of verification results. In: Tests and Proofs, pp. 3–23, Springer International Publishing (2018), https://doi.org/10.1007/ 978-3-319-92994-1
- [19] Beyer, D., Kanav, S.: CoVeriTeam: On-demand composition of cooperative verification systems. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 561–579, Springer International Publishing (2022), https://doi.org/10.1007/978-3-030-99524-9_31
- [20] Beyer, D., Keremoglu, M.E.: CPAchecker: A tool for configurable software verification. In: Computer Aided Verification, pp. 184–190, Springer Berlin Heidelberg (2011), https://doi.org/10.1007/978-3-642-22110-1 16
- [21] Beyer, D., Löwe, S., Wendler, P.: Reliable benchmarking: requirements and solutions. International Journal on Software Tools for Technology Transfer 21(1), 1–29 (nov 2017), https://doi.org/10.1007/s10009-017-0469-y
- [22] Beyer, D., Spiessl, M., Umbricht, S.: Cooperation between automatic and interactive software verifiers. In: Software Engineering and Formal Methods, pp. 111–128, Springer International Publishing (2022), https://doi.org/10. 1007/978-3-031-17108-6 7
- [23] Beyer, D., Strejček, J.: Case study on verification-witness validators: Where we are and where we go. In: Static Analysis, pp. 160–174, Springer Nature Switzerland (2022), https://doi.org/10.1007/978-3-031-22308-2
- [24] Beyer, D., Wehrheim, H.: Verification artifacts in cooperative verification: Survey and unifying component framework. In: Leveraging Applications of Formal Methods, Verification and Validation: Verification Principles,

pp. 143–167, Springer International Publishing (2020), https://doi.org/10. 1007/978-3-030-61362-4_8

- [25] Blanchet, B., Cousot, P., Cousot, R., Feret, J., Mauborgne, L., Miné, A., Monniaux, D., Rival, X.: A static analyzer for large safety-critical software. ACM SIGPLAN Notices 38(5), 196–207 (may 2003), https://doi.org/10. 1145/780822.781153
- [26] Boutonnet, R., Halbwachs, N.: Improving the results of program analysis by abstract interpretation beyond the decreasing sequence. Formal Methods in System Design 53(3), 384–406 (dec 2017), https://doi.org/10.1007/ s10703-017-0310-y
- [27] Cousot, P.: The calculational design of a generic abstract interpreter. In: Calculational System Design, NATO ASI Series F. IOS Press, Amsterdam (1999), URL https://www.di.ens.fr/~cousot/COUSOTpapers/publications. www/Cousot-Marktoberdorf98.pdf.gz
- [28] Cousot, P.: Abstracting induction by extrapolation and interpolation. In: Verification, Model Checking, and Abstract Interpretation, pp. 19–42, Springer Berlin Heidelberg (2015), https://doi.org/10.1007/ 978-3-662-46081-8 2
- [29] Cousot, P., Cousot, R.: Abstract interpretation: A unified lattice model for static analysis of programs by construction or approximation of fixpoints. In: Proceedings of the 4th ACM SIGACT-SIGPLAN symposium on Principles of programming languages, pp. 238–252, ACM Press (1977), https://doi. org/10.1145/512950.512973
- [30] Cousot, P., Cousot, R.: Abstract interpretation frameworks. Journal of Logic and Computation 2(4), 511–547 (1992), https://doi.org/10.1093/ logcom/2.4.511
- [31] Cousot, P., Cousot, R., Feret, J., Mauborgne, L., Miné, A., Rival, X.: Why does Astrée scale up? Formal Methods in System Design 35(3), 229–264 (nov 2009), https://doi.org/10.1007/s10703-009-0089-6
- [32] Dangl, M., Löwe, S., Wendler, P.: CPAchecker with support for recursive programs and floating-point arithmetic. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 423–425, Springer Berlin Heidelberg (2015), https://doi.org/10.1007/978-3-662-46681-0_34
- [33] Farzan, A., Kincaid, Z.: Verification of parameterized concurrent programs by modular reasoning about data and control. In: Proceedings of the 39th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, p. 297–308, ACM (jan 2012), https://doi.org/10.1145/ 2103656.2103693
- [34] Flanagan, C., Saxe, J.B.: Avoiding exponential explosion: Generating compact verification conditions. In: Proceedings of the 28th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, p. 193–205, ACM (jan 2001), https://doi.org/10.1145/360204.360220
- [35] Gopan, D., Reps, T.: Lookahead widening. In: Computer Aided Verification, pp. 452–466, Springer Berlin Heidelberg (2006), https://doi.org/10.1007/ 11817963_41

- 26 S. Saan et al.
- [36] Goualard, F., Granvilliers, L.: Controlled propagation in continuous numerical constraint networks. In: Proceedings of the 2005 ACM symposium on Applied computing, ACM (mar 2005), https://doi.org/10.1145/1066677. 1066765
- [37] Halbwachs, N., Henry, J.: When the decreasing sequence fails. In: Static Analysis, pp. 198–213, Springer Berlin Heidelberg (2012), https://doi.org/ 10.1007/978-3-642-33125-1 15
- [38] Haltermann, J., Wehrheim, H.: Information exchange between over- and underapproximating software analyses. In: Software Engineering and Formal Methods, pp. 37–54, Springer International Publishing (2022), https://doi. org/10.1007/978-3-031-17108-6 3
- [39] Heizmann, M., Barth, M., Dietsch, D., Fichtner, L., Hoenicke, J., Klumpp, D., Naouar, M., Schindler, T., Schüssele, F., Podelski, A.: Ultimate Automizer and the CommuHash normal form. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 577–581, Springer Nature Switzerland (2023), https://doi.org/10.1007/978-3-031-30820-8 39
- [40] Heizmann, M., Hoenicke, J., Podelski, A.: Software model checking for people who love automata. In: Computer Aided Verification, pp. 36–52, Springer Berlin Heidelberg (2013), https://doi.org/10.1007/ 978-3-642-39799-8 2
- [41] Jeannet, B., Miné, A.: Apron: A library of numerical abstract domains for static analysis. In: Computer Aided Verification, pp. 661–667, Springer Berlin Heidelberg (2009), https://doi.org/10.1007/978-3-642-02658-4 52
- [42] Journault, M., Miné, A., Ouadjaout, A.: An abstract domain for trees with numeric relations. In: Programming Languages and Systems, pp. 724– 751, Springer International Publishing (2019), https://doi.org/10.1007/ 978-3-030-17184-1_26
- [43] de León, H.P., Haas, T., Meyer, R.: Dartagnan: SMT-based violation witness validation. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 418–423, Springer International Publishing (2022), https:// doi.org/10.1007/978-3-030-99527-0 24
- [44] Mihaila, B., Sepp, A., Simon, A.: Widening as abstract domain. In: Lecture Notes in Computer Science, pp. 170–184, Springer Berlin Heidelberg (2013), https://doi.org/10.1007/978-3-642-38088-4_12
- [45] Miné, A.: The octagon abstract domain. Higher-Order and Symbolic Computation 19(1), 31-100 (mar 2006), https://doi.org/10.1007/s10990-006-8609-1
- [46] Miné, A.: Static analysis of run-time errors in embedded real-time parallel C programs. Logical Methods in Computer Science 8(1), 1–63 (mar 2012), https://doi.org/10.2168/lmcs-8(1:26)2012
- [47] Miné, A.: Tutorial on static inference of numeric invariants by abstract interpretation. Foundations and Trends® in Programming Languages 4(3-4), 120–372 (2017), https://doi.org/10.1561/2500000034, URL https://hal.sorbonne-universite.fr/hal-01657536/document
- [48] Monat, R., Miné, A.: Precise thread-modular abstract interpretation of concurrent programs using relational interference abstractions. In: Verification,

Model Checking, and Abstract Interpretation, pp. 386–404, Springer International Publishing (2017), https://doi.org/10.1007/978-3-319-52234-0 21

- [49] Saan, S., Schwarz, M., Apinis, K., Erhard, J., Seidl, H., Vogler, R., Vojdani, V.: Goblint: Thread-modular abstract interpretation using side-effecting constraints. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 438–442, Springer International Publishing (2021), https:// doi.org/10.1007/978-3-030-72013-1 28
- [50] Saan, S., Schwarz, M., Erhard, J., Pietsch, M., Seidl, H., Tilscher, S., Vojdani, V.: Goblint: Autotuning thread-modular abstract interpretation. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 547–552, Springer Nature Switzerland (2023), https://doi.org/10.1007/ 978-3-031-30820-8 34
- [51] Saan, S., Schwarz, M., Erhard, J., Seidl, H., Tilscher, S., Vojdani, V.: Correctness witness validation by abstract interpretation (2023), https://doi. org/10.5281/zenodo.8253000, artifact
- [52] Schwarz, M., Saan, S., Seidl, H., Apinis, K., Erhard, J., Vojdani, V.: Improving thread-modular abstract interpretation. In: Static Analysis, pp. 359– 383, Springer International Publishing (2021), https://doi.org/10.1007/ 978-3-030-88806-0 18
- [53] Schwarz, M., Saan, S., Seidl, H., Erhard, J., Vojdani, V.: Clustered relational thread-modular abstract interpretation with local traces. In: Programming Languages and Systems, pp. 28–58, Springer Nature Switzerland (2023), https://doi.org/10.1007/978-3-031-30044-8 2
- [54] Seidl, H., Vogler, R.: Three improvements to the top-down solver. Mathematical Structures in Computer Science 31(9), 1090–1134 (oct 2021), https://doi.org/10.1017/s0960129521000499
- [55] SoSy-Lab: YAML-based exchange format for correctness witnesses (2021), https://gitlab.com/sosy-lab/benchmarking/sv-witnesses/-/blob/main/ README-YAML.md
- [56] Strejček, J.: Issues related to the fact that the semantics of witnesses are defined over CFAs and the translation from C programs to CFAs is undefined (2022), URL https://gitlab.com/sosy-lab/benchmarking/sv-witnesses/-/ blob/main/GraphML witness format issues.pdf
- [57] SV-COMP community: Community meeting (apr 2023)
- [58] Švejda, J., Berger, P., Katoen, J.P.: Interpretation-based violation witness validation for C: NITWIT. In: Tools and Algorithms for the Construction and Analysis of Systems, pp. 40–57, Springer International Publishing (2020), https://doi.org/10.1007/978-3-030-45190-5_3
- [59] Vojdani, V., Apinis, K., Rõtov, V., Seidl, H., Vene, V., Vogler, R.: Static race detection for device drivers: the Goblint approach. In: Proceedings of the 31st IEEE/ACM International Conference on Automated Software Engineering, ACM (aug 2016), https://doi.org/10.1145/2970276.2970337
- [60] Ziat, G.: A combination of abstract interpretation and constraint programming. Theses, Sorbonne Université (jul 2019), URL https://theses.hal. science/tel-03987752

A Pointers

In Section 5.1 we claim that the algorithms generalize from variables to lvalues. The key machinery for this is the *variable set variance* described in the paper. While the original HC4-revise algorithm directly refines variables at leaf nodes, we handle pointers by letting leaf nodes create partial states containing the dynamically resolved variables.

Let $l ::= x | \star x$ be the grammar for lvalues, which can either be variables or dereferencing of variables. Let abstract value domain for pointer variables be may-point-to sets of variables $2^{\mathcal{V}}$. Let constants include address-taking as &x for a variable $x \in \mathcal{V}$.

Evaluation. The forward evaluation of address-taking is defined by $(\&x)^{\sharp} = \{x\}$. Forward evaluation of lvalues uses the following helper function for evaluating lvalues to may-point-to sets:

$$\mathbb{L}\llbracket l \rrbracket^{\sharp} : \mathbb{D} \to 2^{\mathcal{V}}$$
$$\mathbb{L}\llbracket x \rrbracket^{\sharp} d = \{x\}$$
$$\mathbb{L}\llbracket \star x \rrbracket^{\sharp} d = d x$$

The forward evaluation of lvalues is then a join over all the possibilities:

$$\mathbb{E}\llbracket l \rrbracket^{\sharp} d = \bigsqcup_{x \in \mathbb{L}\llbracket l \rrbracket^{\sharp} d} dx.$$

Assume. The backward evaluation of lvalues is defined analogously:

$$\overleftarrow{\mathbb{E}} \llbracket l \rrbracket^{\sharp} v' d = \bigsqcup_{x \in \mathbb{L} \llbracket l \rrbracket^{\sharp} d} d[x \mapsto d x \sqcap v'].$$

However, it is easier understood using the following equivalent formulation:

$$\overleftarrow{\mathbb{E}} \llbracket l \rrbracket^{\sharp} v' d = \begin{cases} d[x \mapsto d \, x \sqcap v'], & \text{if } \mathbb{L} \llbracket l \rrbracket^{\sharp} d = \{x\}, \\ d, & \text{otherwise.} \end{cases}$$

Thus, in the case of an ambiguous may-point-to set, no refinement takes place. This is because each variable is refined in one of the joinees, while left untouched in others, leaving the joined value also unchanged.

Unassume. Unassuming of lvalues is also defined analogously:

$$\widetilde{\mathbb{E}}\llbracket l \rrbracket^{\sharp} v' d = \bigsqcup_{x \in \mathbb{L}\llbracket l \rrbracket^{\sharp} d} \{x \mapsto v'\}.$$

However, it is also easier understood using the following equivalent formulation:

$$\tilde{\mathbb{E}}\llbracket l \rrbracket^{\sharp} v' d = \{ x \mapsto v' \mid x \in \mathbb{L}\llbracket l \rrbracket^{\sharp} d \}.$$

This can be seen as a special case of abstract assignment:

$$\tilde{\mathbb{E}}\llbracket l \rrbracket^{\sharp} v' d = \llbracket l \leftarrow \gamma v' \rrbracket^{\sharp} \{ x \mapsto \bot \mid x \in \mathbb{L}\llbracket l \rrbracket^{\sharp} d \},\$$

where $\llbracket l \leftarrow e \rrbracket^{\sharp} d$ is the abstract operator for assigning the value of expression e to the lvalue l. In this special case the assigned abstract value is v', not $\mathbb{E}\llbracket e \rrbracket^{\sharp} d$.

Therefore, propagating unassume for lvalues can be implemented using existing primitives: backward operators and abstract assignment. This is not limited to just simple pointers but works the same way for C lvalues, which also include array indexing and structure field offsets.