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Abstract We consider a strong variant of the crash fault-tolerant gathering problem called stand-up indulgent gathering (SUIG), by robots endowed with limited visibility sensors and lights on line-shaped networks. In this problem, a group of mobile robots must eventually gather at a single location, not known beforehand, regardless of the occurrence of crashes. Differently from previous work that considered unlimited visibility, we assume that robots can observe nodes only within a certain fixed distance (that is, they are myopic), and emit a visible color from a fixed set (that is, they are luminous), without multiplicity detection. We consider algorithms depending on two parameters related to the initial configuration: M_{init} , which denotes the number of nodes between two border nodes, and O_{init} , which denotes the number of nodes hosting robots. Then, a border node is a node hosting one or more robots that cannot see other robots on at least one side. Our main contribution is to prove that, if M_{init} or O_{init} is odd, SUIG can be solved in the fully synchronous model.

Key words: Crash failure, fault-tolerance, LCM robot model, limited visibility, light

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1 Introduction

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The Distributed Computing research community actively studies mobile robot swarms, aiming to characterize what conditions make it possible for robots that are confused (each robot has its own ego-centered coordinate system), forgetful (robots may not remember all their past actions) to autonomously move around and solve global problems [17]. One of these conditions is about how the robots coordinate their actions [12]: robots can either act all together (FSYNC), act whenever they want (ASYNC), or act in subsets (SSYNC).

One of the problems that researchers have explored is the *gathering problem*, which serves as a standard for comparison [17]. It is easy to state (robots must meet at the same place in a finite amount of time, without knowing where it is beforehand), but hard to solve (two robots that move according to SSYNC scheduling cannot meet in finite time [9], unless there are more assumptions).

Robot failures become more likely as the number of robots increases, or if robots are deployed in dangerous environments, but few studies address this issue [11]. A crash fault is a simple type of failure, where a robot stops following its protocol unexpectedly. For the gathering problem, the desired outcome in case of crash faults must be specified. There are two options: *weak gathering* requires all non-faulty robots to meet, ignoring the faulty ones, while *strong gathering* (or *stand-up indulgent gathering – SUIG*) requires all non-faulty robots to meet at the unique crash location. We believe that SUIG is an attractive task for difficult situations such as dangerous environments: for example, various repair parts could be transported by different robots, and if a robot crashes, the other ones may rescue and repair it after robots carrying relevant parts are gathered at the crash location.

In continuous Euclidean space, weak gathering is solvable in the SSYNC model [1, 3, 8, 10], while SUIG (and its variant with two robots, stand up indulgent rendezvous – SUIR) is only solvable in the FSYNC model [5, 6, 7].

Some researchers have recently switched from studying robots in a continuous space to a discrete one [12]. In a discrete space, robots can only be in certain locations and move to adjacent ones. This can be modeled by a graph where the nodes are locations, hence the term "robots on graphs". A discrete space is more realistic for modeling physical constraints or discrete sensors [2]. However, it is not equivalent to a continuous space in terms of computation: a discrete space has fewer possible robot positions, but a continuous space gives more options to resolve difficult situations (e.g., by moving slightly to break a symmetry).

To our knowledge, SUIG in a discrete setting was only considered under the assumption that robots have infinite range visibility (that is, their sensors are able to obtain the position of all other robots in the system that participate to the gathering) in line-shaped networks. Such powerful sensors may seem unrealistic, paving the way for more practical solutions. With infinite visibility, Bramas et al. [4] showed that the SUIG problem is solvable in the FSYNC model only.

When infinite range visibility is no longer available, robots become unable to distinguish global configuration situations and act accordingly, in particular, robots may react differently to different local situations, yielding in possible synchronization

issues [13, 14]. In this paper, we consider the discrete setting, and aim to characterize the solvability of the SUIG problem when robots have limited visibility (that is, they are myopic) yet are endowed with visible lights taking colors from a finite set (that is, they are luminous). In particular, we are interested in the trade-off between the visibility range (how many hops away can we see other robots positions) and the memory and communication capacity of the robots (each robot can have a finite number of states that may be communicated to other robots in its visibility range). In more details, we study SUIG algorithms that depend on two parameters of the initial configuration: M_{init} and O_{init} . The former is the number of nodes between two border nodes, and the latter is the number of nodes with robots between two border nodes. A border node has at least one robot and no robots on one or both sides. We show that SUIG is solvable in the FSYNC model when either M_{init} or O_{init} is odd.

2 Model

We consider robots that evolve on a line shaped network. The length of the line is infinite in both directions, and consists of an infinite number of nodes $\dots, u_{-2}, u_{-1}, u_0, u_1, u_2, \dots$, such that a node u_i is connected to both $u_{(i-1)}$ and $u_{(i+1)}$. Let $\mathscr{R} = \{r_1, r_2, \dots, r_n\}$ be the set of $n \ge 2$ autonomous robots. Robots are assumed to be anonymous (i.e., they are indistinguishable), uniform (i.e., they all execute the same program, and use no localized parameter such as a particular orientation), oblivious (i.e., they cannot remember their past actions), and disoriented (i.e., they cannot distinguish left and right). Then, we assume that robots do *not* know the number of robots.

A node is considered *occupied* if it contains at least one robot; otherwise, it is *empty*. If a node contains more than one robot, it is said to have a *tower* or *multiplicity*.

The *distance* between two *nodes* u_i and u_j is the number of edges between them. The *distance* between two *robots* r_p and r_q is the distance between two nodes occupied by r_p and r_q . Two robots or two nodes are *adjacent* if the distance between them is one. Two robots are *neighboring* if there is no robot between them.

Each robot r_i maintains a variable L_i , called *light*, which spans a finite set of states called *colors*. We call such robots *luminous robots*. A light is *persistent* from one computational cycle to the next: the color is not automatically reset at the end of the cycle. Let *L* denote the number of available light colors. Let $L_i(t)$ be the light color of r_i at time *t*. We assume the *full light* model: each robot r_i can see the light color of other robots, but also its own light color. Robots are unable to communicate with each other explicitly (*e.g.*, by sending messages), however, they can observe their environment, including the positions (i.e., occupied nodes) and colors of other robots.

The ability to detect towers is called *multiplicity detection*, which can be either *global* (any robot can sense a tower on any node) or *local* (a robot can only sense a tower if it is part of it). If robots can determine the number of robots in a sensed tower, they are said to have *strong* multiplicity detection. We assume that robots do *not*

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have multiplicity detection capability even on their current node but still can sense the visible colors: if there are multiple robots $r_1, r_2, ..., r_k$ in a node u, an observing robot r can detect only colors $\{L_i(t)|1 \le i \le k\}$. So, r can detect there are multiple robots at u if and only if at least two robots among $r_1, r_2, ..., r_k$ have different colors. However, r cannot know how many robots are located in u even if it observe a single color or multiple colors at u.

We assume that robots are *myopic*. That is, they have limited visibility: an observing robot *r* at node *u* can only sense the robots that occupy nodes within a certain distance, denoted by ϕ , from *u*. As robots are identical, they share the same ϕ .

Let $\mathscr{X}_i(t)$ be the set of colors of robots located in node u_i at time t. If a robot r_j located at u_i takes a snapshot at t, the sensor of r_j outputs a sequence, \mathscr{V}_j , of $2\phi + 1$ sets of colors:

$$\mathscr{V}_{i} \equiv \mathscr{X}_{i-\phi}(t), \dots, \mathscr{X}_{i-1}(t), [\mathscr{X}_{i}(t)], \mathscr{X}_{i+1}(t), \dots, \mathscr{X}_{i+\phi}(t).$$

This sequence \mathscr{V}_j is the view of r_j at u_i . To distinguish the sequence center, we use square brackets. If the sequence $\mathscr{X}_{i+1}, \ldots, \mathscr{X}_{i+\phi}$ is equal to the sequence $\mathscr{X}_{i-1}, \ldots, \mathscr{X}_{i-\phi}$, then the view \mathscr{V}_j of r_j is symmetric. Otherwise, it is asymmetric. In \mathscr{V}_j , a node u_k is occupied at time t whenever $|\mathscr{X}_k(t)| > 0$. Conversely, if u_k is empty at t, then $\mathscr{X}_k(t) = \emptyset$ holds.

If there exists a node u_i such that $|\mathscr{X}_i(t)| = 1$ holds, u_i is *singly-colored*. Note that $|\mathscr{X}_i(t)|$ denotes the number of colors at node u_i , thus even if u_i is singly-colored, it may be occupied by multiple robots (sharing the same color). Now, if a node u_i is such that $|\mathscr{X}_i(t)| > 1$ holds, u_i is *multiply-colored*. As each robot has a single color, a multiply-colored node always hosts more than one robot.

In the case of a robot r_j located at a singly-colored node u_i , $[\mathscr{X}_i(t)]$ in r_j 's view \mathscr{V}_j can be written as $[L_j]$. Then, without loss of generality, if the left adjacent node of u_i contains one or more robots with color L_k , and the right adjacent node of u_i contains one or more robots with color L_l , while u_i only hosts r_j , then \mathscr{V}_j can be written as $L_k[L_j]L_l$. Now, if robot r_j at node u_i occupies a multiply-colored position (with two other robots r_k and r_l having distinct colors), then $|\mathscr{X}_i(t)| = 3$, and we can

write $\mathscr{X}_i(t)$ in \mathscr{V}_j as $\begin{bmatrix} L_k \\ L_l \\ [L_j] \end{bmatrix}$. When the observed node in the view is with multiple

colors, we use brackets to distinguish the current position of the observing robot in the view and the inner bracket to explicitly state the observing robot's color. Note that, because we assume that robots do not have multiplicity detection capability, at u_i , there may be two or more robots with L_k and L_l respectively, and there may be two or more robots with L_i other than r_i .

Our algorithms are driven by observations made on the current view of a robot, so we use *view predicates*: a Boolean function based on the current view of the robot. The predicate L_j matches any set of colors that includes color L_j , while predicate

 (L_j, L_k) matches any set of colors that contains L_j or L_k . Now the predicate $\begin{pmatrix} L_1 \\ L_2 \end{pmatrix}$

matches any set that contains both L_1 and L_2 . Some of our algorithm rules expect

that a node is singly-colored, *e.g.*, with color L_k , in that case, the corresponding predicate is denoted by L_k !. To express predicates in a less explicit way, we use character '?' to represent any set, including the empty set. The \neg operator is used to negate a particular predicate P (so, $\neg P$ returns *false* whenever P returns *true* and vice versa). Then, the predicate $\begin{pmatrix} \neg L_1 \\ \neg L_2 \end{pmatrix}$ matches any set that is neither singly-colored L_1 nor singly-colored L_2 . Also, the superscript notation P^y represents a sequence of y consecutive sets of colors, each satisfying predicate P. Observe that $y \leq \phi$. In a given configuration, if the view of a robot r_j at node u_i satisfies predicate $\emptyset^{\phi}[?]$ or predicate $[?] \emptyset^{\phi}$, then r_j is a *border robot* and u_i a *border node*.

At each time instant *t*, robots occupy nodes, and their positions and colors form a *configuration* C(t) of the system. Then, each robot *r* executes Look-Compute-Move cycles infinitely many times: (*i*) first, *r* takes a snapshot of the environment and obtains an ego-centered view of the current configuration (Look phase), (*ii*) according to its view, *r* decides to move or to stay idle and possibly changes its light color (Compute phase), (*iii*) if *r* decided to move, it moves to one of its adjacent nodes depending on the choice made in the Compute phase (Move phase). We consider the *FSYNC* model in which at each *round*, each robot *r* executes an LCM cycle synchronously with all the other robots. We also consider the *SSYNC* model where a nonempty subset of robots chosen by an adversarial scheduler executes an LCM cycle synchronously, at each round. At time instant t = 0, let H_{init} be the maximum distance between neighboring occupied nodes, M_{init} be the number of nodes between two borders including border nodes, and $O_{init}(\leq M_{init})$ be the number of occupied nodes. We assume that $\phi \geq H_{init} \geq 1$, i.e., the visibility graph is connected. As previously stated, no robot is aware of H_{init} , M_{init} and O_{init} .

In this paper, each rule in the proposed algorithms is presented in the similar notation as in [16]: $\langle Label \rangle : \langle Guard \rangle :: \langle Statement \rangle$. The guard is a predicate on the view $\mathcal{V}_j = \mathscr{X}_{i-\phi}, \ldots, \mathscr{X}_{i-1}, [\mathscr{X}_i], \mathscr{X}_{i+1}, \ldots, \mathscr{X}_{i+\phi}$ obtained by robot r_j at node u_i during the Look phase. If the predicate evaluates to *true*, r_j is *enabled*, otherwise, r_j is *disabled*. In the first case, the corresponding rule $\langle Label \rangle$ is also said to be *enabled*. If a robot r_j is enabled, r_j may change its color and then move based on the corresponding statement during its subsequent Compute and Move phases. The statement is a pair of (*New color, Movement*). *Movement* can be $(i) \rightarrow$, meaning that r_j moves towards node u_{i+1} , $(ii) \leftarrow$, meaning that r_j moves towards node u_{i+1} , and $(iii) \perp$, meaning that r_j does not move. For simplicity, when r_j does not move (resp. r_j does not change its color), we omit *Movement* (resp. *New color*) in the statement. The label $\langle Label \rangle$ is denoted as R followed by a non-negative integer (*i.e.*, R0, R1, etc.) where a smaller label indicates higher priority. If the integer in the label is followed by an alphabet (*i.e.*, R1a, R1b, etc.), the priority is determined by the lexicographic order.

Problem definition. A robot is said to be *crashed* at time instant *t* if it stops executing at any time $t' \ge t$. That is, a crashed robot stops execution and remains with the same color at the same position indefinitely. We assume that robots cannot identify a crashed robot in their snapshots (i.e., they are able to see the crashed robots but remain unaware of their crashed status). A crash, if any, can occur at any phase

of the execution, and break the LCM-atomic (i.e., it can occur the end of round, but also between Look phase and Compute phase or between Compute phase and Move phase). More than one crash can occur, however we assume that all crashes occur at the same node. In our model, since robots do not have multiplicity detection capability, a node with a single crashed robot and with multiple crashed robots with the same color are indistinguishable. Similarly, multiple robots with the same color at the same node have the same behavior, but some or all of them can crash.

We consider the *Stand Up Indulgent Gathering* (SUIG) problem defined in [6]. An algorithm solves the SUIG problem if, for any initial configuration C_0 (that may contain multiplicities), and for any execution $\mathscr{E} = (C_0, C_1, ...)$, there exists a round t such that all robots (including the crashed robot, if any) gather at a single node, not known beforehand, for all $t' \ge t$. Note that, if there are multiple crashed nodes, the problem cannot be solved. Thus, we need to assume that all the crashes occur at the same node.

Because we assume that robots are anonymous and uniform, all robots have the same color in the initial configuration.

3 Impossibility Results

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Several impossibility results from the literature hint at which situations are solvable for our problem. Theorem 1 and Corollaries 1–2 are for the case where robots have no lights.

Theorem 1 ([15]). *The gathering problem is unsolvable in FSYNC on line networks starting from an edge-symmetric configuration even if robots can see all the positions of the other robots with global strong multiplicity detection.*

Corollary 1 ([4]). *The SUIG problem is unsolvable in FSYNC on line networks starting from an edge-symmetric configuration even for robots with infinite visibility and global strong multiplicity detection.*

Corollary 2 ([14]). Starting from a configuration where M_{init} is even and O_{init} is even, there exist initial configurations that a deterministic algorithm cannot gather for myopic robots.

As above results, we suppose in the following section that either M_{init} or O_{init} is odd, that is, the initial configurations are not edge-symmetric. The following lemma is also for the case where robots have no lights.

Lemma 1 ([4]). Even starting from a configuration that is not edge-symmetric, the SUIG problem is unsolvable in SSYNC for robots with infinite visibility and global strong multiplicity detection.

Lemma 2. Even starting from a configuration that is not edge-symmetric, the SUIG problem is unsolvable in SSYNC for infinite visibility, global strong multiplicity detection, infinite colors luminous robots.

Proof. Let us suppose for the purpose of contradiction that there exists an algorithm *A* that solves SUIG for infinite visibility and global strong multiplicity detection luminous robots with an infinite number of colors in SSYNC. Consider the configuration *C* that occurs just before gathering is achieved. Now, configuration *C* has either three consecutive occupied nodes (let us call this configuration class C_3) or two consecutive occupied nodes (let us call this configuration class C_2). In a configuration in C_3 , the border robots must be ordered to move inwards by *A*, otherwise gathering is not achieved in the next configuration. From a configuration in C_3 , the SSYNC scheduler may select only one of the border robots for execution, then reaching a configuration in C_2 . So, for any algorithm *A* that solves *SUIG*, an SSYNC scheduler can reach a configuration in C_2 . In the sequel, we show that we may never reduce the number of occupied nodes in any execution that starts from a configuration in C_2 , and hence the gathering is not solved.

Assume that we are in a configuration in C_2 . Let k_1 denote the number of robots on the first occupied node u_1 , and k_2 the number of robots on the second occupied node u_2 . Suppose now that the particular combination of colors at both nodes yields all robots at u_1 not to move. Then, we can crash robots at u_2 . As a result, gathering is never achieved, as the configuration remains in C_2 forever. The same argument holds for robots at u_2 . As a result, algorithm A must command at least one robot at each node to move. Now, the scheduler executes those two robots (from the two nodes) that move. Either they both move inwards (exchanging their nodes) and the configuration remains in C_2 , or at least one of them moves inwards and the resulting configuration remains in C_2 , or another configuration with more occupied nodes and possibly holes. In any case, the number of occupied nodes is not reduced from two to one, so one can again construct an execution that reaches a configuration in C_2 , and repeat the argument forever. Hence, algorithm A does not solve SUIG, a contradiction.

As per Lemma 2, we assume the FSYNC model in the following section.

4 Possibility Results for Myopic Robots

4.1 The case where M_{init} is odd

In this case, we show that the gathering is achieved even if robots do not have lights. For this purpose, in the following we assume that all robots have a single color W (White) which they do not change. The strategy of our algorithm is as follows: The robots on two border nodes move towards other occupied node. The formal description is shown in Algorithm 1.

Lemma 3. Starting from a configuration C where M_{init} is odd, even if there is a crashed robot, all robots gather in $O(M_{init})$ rounds by Algorithm 1.

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Algorithm 1: Algorithm for the case where M_{init} is odd.
/* Do nothing after gathering. */
R0: $\emptyset^{\phi}[W!] \emptyset^{\phi} :: \bot$
/* Border robots move. */
$\mathbf{R1:} \mathfrak{O}^{\phi}[W!](\neg(\mathfrak{O}^{\phi}))::\to$

Proof. Because we assume FSYNC model, if there is no crashed robot, it is clear that all robots gather on the central node between initial borders in $\lfloor M_{init}/2 \rfloor$ rounds. If a border robot r_b crashed at time $t < \lfloor M_{init}/2 \rfloor$ on a node u_i , it stops at u_i . If there are other (non-crashed) border robots on u_i at t, they move toward other occupied node, thus they are in u_{i+1} at t + 1. After that, they cannot move before they become border robots. On the other hand, the other border robots move towards u_i . Thus, eventually, they arrive at u_{i+1} at $M_{init} - 2(t+1) + t$ -th round, and robots on u_{i+1} become a border. After that, all robot at u_{i+1} move to u_i , and the gathering is achieved.

4.2 The case where O_{init} is odd

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In this section, we show that the gathering is achieved if robots have lights with three colors: W (White), R (Red) and B (Blue). We assume that all robots have the same color White in the initial configuration. The formal description is shown in Algorithm 2.

The transition diagram of configurations by the algorithm in the case that no crash occurs represents in Figs. 1-5. In these figures, each small blue box represents a node, and each circle represents the set of robots with the color W, R and B. The robots represented by doubly lined circles are enabled. If no crash occurs during the execution, the strategy of our algorithm is as follows: Initially, all robots are White, and robots on two border nodes become Red in the first round by rule R1 (See Fig. 1(a)). The robots on two border nodes move towards other occupied node. Then, the border robots keep their lights Red or Blue, then the algorithm can recognize that they are border robots. If there exists a White robot in the adjacent node, the border robot changes its color to Blue or Red by rule R2b or R3b (See Fig. 1(b) \rightarrow (c),(e) \rightarrow (f)). Otherwise, it just moves without changing its color by rule R2a or R3a (See Fig. 1(a),(d)). When non-border White robots become border, they change their color to Red (resp. Blue) by rule R4a (resp. R4b) if borders that join the node have Red (resp. Blue) (See Fig. $1(f) \rightarrow (a)$ or (b) (resp. $(c) \rightarrow (d)$ or (e))). We say that White robots are *captured* by a border if a border moves to the node occupied by the White robots. When a border node becomes singly-colored, the border robot moves toward other occupied nodes. Eventually, two borders are neighboring and they have Blue and Red respectively because O_{init} is odd. To achieve the gathering, depending on the initial occupied nodes, one of the followings occurs.

- Case 1: If two borders are singly-colored, the distance between them is two and the central node is empty, both borders move to the central node by rules R2a and R3a (See Fig. 2(b))
- Case 2: If two borders are adjacent and singly-colored, then Blue robots join Red robots by rule R3a at the same time that Red robots become Blue by rule R4c (See Fig. 3(c)).
- Case 3: If two borders are adjacent, one border has White and Red (resp. Blue) robots and the other border is singly-colored Blue (resp. Red), then White robots become Red (resp. Blue) by rule R4a (resp. R4b) and the singly-colored Blue (resp. Red) border moves to the adjacent border by R3a (resp. R2a) (See Fig. 4(b) (resp. (c)).
- Case 4: If two borders are singly-colored Blue (resp. Red), the distance between them is two and the central node is occupied by White robots, then both borders move to the central node by rule R3b (resp. R2b) (See Fig. 5(b) (resp. (d))).

During the execution, if White robot crashes, one border eventually stops executing at the crashed node, but the other border can join the crashed border by rule R2a or R3a. For the case where Red or Blue robots crash, by special rules R5a–R5c, we are able to respond to various failure patterns.

We prove the correctness of Algorithm 2.

Lemma 4. Starting from a configuration C where O_{init} is odd, if no robot crashes, all robots gather in $O(M_{init})$ rounds by Algorithm 2.

Proof. Let r_i and r_j be the initial border robots on different sides in the initial configuration C. Even if they are towers, we can recognize each of them as a robot because we assume they do not crash in the FSYNC model. Because all robots have White color in C, only r_i and r_j execute rule R1 and become Red in the first round (Fig. 1(a)). After that, r_i and r_j with Red execute R2a or R2b. If the adjacent node is occupied by White robots (Fig. 1(b)), border robots execute R2b, change their color to Blue and move to the adjacent occupied node (Fig. 1(c)). Then, the White robots captured by the border execute R4b and the border becomes singly-colored Blue (Fig. 1(d) or (e)). Otherwise, they execute R2a and move to their adjacent node, keeping their color (Fig. 1(a)). After the border robots become Blue, they execute R3a or R3b. If the adjacent node is occupied by White robot (Fig. 1(e)), border robots execute R3b, change their color to Red and move to the adjacent occupied node (Fig. 1(f)). Then, the White robots captured by the border execute R4a and the border becomes singly-colored Red (Fig. 1(a) or (b)). Otherwise, they execute R3a and move to their adjacent node, keeping their color (Fig. 1(d)). Thus, r_i and r_j move toward each other, changing their colors Red and Blue repeatedly when they move to an occupied node. Note that, borders can only move when they are singly-colored.

Let *t* be the round when the distance between r_i and r_j becomes two, and C_t be the configuration at *t*. Let d_i (resp. d_j) be the distance that r_i (resp. r_j) moved before *t*. Then, it is clear that $M_{init} - 3 = d_i + d_j$. In addition, let c_i (resp. c_j) be the number of nodes occupied by White robots such that r_i (resp. r_j) captured before *t*. Then, $c_i + c_j$ is at most O_{init} . Because $O_{init} \le M_{init}$, *t* is $O(M_{init})$ rounds.

Algorithm 2: Algorithm for the case where O_{init} is odd.

Colors W (White), R (Red), B (Blue) Rules /* Do nothing after gathering. */ R0: $\emptyset^{\phi}[?] \emptyset^{\phi} :: \bot$ /* Start by the initial border robots. */ R1: $\emptyset^{\phi}[W!](\neg \emptyset^{\phi}) :: R$ /* Border robots on singly-colored nodes move inwards. */ R2a: $\emptyset^{\phi}[R!] \begin{pmatrix} \neg W! \\ \neg B! \end{pmatrix} (?^{\phi-1}) :: \rightarrow$ R2b: $\emptyset^{\phi}[R!](W!)(?^{\phi-1}) :: B, \rightarrow$ R3a: $\emptyset^{\phi}[B!](\neg W!)(?^{\phi-1}) :: \rightarrow$ R3b: $\emptyset^{\phi}[B!](W!)(?^{\phi-1})$:: R, \rightarrow /* When White robots become border robots, they change their color to the same color as the border robots. */ R R4a: Ø[¢] $(?^{\phi}) :: R$ [W]B $(?^{\phi}) :: B$ R4b: \emptyset^{ϕ} [W]R4c: $\emptyset^{\phi}[R!](B!)(\emptyset^{\phi-1}) :: B$ /* Only for the case that Blue or Red robot crashes. */ $(R!,B!)(\emptyset^{\phi-1})::R$ R5a: \emptyset^{ϕ} В $\begin{bmatrix} W \\ W \end{bmatrix}$ $\begin{bmatrix} B \\ R \end{bmatrix} ($ $(R!,B!)(\emptyset^{\phi-1})::\to$ R5b: Ø[¢] R R5c: Ø[¢] $(R!, B!)(\emptyset^{\phi-1})$

Consider the execution starting from C_t . First, consider the case that there is no White robot between two borders in C_t , i.e., the node between two borders is empty. Because O_{init} is odd, robots in a border have Red and robots in the other border have Blue.

- If both borders are singly-colored in C_t , then they move toward each other by R2a and R3a respectively (Fig. 2(b)). Then, the gathering is achieved.
- If both borders include White robots in C_t , then White robots in both borders execute R4a or R4b respectively (Fig. 2(a)) and both border becomes singly-colored at t + 1.
- Consider the case that a border includes White and Red robots and the other has only Blue robots in C_t (Fig. 3(a)). At t + 1, White border robots change their color to Red by R4a and Blue border robots move toward the Red border by R3a. Then, a singly-colored Red border and a singly-colored Blue border are adjacent (Fig. 2(c)). Then, while Red border robots change their color to Blue by R4c, Blue border robots execute R3a, and the gathering is achieved.
- Consider the case that a border includes White and Blue robots and the other has only Red robots in C_t (Fig. 3(b)). At t + 1, White border robots change their



Fig. 1 Execution of border robots before the number of occupied nodes becomes three.



Fig. 5 Execution of Case 4.

color to Blue by R4b and Red border robots move toward the Blue border by R2a. Then, a singly-colored Red border and a singly-colored Blue border are adjacent (Fig. 2(c)). Then, while Red border robots change their color to Blue by R4c, Blue border robots execute R3a, and the gathering is achieved.

Next, consider the case that there is a White robot r_w between two borders in C_t . Because O_{init} is odd, both borders have Blue or both borders have Red.

- If both borders are singly-colored Red (resp. Blue) at t (Fig. 5(d) (resp. (b))), they moves toward r_w by R2b (resp. R3b). Then, the gathering is achieved.
- If both borders include White robots at t (Fig. 5(a),(c)), the White border robots change their color to the same color as other border robots by R4a or R4b. Then, we finished the discussion about this case.

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- Consider the case that a border is singly-colored Red robots and the other border includes White robots and Red robots at t (Fig. 4(d)). Then, the White border robots execute R4a and the border becomes singly-colored Red at t + 1. At the same time, the singly-colored Red border executes R2b, changes its color to Blue and moves to the node occupied by r_w at t + 1 (Fig. 4(c)). After that, the singly-colored Red border moves to the other border including r_w by R2a and the gathering is achieved, while r_w changes its color to Blue by R4b.
- Consider the case that a border is singly-colored Blue robots and the other border includes White robots and Blue robots at t (Fig. 4(a)). Then, the White border robots execute R4b and the border becomes singly-colored Blue at t + 1. At the same time, the singly-colored Blue border executes R3b, changes its color to Red and moves to the node occupied by r_w at t + 1 (Fig. 4(b)). After that, the singly-colored Blue border moves to the other border including r_w by R3a and the gathering is achieved, while r_w changes its color to Red by R4a.

Therefore, in any case, the gathering is achieved in $O(M_{init})$ rounds.

Lemma 5. Starting from a configuration C where O_{init} is odd, even if a White robot crashes, all robots gather in $O(M_{init})$ rounds by Algorithm 2.

Proof. We assume that all robots have White initially, and there is no rule such that White robot moves in Algorithm 2. Thus, we can discuss the case where the crash of the White robot occurs during the execution by the same way as the case the crash occurs initially.

If a robot r_k at a border u_i of the initial configuration crashes, (1) other robots at u_i becomes Red or (2) the border remains White (i.e., all the robots at u_i crashed). In both cases, the border at u_i cannot move because r_k remains White. On the other hand, the robots r_j at the other border change their color to Red by R1 and move toward u_i by R2a or R2b. By repeating executions of R2a, R2b, R3a and R3b, the border r_j eventually reaches the adjacent node u_{i+1} of u_i (Of course, White robots at other nodes than u_i change their color with r_j by R4a or R4b and move as the border).

- In the case (1), the border at u_i has Red and White robots, and the other border at u_{i+1} is Blue and White, or singly-colored Blue because O_{init} is odd. In the former case, the White robots at u_{i+1} change their color to Blue by R4b, and the border at u_{i+1} becomes singly-colored Blue. Then, the singly-colored Blue border moves to u_i by R3a, and the gathering is achieved.
- In the case (2), the border at u_{i+1} also becomes singly-colored Blue by the same discussion as above. After that, the Blue border robots at u_{i+1} move to u_i by R3b, and the gathering is achieved.

Next, consider the case that a robot at a non-border node of the initial configuration *C* crashes. Let o_1 be a border node u_i , o_2 be its neighboring occupied node, o_k be the *k*-th occupied node from u_i , $o_{(O_{init})}$ be the other border node $u_{(i+M_{init}-1)}$ in *C*. Let r_i (resp. r_j) be the (sets of) initial border robots at u_i (resp. $u_{(i+M_{init}-1)}$) in *C*. Without loss of generality, the crash occurs at o_k . Starting from *C*, both borders move toward

 o_k , and eventually at least one border becomes adjacent to o_k . Let t be the round when at least one border becomes adjacent to o_k . Without loss of generality, then r_i is adjacent to o_k at t.

- Consider the case that k is odd. Then, the border r_i includes Blue robots.
 - Consider the case that r_j is also adjacent to o_k at t, and both of r_i and r_j include White robots or both do not include White robots. Then, r_j also includes Blue robots because O_{init} is odd. When both borders include White robots, White border robots execute R4b and change their color to Blue. Thus, both borders become singly-colored Blue. Then, they move to o_k by R3b, and the gathering is achieved.
 - Consider the case that r_j is also adjacent to o_k at t, and one of borders includes White robots. Without loss of generality, assume that r_i includes White robot at t. Then, r_j is singly-colored Blue. The White robot occupied with r_i executes R4b at t + 1. At the same time, r_j executes R3b, changes its color to Red and moves to o_k . After that, r_i moves to o_k by R3a, and the gathering is achieved.
 - Consider the case that r_j is not adjacent to o_k at t. Then, r_i executes R3b, changes its color to Red and moves to o_k . After that, because the White robot on o_k crashes, it cannot change its color. Thus, the border r_i cannot move from o_k . Eventually, r_j with Blue robots arrives at the adjacent node of o_k (If the node is o_{k+1} , the White robots execute R4b and the border r_j becomes singly-colored Blue). Then, r_j moves to o_k by R3a, and the gathering is achieved.
- Consider the case that k is even. Then, the border r_i includes Red robots.
 - Consider the case that r_j is also adjacent to o_k at t, and both of r_i and r_j include White robots or both do not include White robots. Then, r_j also includes Red robots because O_{init} is odd. When both borders include White robots, White border robots execute R4a and change their color to Red. Thus, both borders become singly-colored Red. Then, they move to o_k by R2b, and the gathering is achieved.
 - Consider the case that r_j is also adjacent to o_k at t, and one of borders includes White robots. Without loss of generality, assume that r_i includes White robot at t. Then, r_j is singly-colored Red. The White robot occupied with r_i executes R4a at t + 1. At the same time, r_j executes R2b, changes its color to Blue and moves to o_k at t + 1. After that, r_i moves to o_k by R2a, and the gathering is achieved.
 - Consider the case that r_j is not adjacent to o_k at t. Then, r_i executes R2b, changes its color to Blue and moves to o_k . After that, because the White robot on o_k crashes, it cannot change its color. Thus, the border r_i cannot move from o_k . Eventually, r_j with Red robots arrives at the adjacent node of o_k (If the node is o_{k+1} , the White robots execute R4a and the border r_j becomes singly-colored Red). Then, r_j moves to o_k by R2a, and the gathering is achieved.

Thus, the lemma holds.

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Lemma 6. Starting from a configuration C where O_{init} is odd, even if a Red robot crashes during the execution, all robots gather in $O(M_{init})$ rounds by Algorithm 2.

Proof. By the definition of Algorithm 2 and the proof of Lemma 4, if a Red robot crashes, it occurs at a border node during the execution. Let r_k be the crashed robot with Red, and u_k be the occupied node by r_k . Let r_j be the border robots at the other (non-crashed) border node.

- Consider the case that the crash occurs just after the execution of R1.
 - If all robots at u_k crash, the border is singly-colored Red and stops its execution completely. The other border r_j moves toward r_k . The neighboring White robot r_i of r_k eventually becomes neighboring to r_j , then r_j becomes singly-colored Red. After that, r_j moves to the node occupied by r_i by R2b. Then, r_i changes its color to Blue by R4b. The Blue border including r_i and r_j moves toward r_k by R3a and eventually moves to u_k . The gathering is achieved.
 - If there is a non-crashed robot r_i in u_k when r_k crashed, then r_i continue to execute, and move toward the other border r_j by R2a or R2b. Let u_i be the node adjacent to u_k where r_i moves. Then, r_i cannot execute any rule before it becomes border.
 - Consider the case that r_i executes R2a. Because O_{init} is odd, when r_j reaches to the neighboring White robot of r_i , r_j becomes Blue and the border becomes singly-colored Blue. After that, r_j moves to u_i holding Blue by R3a. Then, r_i and r_j execute R5b and R5c respectively and move to u_k . The gathering is achieved.
 - Consider the case that r_i executes R2b. Then, r_i is Blue, and there are White robots at u_i . If r_j moves to u_i at the same time as r_i moves, White robots at u_i change their color to Blue by R4b, and all robots at u_i move to u_k by R3a. Then, the gathering is achieved. Otherwise, because O_{init} is odd, when r_j is adjacent to r_i , r_j has Red. Then, r_j moves to u_i by R2a and u_i has White, Blue and Red robots. After that, White robot in u_i executes R5a and changes its color to Red, and robots on u_i becomes Blue and Red. Because r_k is singly-colored Red, robots on u_i execute R5b and R5c, and move to u_k . The gathering is achieved.
- Consider the case that the crash occurs just before the execution of R2a. Then, *u_k* is a border with singly-colored Red robots.
 - Consider the case that its adjacent node u_{k+1} is a border with Blue robots and White robots.
 - Consider the case that all robots at u_k crash. Then, White robots at u_{k+1} change their color to Blue by R4b and the border at u_{k+1} becomes singly-colored Blue. After that, robots at u_{k+1} executes R3a, and the gathering is achieved.
 - Consider the case that there are non-crashed robots r_i at u_k . Then, r_i moves to u_{k+1} . At the same time, White robots at u_{k+1} change their color to Blue by R4b. Thus, the border u_{k+1} becomes Blue and Red. In the next round, they moves to u_k by R5b and R5c. Then, the gathering is achieved.

- If the adjacent node u_{k+1} is empty, we can discuss the same way as the case just after the execution of R1.
- Consider the case that the crash occurs just before the execution of R2b or just after the execution of R2a. Then, we can discuss the same way as the case just after the execution of R1.
- Consider the case that the crash occurs just before the execution of R4c. Then, r_k is singly-colored Red border, and the other border r_j is adjacent to u_k and singly-colored Blue. Thus, r_j moves to u_k by R3a, and the gathering is achieved.
- Consider the case that the crash occurs just after the execution of R4a. Then, r_k is a singly-colored Red border, and it is just before the execution of R2a, R2b, or R4c.
- Consider the case that the crash occurs just after the execution of Compute phase of R3b. Then, r_k is adjacent to singly-colored node u_{k+1} with White robots, and r_k changes its color to Red, but does not move.
 - If all robots at u_k crash, they are Red robots and u_{k+1} is occupied by White robots. Because O_{init} is odd, when the other borders r_j become adjacent to u_{k+1} , then r_j has Blue. After that, r_j changes its color to Red and moves to u_{k+1} by R3b, and then, White robots at u_{k+1} changes its color to Red by R4a. Then, u_{k+1} becomes singly-colored Red. Thus, all robots at u_{k+1} move to u_k by R2a, the gathering is achieved.
 - If there is non-crashed other robots r_i at u_k , r_i moves to u_{k+1} with Red color. The other borders r_j move toward u_{k+1} . If r_j moves to u_{k+1} at the same time as r_i moves, White robots at u_{k+1} changes their color to Red by R4a, and all robots at u_{k+1} moves to u_k by R2a. Then, the gathering is achieved. Otherwise, because O_{init} is odd, when r_j becomes adjacent to u_{k+1} , r_j have Blue (Even if there are White robots with them, they eventually become singly-colored Blue by R4b). After that, r_j moves to u_{k+1} by R3a, then u_{k+1} is occupied by White, Red and Blue robots. Then, White robots at u_{k+1} changes its color to Red by R5a. Thus, because all robots at u_{k+1} move to u_k by R5b and R5c, the gathering is achieved.
- Consider the case that the crash occurs just after the execution of Move phase of R3b. Then, rk moved to an adjacent singly-colored node uk with White robots at round t.
 - If r_j is also adjacent to u_k and is singly-colored Blue at t, then r_j also moves to u_{k+1} by R3b at the same time, and the gathering is achieved.
 - If r_j is adjacent to u_k and is with White robots at t + 1, White robots in both borders execute R4a and R4b, then both borders becomes singly-colored. Then, r_j moves to u_k by R3a and the gathering is achieved.
 - If r_j is adjacent to u_k and is singly-colored Blue at t + 1, then it moves to u_k by R3a. Then, the gathering is achieved.
 - If r_j is not adjacent to u_k at t + 1, White robots at u_k changes its color to Red by R4a and all robots at u_k becomes Red. After that, we can discuss this case

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by the same way as the above cases such that crash occurs in a singly-colored Red border.

Thus, the lemma holds.

Lemma 7. Starting from a configuration C where O_{init} is odd, even if a Blue robot crashes during the execution, all robots gather in $O(M_{init})$ rounds by Algorithm 2.

Proof. By the definition of Algorithm 2 and the proof of Lemma 4, if a Blue robot crashes, it occurs at a border node during the execution. Let r_k be the crashed robot with Blue, and u_k be the occupied node by r_k . Let r_j be the other (non-crashed) border robots.

- Consider the case that the crash occurs just before the execution of R3a. Then, r_k is a singly-colored Blue border.
 - Consider the case that the adjacent node u_{k+1} is a singly-colored Red border.
 - If all robots in u_k crash, Red robots at u_{k+1} change their color to Blue by R4c, and move to u_k by R3a. Then, the gathering is achieved.
 - If there are non-crashed Blue robots r_i in u_k , r_i executes R3a, and moves to u_{k+1} . At the same time, Red robots at u_{k+1} changes their color to Blue by R4c. Then, all robots at u_{k+1} are only non-crashed Blue robots, and all robots at u_k are only crashed Blue robots. Thus, all robots at u_{k+1} moves to u_k by R3a. Thus, the gathering is achieved.
 - Consider the case that the adjacent node u_{k+1} has White and Red robots.
 - If all robots at u_k crash, White robots at u_{k+1} change their color by R4a and the border at u_{k+1} becomes singly-colored Red. After that, the robots at u_{k+1} execute R4c and R3a in sequence, then the gathering is achieved.
 - If there are non-crashed Blue robots r_i in u_k , r_i executes R3a, and moves to the node u_{k+1} . At the same time, White robots at u_{k+1} change their color to Red by R4b. Thus, the border becomes Blue and Red. In the next round, they moves to u_k by R5b and R5c, and the gathering is achieved.
 - Consider the case that the adjacent node u_{k+1} is empty.
 - If all robots r_k at u_k crash, r_j moves toward u_k . Eventually, r_j becomes adjacent to u_k and singly-colored Red. After that, r_j becomes singly-colored Blue by R4c, and moves to u_k by R3a. Then, the gathering is achieved.
 - If there are non-crashed robots r_i at u_k when r_k crashes, r_i moves to the adjacent node u_{k+1} by R3a toward r_j . After that, r_i cannot execute any rule before it becomes border. When r_j becomes adjacent to u_{k+1} , r_j has Red because O_{init} is odd. Then, r_j moves to u_{k+1} by R2a, and the border robots at u_{k+1} have Red and Blue robots. They move to u_k by R5b and R5c, and the gathering is achieved.
- Consider the case that the crash occurs just before the execution of R3b. In this case, r_k is also a singly-colored Blue border, and the adjacent occupied node u_{k+1} has singly-colored White robots r_w .

- Consider the case that all robots at u_k crash, then r_j eventually becomes adjacent to u_{k+1} . Then, r_j has Blue because O_{init} is odd. If r_j includes White robots, the White robots executes R4b and r_j becomes singly-colored Blue. Then, r_j executes R3b and moves to u_{k+1} . After that, r_w executes R4a and the border at u_{k+1} becomes singly-colored Red. Then, robots at u_{k+1} execute R4c and R3a in sequence, and the gathering is achieved.
- Consider the case that there are non-crashed border robots r_i in u_k when r_k crashes, r_i continues to execute and moves to u_{k+1} by R3b. Then, r_i become Red and move to u_{k+1} at time t. Because O_{init} is odd, when r_j is neighboring to r_w , r_j has Blue. If r_j is also adjacent to u_{k+1} and moves to u_{k+1} at t, then we can discuss the case in the same way as above. Otherwise, r_i cannot execute any rule before it becomes border. After r_j becomes singly-colored Blue, it moves to u_{k+1} by R3a, and u_{k+1} becomes a border with White, Red and Blue robots. After that, White robots r_w at u_{k+1} change their color to Red by R5a, and then, there are only Blue and Red robots at u_{k+1} . Then, all robots at u_{k+1} move to u_k by R5b and R5c, and the gathering is achieved.
- Consider the case that the crash occurs just after the execution of R3a or R4b. Then, *r_k* is a singly-colored Blue border. We can discuss it by the same way as the case just before the execution of R3a or R3b.
- Consider the case that the crash occurs just after the execution of R4c. By the proof of Lemma 4, the gathering is achieved.
- Consider the case that the crash occurs just after the execution of Compute phase of R2b. Then, just before the execution, the color of r_k is Red, and r_k is adjacent to singly-colored node u_{k+1} with White robots. After the execution, r_k changes its color to Blue, but does not move. The other borders r_j move toward u_{k+1} . Because O_{init} is odd, when r_j become neighboring to u_{k+1} , r_j have Red (Even if there are White robots with them, they eventually become singly-colored Red by R4a).
 - If all robots at u_k crash, when r_j moves to u_{k+1} by R2b, then u_{k+1} is occupied by White and Blue robots. Then, White robots at u_{k+1} change their color to Blue by R4b. Thus, because all robots at u_{k+1} move to u_k by R3a, the gathering is achieved.
 - If there are non-crashed other robots r_i at u_k just before the execution of R2b, r_i moves to u_{k+1} with Blue color. If r_j moves to u_{k+1} at the same time as r_i moves, White robots at u_{k+1} change their color to Blue by R4b, and all robots at u_{k+1} move to u_k by R3a. Then, the gathering is achived. Otherwise, r_j moves to u_{k+1} by R2a, then u_{k+1} is occupied by White, Red and Blue robots. Then, White robots at u_{k+1} change their color to Red by R5a. Thus, because all robots at u_{k+1} move to u_k by R5b and R5c, the gathering is achieved.
- Consider the case that the crash occurs just after the execution of Move phase of R2b. Then, rk moved to an adjacent singly-colored node with White robots. Then, White robots at uk changes its color to Blue by R4b, and let t be the round. If rj is adjacent to uk at t, rj moves to uk by R2a and the gathering is achieved at t. If rj is not adjacent to uk at t, uk becomes singly-colored Blue. After that, we can

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discuss this case by the same way as the above cases such that crash occurs in a singly-colored Blue border.

Thus, the lemma holds.

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From Lemmas 4–7, we can deduce:

Theorem 2. Starting from a configuration C where O_{init} is odd, Algorithm 2 solves the SUIG problem on line-shaped networks without multiplicity detection in $O(M_{init})$ rounds.

5 Conclusion

We presented the first stand-up indulgent gathering algorithms for myopic luminous robots on line graphs. One is for the case where M_{init} is odd, while the other is for the case where O_{init} is odd. The hypotheses used for our algorithms closely follow the impossibility results found for the other cases.

Some interesting questions remain open:

- 1. Are there any algorithms for the case where crashed robots are located at different nodes? (In that case, one has to weaken the gathering specification, e.g., by requiring each correct robot to eventually gather at a crashed location, if any)
- 2. Are there any deterministic algorithms for the case where M_{init} and O_{init} are even (Such solutions would have to avoid starting or ending up in edge-view-symmetric situations)?
- 3. Are there any algorithms for the case where *O_{init}* is odd that use fewer colors than ours?
- 4. We present distinct solutions for the cases where M_{init} is odd and O_{init} is odd. It would be interesting to design a single algorithm that handles both cases.

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