Guido Herrmann Martin J. Pearson Alexander Lenz Paul Bremner Adam Spiers Ute Leonards (Eds.)

# **Social Robotics**

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# A Low-Cost Classroom-Oriented Educational Robotics System

Mário Saleiro<sup>1</sup>, Bruna Carmo<sup>2</sup>, Joao M.F. Rodrigues<sup>1</sup>, and J.M.H. du Buf<sup>1</sup>

 $^1\,$  Vision Laboratory, LARSyS, University of the Algarve, 8005-139 Faro, Portugal,  $^2\,$  Escola Superior de Educação e Comunicação, University of the Algarve, 8005-139

Faro, Portugal

{masaleiro,jrodrig,dubuf}@ualg.pt, b.santos.carmo@gmail.com http://w3.ualg.pt/~dubuf/vision.html

Abstract. Over the past few years, there has been a growing interest in using robots in education. The use of these tangible devices in combination with problem-based learning activities results in more motivated students, higher grades and a growing interest in the STEM areas. However, most educational robotics systems still have some restrictions like high cost, long setup time, need of installing software in children's computers, etc. We present a new, low-cost, classroom-oriented educational robotics system that does not require the installation of any software. It can be used with computers, tablets or smartphones. It also supports multiple robots and the system can be setup and is ready to be used in under 5 minutes. The robotics system that will be presented has been successfully used by two classes of 3rd and 4th graders. Besides improving mathematical reasoning, the system can be employed as a motivational tool for any subject.

Keywords: Education, Robotics.

#### 1 Introduction

In the last few years, the use of robots in education has become a very popular way of providing an interdisciplinary, project-based learning activity, with special focus on STEM (science, technology, engineering and mathematics) areas [16]. Robots offer major new benefits to education at all levels [6]. Long-term experiments that involve robots in classrooms have resulted in an increase of students enrolled in such classes, as well as an improvement in general learning, increased motivation and higher performance [5]. Considering the current shortage of student interest in STEM topics, increasing attention has been paid to developing innovative tools for improved teaching of STEM, including robotics [16]. Especially kids from elementary schools (6-10 years) tend to show a big interest in robots, making it an excellent motivational tool for low-grade education [9,15]. Moreover, it is a current belief that getting kids in touch with robotics will spark interest in natural sciences, engineering and computer science [15,19]. The success of educational robotics has attracted so much attention in some countries that robotics can officially be a part of a primary school's curriculum [14].

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Much effort in introducing robots in classrooms is focused on introducing robot technologies to the student and underestimate the role of pedagogy. However, robots can be used as an educational tool [17]. They must also be seen as potential vehicles of new ways of thinking about teaching, learning and education at large. In this perspective, teachers can stop functioning as an intellectual "authority" who transfers ready knowledge to students, but rather act as an organizer, coordinator and facilitator of learning for students by raising the questions and problems to be solved and offering the necessary tools for the students to work with creativity, imagination and independence [6].

Using robotics in classrooms is also a good way of teaching programming, since kids are able to view the result of their command sequences in the real world. Success in teaching programming to kids has been reported using Scratch (a virtual programming environment by blocks) with the WeDo kits, enabling children to program simple robotic models [18].

Several robotics systems for education, like the LEGO NXT kits, have been widely used in classrooms all over the world [10,17]. However, they can be quite expensive, they require the installation of specific software and the configuration of the programming connections. This can be difficult for teachers without proper training [16]. When it is not possible to buy ready-to-use robotics kits, some skilled teachers find a way of making their own robot kits [8,13] or buy cheap modules, easily available at online shops, and modify them to better suit their needs [7]. The growing popularity of electronic boards like Arduino [1], Raspberry Pi [3] and tools like 3D printers in the past few years has enabled more and more persons to create robots for purposes like education.

The system that we present here aims at enabling young students, starting at 8 years, to control the behaviour of a tangible model by means of a virtual environment, so that it can be used to learn programming and also as a motivational element when teaching other subjects. With such an approach, kids develop mathematical reasoning and creativity and also become interested in other subjects, as long as knowledge in those areas is embedded in the activity as a requirement for the robot to complete a task. The system that will be described is low cost, it does not require the installation of any specific software, it can be used with computers, tablets or smartphones, it can be setup in under 5 minutes, and it is extremely easy to use. Moreover, young kids need at most 5 minutes to get acquainted with the robotics system. The main difference between the system that will be presented and other commercial systems is that professors and educators can use them without spending too much time in preparing the activities and they do not need to know anything about robotics.

This paper is organised as follows. Section 2 describes the hardware and software of the complete educational robotics system. Section 3 describes two experiments using the system in 3rd and 4th grade classrooms and the educational components involved. Section 4 addresses conclusions and further work.

# 2 The Educational Robotics System

The system that is going to be presented in this section was designed to be extremely simple to use and fast to set up. When it was designed we had two major criteria in mind: if it cannot be set up in under five minutes, it will not be suitable for teachers, and if it cannot be explained to children in under five minutes, it will not be suitable for kids. Also, the system has been designed to have an extremely low cost. The system is composed of five robots which were named "Infantes" and a single multi-robot controller. This architecture (see Fig. 1) allows the robots to be built with simple and cheap components, while maintaining the usability of a web-based graphical programming interface through a Wi-Fi connection. This architecture also makes the system extremely easy to use at science fairs and exhibitions, allowing visitors to interact with the robots with their own smartphones or tablets.



Fig. 1. Architecture of the educational robotics system, composed of Infante robots, a multi-robot controller, a web-based programming interface that can be accessed by any device with a Wi-Fi connection, and a Javascript-enabled web browser

#### 2.1 The Infante Robots

The robots are very small, each one fitting in a box with size  $7.5 \times 7.5 \times 7.5$  cm, which makes them ideal to work on table tops (see Fig. 2). They are differential drive robots, with two motorised wheels and a ball caster. Having the minimisation of cost in mind, we used the cheapest RC servos on the market (SG90 9g servos,  $\approx 1.80 \in$  each) and modified them for continuous rotation, which is a simple process that consists of replacing the internal potentiometer connections with two resistors. The wheels were scavenged from old printers, but cheap wheels can also be bought without a major increase in price. Concerning the electronics, we designed a custom board with a PIC16F88 microcontroller ( $\approx 1.80 \in$  per microcontroller), which is also one of the cheapest microcontrollers on the

market. The custom board umentation support and cost. For line following sensors (0.15€ each), and in an HC-05 Bluetooth module could find. For the structure terials like expanded PVC the building materials and each robot cost about 15€ and to put the first one tested, the others were require 4 AAA batteries.



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market. The custom board could be replaced by an Arduino for a better documentation support and easier expandability, but this would also increase the cost. For line following and crossings detection we added 4 TCRT5000 infrared sensors (0.15  $\!\!\!\in$  each), and for communications with the main controller we used an HC-05 Bluetooth module (4.5  $\!\!\!\in$  each) which was also the cheapest that we could find. For the structure of the robot, we used cheap and easy to work materials like expanded PVC and plexiglass. We estimate that taking into account the building materials and some other discrete components, the parts to make each robot cost about 15  $\!\!\!\!\in$ , although some work was required to develop them and to put the first one together. However, once the first robot was designed and tested, the others were assembled rather quickly. As power source, the robots require 4 AAA batteries.

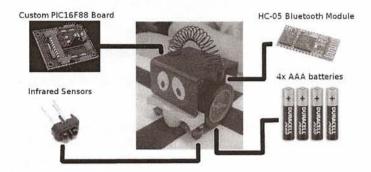


Fig. 2. Infante robot and its electronic parts

As mentioned above, the robots have four infrared sensors which allow them to follow lines and detect line crossings. This is because they were designed to be used on a grid of black lines on a white background, which are considered to be the "roads" for the robot to navigate. The PIC microcontroller has simple firmware that takes care of driving the motors and reading the sensors, so that kids do not have to worry about the low-level part of the robot. The firmware also contains a single communication protocol with only three commands: go forward, rotate 90 degrees to the left, and rotate 90 degrees to the right. Using the programming interface that will be described in Section 2.3, kids are able to build sequences of commands and send the whole sequences to the robot. Every time the robot detects a line crossing, it concludes the current action and it proceeds to execute the action that corresponds to the next command.

#### 2.2 The Multi-robot Controller

The multi-robot controller is an electronic system that acts as a hub for robots and also as a hub for users. It consists of a Raspberry Pi mini computer (35€) running Raspbian Wheezy installed on an SD card, with Bluetooth and Wi-Fi dongles (see Fig. 3). The Raspberry Pi is configured to act as a wireless access

point, creating an open Wi-Fi network that users can connect to using their laptops, tablets or smartphones. Besides that, the Raspberry Pi also runs a web server, providing the web-based robot programming interface at a specific address.

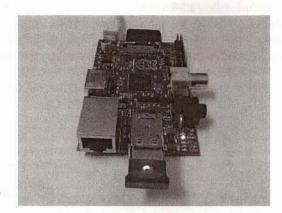


Fig. 3. The multi-robot controller, which consists of a Raspberry Pi mini computer, a Bluetooth dongle, a Wi-Fi dongle and a power supply

The web server being used is based on Tornado [4], which is a Python web framework with asynchronous networking library. By using non-blocking network I/O, Tornado can maintain tens of thousands of open connections, making it ideal for long polling, WebSockets, and other applications that require a long-living connection to each user. Since it is a Python framework, it is rather easy to establish a data flow between the web server and the Bluetooth serial connections to the robots. Whenever a user sends a sequence of actions to a specific robot, the multi-robot controller establishes the Bluetooth serial connection to the desired robot, sends the action sequence and exits the connection. This allows to use multiple robots with a single controller. When only a few robots are used, the Bluetooth serial connections to the robots may be kept alive to avoid the delays due to the establishment of the connections.

#### 2.3 Web-Based Robot Graphical Programming Interface

The use of a web-based interface is a big advantage over other existing educational robotics systems, since it allows the users to program the robot using any device with a Wi-Fi connection and a Javascript-enabled web browser, without having to install any additional software.

The programming interface is based on Blockly [2], a google project that allows one to make short programs by dragging and dropping programming blocks to interact with virtual objects on the computer screen. It is similar to Scratch, but works in a web-browser. Since Blockly is an open-source project, we adapted it to contain only three basic operations: go forward, rotate 90° to the left, and rotate 90° to the right (see Fig. 4).

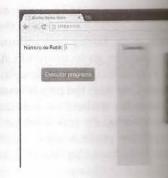


Fig. 4. Example of the tions translate into real robust with Portuguese children.

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# 3 Results

In this section we present were used in classrooms. 4th graders from an element developing mathematical features an activity with a also in Faro, in which the waste by using the robots.

# 3.1 Developing Mathematical

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Fig. 4. Example of the programming interface based on blockly and how the instructions translate into real robot actions. The interface is in Portuguese, since it was used with Portuguese children.

This limitation of the instructions was adopted to make it suitable for the young children who would participate in the experiments that we planned to conduct. We also enabled Blockly to interact with real devices, the Infante robots, by using a Python web server based on the Tornado framework, as previously mentioned. Apart from these modifications, we also added a simple input box to the interface in order to select the number of the robot to be programmed.

#### 3 Results

In this section we present the results of two different situations in which robots were used in classrooms. The first case concerns an activity with a class of 4th graders from an elementary school in Faro, Portugal, with the objective of developing mathematical reasoning while teaching geography. The second case features an activity with a class of 3rd graders from another elementary school, also in Faro, in which they were taught about recycling and reusing electronic waste by using the robots.

# 3.1 Developing Mathematical Reasoning in Geography

Mathematical reasoning may be understood as an activity where a pupil participates while interacting with others in solving a mathematical problem [20]. In this kind of activities, pupils must be guided to explain their reasoning in order to reach a certain conclusion or to justify the way of approaching a specific problem [11]. In order to develop this kind of reasoning, professors must provide activities that allow the pupils to (a) develop methods of mathematical thinking, be encouraged to explore, try and make mistakes, (c) formulate hypotheses, them and provide arguments about their validity, and (d), question their and others' reasonings. This kind of activities leads to the development of beductive, inductive and abductive reasoning [12].

Having the development of this kind of thinking and reasoning in mind, we designed an activity with several tasks where the pupils would have to plan the paths of a robot, on a grid, in order to reach certain goals, taking into account that in some tasks there are some restrictions. For example: go to mountain A, pass by mountain B but avoid mountain C. This activity also includes geography, since the goals were always mountains from Portugal. This way, the pupils learn the relative positions of the major Portuguese mountains without even being aware of it. Teaching this kind of things is usually achieved by repetition and memorisation. By using the robots we were able to get the pupils actually interested and motivated to learn the locations by themselves, since they wanted the robot to complete all the tasks.



Fig. 5. 4th graders using the robots. Left: one pupil explains the programming sequences while another uses a computer to program the robot. Right: pupils use a tablet computer to program the robots.

To perform the activity, the class was split into groups of 5 to 6 pupils, with one robot for each group. After a short setup and explanation, the groups were left to work on their own for an hour and a half, during which we observed their reactions, discussions and explanations. As we already expected, getting started with the robots was no problem for them, irrespective of using a laptop or a tablet. We noticed that, at start, most of them could not distinguish left from right. They also forgot to make the robot go forward after a rotation, making it spin around, therefore questioning the programming they had just made in order to find out what was wrong. Another problem that we noticed at start was that the pupils had some difficulty in assuming the point of view of the robot in order to to decide the correct turns. However, after some brief discussions, as one or two of them had already learned how to do it, they taught each other. justifying their way of solving that problem (see Fig. 5). We also noticed that, whenever something did not work as expected, they discussed with each other to try to find the errors, explaining their thoughts. If there were several solutions to complete a task, they would discuss which path to follow, either preferring the shortest path for easier programming, or by choosing a more challenging path, using their creativity. The pupils were so motivated that they finished all tasks sooner than we expected and autonomously. They then used their creativity to invent new tasks themselves and kept playing with the robots. By the end of the activity, all the pupils knew the relative positions of the mountains, they knew how to program the robots, and but all pupils loved it and

#### 3.2 Teaching Pupils about

In the activity that we describe the ease with which 4th grades further to test the system will Fig. 6). The activity was them aware of recycling and given to groups of 5 to 6 pupils were placed in the robot's name also placed in the grid. The that was then placed on top of We left them again to work the previous activity, the puril regardless of the programming problems occurred: not making the same place, confusing left will of view of the robot. However, were working as a group, trying other why they thought the moter could fix it. Once more, the pure tasks rapidly and then invest robots until the class ended.



Fig. 6. Third graders using the carrying the lamp to the proper the robot while others analyse approgramming sequence.

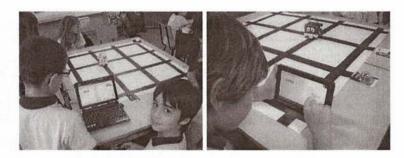
# 4 Conclusions and F

Confirmed by the activities that that our goal of designing an education

how to program the robots, and they could justify their options. Needless to say, but all pupils loved it and did not want the class to end.

#### 3.2 Teaching Pupils about Recycling and Reusing Electronic Waste

In the activity that we described in the previous section, we were amazed by the ease with which 4th graders used the educational robot system. So, we went further to test the system with a class of 3rd graders from another school (see Fig. 6). The activity was similar to the previous one, but was designed to make them aware of recycling and reusing electronic waste. Again, a list of tasks was given to groups of 5 to 6 pupils and some electronic parts like lamps and batteries were placed in the robot's navigation grid. The corresponding recycling bins were also placed in the grid. The tasks were to make the robot reach a specific item, that was then placed on top of the robot, and then carried to the proper bin. We left them again to work on their own and observed their behaviour. As in the previous activity, the pupils learned how to use the system very quickly regardless of the programming device used. We verified that the same initial problems occurred: not making the robot go forward, making it spin around at the same place, confusing left with right, and not being able to assume the point of view of the robot. However, like in the other activity, after a short time they were working as a group, trying to pinpoint their mistakes, and explaining each other why they thought the robot was not doing what they wanted and how they could fix it. Once more, the pupils were extremely motivated. They finished the tasks rapidly and then invented their own tasks and kept on programming the robots until the class ended.



**Fig. 6.** Third graders using the robots. Left: pupils use a laptop to control the robot in carrying the lamp to the proper recycling bin. Right, a pupil uses a tablet to program the robot while others analyse the solution and say which commands to put in the programming sequence.

#### 4 Conclusions and Further Work

Confirmed by the activities that we realised with 3rd and 4th graders, we conclude that our goal of designing an educational robotics system with an extremely fast

setup and which is extremely easy to use has been achieved. The setup time was very short and the pupils learned how to use the robots very quickly. The low-cost robots executed the commands that were sent to them as expected, and the communication between them and the multi-robot controller also worked without any problems. The system proved to be reliable and very easy to use in noisy classroom environments.

As we already mentioned, our objective was also to create a platform that would allow the robots to be used as a tool in teaching any other subject, and not to focus on the process of robot building. Throughout our experiments, we verified that the deployment of robots can be so motivating that pupils actually want to learn things they consider difficult and boring to memorise. Also, pupils perceived the robots as toys and harmoniously played together, confidently discussing how to tackle the challenges and problems they were facing.

We also verified that, regardless of the subject being taught in the activity in which the robots were used, there was a constant development of mathematical reasoning, deductive, abductive as well as inductive. This is extremely important since it enables pupils to formulate hypotheses, to test them, to find out what's wrong, and then formulate new hypotheses that may solve a problem. It also encourages them to work in a group, discussing with each other and communicating their hints to the other group members, who can learn through positive and negative feedback. The development of spatial relations between objects also helps to build mental representations of the world and to use these in reasoning. By deploying the robots as a motivational learning tool in teaching other subjects, the pupils become interested in learning them, since they feel a need to learn to accomplish the tasks that are given to them. By designing activities with robots where there are multiple correct solutions pupils are stimulated to deploy their creativity in pursuit of a solution for a given problem.

As further work we intend to make some improvements of the robots, like changing the power source to Li-ion batteries and adding more features to the communication protocol embedded in the robot's firmware. Adding some distance and other sensors, or a small gripper might also be an option to allow for more creative uses and to increase the user's motivation. We also intend to extend the graphical programming interface by re-adding all the programming blocks that Blockly originally had, such that the system can be more challenging and be used by persons of any age.

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