

# Blind Image Deconvolution



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Methods and Convergence



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*To*

*Two very special renunciants – Kalipada Maharaj  
and Swami Shantananda.*

*SC.*

*Karthi chitappa.*

*VRB.*

*All from whom I have learned.*

*RMR.*



# Preface

The issue of blur in images has kept researchers in the area of image processing busy for over half a century, and the researchers at the Vision and Image Processing Laboratory, IIT Bombay, were no exceptions. We have been working in this area for the last two decades. Earlier our efforts were confined to investigating the beneficial aspects of blur in images wherein we had used defocus blur to estimate depth in a scene. A monograph entitled *Depth from Defocus: A Real Aperture Imaging Approach*, published by the same publisher in 1999, dealt with the problem of deblurring when there are multiple observations. We derived performance bounds as a function of relative blurring among multiple observations. Subsequently, in the monograph entitled, *Motion-Free Super-Resolution*, again published by Springer in 2005, we demonstrated how the relative blurring among various observations could be used to super-resolve an image very efficiently.

In this monograph we investigate the deblurring problem for a single observation. This problem is well known in literature as blind deconvolution, and there has been a large volume of work by various researchers as regards how one can perform deconvolution when the blur is not known. So the question does arise what purpose then does this monograph serve? Is this yet another book on image deblurring? Publication of this monograph does require a justification. A review of the published literature on blind deconvolution suggests that these works concentrate mostly on accuracy and efficiency of various proposed algorithms.

Since blind deconvolution has both the point spread function (PSF) and the image unknown, both of them are required to be estimated. Very often this requires solving an alternate minimization problem. However, solving an alternate minimization problem does not always lead to useful solutions. Sometimes it leads to trivial solutions. It is no wonder that a method proposed by one group does not work for another group as it requires several parameters to be tweaked. Hence there is a genuine need for analyzing convergence properties of alternate minimization methods. We perform precisely the same in this monograph so that the end user can pre-judge the applicability of a given blind deconvolution algorithm for a specific

problem. This also helps in understanding the behavior of an algorithm as iterations proceed. We hope that the readers will find the analysis useful.

In order to make the monograph useful to the practitioners in the industry, we also investigate one of the current techniques in detail for blind deconvolution. This involves usage of sparsity as a constraint as opposed to typical image PSF priors such as Gibbs distribution. It can be seen that sparsity is, indeed, a good candidate method for blind deconvolution.

The intended primary audience of this monograph are the graduate students in mathematics, EE, and CS departments. This should also serve as a good reference book for researchers in the area of image processing. The practitioners in image processing area would also find the monograph useful as it will provide them with a good theoretical insight to the problem. We have tried to make the book self-contained and hence there is no pre-requisite. However, familiarity with basics of signal and image processing and linear algebra would be helpful to the readers.

We sincerely hope that the readers will find the monograph useful and we welcome comments and suggestions from the readers.

Mumbai, India  
April 2014

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# List of Symbols

$\  \cdot \ _2, \ell_2$	L2 norm
$\  \cdot \ _1, \ell_1$	L1 norm
$\otimes$	Kronecker product
$y, \underline{y}$	Observed blurred and noisy image (matrix and vector form, respectively)
$x, \underline{x}$	Original sharp image (matrix and vector form, respectively)
$k$	Point spread function
$\mathcal{N}, \underline{n}$	White Gaussian noise
$\oplus$	Convolution operation
$K$	Convolution matrix
$R_x(\cdot)$	Image regularizer
$\lambda_x$	Image regularization factor
$R_k(\cdot)$	PSF regularizer
$\lambda_k$	PSF regularization factor
$\eta_{min}$	Minimum eigenvalue of sum of convolution matrices arising from PSF and regularizer
$\gamma_{min}$	Minimum eigenvalue of sum of convolution matrices arising from image and regularizer
$d(.,.)$	non-negative function used to define three- and four-point properties
$g_{h,i}, g_{v,i}$	First order horizontal and vertical difference at pixel $i$
$D$	First order difference matrix
$y'$	First order difference of $y$