Using Flow Specifications of Parameterized Cache Coherence Protocols for Verifying Deadlock Freedom

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Abstract. We consider the problem of verifying deadlock freedom for symmetric cache coherence protocols. While there are multiple definitions of deadlock in the literature, we focus on a specific form of deadlock which is useful for the cache coherence protocol domain and consistent with the internal definition of deadlock in the Murphi model checker: we refer to this deadlock as a *system-wide deadlock (s-deadlock)*. In s-deadlock, the entire system gets blocked and is unable to make any transition. Cache coherence protocols consist of N symmetric cache agents, where N is an unbounded parameter; thus the verification of s-deadlock freedom is naturally a parameterized verification problem.

Parametrized verification techniques work by using sound abstractions to reduce the unbounded model to a bounded model. Efficient abstractions which work well for industrial scale protocols typically bound the model by replacing the state of most of the agents by an abstract environment, while keeping just one or two agents as is. However, leveraging such efficient abstractions becomes a challenge for s-deadlock: a violation of s-deadlock is a state in which the transitions of all of the unbounded number of agents cannot occur and so a simple abstraction like the one above will not preserve this violation. Authors of a prior paper, in fact, proposed using a combination of over and under abstractions for verifying such properties. While quite promising for a large class of deadlock errors, simultaneously tuning over and under abstractions can become complex.

In this work we address this challenge by presenting a technique which leverages high-level information about the protocols, in the form of message sequence diagrams referred to as *flows*, for constructing invariants that are collectively stronger than s-deadlock. Further, violations of these invariants can involve only one or two interacting agents: thus they can be verified using efficient abstractions like the ones described above. We show how such invariants for the German and Flash protocols can be successfully derived using our technique and then be verified.

1 Introduction

We consider the problem of verifying deadlock freedom for symmetric cache coherence protocols. Consider a cache coherence protocol $\mathcal{P}$ (N) where the parameter N represents an unbounded number of cache agents. The protocol implements requests sent by the agents using messages exchanged in the protocol. For a protocol designer, the main property of interest is the request-response property, i.e., every request from an agent eventually gets a response. Since this property is a liveness property which is hard for

existing model checking tools, designers resort to identifying causes for response property failure, such as deadlock-style failures, and verify against them.

The literature is abundant with various definitions of deadlock [8, 22]. We focus on deadlock errors in which the entire protocol gets blocked, i.e., no agent of the protocol can make any transition. We refer to such an error as a *system-wide deadlock (s-deadlock)*. If we model each transition τ of the protocol to have a guard $\tau.g$, which is $false$ if the transition is not enabled, the s-deadlock error occurs if the guards of all the transitions are *false*, i.e., $\bigwedge_\tau \neg(\tau.g)$ is *true*. This kind of failure, while weaker than other broader classes of deadlock failures, is commonly observed in industrial computer system designs and is consistent with the internal definition for deadlock used by the Murphi model checker as well [23]. This class of deadlocks is well motivated for parameterized cache coherence protocols as these use a centralized synchronization mechanism (e.g. a directory) and thus any deadlock results in the directory getting blocked. It is highly likely that such a deadlock in the shared directory will end up involving all of the agents of the protocol getting blocked, i.e., unable to make any transition.

Since an s-deadlock error involves all of the unbounded number of agents getting blocked and unable to make any transition, verification of s-deadlock freedom naturally is a parameterized verification problem. Parameterized verification techniques work by using sound abstractions to reduce the unbounded model to a finite bounded model that preserves the property of interest. These abstractions typically tend to be simple over-abstractions such as *data-type reduction* [30]. This abstraction keeps a small number of agents (1 or 2) as is and replaces all the other agents with an abstract environment. Such abstractions along with parameterized techniques like the CMP (CoMPositional) method [11] have had considerable success in verifying key safety properties like mutual exclusion and data integrity even for industrial scale protocols [11, 32, 37].

1.1 Challenge in Verifying S-deadlock

While parameterized techniques are successful for safety properties such as mutual exclusion and data integrity, the application of such abstractions for parameterized verification of properties such as s-deadlock is hard. The key challenge arises from the fact that an s-deadlock violation is a state in which all the guards are *false*, i.e., when $\bigwedge_\tau \neg(\tau.g)$ holds; simple over-abstractions such as data-type reduction will easily mask this violation due to the discarded state of agents other than 1 and 2 and the extra transitions of the environment.

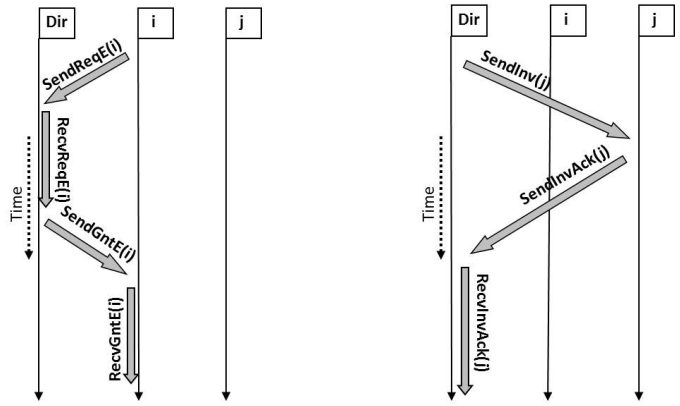
One approach to address the above issue is to use a combination of over and under abstractions (i.e., a *mixed* abstraction) instead of data-type reduction, as described in a prior deadlock verification work [8]. While promising for verifying a large class of deadlock errors, the use of mixed abstraction requires reasoning about over and under abstraction simultaneously and easily becomes fairly complex.

In this paper we take a different approach. We show how high-level information about the protocols, in the form of message sequence diagrams referred to as *flows*, can be leveraged to construct invariants which are collectively stronger than the s-deadlock freedom property. These invariants are amenable to efficient abstractions like data-type reduction which have been used in the past for verifying industrial scale protocols.

1.2 Leveraging Flows for Deadlock Freedom

Cache coherence protocols implement high-level requests for read (termed $Shared$) or write (termed $Exclusive$) access from cache agents, or for invalidating access rights (termed $Invalidate$) of some agent from the central directory. The implementation of these requests is done by using a set of transitions which should occur in a specific protocol order. This ordering information is present in diagrams referred to as *message flows* (or *flows* for brevity). These flows are readily available in industrial documents in the form of message sequence charts and tables [37].

Fig. 1 shows two of the flows for the German cache coherence protocol describing the processing of the $Exclusive$ and $Invalidate$ requests. Each figure has a directory Dir, and two agents i and j. The downward vertical direction indicates the passage of time. The $Exclusive$ request is sent by the cache agent i to the directory Dir to request a write access. The $Exclusive$ flow in Fig. 1(a) describes the temporal sequence of transitions which occur in the implementation in order to process this request: each message is a transition of the protocol. The message $SendReqE(i)$ is sent by the agent i to Dir which receives this message by executing the transition $RecvReqE(i)$. Next, if the directory is able to grant $Exclusive$ access, it sends the message $SendGntE(i)$ to agent i which receives this grant by executing $RecvGntE(i)$. However, in case the directory is unable to send the grant since another agent j has access to the cache line, the directory sends a request to invalidate the access rights of j. The temporal sequence of transitions which occur in the implementation in this case are shown in the $Invalidate$ flow in Fig. 1(b). This flow proceeds by the directory sending the $SendInv(j)$ message, the agent j sending the acknowledgment message $SendInvAck(j)$, and the directory receiving it by executing $RecvInvAck(j)$ transition.



(a) $Exclusive$ flow (b) $Invalidate$ flow

Fig. 1: Flows for the German protocol.

Freedom from S-deadlock At a high-level, our method tries to exploit the fact that if the protocol is s-deadlock free, when none of the transitions of an agent are enabled, another agent can be identified which must have a transition enabled. This identification leverages the key insight that in any state of the protocol, if all the transitions of some agent, say a_1, cannot occur, then, some flow of that agent must be blocked since it depends on another flow of another agent, say a_2, to finish. Then, there are two possibilities: (1) the agent a_2 is enabled, in which case the state is not an s-deadlock state, or (2) the agent a_2 is blocked as well, in which case it depends on another agent a_3. If this dependence chain is acyclic, with the final agent in the chain enabled, the protocol is s-deadlock free. However, if the final agent is not enabled, or if the dependence chain has a cycle, the protocol may either have an s-deadlock error or there may be an error in the flow diagrams used.

As an example, for the German protocol, if the $Exclusive$ flow of agent i is blocked since the transition $SendGntE(i)$ cannot occur, it is waiting for j to get invalidated. In the protocol, at least some transition of the $Invalidate$ flow on agent j can occur. This enables proving freedom from s-deadlock for the protocol.

Using the above insight, by analyzing the dependence between blocked agents, our method is able to point to an agent which must have at least one transition enabled in every reachable state of the protocol. Specifically, our method enables the derivation of a set of invariants $\mathcal{I}$ which collectively partition the reachable state of the protocol. Each invariant then points to the agents which must have at least one transition enabled when the protocol is in a state belonging to its partition. These invariants are derived in a loop by iteratively model checking them on a protocol model with c agents, where c is heuristically chosen as discussed in Section 3.

Verifying for an Unbounded Number of Agents Once the invariants in $\mathcal{I}$ are derived, they hold for a model with c agents. These invariants use just one index (i.e., they are of the form $\forall i : \phi(i)$) and thus, they can be verified for an unbounded number of agents by using efficient parameterized verification techniques such as data-type reduction along with the CMP (CoMPositional) method [11]. This technique has previously been successful for verifying mutual exclusion for industrial protocols [32]. We note that our approach is not limited to the CMP method: the invariants derived may be verified by using any parameterized safety verification technique [15, 27, 34, 35].

1.3 Key Contributions

Our method proves s-deadlock freedom for parameterized protocols (formalized in Section 2). It takes a Murphi model of the protocol as input. As shown in Fig. 2, first, a set of invariants $\mathcal{I}$ which collectively imply s-deadlock freedom are derived on a model with c agents (Section 3). These invariants are verified for an unbounded number of agents by using state-of-the-art parameterized verification techniques (Section 4). We verified Murphi implementations of two challenging protocols, the German and Flash protocols using our method (Section 5).

Limitation: The key limitation of our approach is that the invariants have to be derived manually by inspecting counterexamples. This can be automated if additional information about conflicting flows is available in the flow diagram itself.

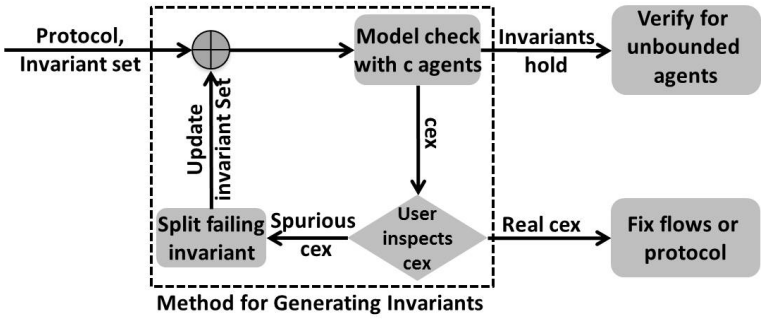


Fig. 2: Experimental Flow

1.4 Relevant Related Work

Deadlock Verification: The work closest to ours is by Bingham *et al.* [7, 8]. They formally verify deadlock as a safety property for protocols by specifying it using user-identified Quiescent states (*i.e.*, a state in which no resources are held): they specify a protocol state to be a deadlock state if no Quiescent state is reachable from it. They prove freedom from such a deadlock by using a combination of over and under abstractions (also referred to as a *mixed* abstraction [16]). Their approach is a promising way to verify deadlock freedom which scales to protocols like the Flash protocol. However, the required tuning of both under and over abstractions simultaneously can be complex. In contrast, we take the flow-based alternative to enable simpler abstractions like data-type reduction.

Since the ultimate goal of any deadlock verification effort is to verify the response property (i.e. every high-level request eventually gets a response), we contrast our work with liveness verification efforts as well. Among techniques for parameterized verification of liveness, McMillan has verified liveness properties of the Flash protocol [28,29]. The proof is manual and works on the basis of user supplied lemmas and fairness assumptions. In contrast, our method reduces manual effort by leveraging information from flows along with the CMP method. Among automatic approaches for verifying liveness properties, Baukus *et al.* verified liveness properties of the German protocol [6] using a specialized logic called WSIS. Fang *et al.* used automatically deduced ranking functions [21] and, in a prior work, counter abstraction [35] to verify liveness properties. While fully automatic, these approaches tend to exhibit limited scalability for larger protocols such as Flash, due to the inherent complexity of the liveness verification problem. In contrast to these, our approach, while requiring some user guidance, achieves much greater scalability and enables us to verify the Flash protocol.

Parameterized Verification Techniques: We note that the invariants derived using our method can be verified for an unbounded number of caches by any parameterized safety verification technique, it is not dependent on the CMP method which we used. Our choice of using the CMP method was motivated by the fact that it is the only state-of-the-art method we are aware of which has been used successfully for verifying protocols like Flash and other industrial scale protocols. Among other techniques, an

important technique is by Conchon *et al.* [15] which uses a backward reachability algorithm to automatically prove a simplified version of the Flash protocol. Next, there are numerous other prior approaches in literature for parameterized verification of safety properties. The CMP method falls in the broad category of approaches which use *compositional reasoning* [1, 2] and *abstraction* based techniques to verify parameterized systems; the literature is abundant with examples of these [13, 14, 17, 20, 27, 28, 34, 35]. Next, another category of approaches work by computing a *cutoff* bound k and showing that if the verification succeeds for k agents, then the protocol is correct for an arbitrary number of agents [3, 5, 12, 18, 19, 24]. Finally, there are approaches based on *regular model checking* which use automata-based algorithms to verify parameterized systems [4, 9, 10, 36]. To the best of our knowledge, the CMP method is the state-of-the-art for protocol verification in contrast to these methods and has been used to successfully verify larger protocols such as Flash with minimal manual effort. (Other methods which verify Flash protocol in full complexity are by Park *et al.* [33] and Park *et al.* [17]. As described by Talupur *et al.* [37], these are significantly manual and take much more time to finish verification of the Flash protocol compared to the CMP method.)

2 Protocols, Flows and S-deadlock Freedom: Background

2.1 Preliminaries

A protocol $\mathcal{P}$ (N) consists of N symmetric cache agents, with *ids* from the set $\mathbb{N}_N = \{1, 2, 3, \ldots, N\}$. We follow our prior approach [37] (which was inspired by the approach of Kristic [25]) in formalizing cache coherence protocols.

Index Variables: The protocol uses index variables quantified over the set of index values $\mathbb{N}_N$. Thus, if i is an index variable, then i takes values from the domain $\mathbb{N}_N$.

State Variables: The state of the protocol consists of local variables, and global variables shared between the agents. Each of these types of variables can either hold values from the Boolean domain (variables with values from generic finite domains can be represented as a set of Boolean variables) or pointers which can hold agent *ids*. We represent the Boolean variables in the global state as G_B, and the pointers as G_P. The Boolean local state variables of each agent i are encoded as $L_B[i]$, and the local pointer variables as $L_P[i]$. The pointer variables have values from the domain $\mathbb{N}_N \cup \{null\}$, where $null$ represents that the variable does not hold any index value.

Expressions: An expression is a, possibly quantified, propositional formula with atoms G_B, $G_P = j$, $L_B[i]$ and $L_P[i] = j$, where, i and j are index variables.

Assignments: Assignments are of the form $G_B := b$, or $G_P := j$, $L_B[i] := b$ or $L_P[i] := j$, where, b is a variable with Boolean value and i, j are index variables.

Rules: Each agent i consists of a set of rules $rl_1(i), rl_2(i), rl_3(i), \ldots, rl_k(i)$. Each rule $rl_j(i)$ can be written as: $rl_j(i) : rl_j(i).\rho \rightarrow rl_j(i).a$, where, $rl_j(i)$ is the rule name, $rl_j(i).\rho$, the guard, is an expression, and $rl_j(i).a$ is a list of assignments, such that these assignments are restricted to only update the global variables or the local variables of agent i. The local variables and rules for all agents i are symmetric.

Protocol: The above defined variables and rules naturally induce a state transition system. A *protocol*, then, is a state transition system (S, Θ, T), where S is the set of

protocol states, $\Theta \subseteq S$ is the set of initial states, and $T \subseteq S \times S$ is the transition relation. Each protocol state $s \in S$ is a valuation of the variables G_B, G_P, and $L_B[i]$, $L_P[i]$ for each agent i. There exists a transition $\tau(i_v) = (s, s'), (s, s') \in T$ from state s to s' if there is a rule $rl_j(i)$ and value of index variable $i = i_v$, s.t. $rl_j(i_v).\rho$ holds in s, and s' is obtained by applying $rl_j(i_v).a$ to s. In state s, we say that the rule $rl_j(i)$ is *enabled* for agent with *id* i_v if the guard $rl_j(i_v).\rho$ is *true*. When the enabled rule is executed, its action is applied to update the state and we say that the rule $rl_j(i)$ has *fired* for agent i_v. The action is applied atomically to update the state, thus the transitions of the protocol have interleaving semantics. Finally, we define an execution *trace* of the protocol as a series of transitions where each transition is a fired rule. Thus, a trace can be represented by a series ($rl_a(i_0)$, $rl_b(i_1)$, ..., $rl_s(i_k)$), where the transition $rl_m(i_n)$ is the rule rl_m fired for the agent with *id* i_n.

S-deadlock Definition We define a protocol state s to be an *s-deadlock state* if no rule in that state is enabled. Then, a protocol is s-deadlock free if in all states, there exists at least one rule which is enabled. This can be expressed as the invariant: $\bigvee_i \bigvee_j rl_j(i).\rho$, i.e., the protocol is s-deadlock free if the disjunction of the guards of all the rules of all the agents is *true* for all the reachable states.

Flows Flows describe the basic organization of rules for implementing the high-level requests in a protocol (for example a request for $Exclusive$ access or an $Invalidate$). We model a flow as a set of rules $\mathcal{F}(i)$ of the form $\{rl_a(i), rl_b(i), rl_c(i), \ldots, rl_n(i)\}$ which accomplish a high-level request of agent i.[3] The rules in a flow are partially ordered, with the partial order relation denoted as $\prec_{\mathcal{F}(i)}$. For example, in the $Exclusive$ flow in Fig. 1(a), the rules (arrows) are totally ordered along the downward direction. Thus $SendReqE(i) \prec_{\mathcal{F}_E(i)} RecvReqE(i)$, where $\mathcal{F}_E$ denotes the set of rules for $Exclusive$ flow. For every rule $rl_k(i)$ in the flow $\mathcal{F}(i)$, the partial order naturally induces the following *precondition*: for the rule $rl_k(i)$ to fire, all the rules preceding that rule in the partial order of the flow $\mathcal{F}(i)$ must have already been fired. This precondition is denoted by $rl_k(i).p_{\mathcal{F}(i)}$ and, formally, can be written as:

$$rl_k(i).p_{\mathcal{F}(i)} = \forall j : \big(\{(rl_j(i) \in \mathcal{F}(i)) \wedge (rl_j(i) \prec_{\mathcal{F}(i)} rl_k(i))\} \Rightarrow (rl_j(i).fired = true)\big),$$

where $rl_j(i).fired$ is an auxiliary variable which is initially set to *false* when the flow $\mathcal{F}(i)$ starts and is set to *true* when the rule $rl_j(i)$ has fired for that flow.

Designs of protocols are presented in industrial documents as a set of flows $\mathcal{F}_1(i)$, $\mathcal{F}_2(i)$, $\mathcal{F}_3(i)$, ..., $\mathcal{F}_k(i)$. In order to process a high-level request, a protocol may use a combination of these flows, e.g. in order to execute a request for $Exclusive$ access the German protocol uses the $Exclusive$ and $Invalidate$ flows. Each flow in a protocol represents an execution scenario of the protocol for processing some high-level request. Thus many of the flows of a protocol tend to exhibit a lot of similarity as they are different execution scenarios of the same high-level request. This makes them fairly easy to understand. In Section 3, we show how a set of invariants collectively implying s-deadlock freedom can be derived from these flows.

[3] For ease of exposition we assume that the guard and action of a rule are over the variables of a single agent. Thus, a flow containing such rules also involves a single agent. In general, a rule and thus a flow can involve a larger but fixed number of interacting agents as well. Our approach can be easily generalized to that case.

Some definitions: We define the union of all the flows of agent i by $\mathcal{R}(i)$, i.e., $\mathcal{R}(i) = \bigcup_k \mathcal{F}_k(i)$. Next, we define the operator $\widehat{en}$ which is *true* for a set of rules, if at least one rule in the set is enabled, else it is *false*. Thus, for example, $\widehat{en}(\mathcal{R}(i))$ holds if at least one of the rules in $\mathcal{R}(i)$ is enabled. In this case, we say that the agent i is enabled. Similarly, we say that a flow $\mathcal{F}(i)$ is enabled if at least one of its rules is enabled, i.e., $\widehat{en}(\mathcal{F}(i))$ holds. In case a flow $\mathcal{F}(i)$ is not enabled, we say that it is *blocked* on some rule $rl_j(i) \in \mathcal{F}(i)$ if the precondition of the rule $rl_j(i).p_{\mathcal{F}(i)}$ holds but the guard of the rule $rl_j(i).\rho$ is *false*.

2.2 German Protocol Implementation

The German protocol consists of agents such that each agent can have $Exclusive$ (E), $Shared$ (S) or $Invalid$ (I) access to a cache line, as stored in the variable $Cache[i].State$. An agent i requests these access rights by sending messages on a channel $ReqChannel[i]$ to a shared directory which sends corresponding grants along the channel $GntChannel[i]$. The directory is modeled as a set of global variables which serves one agent at a time: it stores the *id* of the agent being served in the variable $CurPtr$. It also stores the nature of the request in the variable $CurCmd$ with values in $\{ReqE, ReqS, Empty\}$, where $ReqE$ represents a request for $Exclusive$ access, $ReqS$ for $Shared$ and $Empty$ for no request. Finally, the directory tracks if $Exclusive$ access is granted to some agent or not using the variable $ExGntd$: it is *true* if access is granted and *false* otherwise. A simplified version of the code for the $Exclusive$ request is shown in Fig. 3, with the original Murphi implementation [11] presented in Appendix A.

In processing the $Exclusive$ request, before sending the grant $SendGntE(i)$, the directory checks if there are any sharers of the cache line (by checking $ShrSet$ = $\{\}$). If there are sharers, the $Invalidate$ flow is invoked for each agent in $ShrSet$. Upon invalidation of all the agents in $ShrSet$, the $ShrSet$ becomes empty and so the $SendGntE(i)$ rule becomes enabled for execution. We show the code for the $SendInv(i)$ rule below.

```
∀ i : ℕ_N ; do Rule SendInv(i)
  InvChannel[i].cmd = Empty ∧ i ∈ ShrSet ∧
    ((CurCmd = ReqE) ∨ (CurCmd = ReqS ∧ ExGntd = true))
 →
  InvChannel[i].cmd := Invalidate;
End;
```

We note a condition Inv_Cond, which must be *true* for invoking the $Invalidate$ flow and can be identified from the guard of $SendInv(i)$; $Inv\_Cond : \big(\big((CurCmd = ReqE) \vee ((CurCmd = ReqS) \wedge (ExGntd = true))\big) \wedge (ShrSet \neq \{\})\big)$.

3 Deriving Invariants for Proving S-deadlock Freedom

In this section, we show how a set of invariants $\mathcal{I}$ can be derived from flows such that the invariants in $\mathcal{I}$ collectively imply s-deadlock freedom. At a high-level, our method tries to show s-deadlock freedom by partitioning the global state of the protocol using predicates, such that for each partition, some agent i has at least one transition

```
∀ i : ℕ_N; do Rule SendReqE(i)
  ReqChannel[i].cmd=Empty ∧
   (Cache[i].State=I ∨ Cache[i].State=S)
 →
  ReqChannel[i].cmd := ReqE;
End;

∀ i : ℕ_N; do Rule RecvReqE(i)
  ReqChannel[i].cmd=ReqE ∧ CurCmd=Empty
 →
  CurCmd := ReqE; CurPtr := i;
   ReqChannel[i].cmd := Empty;
End;

∀ i : ℕ_N; do Rule SendGntE(i)
  CurCmd=ReqE ∧ CurPtr=i ∧
    GntChannel[i]=Empty ∧ Exgntd=false
   ∧ ShrSet={}
 →
  GntChannel[i] := GntE; ShrSet := {i};
  ExGntd := true; CurCmd := Empty;
  CurPtr := NULL;
End;

∀ i : ℕ_N; do Rule RecvGntE(i)
  GntChannel[i]=GntE
 →
  Cache[i].State := E; GntChannel[i] := Empty;
End;
```

Fig. 3: Implementation of the $Exclusive$ Request.

enabled. Each invariant inv is of the form $inv.pred \Rightarrow \big(\forall i \in In^{inv} : \widehat{en}(\mathcal{R}(i))\big)$, where $inv.pred$ is a predicate on the global variables of the protocol, $In^{inv} \subseteq \mathbb{N}_N$ s.t. $\neg(In^{inv} = \{\})$ (this is discharged as a separate assertion for model checking) and $\widehat{en}(\mathcal{R}(i))$ denotes a disjunction of the guards of the rules in $\mathcal{R}(i)$. *The key insight is that since $\widehat{en}(\mathcal{R}(i))$ has transitions from a single agent, the abstractions required for model checking inv for an unbounded number of agents are significantly simpler than those for checking the original s-deadlock property,*[4] *as discussed in Section 4.*

Our method iteratively model checks each invariant in $\mathcal{I}$ to refine it. Suppose, the invariant $inv \in \mathcal{I}$ fails on model checking with the state of the protocol at failure being s_f. Then, there exists some agent i_f such that when $inv.pred$ holds in s_f, $i_f \in In^{inv}$ is *true* and $\widehat{en}(\mathcal{R}(i_f))$ is *false* in s_f. This can happen due to two reasons: first, there may be a mismatch between the flow specification and the rule-based protocol description. This can be due to a missing rule in some flow, a missing flow all together,

[4] In the case of rules involving more than one agent (say c), the corresponding invariants may involve transitions from c agents as well. Since c is small for practical protocols, the abstraction constructed for verifying such invariants will be simple as well.

or an implementation error: the cause for the mismatch can be discovered from the counterexample. As an example for this case, the counterexample may show that all flows of the agent i_f are not enabled, however the agent still has some rule $rl_e(i_f)$ enabled: this rule may be a part of a missing flow. However, typically the invariant inv fails due to the second reason: there must exist some flow $\mathcal{F}$ of the agent i_f which is blocked (i.e. it has a rule which is expected to be enabled and so has precondition *true* but has its guard *false*). This blocked flow is waiting for another flow $\mathcal{F}$' of another agent i_s to complete. As an example, for the German protocol, the $Exclusive$ flow may be blocked for agent i_f with the rule $SendGntE(i_f)$ having precondition *true* but guard *false* and waiting for an $Invalidate$ request to complete for another agent i_s in the set $Sharers$. In this case, the set $\mathcal{I}$ is refined by splitting the invariant inv.

The invariant inv is split by, (1) splitting the predicate $inv.pred$ to further partition the global state, and (2) updating the set In^{inv} for each partition. To accomplish this, the user identifies a pointer variable from G_P or $L_P[i]$ (or an auxiliary variable) $\hat{w}$, such that it has the value i_s in the failing state s_f (and so acts as a *witness* variable for i_s). The user also identifies a conflict condition $conf$ on the global state which indicates when i_s is enabled and i_f fails. This is done by using the heuristic that if the rule $rl_f(i_f)$ of flow $\mathcal{F}$ of agent i_f is blocked, $conf$ can be derived by inspecting the guard of $rl_f(i_f)$; the condition $conf$ generally is the cause for falsification of $rl_f(i_f).\rho$. For example, for the German protocol, $conf$ is derived from the guard of $SendGntE$ and $\hat{w}$ points to some sharer which is being invalidated.

Using $conf$ and $\hat{w}$, the invariant can be split into two invariants. (1) The first invariant excludes the case when conflict happens from the original invariant, i.e., $inv1 : (inv.pred \wedge \neg conf) \Rightarrow \big(\forall i \in In^{inv1} : \widehat{en}(\mathcal{R}(i))\big)$, where $In^{inv1} = In^{inv}$. (2) The second invariant shows that when a conflict happens, the agent pointed to by $\hat{w}$ must be enabled and so the protocol is still s-deadlock-free, i.e., $inv2 : (inv.pred \wedge conf) \Rightarrow \big(\forall i \in In^{inv2} : \widehat{en}(\mathcal{R}(i))\big)$, where $In^{inv2} = \{i | \, (i \in \mathbb{N}_N) \wedge (i = \hat{w})\}$. For both the invariants, assertions which check that the corresponding set of indices are non-empty are also verified. For example, for $inv1$, this assertion is $(inv.pred \wedge \neg conf) \Rightarrow In^{inv1}$.

Our method derives these invariants by iteratively model checking with a small number c (3 for German protocol) of agents. (Once the invariants are derived for c agents, they are verified for an unbounded number of agents, as shown is Section 4.) This number c needs to be chosen to be large enough such that the proof of s-deadlock freedom is expected to generalize to an unbounded number of agents. For the protocols we verified, we found that as a heuristic, c should be one more than the maximum number of agents involved in processing a high-level request. For the German protocol, an $Exclusive$ request may involve two agents, a requesting agent i and an agent j getting invalidated, so we chose c to be equal to 3.

Fig. 4 shows the details of the method. It starts with an initial broad guess invariant, *true* $\Rightarrow \big(\forall i \in \mathbb{N}_N : \widehat{en}(\mathcal{R}(i))\big)$ (line 1). This indicates that in all reachable states, every agent has at least one transition enabled. As this invariant is *false*, this broad guess invariant is refined into finer invariants, using the loop. On finishing, the user is able to derive a set of invariants, $\mathcal{I}$, which collectively imply s-deadlock freedom. Further, the user is also able to derive an assertion set, $\mathcal{A}$, such that for each invariant inv in $\mathcal{I}$, an assertion in $\mathcal{A}$ checks if the set of indices In^{inv} is non-empty when $inv.pred$ holds.

DERIVE_INVARIANTS($\mathcal{P}(c)$):
1: $\mathcal{I} = \{\mathit{true} \Rightarrow \big(\forall i \in \mathbb{N}_N : \widehat{en}(\mathcal{R}(i))\big)\}$
2: $\mathcal{A} = \{\}$
3: while $\mathcal{P}(c) \not\models \mathcal{I}$ do
4: Let $inv \in \mathcal{I} : \mathcal{P}(c) \not\models inv$ and
$inv : inv.pred \Rightarrow \big(\forall i \in In^{inv} : \widehat{en}(\mathcal{R}(i))\big)$, where, $In^{inv} \subseteq \mathbb{N}_N$
5: Inspect counterexample cex and failing state s_f:
6: Case 1: mismatch between flows and protocol
7: Exit loop and fix flows or protocol
8: Case 2: identify conflicting agents i_f and i_s s.t.
9: (1) $i_f : \big((i_f \in In^{inv}) \wedge (\neg\widehat{en}(\mathcal{R}(i_f)))\big)$ holds in s_f.
10: (2) $\exists rl_f \in \mathcal{F}(i_f)$ s.t. $\big(rl_f(i_f).p_{\mathcal{F}} \wedge \neg(\widehat{en}(\mathcal{F}(i_f)))\big)$ holds in s_f.
11: (3) $\widehat{en}(\mathcal{R}(i_s))$ holds in s_f.
12: Identify $conf$ and witness $\hat{w}$ from above information
13: $inv1 : (\neg conf \wedge inv.pred) \Rightarrow \big(\forall i \in In^{inv} : \widehat{en}(\mathcal{R}(i))\big)$
14: $inv2 : (conf \wedge inv.pred) \Rightarrow \big(\forall i \in In^{inv2} : \widehat{en}(\mathcal{R}(i))\big)$, where,
$In^{inv2} = \{i | \, i = \hat{w}\}$
15: $\mathcal{I} = \{\mathcal{I} \setminus inv\} \cup \{inv1, inv2\}$
16: $\mathcal{A} = \big(\mathcal{A} \setminus \big(inv.pred \Rightarrow (In^{inv} \neq \{\})\big)\big) \cup$
$\{\big(inv1.pred \Rightarrow (In^{inv1} \neq \{\})\big), \big(inv2.pred \Rightarrow (In^{inv2} \neq \{\})\big)\}$

Fig. 4: Method for Deriving Invariants from Flows.

Soundness of the Method The following theorem shows that the invariants in $\mathcal{I}$ along with the assertions in $\mathcal{A}$ collectively imply s-deadlock freedom, with proof in Appendix B.

Theorem. *If the set of invariants $\mathcal{I}$ along with the set of assertions $\mathcal{A}$ hold, they collectively imply s-deadlock freedom, i.e.,* $\big(\big(\bigwedge_{inv \in \mathcal{I}}(\mathcal{P} \models inv)\big) \wedge \big(\bigwedge_{asrt \in \mathcal{A}}(\mathcal{P} \models asrt)\big)\big) \Rightarrow \big(\mathcal{P} \models (\bigvee_i \bigvee_j rl_j(i).\rho)\big)$.

3.1 Specifying Invariants for the German Protocol

We derive the invariants for a model of the German protocol with 3 cache agents. We start with the initial invariant that for all agents, some flow is enabled, i.e., INV-1: $true \Rightarrow \big(\forall i \in \mathbb{N}_N : \widehat{en}(\mathcal{R}(i))\big)$.

Iteration 1: Model checking the invariant INV-1 returns a counterexample trace ($SendReqE(1)$, $RecvReqE(1)$, $SendReqE(2)$). Since the index of the last rule in the trace is 2, $\widehat{en}(\mathcal{R}(2))$ must be *false*. This is because the rule $RecvReqE(2)$ of the $Exclusive$ flow of cache 2 is not fired and thus has precondition *true* but guard *false*. The user identifies the conflict condition $conf = \neg(CurCmd = Empty)$ from the guard of the blocked rule $RecvReqE(2)$. Since $CurPtr$ is the witness pointer in the protocol for the variable $CurCmd$, the witness $\hat{w}$ is set to $CurPtr$. Thus, the invariant is split as follows:

- INV-1.1: $(CurCmd = Empty) \Rightarrow (\forall i \in \mathbb{N}_N : \widehat{en}(\mathcal{R}(i)))$.

– INV-1.2: $\neg(CurCmd = Empty) \Rightarrow (\forall i \in In^{inv-1.2} : \widehat{en}(\mathcal{R}(i)))$, where $In^{inv-1.2} = \{i | (i \in \mathbb{N}_N) \wedge (i = CurPtr)\}$. The assertion $\neg(CurCmd = Empty) \Rightarrow \neg(In^{inv-1.2} = \{\})$ is also checked.

Iteration 2: Next, on model checking the invariants INV-1.1 and INV-1.2, the invariant INV-1.2 fails. The counterexample trace returned is ($SendReqE(1)$, $RecvReqE(1)$, $SendGntE(1)$, $SendReqE(2)$, $RecvReqE(2)$, $SendReqE(2)$). Since the last rule of the counterexample is from cache 2, $\widehat{en}(\mathcal{R}(2))$ must be *false* even when $CurPtr = 2$. Further, there are two flows for two $Exclusive$ requests by cache 2 active in the counterexample, the first with $SendReqE(2)$ fired and the second with $SendReqE(2)$, $RecvReqE(2)$ fired. Since the first flow is blocked on the rule $RecvReqE(2)$, the guard of this rule is inspected. The guard is *false* as $CurCmd$ is not empty. However, since the corresponding witness variable for $CurCmd$ is $CurPtr$ which is already 2 (due to the processing of the second flow), this is not a conflict with another cache. The conflict must then be for the second $Exclusive$ flow. The second flow is blocked on the rule $SendGntE(2)$ with precondition *true* but guard *false*: the user identifies the conflict condition $conf$ from the guard of $SendGntE$ to be Inv_Cond. Now, if Inv_Cond is *true*, the $Invalidate$ flow for some sharer cache (cache 1 in this trace) must be active. Thus, the user identifies $\hat{w}$ to point to a sharer which must be invalidated: this is done using the auxiliary variable $Sharer$, which points to the last sharer to be invalidated in $ShrSet$. Thus, the invariant INV-1.2 is split as follows:

– INV-1.2.1: $\big(\neg(CurCmd = Empty) \wedge (\neg Inv\_Cond)\big) \Rightarrow (\forall i \in In^{inv-1.2.1} : \widehat{en}(\mathcal{R}(i)))$, where, $In^{inv-1.2.1}$ = $In^{inv-1.2}$. An assertion that the precondition implies the index set is non-empty is also checked.
– INV-1.2.2: $\big(\neg(CurCmd = Empty) \wedge (Inv\_Cond)\big) \Rightarrow (\forall i \in In^{inv-1.2.2} : \widehat{en}(\mathcal{R}(i)))$, where, $In^{inv-1.2.2} = \{i | (i \in \mathbb{N}_N) \wedge (i \in ShrSet)\}$. An assertion that the precondition implies the index set is non-empty is also checked.

Iteration 3: Next, on model checking, the invariants INV-1.1, INV-1.2.1, INV-1.2.2, along with the added assertions hold for a model with 3 caches. Then, to prove s-deadlock freedom, this set of invariants form a candidate set to verify a protocol model with an unbounded number of agents. The property is checked for unbounded agents using techniques described in Section 4.

4 Verifying Flow Properties for Unbounded Agents

We now show how to verify the invariants in $\mathcal{I}$ for an unbounded number of agents by leveraging the data-type reduction abstraction along with the CMP method.

Abstraction: Data-type Reduction Since the invariant is of the form $inv.pred \Rightarrow \big(\forall i \in In^{inv} : \widehat{en}(\mathcal{R}(i))\big)$, by symmetry, it is sufficient to check: $inv.pred \Rightarrow \big((1 \in In^{inv}) \Rightarrow \big(\widehat{en}(\mathcal{R}(1))\big)\big)$. In order to verify this invariant, just the variables of agent 1 are required. Then, our abstraction keeps just the agent 1, and discards the variables of all the other agents by replacing them with a state-less *environment* agent. We refer to agent 1 as a *concrete* agent and the environment as *Other* with *id o*.

In the original protocol, since all the agents other than agent 1 interact with it by updating the global variables, the actions of these agents on the global variables are over-approximated by the environment agent. This environment agent does not have any local state. The construction of this agent *Other* is automatic and accomplished syntactically: further details on the automatic construction are available in [37]. The final constructed abstraction then consists of: (1) a concrete agent 1, (2) an environment agent *Other* with *id* o, and (3) invariants specified on variables of agent 1 and global variables. This abstraction is referred to as *data-type reduction*. If the original protocol is $\mathcal{P}$, and invariant set $\mathcal{I}$, we denote this abstraction by $data\_type$ and thus the abstract model by $data\_type(\mathcal{P})$ and the abstracted invariants on agent 1 by $data\_type(\mathcal{I})$.

Abstraction for German Protocol We now describe how the rule $SendGntE(i)$ gets abstracted in $data\_type(\mathcal{P})$. In the abstract model, there is one concrete agent 1, which has the rule $SendGntE(1)$. Next, $SendGntE(o)$ is constructed as follows. (1) The guard is abstracted by replacing all atoms consisting of local variables (e.g. $GntChannel[i] = Empty$) with *true* or *false* depending on which results in an over-abstraction and by replacing any usage of i in atoms with global variables (e.g. $CurPtr = i$) with o (i.e. $CurPtr = o$). (2) The action is abstracted by discarding any assignments to local variables. Further, assignments to global pointer variables are abstracted as well: any usage of i (e.g. $CurPtr := i$) is replaced by o (i.e. $CurPtr := o$). The rule for agent *Other* is shown below:

```
Rule SendGntE(o)
   CurCmd = ReqE ∧ CurPtr = o ∧ true ∧ Exgntd = false ∧
    ShrSet = {}
  →
   ShrSet := {o}; ExGntd := true; CurCmd := Empty;
   CurPtr := NULL;
End;
```

The Abstraction-Refinement Loop of the CMP Method The CMP method works as an abstraction-refinement loop, as shown in Fig. 5. In the loop, the protocol and invariants are abstracted using data-type reduction. If the proof does not succeed, the user inspects the returned counterexample cex and following possibilities arise. (1) Counterexample cex is real, in which case an error is found and so the loop exits. (2) Counterexample cex is spurious and so the user refines the protocol by adding a *non-interference lemma* lem. The function *strengthen* updates the guard $rl_j(i).\rho$ of every rule $rl_j(i)$ of the protocol to $rl_j(i).\rho \wedge lem(j)$; this way, on re-abstraction with $data\_type$ in line 1, the new abstract protocol model is refined. Additional details on the CMP method are available in [11, 25].

5 Experiments

Using our approach, we verified Murphi (CMurphi 5.4.6) implementations of the German and Flash protocols (available online [31]). Our experiments were done on a 2.40 GHz Intel Core 2 Quad processor, with 3.74 GB RAM, running Ubuntu 9.10.

German Protocol We verified the invariants discussed in Section 3.1, in order to prove s-deadlock freedom. We chose to use an abstraction with 2 agents and an environment agent, so that the mutual exclusion property can also be checked.

CMP($\mathcal{P}(N), \mathcal{I}$)
1: $\mathcal{P}^{\#} = \mathcal{P}(N)$;$\mathcal{I}^{\#} = \mathcal{I}$
2: *while* $data\_type(\mathcal{P}^{\#}) \not\models data\_type(\mathcal{I}^{\#})$ *do*
3: examine counterexample cex
4: if cex is real, exit
5: if spurious:
6: find lemma $lem = \forall i.lem(i)$
7: $\mathcal{P}^{\#} = strengthen(\mathcal{P}^{\#}, lem)$
8: $\mathcal{I}^{\#} = \mathcal{I}^{\#} \cup lem$

Fig. 5: The CMP method

The proof finished in 217s with 7M states explored. No non-interference lemmas were required to refine the model, in order to verify the invariants presented in Section 3.1. Since typically protocols are also verified for properties like data integrity (i.e. the data stored in the cache is consistent with what the processors intended to write) and mutual exclusion, we model checked the above invariants along with these properties. In this case, the abstract model was constrained and model checking this model was faster and took 0.1 sec with 1763 states explored.

Buggy Version We injected a simple error in the German protocol in order to introduce an s-deadlock. In the bug, an agent being invalidated drops the acknowledgement $SendInvAck$ it is supposed to send to the directory. This results in the entire protocol getting blocked, hence an s-deadlock situation. This was detected by the failing of the invariant INV-1.2.2, discussed in Section 3.1.

Flash Protocol Next, we verified the Flash protocol [26] for deadlock freedom. The Flash protocol implements the same high-level requests as the German protocol. It also uses a directory which has a Boolean variable $Pending$ which is *true* if the directory is busy processing a request from an agent pointed to by another variable $CurSrc$ (name changed from original protocol for ease of presentation). However, the Flash protocol uses two key optimizations over the German protocol. First, the Flash protocol enables the cache agents to directly forward data between each other instead of via the directory, for added speed. This is accomplished by the directory by forwarding incoming requests from the agent i to the destination agent, $FwDst(i)$, with the relevant data. Second, the Flash protocol uses non-blocking invalidates, i.e, the $Exclusive$ flow does not have to wait for the $Invalidate$ flow to complete for the sharing agents in $ShrSet$. Due to these optimizations, the flows of the Flash protocol are significantly more complex than those of German protocol. Further, due to forwarding, some rules involve two agents instead of one for the German protocol: thus the flows involve two agents as well. Each flow then is of the form $\mathcal{F}_k(i, j)$, where i is the requesting agent for a flow and $j = FwDst(i)$ is the destination agent to which the request may be forwarded by the directory. Then, we define $\mathcal{R}(i)$ to be equal to $\bigcup_k \mathcal{F}_k(i, FwDst(i))$.

We derived the invariants from the flows by keeping c to be equal to 3, as each request encompasses a maximum of 2 agents (forwarding and invalidation do not happen simultaneously in a flow). The final invariants derived using our method are as follows:

Directory Not Busy: If the directory is not busy (i.e., $Pending$ is *false*), any agent i can send a request. Thus the invariant INVF-1: $\neg(Pending) \Rightarrow \big(\forall i \in \mathbb{N}_N : \widehat{en}(\mathcal{R}(i))\big)$.

However, if the directory is busy (i.e., $Pending$ is *true*), two possibilities arise. (1) It may be busy since it is processing a request from agent $CurSrc$. Or, (2) in case the request from $CurSrc$ requires an invalidate, the directory may remain busy with invalidation even after the request from $CurSrc$ has been served. This is because Flash allows the request from $CurSrc$ to complete before invalidation due to non-blocking invalidates. Hence the following invariants:

Directory Busy with Request: Invariant INVF-2: $\big((Pending) \wedge (ShrSet = \{\})\big) \Rightarrow \big(\forall i \in In^{invF-2}\widehat{en}(\mathcal{R}(i))\big)$, where $In^{invF-2} = \{i|\,(i \in \mathbb{N}_N) \wedge (i = CurSrc)\}$.

Directory Busy with Invalidate: Invariant INVF-3: $\big((Pending) \wedge \neg(ShrSet = \{\})\big) \Rightarrow \big(\forall i \in In^{invF-3}\widehat{en}(\mathcal{R}(i))\big)$, where $In^{invF-3} = \{i|\,(i \in \mathbb{N}_N) \wedge (i \in ShrSet)\}$.

Runtime: We verified the above invariants along with the mutual exclusion and the data integrity properties for an unbounded model abstracted by keeping 3 concrete agents (one agent behaves as a directory) and constructing an environment agent *Other*. The verification took 5127s with about 20.5M states and 152M rules fired. In this case we reused the lemmas used in prior work by Chou *et al.* [11] for verifying the mutual exclusion and data integrity properties in order to refine the agent *Other*.

Verifying Flash vs German Protocol: The flows of the Flash protocol involve two indices: we eliminated the second index by replacing it with the variable $FwDst(i)$ which stores information of the forwarded cache and thus made the verification similar to the German protocol case. Next, Flash protocol uses lazy invalidate: even if the original request has completed, the directory may still be busy with the invalidate. As explained above, this was in contrast to the German protocol and resulted in an additional invariant INVF-3.

Comparison with Other Techniques: The only technique we are aware of which handles Flash with a high degree of automation is by Bingham *et al.* [8]. While a direct comparison of the runtime between their approach and ours is infeasible for this paper, we note that the invariants generated using our approach only require an over-abstraction in contrast to theirs which requires a mixed-abstraction. This is an advantage since development of automatic and scalable over-abstraction based parameterized safety verification techniques is a promising area of ongoing research (e.g. [15]) which our approach directly benefits from.

6 Conclusions and Future Work

In this paper we have presented a method to prove freedom from a practically motivated deadlock error which spans the entire cache coherence protocol, an s-deadlock. Our method exploits high-level information in the form of message sequence diagrams—these are referred to as *flows* and are readily available in industrial documents as charts and tables. Using our method, a set of invariants can be derived which collectively imply s-deadlock freedom. These invariants enable the direct application of industrial scale techniques for parameterized verification.

As part of future work, we plan to take up verification of livelock freedom by exploiting flows. Verifying livelock requires formally defining a notion of the protocol doing useful work. This information is present in flows—efficiently exploiting this is part of our ongoing research.

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A The German Protocol Code (Chou *et al.* [11])

```
const  ---- Configuration parameters ----

  NODE_NUM : 4;
  DATA_NUM : 2;

type   ---- Type declarations ----

  NODE : scalarset(NODE_NUM);
  DATA : scalarset(DATA_NUM);

  CACHE_STATE : enum {I, S, E};
  CACHE : record State : CACHE_STATE; Data : DATA; end;

  MSG_CMD : enum {Empty, ReqS, ReqE, Inv, InvAck, GntS, GntE};
  MSG : record Cmd : MSG_CMD; Data : DATA; end;

var   ---- State variables ----

  Cache : array [NODE] of CACHE;      -- Caches
  Chan1 : array [NODE] of MSG;        -- Channels for Req*
  Chan2 : array [NODE] of MSG;        -- Channels for Gnt* and Inv
  Chan3 : array [NODE] of MSG;        -- Channels for InvAck
  InvSet : array [NODE] of boolean;   -- Nodes to be invalidated
  ShrSet : array [NODE] of boolean;   -- Nodes having S or E copies
  ExGntd : boolean;                   -- E copy has been granted
  CurCmd : MSG_CMD;                   -- Current request command
  CurPtr : NODE;                      -- Current request node
  MemData : DATA;                     -- Memory data
  AuxData : DATA;                     -- Latest value of cache line

---- Initial states ----

ruleset d : DATA do startstate "Init"
  for i : NODE do
    Chan1[i].Cmd := Empty; Chan2[i].Cmd := Empty; Chan3[i].Cmd := Empty;
    Cache[i].State := I; InvSet[i] := false; ShrSet[i] := false;
  end;
  ExGntd := false; CurCmd := Empty; MemData := d; AuxData := d;
end end;

---- State transitions ----

ruleset i : NODE do rule "SendReqS"
  Chan1[i].Cmd = Empty & Cache[i].State = I
==>
  Chan1[i].Cmd := ReqS;
end end;

ruleset i : NODE do rule "SendReqE"
  Chan1[i].Cmd = Empty & (Cache[i].State = I | Cache[i].State = S)
==>
  Chan1[i].Cmd := ReqE;
end end;

ruleset i : NODE do rule "RecvReqS"
  CurCmd = Empty & Chan1[i].Cmd = ReqS
==>
  CurCmd := ReqS; CurPtr := i; Chan1[i].Cmd := Empty;
  for j : NODE do InvSet[j] := ShrSet[j] end;
end end;

ruleset i : NODE do rule "RecvReqE"
  CurCmd = Empty & Chan1[i].Cmd = ReqE
==>
  CurCmd := ReqE; CurPtr := i; Chan1[i].Cmd := Empty;
  for j : NODE do InvSet[j] := ShrSet[j] end;
end end;

ruleset i : NODE do rule "SendInv"
  Chan2[i].Cmd = Empty & InvSet[i] = true &
  ( CurCmd = ReqE | CurCmd = ReqS & ExGntd = true )
==>
  Chan2[i].Cmd := Inv; InvSet[i] := false;
end end;

ruleset i : NODE do rule "SendInvAck"
  Chan2[i].Cmd = Inv & Chan3[i].Cmd = Empty
==>
  Chan2[i].Cmd := Empty; Chan3[i].Cmd := InvAck;
  if (Cache[i].State = E) then Chan3[i].Data := Cache[i].Data end;
  Cache[i].State := I; undefine Cache[i].Data;
end end;

ruleset i : NODE do rule "RecvInvAck"
  Chan3[i].Cmd = InvAck & CurCmd != Empty
==>
  Chan3[i].Cmd := Empty; ShrSet[i] := false;
  if (ExGntd = true)
  then ExGntd := false; MemData := Chan3[i].Data; undefine Chan3[i].Data end;
end end;

ruleset i : NODE do rule "SendGntS"
  CurCmd = ReqS & CurPtr = i & Chan2[i].Cmd = Empty & ExGntd = false
==>
  Chan2[i].Cmd := GntS; Chan2[i].Data := MemData; ShrSet[i] := true;
  CurCmd := Empty; undefine CurPtr;
end end;

ruleset i : NODE do rule "SendGntE"
  CurCmd = ReqE & CurPtr = i & Chan2[i].Cmd = Empty & ExGntd = false &
  forall j : NODE do ShrSet[j] = false end
==>
  Chan2[i].Cmd := GntE; Chan2[i].Data := MemData; ShrSet[i] := true;
  ExGntd := true; CurCmd := Empty; undefine CurPtr;
end end;

ruleset i : NODE do rule "RecvGntS"
  Chan2[i].Cmd = GntS
==>
  Cache[i].State := S; Cache[i].Data := Chan2[i].Data;
  Chan2[i].Cmd := Empty; undefine Chan2[i].Data;
end end;

ruleset i : NODE do rule "RecvGntE"
  Chan2[i].Cmd = GntE
==>
  Cache[i].State := E; Cache[i].Data := Chan2[i].Data;
  Chan2[i].Cmd := Empty; undefine Chan2[i].Data;
end end;

ruleset i : NODE; d : DATA do rule "Store"
  Cache[i].State = E
==>
  Cache[i].Data := d; AuxData := d;
end end;

---- Invariant properties ----

invariant "CtrlProp"
  forall i : NODE do forall j : NODE do
    i != j -> (Cache[i].State = E -> Cache[j].State = I) &
              (Cache[i].State = S -> Cache[j].State = I | Cache[j].State = S)
  end end;

invariant "DataProp"
  ( ExGntd = false -> MemData = AuxData ) &
  forall i : NODE do Cache[i].State != I -> Cache[i].Data = AuxData end;
```

B Proof of Soundness

Before proving the theorem, we first establish the following lemma:

Lemma. *The disjunction of predicates of all invariants in $\mathcal{I}$ holds, i.e., $\bigvee_{inv \in \mathcal{I}} inv.pred$ holds.*

Proof. We prove this by induction over the splitting step in our method.

Base Case: Our method starts with the initial invariant $true \Rightarrow \big(\forall i \in IndexSet : \widehat{en}(\mathcal{R}(i))\big)$ in $\mathcal{I}$. Thus, it trivially satisfies the lemma.

Induction Step: Next, suppose at some point during the generation of invariants, the set of candidates is $\mathcal{I}$. On model checking, invariant inv in $\mathcal{I}$ fails with $\widehat{en}(\mathcal{R}(i_f))$ being *false* for agent i_f. In case there is an error in the protocol or flows due to a rule $rl(i_f)$ being enabled for agent i_f in the failing state, the loop exits without modifying $\mathcal{I}$ and so the lemma holds trivially. In the second case, the invariant is split into invariants $inv1$ and $inv2$ by using conflict condition $conf$.

Now for this case, $inv1.pred = (inv.pred \wedge \neg conf)$ and $inv2.pred = (inv.pred \wedge conf)$. Clearly, the disjunction of predicates $inv1$ and $inv2$ equals to $inv.pred$, the predicate of inv. Thus, the disjunction of predicates of the new and old set of invariants is the same, i.e., $\bigvee_{inv \in \mathcal{I}} inv.pred = \bigvee_{inv \in \mathcal{I}'} inv.pred$, where the new set of invariants $\mathcal{I}' = \mathcal{I} \setminus \{inv\} \cup \{inv1, inv2\}$.

Hence, by induction, the above lemma holds. □

Now, using the above lemma, we prove the following theorem to establish soundness of our method:

Theorem. *If the set of invariants $\mathcal{I}$ along with the set of assertions $\mathcal{A}$ hold, they collectively imply s-deadlock freedom, i.e.,* $\big(\big(\bigwedge_{inv \in \mathcal{I}} (\mathcal{P} \models inv)\big) \wedge \big(\bigwedge_{asrt \in \mathcal{A}} (\mathcal{P} \models asrt)\big)\big) \Rightarrow \big(\mathcal{P} \models (\bigvee_i \bigvee_j rl_j(i).\rho)\big)$.

Proof. Let the protocol be in some reachable state s. We argue that some agent has at least one rule enabled in every such reachable state. By the above lemma, $\bigvee_{inv \in \mathcal{I}} inv.pred$ holds in state s. Thus, there must exist some invariant inv such that its predicate holds in s, i.e., $\exists inv \in \mathcal{I} : inv.pred = true$.

Now, let inv be $inv.pred \Rightarrow (\forall i \in In^{inv} : \widehat{en}(\mathcal{R}(i)))$. Then, since the assertion $inv.pred \Rightarrow In^{inv} \neq \{\}$ is in the set $\mathcal{A}$, which holds as well, there is some agent i_0 such that it is in In^{inv} and $\widehat{en}(\mathcal{R}(i_0))$ holds, i.e., $\exists i_o \in In^{inv} : \widehat{en}(\mathcal{R}(i_0))$. Thus, agent i_0 is enabled in the state s, and so the state is not an s-deadlock state.

□