

Strategies for Parallel Markup

Bruce R. Miller¹ *

National Institute of Standards and Technology, Gaithersburg, MD, USA

Abstract. Cross-referenced parallel markup for mathematics allows the combination of both presentation and content representations while associating the components of each. Interesting applications are enabled by such an arrangement, such as interaction with parts of the presentation to manipulate and querying the corresponding content, and enhanced search indexing. Although the idea of such markup is hardly new, effective techniques for creating and manipulating it are more difficult than it appears. Since the structures and tokens in the two formats often do not correspond one-to-one, decisions and heuristics must be developed to determine in which way each component refers to and is referred to by components of the other representation. Conversion between fine and coarse grained parallel markup complicates ID assignments. In this paper, we will describe the techniques developed for \LaTeX XML, a \TeX / \LaTeX to XML converter, to create cross-referenced parallel MathML. While we do not yet consider \LaTeX XML's content MathML to be useful, the current effort is a step towards that continuing goal.

1 Introduction

Parallel markup for mathematics provides the capability of providing alternative representations of the mathematical expression, in particular, both the presentation form of the mathematics, i.e. its appearance, along with the content form, i.e. it's meaning or semantics. Cross-linking between the two forms provides the connection between them such that one can determine the meaning associated with every visible fragment of the presentation and, conversely, the visible manifestation of each semantic sub-expression. Thus cross-linked parallel markup provides not only the benefits of the presentation and content forms, individually, but support many other applications such as: hybrid search where both the presentation and content can be taken into account simultaneously; interactive applications where the visual representation forms part of the user-interface, but supports computations based on the content representation.

Of course, the *idea* of parallel markup is hardly new. The `m:semantics` element has been part of the MathML specification[?] since the first version, in 1998! What seems to be missing are effective strategies for creating, manipulating and using this markup. Fine-grained parallelism is when the smallest sub-expressions are represented in multiple forms, whereas with coarse-grained

* The final publication is available at <http://link.springer.com>

parallelism the entire expression appears in the several forms. Fine-grained parallelism is generally easier to create initially, and particularly when one deals with complex ‘transfix’ notations, or wants to preserve the appearance, but can infer the semantic intent, of each sub-expression. Coarse-grained is often required by applications which may understand only a single format, or are unable to disentangle the fine-grained structure. HTML5 only just barely accepts coarse-grained parallel markup, for example. Conversion from fine to coarse grained is not inherently difficult, it can be carried out by a suitable walk of the expression tree for each format. But what isn’t so clear is how to maintain the associations between the symbols (or more generally, the nodes) in the two trees. Indeed, since there is typically no one-to-one correspondence between the elements of each format. Fine-grained parallelism, by itself, doesn’t guarantee a clear association between all the symbols between the branches.

Our context here is $\text{\LaTeX}\text{XML}$, a converter from $\text{\TeX}/\text{\LaTeX}$ to XML, and thence to web appropriate formats such as HTML, MATHML and OpenMath. Input documents range from highly semantic markup such as sTeX[?], to intermediate such as used in DLMF[?], to fairly undisciplined, purely presentational, markup as found on arXiv[?]. \TeX induces high expectations for quality formatting forcing us to preserve the presentation of math. Meanwhile, the promise of global digital mathematics libraries and the potential reuse of a legacy of mathematics material encourages us to push as far as possible the extraction of content from such documents. At the very least, we should preserve whatever semantics is available in order to enable other technologies and research, such as LLaMaPuN[?], to resolve the remaining ambiguities.

In this paper, we describe the markup used in $\text{\LaTeX}\text{XML}$ both for macros with known semantics, and for the result of parsing, and strategies for conversion to cross-linked, parallel markup combining Presentation MathML (pMML) and Content MathML (cMML). It should be noted that this does not mean that $\text{\LaTeX}\text{XML}$ is producing useful quality cMML; the current work is a stepping stone towards that long-term goal.

2 Motivation

Before diving into examples, a brief introduction to $\text{\LaTeX}\text{XML}$ ’s internal mathematics markup, informally called XMath, is in order. This markup, inspired by OpenMath and both pMML and cMML, is intentionally hybrid in order to capture both the presentation and content properties of the mathematical objects throughout the step-wise processing from raw \TeX markup, through parsing and, ultimately, semantic annotation. Please see the online manual¹ for more details.

XMAp generalized application (think `m:apply` or `om:OMA`);

XMTok generalized token (think `m:mi`, `m:mo`, `m:mn`, `m:csymbol`);

XMDual parallel markup container of the content and presentation branches;

¹ <http://dlmf.nist.gov/LaTeXXML/manual/>

Listing 1.1. Internal representation of $a + F(a, b)$, after parsing (assuming F as a function)

```

<XMAp>
  <XMTok meaning=" plus" role="ADDOP">+</XMTok>
  <XMTok role="ID" font=" italic">a</XMTok>
  <XMDual>
    <XMAp>
      <XMRef idref="m1.1"/>
      <XMRef idref="m1.2"/>
      <XMRef idref="m1.3"/>
    </XMAp>
    <XMAp>
      <XMTok role="FUNCTION" xml:id="m1.1" font=" italic">F</XMTok>
      <XMWrap>
        <XMTok role="OPEN" stretchy=" false">(</XMTok>
        <XMTok role="ID" xml:id="m1.2" font=" italic">a</XMTok>
        <XMTok role="PUNCT">,</XMTok>
        <XMTok role="ID" xml:id="m1.3" font=" italic">b</XMTok>
        <XMTok role="CLOSE" stretchy=" false">)</XMTok>
      </XMWrap>
    </XMAp>
  </XMDual>
</XMAp>

```

XMRef shares nodes between branches of **XMDual**, via `xml:id` and `idref` attributes; **XMWrap** container unparsed sequences of tokens or subtrees (think `m:mrow`).

By way of motivation, consider the simple example in Listing 1.1. The `role` attribute on tokens indicates the syntactic role that it plays in the grammar; in this case, we’ve asserted that F is a function, allowing the expression to be parsed. At the top-level, the sum requires no special parallel treatment since the presentation for infix operators is trivially derived from the content form (i.e. the application of ‘+’ to its arguments). The application of F to its arguments benefits somewhat from parallel markup. This is a typical situation with the fine-grained **XMDual**: the content branch is the application of some function or operator (here F) to arguments (here a, b), but they are represented indirectly using **XMRef** to point to the corresponding sub-expressions within the presentation. While one could represent the delimiters and punctuation as attributes (as in MathML’s `m:mfenced`), that loses attributes of those attributes such as stretchiness, size or even color. A more compelling case is made when more complex prefix notations or semantic macros are involved, as we will shortly see.

However, this simple example already hints at a hidden complexity. Converting to either pMML and cMML is straightforward (given rules for mapping **XMath** elements to **MATHML**): simply walk the tree, following each **XMRef** to the referenced node and choosing the first or second branch of **XMDual** for content or presentation, respectively. Even cross-linking is straightforward in the absence of **XMDual**, when the generated content or presentation nodes are ‘sourced’ from the same **XMath** node (F , a , and b , in the example): we simply assign `ID`’s to the source **XMath** node and the generated nodes and record the association between the two; afterwards, the presentation and content nodes that were sourced from the same `ID` get an `xref` attribute referring to the other, in order to connect them.

Listing 1.2. MATHML representation of $a + F(a, b)$

```

<math display="block" alttext="a+F(a,b)" class="ltx_Math" id="m1">
  <semantics id="m1a">
    <mrow xref="m1.7.cmml" id="m1.7">
      <mi xref="m1.4.cmml" id="m1.4">a</mi>
      <mo xref="m1.5.cmml" id="m1.5">+</mo>
      <mrow xref="m1.6.cmml" id="m1.6d">
        <mi xref="m1.1.cmml" id="m1.1">F</mi>
        <mo xref="m1.6.cmml" id="m1.6e">&ApplyFunction;</mo>
        <mrow xref="m1.6.cmml" id="m1.6c">
          <mo xref="m1.6.cmml" id="m1.6" stretchy="false">(</mo>
            <mi xref="m1.2.cmml" id="m1.2">a</mi>
            <mo xref="m1.6.cmml" id="m1.6a">,</mo>
            <mi xref="m1.3.cmml" id="m1.3">b</mi>
            <mo xref="m1.6.cmml" id="m1.6b" stretchy="false">)</mo>
          </mrow>
        </mrow>
      </mrow>
    </semantics>
    <annotation-xml id="m1b" encoding="MathML-Content">
      <apply xref="m1.7" id="m1.7.cmml">
        <plus xref="m1.5" id="m1.5.cmml"/>
        <ci xref="m1.4" id="m1.4.cmml">a</ci>
        <apply xref="m1.6d" id="m1.6.cmml">
          <ci xref="m1.1" id="m1.1.cmml">F</ci>
          <ci xref="m1.2" id="m1.2.cmml">a</ci>
          <ci xref="m1.3" id="m1.3.cmml">b</ci>
        </apply>
      </apply>
    </annotation-xml>
    <annotation id="m1c" encoding="application/x-tex">a+F(a,b)</annotation>
  </semantics>
</math>

```

But with XMDual one has not only to determine when the generated nodes are related, one has to contend with extra tokens; in the example, the parentheses and comma appear only in the presentation. Presumably, those tokens should be associated with the *application* of F , as would the containing `m:mrow`? The desired result is shown in Listing 1.2.

A fuller illustration of the issues encountered in typical L^AT_EX markup combines complex prefix notations and semantic macros, such as:

```

\left\langle\!\!\left\langle\Psi\middle|\mathcal{H}\middle|\Phi\right\rangle\!\!\right\rangle
+ \defint{a}{b}{F(x)}{x}

```

This example, whose internal form is shown in Listing 1.3, involves quantum-mechanics notations, which L^AT_EXML’s parser is happily able to recognize. Additionally, we’ve introduced a semantic macro `\defint` to represent definite integration, which will be transformed to so-called ‘Pragmatic’ Content MathML form, to enhance the illustration with a many-to-many correspondence. (The implementation of `\defint` is not difficult, but outside the scope of this article)

Listing 1.3. Internal representation of $\langle \Psi | \mathcal{H} | \Phi \rangle + \int_a^b F(x) dx$

```

<XMAApp>
  <XMTok meaning=" plus" role="ADDOP">+</XMTok>
  <XMDual>
    <XMAApp>
      <XMTok meaning=" quantum-operator-product" />
      <XMRef idref="m2.5" />
      <XMRef idref="m2.6" />
      <XMRef idref="m2.7" />
    </XMAApp>
    <XMWrap>
      <XMTok role="OPEN"></XMTok>
      <XMTok role="ID" xml:id="m2.5"> $\Psi$ </XMTok>
      <XMTok role="CLOSE" stretchy="true">|</XMTok>
      <XMTok role="ID" xml:id="m2.6" font="caligraphic"> $\mathcal{H}$ </XMTok>
      <XMTok role="OPEN" stretchy="true">|</XMTok>
      <XMTok role="ID" xml:id="m2.7"> $\Phi$ </XMTok>
      <XMTok role="CLOSE"></XMTok>
    </XMWrap>
  </XMDual>
  <XMDual>
    <XMAApp>
      <XMTok meaning=" hack-definite-integral" role="UNKNOWN" />
      <XMRef idref="m2.1" />
      <XMRef idref="m2.2" />
      <XMRef idref="m2.3" />
      <XMRef idref="m2.4" />
    </XMAApp>
  </XMDual>
  <XMAApp>
    <XMAApp>
      <XMTok role="SUPERSCRIPTOP" scriptpos="post2" />
      <XMAApp>
        <XMTok role="SUBSCRIPTOP" scriptpos="post2" />
        <XMTok mathstyle="display" meaning="integral" role="INTOP"> $\int$ </XMTok>
        <XMTok role="ID" xml:id="m2.1" font="italic">a</XMTok>
      </XMAApp>
      <XMTok role="ID" xml:id="m2.2" font="italic">b</XMTok>
    </XMAApp>
    <XMAApp>
      <XMTok meaning=" times" role="MULOP"> $\times$ </XMTok>
      <XMDual xml:id="m2.3">
        <XMAApp>
          <XMRef idref="m2.3.1" />
          <XMRef idref="m2.3.2" />
        </XMAApp>
        <XMAApp>
          <XMTok role="FUNCTION" xml:id="m2.3.1" font="italic">F</XMTok>
          <XMWrap>
            <XMTok role="OPEN" stretchy="false">(</XMTok>
            <XMTok role="UNKNOWN" xml:id="m2.3.2" font="italic">x</XMTok>
            <XMTok role="CLOSE" stretchy="false">)</XMTok>
          </XMWrap>
        </XMAApp>
      </XMDual>
    </XMAApp>
    <XMAApp>
      <XMTok meaning=" differential-d" role="DIFFOP" font="italic">d</XMTok>
      <XMTok role="UNKNOWN" xml:id="m2.4" font="italic">x</XMTok>
    </XMAApp>
  </XMAApp>
</XMDual>
</XMAApp>

```

3 Algorithm

The goal is to associate each of the generated target pMML (or cMML) nodes with some node(s) in the generated cMML (or pMML, respectively). We do this by ascribing to each generated node a *source* XMath node, not necessarily the *current* node, the one that directly generated the target node. Once this is done, the cross-referencing is easily established: the xref of a pMML (cMML) node is the cMML (pMML, respectively) node that was ascribed to the same source XMath node; if multiple nodes were ascribed to that source node, the first target node, in document order, is the sensible choice.

A key to deciding which XMath node to ascribe as the source is whether the node is visible to either or both branches. The common, simple, case is an XMath node, visible to both branches, that generates a token node in the target; in that case the current node is used as the source. Node visibility can be determined by an algorithm such as the marking part of mark-and-sweep garbage collection.

However, MATHML elements which are containers generally do not correspond to symbols, and ought to be associated with the nearest application (think `m:apply` or `m:mrow`)². In this case, the source should be the nearest ancestor XM-Dual of the current XMath node, which we'll call the *current container*.

Similar reasoning applies in the special case when a token symbol (non-container) is generated from an XMath token which is not visible to the opposite branch; it may simply be notational icing of some transfix, or it may be the only visible manifestation of what we'll call the *current operator*. The current operator is the top-most operator being applied within the current container. In the example, the angle brackets and vertical bars are the only visible manifestation of the **quantum-operator-product** operator.

In summary, the *source* node for a given *target* is

```
if target is a container
  if current container exists
    current container
  else
    current
else target is visible in both branches
  current
else if current container exists
  if current operator is hidden from presentation
    current operator
  else
    current container
else
  current
```

² Exceptions are `m:msqrt` or `m:menclase` where they tend to represent *both* the application of an operation and yet are the only visible manifestation of the operator! However, we also note that a common use of cross-linking in HTML is to turn them into href links; but HTML does not allow nested links!

Applying this method to the example from Listing 1.3 yields 1.4, where we can see, for example, that the angle brackets and vertical bars are associated with the `quantum-operator-product` operator while the various `m:bvar`, `m:lowlimit`, etc, are properly associated with the integral, *not* the integral operator.

4 Outlook

The Digital Library of Mathematical Functions³ had from the outset linkage from (most) symbols to their definitions. However this new approach to the problem provides a much cleaner implementation, and allowed the mechanism to be extended to less textual operators such as binomials, floor, 3-j symbols, etc.

Parallel markup must also be adapted to larger structures such as `eqnarray`, and AMS alignments with intertext containing multiple formula and/or document-level text markup. While the fundamental issue is the same — separating presentation and content forms — this seems to demand a distributed markup that separates the presentation and content forms into distinct math containers. `LATEXML` currently has an ad-hoc, but not entirely satisfactory solution for this, but we will experiment with adapting the methods described here. However, it remains to be seen whether cross-referencing across separate math containers can be made useful.

And, now that generating Content MathML is more fun, we must continue work towards generating *good* Content MathML. Ongoing work will attempt to establish appropriate OpenMath Content Dictionaries, probably in a FlexiFormal sense[?], improved math grammar, and exploring semantic analysis.

³ <http://dlmf.nist.gov/>

Listing 1.4. MATHML representation of $\langle \Psi | \mathcal{H} | \Phi \rangle + \int_a^b F(x) dx$

```

<math display="block" alttext="..." class="ltx_Math" id="m2">
  <semantics id="m2a">
    <mrow xref="m2.13.cmml" id="m2.13">
      <mrow xref="m2.9.cmml" id="m2.9">
        <mo xref="m2.8.cmml" id="m2.8">&LeftAngleBracket;</mo>
        <mi mathvariant="normal" xref="m2.5.cmml" id="m2.5">&Psi;</mi>
        <mo xref="m2.8.cmml" id="m2.8a" stretchy="true" fence="true">|</mo>
        <mi xref="m2.6.cmml" id="m2.6" class="ltx_font_mathcaligraphic">&HilbertSpace;</mi>
        <mo xref="m2.8.cmml" id="m2.8b" stretchy="true" fence="true">|</mo>
        <mi mathvariant="normal" xref="m2.7.cmml" id="m2.7">&Phi;</mi>
        <mo xref="m2.8.cmml" id="m2.8c">&RightAngleBracket;</mo>
      </mrow>
      <mo xref="m2.10.cmml" id="m2.10">+</mo>
      <mrow xref="m2.12.cmml" id="m2.12c">
        <msubsup xref="m2.12.cmml" id="m2.12">
          <mo xref="m2.11.cmml" id="m2.11" symmetric="true" largeop="true">&int;</mo>
          <mi xref="m2.1.cmml" id="m2.1">a</mi>
          <mi xref="m2.2.cmml" id="m2.2">b</mi>
        </msubsup>
        <mrow xref="m2.12.cmml" id="m2.12b">
          <mrow xref="m2.3.cmml" id="m2.3c">
            <mi xref="m2.3.1.cmml" id="m2.3.1">F</mi>
            <mo xref="m2.3.cmml" id="m2.3d">&ApplyFunction;</mo>
            <mrow xref="m2.3.cmml" id="m2.3b">
              <mo xref="m2.3.cmml" id="m2.3" stretchy="false">(</mo>
              <mi xref="m2.3.2.cmml" id="m2.3.2">x</mi>
              <mo xref="m2.3.cmml" id="m2.3a" stretchy="false">)</mo>
            </mrow>
          </mrow>
          <mo xref="m2.11.cmml" id="m2.11a">&InvisibleTimes;</mo>
          <mrow xref="m2.12.cmml" id="m2.12a">
            <mo xref="m2.11.cmml" id="m2.11b">d</mo>
            <mi xref="m2.4.cmml" id="m2.4">x</mi>
          </mrow>
        </mrow>
      </mrow>
    </mrow>
  </semantics>
  <annotation-xml id="m2b" encoding="MathML-Content">
    <apply xref="m2.13" id="m2.13.cmml">
      <plus xref="m2.10" id="m2.10.cmml" />
      <apply xref="m2.9" id="m2.9.cmml">
        <csymbol xref="m2.8" id="m2.8.cmml" cd="latexml">quantum-operator-product</csymbol>
        <ci xref="m2.5" id="m2.5.cmml">normal-&Psi;</ci>
        <ci xref="m2.6" id="m2.6.cmml">&HilbertSpace;</ci>
        <ci xref="m2.7" id="m2.7.cmml">normal-&Phi;</ci>
      </apply>
      <apply xref="m2.12c" id="m2.12.cmml">
        <int xref="m2.11" id="m2.11.cmml" />
        <bvar xref="m2.12c" id="m2.12a.cmml">
          <ci xref="m2.4" id="m2.4.cmml">x</ci>
        </bvar>
        <lowlimit xref="m2.12c" id="m2.12b.cmml">
          <ci xref="m2.1" id="m2.1.cmml">a</ci>
        </lowlimit>
        <lowupper xref="m2.12c" id="m2.12c.cmml">
          <ci xref="m2.2" id="m2.2.cmml">b</ci>
        </lowupper>
        <apply xref="m2.3c" id="m2.3.cmml">
          <ci xref="m2.3.1" id="m2.3.1.cmml">F</ci>
          <ci xref="m2.3.2" id="m2.3.2.cmml">x</ci>
        </apply>
      </apply>
    </annotation-xml>
  <annotation id="m2c" encoding="application/x-tex">...</annotation>
</math>

```
