

Recent Harmony Search Algorithms for 0–1 Optimization Problems

Broderick Crawford^{1,2,3(✉)}, Ricardo Soto^{1,4,5}, Néstor Guzmán¹,
Franklin Johnson^{1,6}, and Fernando Paredes⁷

¹ Pontificia Universidad Católica de Valparaíso, Valparaíso, Chile
`{broderick.crawford,ricardo.soto}@ucv.cl, nestor.guzman@live.cl`

² Universidad Central de Chile, Santiago, Chile

³ Universidad San Sebastián, Santiago, Chile

⁴ Universidad Autónoma de Chile, Santiago, Chile

⁵ Universidad Científica del Sur, Lima, Peru

⁶ Universidad de Playa Ancha, Valparaíso, Chile

`franklin.johnson@upla.cl`

⁷ Escuela de Ingeniería Industrial, Universidad Diego Portales, Santiago, Chile
`fernando.paredes@udp.cl`

Abstract. The Set Covering Problem (SCP) has long been concentrating the interest of many researchers in the field of Combinatorial Optimization. SCP is a 0–1 integer programming problem that consists in finding a set of solutions which allow to cover a set of needs at the lowest cost possible. There are many applications of these kind of problems, the main ones are: location of services, files selection in a data bank, simplification of boolean expressions, balancing production lines, among others. Different metaheuristics have been proposed to solve it. Here, we present the possibilities to solve Set Covering Problems with Harmony Search.

Keywords: Set covering problem · Metaheuristics · Harmony search algorithm

1 Introduction

The Set Covering Problem (SCP) [9, 10, 19] is a classic problem that consists in finding a set of solutions which allow to cover a set of needs at the lowest cost possible. There are many applications of these kind of problems, the main ones are: location of services, files selection in a data bank, simplification of boolean expressions, balancing production lines, among others.

In the field of Metaheuristics, many algorithms have been developed to solve the SCP. Examples of these optimization algorithms include: Genetic Algorithm (GA) [22, 26], Ant Colony Optimization (ACO) [23], Particle Swarm Optimization (PSO) [2, 11], Artificial Bee Colony (ABC) [8, 9, 13], Cultural Algorithms [10] and 2-level Metaheuristics [12, 27].

Here, we present the possibilities to solve Set Covering Problems with Harmony Search (HS). The HS algorithm is a relatively new population-based metaheuristic optimization algorithm. It imitates the music improvisation process where musicians improvise toning their instruments by searching for a perfect harmony. Since the emergence of this algorithm in 2001, it attracted many researchers from various fields especially those working on solving hard optimization problems.

More specifically, our interest is to solve the column-based representation of the SCP (its binary representation) and to solve binary optimization problems, several instances of Harmony Search have been developed recently [21, 29–31].

2 The Set Covering Problem

The SCP [6] can be formally defined as follows. Let $A = (a_{ij})$ be an m -row, n -column, zero-one matrix. We say that a column j can cover a row if $a_{ij} = 1$. Each column j is associated with a nonnegative real cost c_j . Let $I = \{1, \dots, m\}$ and $J = \{1, \dots, n\}$ be the row set and column set, respectively. The SCP calls for a minimum cost subset $S \subseteq J$, such that each row $i \in I$ is covered by at least one column $j \in S$. A mathematical model for the SCP is

$$v(\text{SCP}) = \min \sum_{j \in J} c_j x_j \quad (1)$$

subject to

$$\sum_{j \in J} a_{ij} x_j \geq 1, \quad \forall i \in I, \quad (2)$$

$$x_j \in \{0, 1\}, \forall j \in J \quad (3)$$

The objective is to minimize the sum of the costs of the selected columns, where $x_j = 1$ if column j is in the solution, 0 otherwise. The restrictions ensure that each row i is covered by at least one column.

The SCP has been applied to many real world problems such as crew scheduling [1], location of emergency facilities [24], production planning in industry [28], vehicle routing [3], ship scheduling [15], network attack or defence [4], assembly line balancing [16], traffic assignment in satellite communication systems [25], simplifying boolean expressions [5], the calculation of bounds in integer programs [7], information retrieval [14], political districting [17], stock cutting, crew scheduling problems in airlines [20] and other important real life situations.

3 The Harmony Search algorithm

Harmony search [18] is a relatively new population-based metaheuristic optimization algorithm, that imitates the music improvisation process where the

musicians improvise toning their instruments by searching for a perfect state of harmony. It was able to attract many researchers to develop HS-based solutions for many optimization problems. HS imitates the behavior of musicians when they cooperate the pitches of their instruments together to obtain a fantastic harmony as measured by aesthetic standards.

HS is a very successful metaheuristic algorithm that can explore the solutions of a given problem, where each solution (harmony) is generated exploring and exploiting intelligently the search space. HS performs optimization analogizing the improvisation process of musicians:

- Each musician corresponds to each decision variable.
- Musical instruments pitch range corresponds to the decision variables value range.
- Musical harmony at a certain time corresponds to a solution at a certain iteration.
- Audiences aesthetics corresponds to the objective function.

HS possesses several advantageous characteristics:

- The generation of a new solution after considering all the existing solutions, rather than considering only some solutions as in other metaheuristics.
- The independent consideration of each decision variable in a solution.
- It does not require any starting values of the decision variables nor does it require complex derivatives as in gradient-based methods.

Other important strengths of HS are their improvisation operators, memory consideration, pitch adjustment and random consideration. All above achieving the desired balance between the two major forces for any metaheuristic: exploitation and exploration.

Besides the good features offered by the original HS algorithm, recently have emerged new HS variants of our special interest to address binary problems [21, 29–31]. Following we describe briefly the Novel Global Harmony Search [31] in our attempt to find new ways of solving the SCP.

3.1 A Novel Global Harmony Search

Inspired by the swarm intelligence of particle swarm, in [31] a Novel Global Harmony Search algorithm (NGHS) is proposed to solve 0–1 knapsack problems. It is unsurprising that the HS algorithms can be used to solve 0–1 problems. However, the HS needs be modified for solving some difficulty binary problems. We intend to rescue these new capabilities to attack the column-based SCP.

The NGHS and the HS are different in the following:

- HS parameters Harmony Memory Considering Rate (HMCR) and Pitch Adjusting Rate (PAR) are excluded from the NGHS, and genetic mutation probability P_m is included in the NGHS.

- The NGHS modifies the Improvisation Step of the HS as follows:

Algorithm 1. *NGHS()*

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1: for ( $i = 1$  to  $n$ ) do
2:    $step_i = |x_i^{best} - x_i^{worst}|$  % calculation of the adaptive step
3:    $x_i^{new} = x_i^{best} \pm \bigcup(0, 1) step_i$  % position updating
4:   if ( $\bigcup(0, 1) \leq P_m$ ) then
5:      $x_i^{new} = x_{iL} + \bigcup(0, 1) (x_{iU} - x_{iL})$  % genetic mutation
6:   end if
7: end for

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Where x_i^{best} and x_i^{worst} denote respectively the best and worst global harmony. $\bigcup(0, 1)$ represents uniformly generated random numbers in $[0, 1]$. Dynamically adjusted $step_i$ keeps a balance between the global search and the local search. Genetic mutation operation with a small probability is carried out for the worst harmony of harmony memory after updating position, preventing the premature convergence.

- After improvisation, the NGHS replaces the worst harmony x^{worst} in HM with the new harmony x_i^{new} even if x_i^{new} is worse than x^{worst} .

4 Conclusion

In this paper, we turn the attention in the Harmony Search algorithms, they have several characteristics that make it a good alternative to tackle the Set Covering Problem, exploring the search space in both intensification and diversification in order to provide a near-optimal solution within a reasonable time.

Some modification and improvements to HS have been implemented in various domains. In order to find satisfactory solutions for the 0–1 optimization problems, several variants of HS have been developed. One of the most promising is the Novel Global Harmony Search Algorithm [31]. Given the good capabilities of NGHS to solve binary problems, it is considered feasible and motivating to devote efforts in its implementation to solve SCP problems.

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