

Optimization of Electricity Markets Participation with Simulated Annealing

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Abstract. The electricity markets environment has changed completely with the introduction of renewable energy sources in the energy distribution systems. With such alterations, preventing the system from collapsing required the development of tools to avoid system failure. In this new market environment competitiveness increases, new and different power producers have emerged, each of them with different characteristics, although some are shared for all of them, such as the unpredictability. In order to battle the unpredictability, the power supplies of this nature are supported by techniques of artificial intelligence that enables them crucial information for participation in the energy markets. In electricity markets any player aims to get the best profit, but is necessary have knowledge of the future with a degree of confidence leading to possible build successful actions. With optimization techniques based on artificial intelligence it is possible to achieve results in considerable time so that producers are able to optimize their profits from the sale of Electricity. Nowadays, there are many optimization problems where there are no that cannot be solved with exact methods, or where deterministic methods are computationally too complex to implement. Heuristic optimization methods have, thus, become a promising solution. In this paper, a simulated annealing based approach is used to solve the portfolio optimization problem for multiple electricity markets participation. A case study based on real electricity markets data is presented, and the results using the proposed approach are compared to those achieved by a previous implementation using particle swarm optimization.

Keywords: artificial intelligence, electricity markets, portfolio optimization, simulated annealing.

1 Introduction

The electric sector has undergone several changes, which caused an increase in competitiveness. These changes are due to the new imposed rules and to the physical limitations, which led to emergence of financial issues [1], [2]. Electricity market participants, mainly sellers and buyers have the need for effective methods that support their actions; the system itself also requires methods to assure the functioning of

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markets [3]. One of the main causes for the changes in electricity markets is the massive integration of renewable energy sources, which has very particular characteristics: intermittence in the production and distributed nature. In this context we can highlight mainly wind power and solar energy. These hold a great influence on how the management of the electricity network is made and but also in how electricity is traded.

One of the most accepted solutions to deal with the introduction of distributed renewable energy sources is the emergence of the concept of Smart Grid [4], which in recent times has evolved from a concept to a visible reality. Smart Grid are small sub-systems capable of maintaining operating independently of each other and together form a working system. The implementation of Smart Grids has been increasing worldwide, as result from the large distributed generation incorporated in the network [5]. With all these changes market, participants are concerned with the forecasting of the behavior of markets, as this knowledge can anticipate and enable them achieving the best results from trading.

Multi-agent simulators have emerged as suitable tools to support players' decision in energy markets. Multi-agent simulation allows modeling different entities, such as independent agents, with specific objectives and characteristics. It also facilitates the expansion of the used models and the integration of new models. MASCEM (Multi-Agent Simulator of Competitive Electricity Markets) [6] is an agent base simulator of electricity markets, which is integrated with AiD-EM (Adaptive Learning Strategic Bidding System), a decision support system that aims at providing market players with appropriate suggestions on what actions should be performed in every time and in different negotiating contexts [7].

Despite all the advances in the electricity markets field, the ability to learn to adapt to new situations and make the best possible outcomes for electricity market players are still far from being achieved. A less explored area is the option of multiple markets participation, which can be optimized to give players greater profitability in their market operations. This work proposes a portfolio optimization model for multiple markets participation. This model offers the possibility to buy and sell electricity in the same period in different markets. A Simulated Annealing approach is proposed to solve the optimization problem, and the achieved results are compared to those using a previous implementation with Particle Swarm Optimization [8].

After this introductory section, section 2 presents the mathematical formulation of the portfolio optimization problem, and section 3 describes the proposed Simulated Annealing approach. Section 4 presents the achieved results using real electricity market data from the Iberian electricity market operator – MIBEL [9]. Finally, section 5 presents the most relevant conclusions of this work.

2 Portfolio optimization for multiple electricity markets participation

2.1 Portfolio optimization

The first recognized work in the portfolio optimization area has been published the first work by Markowitz [10]. The addressed problem was a multiobjective portfolio

optimization that considered: maximizing the profit and minimizing the risk. The work of Markowitz enables finding the balance between the fulfillment of two goals.

The problem addressed by this paper considers a real-time approach, which differs from that presented by Markowitz. With this methodology we intend to support the decision of players on the negotiation of Electricity. For this different scenarios are presented to the player so that it can analyze and make its decisions. With this approach it is also possible to purchase and sale power in the same period in different markets, as introduced in [8], thus building on the Markowitz approach, which does not support such feature. With this, the negotiation methodology adapts itself to the so-called spot market as it no longer considers buyers and sellers as independent players, rather seeing them as players (able to perform both actions).

With the support of these tools it is possible to enable players changing their negotiation profiles (possibility of participating in different types of markets and negotiating different amounts of electricity). The optimization considers real data obtained from different European markets. However, it also enables expanding the optimization to other horizons, making use of several available Electricity market prices forecast and estimation tools [11], [12] and [13].

The optimization process required forecasts of the expected Electricity prices for each period. The work presented in [12] proposes a market prices forecast methodology, which is provided through the use of a neural network, which was used for the same purpose in this work. The participation of the player in different markets is possible, where each market has different rules of trading. For example in bilateral contracts and the smart grids market the negotiated amount may interfere with the asking price, so the price of Electricity depends on the negotiated amount.

One way to try to estimate the variable price of energy is to use a function that calculates the price of electricity in view of the possible amount of electricity traded. The work published in [11] presents an electricity price estimation methodology using fuzzy logic techniques. This paper proposed the application of clustering to split the price profile / quantity. Using these clusters, fuzzy logic is used to create a function for each created interval.

2.2 Mathematical formulation

The formulation presented in (1) is used to represent the optimization problem, as proposed in [8]. In (3) d represents the weekday, $Nday$ represent the number of days, p represents the negotiation period, $Nper$ represent the number of negotiation periods, $Asell_M$ and $Abuy_S$ are boolean variables, indicating if this player can enter in negotiation in each market type, M represents the referred market, $NumM$ represents the number of markets, S represents a session of the balancing market, and $NumS$ represents the number of sessions. Variables $ps_{M,d,p}$ and $ps_{S,d,p}$ represent the expected (forecasted) prices of selling and buying electricity in each session of each market type, in each period of each day. The outputs are $Spow_M$ representing the amount of power to sell in market M and $Bpow_S$ representing the amount of power to buy in session S .

$$f(Spow_{M...NumS}, Bpow_{S1...NumS})$$

$$= Max \left[\sum_{M=M1}^{NumM} (Spow_{M,d,p} \times ps_{M,d,p} \times Asell_M) - \sum_{S=S1}^{NumS} (Bpow_S \times ps_{S,d,p} \times Abuy_S) \right] \quad (1)$$

$$\forall d \in Nday, \forall p \in Nper, Asell_M \in \{0,1\}, Abuy \in \{0,1\}$$

$$ps_{M,d,p} = Value(d, p, Spow_M, M)$$

$$ps_{S,d,p} = Value(d, p, Bpow_S, S)$$

The formulation considers the expected production of a market player for each period of each day. As explained in section 2.1, the price value of electricity in some markets depends on the power amount to trade. With the application of a clustering mechanism it is possible to apply a fuzzy approach to estimate the expected prices depending on the negotiated amount. Equation (2) defines this condition.

$$Value(day, per, Pow, Market) = Data(fuzzy(pow), day, per, Market) \quad (2)$$

Equation (3) represents the main constraint to be applied in this type of problems, and imposes that the total power that can be sold in the set of all markets is never higher than the total expected production (TEP) of the player, plus the total of purchased power [8]. Further constraints depend on the nature of the problem itself, e.g. type of each market, negotiation amount, type of supported player (renewable based generation, cogeneration, etc.).

$$\sum_{M=M1}^{NumM} Spow_M \leq TEP + \sum_{S=S1}^{NumS} Bpow_S \quad (3)$$

3 Proposed simulated annealing approach

This paper proposes a simulated annealing algorithm to solve the electricity market participation portfolio optimization problem defined in section 2. More specifically, the objective is to allocate in an optimal way the resources that provide the best profits for the player in selling its available power in the market. This type of meta-heuristic methods have the particularity of being not accurate, which means that the exact best global solution is hardly achieved.

3.1 SA methodology

Simulated annealing is an optimization method that imitates the annealing process used in metallurgic. The final properties of this substance depend strongly on the

cooling schedule applied, i.e. if it cools down quickly the resulting substance will be easily broken due to an imperfect structure, if it cools down slowly the resulting structure will be well organized and strong. When solving an optimization problem using simulated annealing the structure of the substance represents a codified solution of the problem, and the temperature is used to determine how and when new solutions are perturbed and accepted. The algorithm is basically a three steps process: perturb the solution, evaluate the quality of the solution, and accept the solution if it is better than the new one [14]. Fig. 1 shows the flowchart of the simulated annealing meta-heuristic.

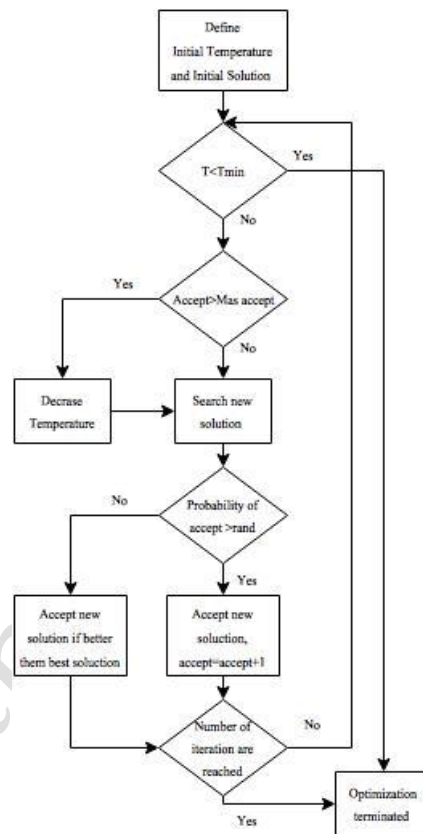


Fig. 1 – SA flowchart

The temperature minimum, the acceptance maximum value and the maximum number of iterations are parameters defined by user. As shown in the diagram of Fig. 1 the algorithm requires an initial solution to start. This is defined through a set of random numbers. When the searching process begins, the search does not stop until the stopping criteria are met. The considered stopping criteria are: the current temperature and the maximum number of iterations. As can be seen by Fig. 1, if the current temperature is minor than the minimum temperature the algorithm stops its search, similarly to what happens if the number of iterations exceeds the maximum number. Simulated annealing is known for two particular factors of this algorithm, namely the decrease of the temperature and the probability of acceptance. As shown by the diagram of by Fig. 1,

the temperature only decreases when the number of acceptances is greater than a stipulated maximum. This acceptance number is only incremented when the probability of acceptance is higher than a random number, which allows some solutions to be accepted even if their quality is lower than the previous. When the condition of acceptance is not satisfied, the solution is compared to the previous one, and if it is better, the best solution is updated.

Each iteration is necessary to seek a new solution, this solution is calculated according to the equation (4).

$$\text{new solution} = \text{solution} + S \times N(0,1) \quad (4)$$

solution in equation (1) refers to the previous solution, because this may not be the best found so far. $N(0,1)$ is a random number with a normal distribution, the variable S is obtained through equation (5).

$$S = 0.01 \times (\text{upbound} - \text{lwbound}) \quad (5)$$

The *upbound* and *lwbound* are the limits of each variable, which prevent from getting out of the limits of the search problem.

3.2 SA parameters

The decisive parameters in SA's research are: the decrease of temperature and the likelihood of acceptance. Considering this, 4 different variants of the simulated annealing algorithm have been implemented in this work, combining different approaches for calculating these two components. It is expected that this will bring different results for different groups, as these components introduce a strong randomness in SA, which makes them reflect in the final results.

Table 1 – Different temperature decreasing and probability of acceptance calculation methods

Group	Temperature decreasing	Probability of acceptance	Ref.
1	$T_i = T_{i-1} \times \alpha$	$P = (2\pi T)^{-\frac{D}{2}} e^{\left(\frac{-\Delta x}{K \times T}\right)}$	[15]
2	$T_i = \frac{T_0}{i}$	$P = \frac{T_0}{(\Delta x^2 + T^2)^{\frac{(D+1)}{2}}}$	[15]
3	$T_i = T_0 e^{-ci^{\frac{1}{D}}}$	$P = \prod_{d=1}^D \frac{1}{2(y_d + Ti) \ln\left(1 + \frac{1}{T_i}\right)}$	[15]
4	$T_i = T_0 \times \alpha^i$	$T_i = \frac{1}{1 + e^{\frac{\Delta x}{T_{max}}}}$	[16]

Where:

- $\alpha = 0.95$;
- i is the current iteration;
- $\Delta x = y(x^{max} - x^i)$ is the difference between best solution and current solution;

- $K = 1$ is the Boltzmann constant ;
- $T_0 = 1$ is the initial temperature;
- D is the number of variables;
- $c = 0.1$;
- $|y_d|$ is the abs of solution current;
- $T_{min} = 1 \times 10^{-10}$;
- $acceptance_{max} = 15$.

Besides from these two main parameters, there are issues that may affect the searching process of this algorithm, taking into account that the process of disturbing the solution can determine the search. When the temperature value is high, the search can easily scroll through the search space and leaving important points without being explored. For example, if the initial temperature is too high the search will fall to a point near the ideal very rapidly. However, it is also very likely that the search process could skip this point to points where the solution is worse than the previous. Then the application of too much disturbance is useless and should be avoided [14].

Another important factor is the decrease of temperature. At high temperatures, the simulated annealing method searches for the global optimum in a wide region; on the contrary, when the temperature decreases the method reduces the search area. This is done to try to refine the solution found in high temperatures. This is a good quality that makes the simulated annealing a good approach for problems with multiple local optima. Simulated annealing, thereby, does not easily converge to solutions near the global optimum; instead this algorithm seeks a wide area always trying to optimize the solution. Thus, it is important to note that the temperature should come down slowly allowing the search method to pass through a large part of the search space [14].

4 Case study

This section presents a case study that illustrates the application of the proposed methodology. The market price forecasts are performed using an artificial neural network (ANN) [12], which is trained using the historic log of electricity market prices from the Iberian Market – MIBEL; further details about this market can be consulted in [9]. With the use of MIBEL data, simulations become realistic because data are taken from a real environment, which makes results reliable. Four different markets based on MIBEL are considered: day-ahead spot market, bilateral contracts, a Smart Grid (SG) market, and the balancing market, with two negotiation sessions, which in the total makes it possible to carry out negotiations in five market sessions. In the spot and balancing markets, the expected prices are forecasted using the ANN, while the expected price in bilateral contracts and SG market are adjusted using the fuzzy logic estimative presented in [11].

Simulations are undertaken concerning 1 day with 24 hourly negotiation periods. The TEP value is 10 MW. Additionally, the supported player can buy up to 10 MW in each market where purchase is allowed to a seller. In the balancing market sessions each player is only able to do one action (buy or sell) in each period. The optimization using the proposed simulated annealing (SA) approach is executed 1000 times, which

can ultimately result in 1000 different optimization results, depending on the random variables. Table 2 and Table 3 present the optimization outputs (respectively purchases and sales of electricity) for the first period of the considered simulation day. These results concern the simulation that has registered the highest objective function value using each of the groups presented in Table 1.

Table 2 - Sales scheduling in the different markets

SA Variation	Sales (MW)				
	Spot	Bilateral	Balancing 1	Balancing 2	Smart Grid
SA Group 1	20,08	11,5168	0	0	8,5464822
SA Group 2	19,24	11,7707	0	0	8,0711197
SA Group 3	19,03	11,6413	0	0	9,0258825
SA Group 4	20,29	11,5001	0	0	8,6843312

As shown by Table 2, which shows the sales made in the different markets, the four implemented variants present very similar results. In this case the balancing sessions assume values of zero because as shown in Table 3, these markets are used to purchase electricity. Table 3 shows the electricity purchase in the various markets.

Table 3 - Sales scheduling in the different markets

SA Variation	Purchases (MW)				
	Spot	Bilateral	Balancing 1	Balancing 2	Smart Grid
SA Group 1	0	4,8456	10	10	5,298138
SA Group 2	0	4,59959	10	10	4,4853613
SA Group 3	0	4,78831	10	10	4,9050412
SA Group 4	0	4,99057	10	10	5,4875652

From Table 3 one can see the results recorded for electricity purchases. As can be observed, since the spot market has been used to sell electricity, it cannot be used to purchase as well, according to the restriction defined in the model. All four SA groups also show very similar results regarding the electricity purchases. Table 4 presents the comparison between the objective function results of the group variants implemented in SA and the results of a previous implementation based on a particle swarm optimization (PSO) approach [8]. The minimum, maximum and mean results are shown, as well as the standard deviation (STD) registered in the 1000 simulations. Additionally, the average execution time of each method variation is also displayed.

Table 4 shows that SA Groups 1, 2 and 3 present very similar objective function results and execution time as well. SA Group 4, on the other hand, presents worse objective function results, but in a much faster execution time (3 times faster than the other SA approaches, and 6 times faster than PSO). SA Groups 1, 2 and 3 also present a higher mean value of objective function, which is around 5% higher than PSO. This is also reflected on the much higher minimum achieved value that SA Groups 1, 2 and 3 are able to achieve when compared to PSO (almost doubling the value of PSO), and also on the STD, which is three times lower. PSO is, however, the algorithm that

records the highest objective function value, with a value about 3 % higher than that achieved by SA Groups 1, 2 and 3. This very small difference is largely compensated by the great gain in execution time and reliability. Fig. 2 expands the explanation on this question.

Table 4 – Objective function results of the proposed SA approach, compared to the PSO

Algorithm	Objective value (€)				Time (seconds)
	Minimum	Mean	Maximum	STD	
PSO	935,0451386	1802,21	2000,6456	160,423489	1,024635318
SA Group 1	1781,480543	1884,04	1927,2421	55,5014797	0,51910964
SA Group 2	1782,445013	1882,49	1933,5664	56,4381753	0,507367551
SA Group 3	1782,519507	1883,28	1930,1467	56,0204514	0,508814344
SA Group 4	980,9189744	1616,23	1925,3661	203,592965	0,174417698

Fig. 2 shows a Box Plot for the implemented algorithms. With this representation it is intended to give the information on which of the algorithms is positioned in the best cost benefit ratio. These plots are built at the expense of five parameters of which three (median, 1st and 3rd quartile) are calculated on the results of the simulations and the other two (maximum and minimum) derive from a simple observation data. With this graph we get insightful information on how the data are distributed to as: greater or lesser concentration, symmetry and the existence of outliers. In the Box Plot, the analysis is done taking into account the length of the line joining the minimum point to the maximum and the size of the box. The median value gives skew indications of the data.

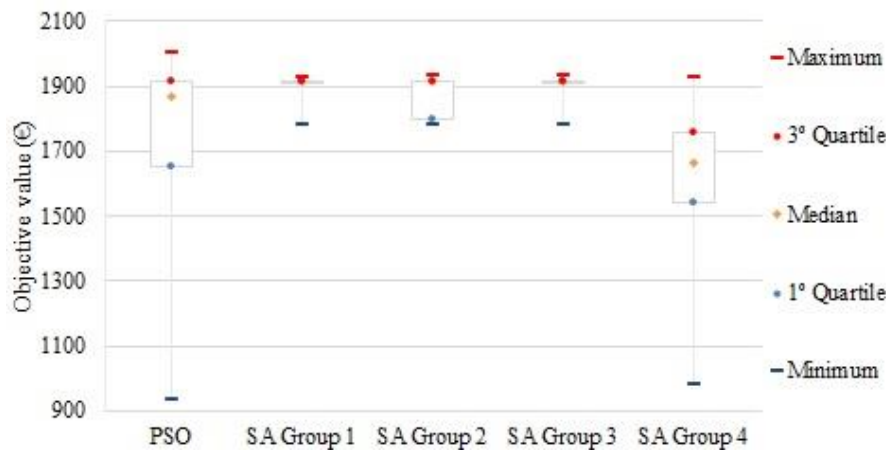


Fig. 2 - Box plots for the different methods

As presented in Fig. 2, the range from the minimum value to the value of the 1st quartile represents 25% of the data. Similarly, from the value of the 3rd quartile to the maximum value are also represented another 25% of the data. Amidst the values of the 1st quartile and 3rd quartile are represented 50% of the data. As can be seen from the

data, the results from SA groups 1, 2 and 3 are much more concentrated than the other algorithms, which means that they are more reliable. Although the figures provided by PSO are not as concentrated as the result of SA, this approach cannot be ruled out because this is the algorithm that presents the highest value of objective function, which represents the possibility of achieving the highest profit.

Fig. 3 shows the 95% confidence interval for the results of SA groups 1, 2 and 3.

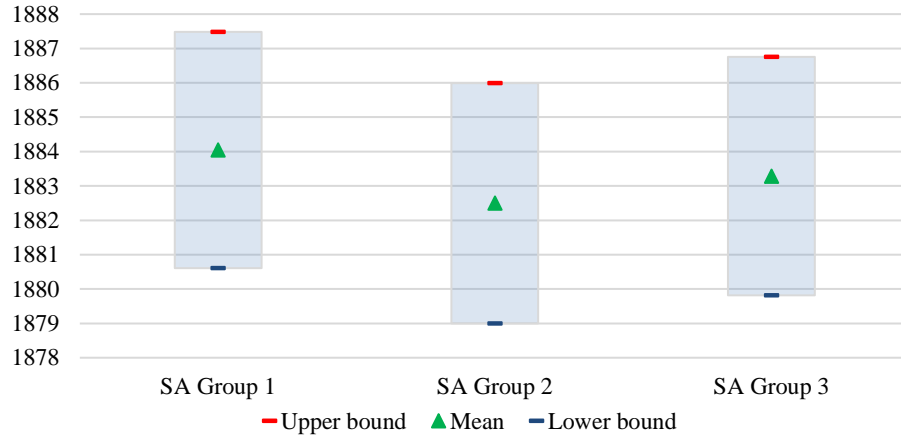


Fig. 3 - Confidence interval of SA Groups 1, 2 and 3

As can be seen by Fig. 3, the confidence intervals of the three SA groups have similar amplitudes. In this case, by performing the analysis of the figure and applying the theory of confidence intervals, there is a 95% chance of a simulation result being between the minimum and maximum with a certain error, in this case SA Group 1 shows a 3.4382 error, SA Group 2 shows 3.4963, and SA Group 3 presents a 3.4704 error; this error can also be called tolerance. Although the presented results regard the first period of the simulation day, the 1000 samples in the other periods for each algorithm keeps a very similar performance.

Fig. 4 shows the convergence performance of the four SA groups.

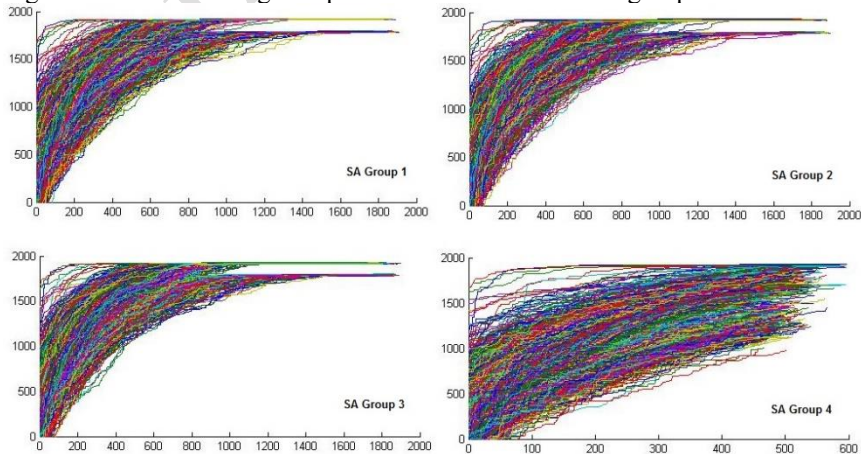


Fig. 4 – Convergence performance of the SA algorithms

Each line in Fig. 4 represents the convergence of each simulation in each of the 1000 simulations. The evolution of the objective function (€) (yy axis) is represented along the iterations (xx axis). In Fig. 4 it is visible that, as seen before, SA Group 4 is the approach that shows the worst results. One important fact is that in SA Groups 1, 2 and 3, in the final part of the convergence process, results are concentrated in two lines, as it is possible to see from the respective graphs of the figure. This strongly indicates the possibility of the existence of a local optimum, in this case a local maximum. The proposed SA approaches, as it is possible to note, have proven to be able to work around this situation and present the best solution.

5 Conclusions

This paper presented a SA approach to solve the portfolio optimization problem, for multiple electricity markets participation. The proposed approach is composed by four different groups regarding the calculation of the most important variables required by SA algorithms. The proposed SA approach has been compared to a previous implementation of a PSO based approach.

Similarly the PSO, SA also has been able to solve the problem of portfolios optimization in the electricity markets, as it was possible to observe the results. By comparing the results of the proposed SA approach with the previous PSO implementation, it is demonstrated that the SA results presented more homogenous results than the PSO, although the highest objective function result was found by PSO. SA has also shown much lower execution times, which, together with the much larger credibility of the SA, as shown by the analysis of the staggering of sales and purchases, supports the conclusion that the SA methods are more reliable, and safer to being used in real cases. The proposed methodology is intended to be used to generate scenarios so that market users can use them in order to maximize their results from negotiating in the market. It should be noted that the results presented are only a period, but the methodology is prepared to be extended to other periods as well as other markets.

As future work other algorithms will also be used to solve this problem, so that results can be compared, such as genetic algorithms and other variants of PSO. A methodology that can measure the risk through the prediction error of electricity prices will also be formulated and integrated in the current approach. As here shows the results for a period of one hour, you can choose other periods where the scenario is completely different, because in electricity markets, and especially in the spot market, there is a lot of volatility in electricity prices. This means that totally different scenarios can be found, which should be studied in order to show the adaptability of algorithms.

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