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Hesitant Fuzzy Methods for Multiple Criteria Decision Analysis

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Preface

Multiple criteria decision making (MCDM) is a process to make an optimal choice that has the highest degree of satisfaction from a set of alternatives that are characterized in terms of multiple conflicting criteria, which is a usual task in human activities. In the classical MCDM, both the criteria values and the weights of criteria are usually expressed by crisp (no-fuzzy) numbers. To address the classical MCDM problems, lots of prominent decision-making methods have been developed, such as the TOPSIS (*Technique for Order Preference by Similarity to Ideal Solution*) method (Hwang and Yoon 1981), the TODIM (*an acronym in Portuguese of interactive and multi-criteria decision making*) method (Gomes and Lima 1991, 1992), the LINMAP (*linear programming technique for multidimensional analysis of preferences*) method (Srinivasan and Shocker 1973), the QUALIFLEX (*qualitative flexible multiple criteria method*) method (Paelinck 1976, 1977, 1978), and the ELECTRE (*ELimination Et Choix Traduisant la REalité*) method (Roy 1968). The TOPSIS method is a kind of simple and useful decision-making method and is suitable to handle the classical MCDM problems in which the weights of criteria are completely known in advance. The basic idea of TOPSIS method is that the optimal alternative should have the shortest distance from the positive ideal solution (PIS) and have the farthest distance from the negative ideal solution (NIS), simultaneously. The TODIM method is a discrete multiple criteria decision analysis method based on prospect theory (Kahneman and Tversky 1979) and has been proven to be a valuable tool for solving the classical MCDM problems in case of considering the decision maker's psychological behavior. The QUALIFLEX method is one of outranking methods and is suitable to deal with the decision-making problems where the number of criteria markedly exceeds the number of alternatives. The most characteristic of the QUALIFLEX method is the correct treatment of cardinal and ordinal information (Rebai et al. 2006). The LINMAP method is a practical and useful approach to address the classical MCDM problems in which the weights of criteria and the PIS are unknown in advance. In the LINMAP approach, the decision maker is required to not only provide the ratings of alternatives with respect to each criterion, but also simultaneously give the incomplete preference relations on pair-wise comparisons of alternatives.

Because of the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, the criteria values and/or weights of criteria involved in the MCDM problems are not always expressed in crisp numbers, but some are more suitable to be denoted by fuzzy numbers and their extensions, such as interval numbers, triangular fuzzy numbers (TFNs), trapezoidal fuzzy numbers (TrFNs), linguistic variables, intuitionistic fuzzy numbers (IFNs), and hesitant fuzzy elements (HFEs). This book mainly investigates the MCDM or multiple criteria group decision-making (MCGDM) problems in which the criteria values and/or the weights of criteria are expressed as HFEs (Xia and Xu 2011). Hesitant fuzzy set (HFS) was originally introduced by Torra (2010) as an extension of fuzzy set (Zadeh 1975). The HFE is the basic element of HFS, which allows the membership of an element to a given set to be represented by a few different values, and is a useful tool to describe and deal with uncertain information in the process of MCDM, especially for the practical MCDM problems in the case of considering the decision maker's hesitation. For example, the individual decision maker may have hesitancy among 0.1, 0.3, and 0.4 when he/she provides the assessment of an alternative with a criterion. Then, this decision maker can employ the HFE $H\{0.1,0.3,0.4\}$ to express his/her assessment provoked by hesitation. In this case, the HFE is used to express the individual decision maker's hesitation. On the other hand, Xu and Xia (2011) gave an example to illustrate how to employ the HFE to express the decision organization's hesitation: A decision organization including several experts is authorized to estimate the degree to which an alternative should satisfy a criterion. Suppose that there are three cases: Some experts provide 0.3, some provide 0.5, and the others provide 0.6, and these three parts cannot persuade each other. Thus, the degree that the alternative satisfies the criterion can be expressed by a HFE $H\{0.3,0.5,0.6\}$. It is noted that the HFE $H\{0.3,0.5,0.6\}$ can describe the above situation more objectively than the IFN $I(0.3,0.4)$ or the interval number [0.3,0.6], because the degree to which the alternative satisfies the criterion is not the convex combination of 0.3 and 0.4, or the interval between 0.3 and 0.6, but just three possible values (Xu and Xia 2011). Obviously, the use of hesitant fuzzy assessments makes the decision makers or the experts' judgments more reliable and informative in decision-making process. The HFEs have also been successfully applied in the fields of decision making (Xia and Xu 2011; Farhadinia 2013; Rodríguez et al. 2014; Zhu and Xu 2014) and clustering analysis (Zhang and Xu 2015c, 2015d), etc.

How to make a scientific decision with hesitant fuzzy information is an interesting and important research topic. In real-life world, one may encounter the hesitant fuzzy decision-making problems with different situations, such as the cases that the weights of criteria and/or experts are completely unknown or known in advance, the cases that the PIS and NIS are known or unknown in advance, the case that the decision maker's psychological behavior should be taken into account, and the case that the number of criteria markedly exceeds the number of alternatives, and the hesitant fuzzy decision-making problems under different situations need different decision-making methods to solve. For this purpose, the authors have recently developed lots of distinct hesitant fuzzy decision-making methods to deal with different MCDM or MCGDM problems effectively, in which the decision data are expressed by hesitant fuzzy information. For example, Xu and Zhang (2013) developed hesitant fuzzy TOPSIS decision analysis method to address the hesitant fuzzy MCDM problems in which the PIS and NIS are known in advance; Zhang and Xu (2014a) proposed the hesitant fuzzy TODIM decision analysis approach to solve the hesitant fuzzy MCDM problems in the case of considering the decision maker's psychological behavior; Zhang and Xu (2015a) put forward the hesitant fuzzy QUALIFLEX decision analysis method to handle the hesitant fuzzy MCDM problems in which the number of criteria markedly exceeds the number of alternatives; Zhang and Xu (2014b) and Zhang et al. (2015b), respectively, developed the hesitant fuzzy LINMAP decision analysis methods to solve the hesitant fuzzy MCGDM problems in which the weights of criteria and the PIS are unknown in advance; we developed the consensus model-based hesitant fuzzy group decision analysis method to investigate the hesitant fuzzy MCGDM problems where the weights of criteria are known in advance but the weights of experts are partially known or completely unknown; Zhang et al. (2015a) proposed the deviation modeling models-based heterogeneous hesitant fuzzy group decision analysis method to solve the MCGDM problems in which the criteria values are expressed as real numbers, interval numbers, linguistic variables, IFNs, HFEs, and hesitant fuzzy linguistic term sets (HFLTSs). It is easily noticed that these aforementioned decision analysis methods possess distinct characteristics and can also be suitable to solve different decision-making problems under various circumstances.

This book provides a thorough and systematic introduction to the above results, which is organized into six chapters that deal with six different but related issues and are listed below:

Chapter 1 introduces a maximizing deviation model-based hesitant fuzzy TOPSIS method for solving the MCDM problems in which the criteria values are expressed by HFEs and the weights of criteria are completely unknown or partially known. There are two key issues being addressed in this approach. The first one is to establish an optimization model on the basis of the idea of the maximizing deviation method, which is mainly used to determine the weights of criteria. According to the idea of the TOPSIS, the second one is to calculate the revised closeness index of each alternative to the hesitant fuzzy positive ideal solution, which can be used to determine the ranking orders of alternatives. On the other hand, this decision-making method is further extended to address the MCDM problems with incomplete weight information in which the criteria values are expressed by interval-valued hesitant fuzzy elements (IVHFEs).

Chapter 2 presents a ranking functions-based hesitant fuzzy TODIM approach to solve the hesitant fuzzy MCDM problems in the case of considering the decision maker's psychological behavior. Firstly, two novel ranking functions are introduced to compare the magnitudes of HFEs and IVHFEs, which are more reasonable and effective compared with the existing ranking functions. Then, a prospect value function is constructed for measuring the dominance degree of each alternative over the others based on novel ranking functions and distance measures. By aggregating these prospect values, the overall prospect value of each alternative is obtained and the corresponding ranking of alternatives is determined. Finally, a practical decision-making problem that concerns the evaluation and ranking of the service quality among domestic airlines is used to illustrate the validity and applicability of the proposed method. On the other hand, as an extension of HFE, hesitant trapezoidal fuzzy number (HTrFN) developed by Zhang et al. (2016) is suitable to tackle the imprecise and ambiguous information in complex decision-making problems and is well enough to represent the uncertainty and vagueness of comparative linguistic expressions. To handle the MCGDM problems in which the decision data are expressed as comparative linguistic expressions based on HTrFNs, this chapter also introduces a hesitant trapezoidal fuzzy TODIM approach developed by Zhang and Liu (2016).

Chapter 3 develops a hesitant fuzzy QUALIFLEX with a signed distance-based comparison method to handle the MCDM problems in which both the assessments of alternatives on criteria and the weights of criteria are expressed by HFEs. We propose a novel concept of hesitancy index for the HFE to measure the degree of hesitancy of the decision maker or the decision organization. By taking the hesitancy indices into account, we present a signed distance-based ranking method to compare the magnitudes of HFEs. Using the signed distance-based comparison approach, we define the concordance/discordance index, the weighted concordance/discordance index, and the comprehensive concordance/discordance index. By investigating all possible permutations of alternatives with respect to the level of concordance/discordance of the complete preference order, the optimal ranking orders of alternatives can be obtained. An application study of the proposed method on green supplier selection is conducted. The study indicates that the proposed method does not require the complicated computation procedures but still yields a reasonable and credible solution. Finally, we extend this technique to manage the heterogeneous information including real numbers, interval numbers, TFNs, IFNs, and HFEs.

Chapter 4 puts forward a hesitant fuzzy LINMAP group decision method with interval programming models for solving the MCGDM problems in which the criteria values are represented by HFEs and the pair-wise comparison judgments over alternatives are taken as interval numbers. The main advantages of this method are that (1) it can sufficiently consider the experts' hesitancy in expressing their assessment information on criteria values by using HFEs and (2) it can take into account the uncertainty of preference information over alternatives by using interval numbers. On the other hand, this chapter also develops a hesitant fuzzy programming model-based LINMAP method for handling the MCDM problems with incomplete and/or inconsistent weight information in which the ratings of alternatives with each criterion are taken as HFEs and the incomplete judgments on pair-wise comparisons of alternatives with hesitant degrees are also represented by HFEs. This proposed approach makes several contributions to the literature and practices. Firstly, the concept of hesitant fuzzy programming model in which both the objective function and some constraints' coefficients take the form of HFEs is defined. Secondly, an effective approach is presented to solve the derived model. Thirdly, a bi-objective programming model is constructed to address the issues of incomplete and inconsistent weights of the criteria under hesitant fuzzy environments.

Chapter 5 investigates the MCGDM problem where the criteria values take the form of HFEs and the weights of criteria are known in advance but the weights of experts are partially known or completely unknown. To solve such a decision-making problem, we propose a consensus model-based hesitant fuzzy group decision-making method, which is motivated by the literature (Zhang and Xu 2014c, 2015b). The proposed method first defines the consensus index from the perspectives of both the ranking and the magnitude of decision information, derives the experts' weights on the basis of the idea of the maximizing consensus, and then utilizes the extended TOPSIS to rank all alternatives. The prominent characteristic of the developed approach is that it can not only take into account the experts' weights and reduce the influence of unjust arguments on the decision results, but also take full advantage of the decision information from both the perspective of ranking and the angle of the sizes of values.

Chapter 6 proposes a deviation modeling method to deal with the heterogeneous MCGDM problems with incomplete weight information. There are three key issues being addressed in this approach. To determine the optimal weights of criteria for each expert, the first one is to construct a maximizing deviation optimal model under heterogeneous fuzzy environment. Borrowing the idea of TOPSIS, the second one is to calculate the relative closeness indices of the alternatives for each expert. The third one is to establish a minimizing deviation optimal model based on the idea that the opinion of the individual expert should be consistent with that of the group to the greatest extent, which is used to determine the weights of experts and identify the optimal alternative. The proposed approach is applied to solve the selection problem of Strategic Freight Forwarder of China Southern Airlines, and a comparison analysis with a similar approach is conducted to demonstrate the advantages of the proposed method.

This book can be used as a reference for researchers and practitioners working in the fields of fuzzy mathematics, operations research, information science, management science and engineering, etc. It can also be used as a textbook for postgraduate and senior undergraduate students.

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