# Integrated Smart Energy System Based on Production-Oriented Consumption

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**Abstract.** Modern power grids are facing a number of challenges, such as ever-increasing consumption, development of alternative energy generators, and decentralization of energy markets. Renewable energy sources are strongly weather-dependent and, therefore, cannot provide reliable production profiles. This leads us to the concept of demand side management, where energy users modify their consumption patterns due to the availability of generation capacities. Additionally, power flow distribution should be carefully studied in order to avoid overloads in the grid. In this work, we describe a multi-supplier multi-consumer grid model and formulate a problem of an optimal energy contract assignment with respect to power flow constraints. Though generally energy consumers are selfish, we assume their readiness for collaboration within a smart energy system. We offer a cooperative solution for this problem on the assumption of appropriate coordination between agents. Finally, we provide an example illustrating the applicability of our methods.

**Keywords:** Demand side management · Multi-agent systems · Power grids · Congestion · Load flow

## 1 Introduction

The structure of power supply has been changing significantly in last two decades. A new vision of power grid structures has appeared due to set of reasons, firstly, such as ever-growing energy consumption and development of alternative energy sources. New requirements for power grid structures with generators of renewable energy exacerbate such a crucial problem in power grid management as imbalance between energy demand and supply. Hence, the power system should turn to the production-oriented consumption that leads us to the idea of demand side management (DSM).

Much work has been done in the field of demand side management and, more specifically, demand response. Basic pricing mechanisms for systems with single


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supplier and multiple consumers are studied by [1, 2]. Distributed generation and storage are considered in [3, 4]. Coalition formation for local networks is examined in various scenarios in [5, 6] with use of methods of cooperative game theory. However, the questions of power grid's topology and congestion in transmission lines are not taken into account in these works. In present paper we fill this gap since transmission network is a significant element of any power grid system.

Actually, the question of collaboration in energy market has long been irrelevant, and, in contrast to smart grid consumer collaboration, competitive market forces of power generation, consumption, and transmission have been deeply investigated [7–9]. Nowadays, the newly established circumstances force us to investigate collaborative mechanisms in power consumption. Due to modern technologies, generators and consumers are now able to be integrated spatially in a smart local (nodal) power system. Smart grid power system could guarantee the conditions of optimal nodal demand-side consumption by virtue of the agreement between consumers to collaborate in energy consumption.

In this paper we formulate a model of cooperating consumers in a power network. We show that an integrated smart energy system could reduce overall costs in congested networks with multiple producers. An example of a network with tree structure is considered in order to show the applicability of this idea.

## 2 Model Description

In this section, we formulate the power grid model with multiple generators and consumers. Each generator has a production cost function, and it depends on the total amount of energy this generator is assigned to produce. Consumers are price-taking agents with energy demand that they need to meet. Both production and transmission costs are covered by consumers. We now discuss all elements of this model in detail.

### 2.1 Network Structure

Consider a directed connected graph $G = (V, A)$, where $V$ is a set of nodes and $A$ is a set of arcs. Since energy current in a power grid can flow in either direction of any link, we assume that for any arc $(k, l) \in A$ there is also an arc $(l, k) \in A$. We also claim that only one of these two arcs can be used at the same time (i.e., energy current cannot proceed in both directions of a link simultaneously).

We enumerate nodes in $V$ a specific way, so that set $V$ consists of three subsets: a set of $m$ consumer nodes $V_Q = \{1, \ldots, m\}$, a set of $n$ producer nodes $V_P = \{m+1, \ldots, m+n\}$, and a set of all other nodes $V_O = \{m+n+1, \ldots, |V|\}$.

Consumers make bilateral energy contracts with multiple producers in order to meet their energy demands. By $e_{ij} \geq 0$ we denote a contract between a consumer $i \in V_Q$ and a producer $j \in V_P$, and $d_i \leq 0$ is an energy demand of consumer $i \in V_Q$. Let us also define $\overline{e}_i = \{e_{ij},\ j \in V_P\}$, consumer $i$'s contract profile, and $\overline{e} = \{e_{ij},\ i \in V_Q,\ j \in V_P\}$, a contract profile of all consumers. Therefore, demand constraints take the following form:

$$d_i + \sum_{j=m+1}^{m+n} e_{ij} = 0, i \in V_Q. \tag{1}$$

For all other nodes $k \in V_P \cup V_O$, we also define $d_k$:

$$\begin{aligned} d_k &= \sum_{i=1}^{m} e_{ik},\ k \in V_P, \\ d_k &= 0,\ k \in V_O \end{aligned}. \tag{2}$$

Let us call $d_k$ an energy input in a node $k \in V$. It is positive in producer nodes, negative in consumer nodes, and equal zero otherwise.

For each producer $j \in V_P$ we define a unit production cost $\alpha_j(d_j) \geq 0$, a function of an energy input $d_j$, the total sum of contracts assigned to this producer. Consumers share production costs proportionally to their contracts, and for a given contract profile $\overline{e}$ consumer $i \in V_Q$ pays to producer $j \in V_P$ the following price:

$$c_{ij}(\overline{e}) = e_{ij} \cdot \alpha_j(d_j) = e_{ij} \cdot \alpha_j\left(\sum_{i=1}^{m} e_{ij}\right). \tag{3}$$

Therefore, we can determine the overall production cost of each consumer $i \in V_Q$ for a given contract profile $\overline{e}$:

$$c_i(\overline{e}) = \sum_{j \in V_P} c_{ij}(\overline{e}) = \sum_{j \in V_P} e_{ij} \cdot \alpha_j\left(\sum_{i=1}^{m} e_{ij}\right). \tag{4}$$

## 2.2 Load Flow

Transmission costs are the second type of expenses covered by consumers. These costs depend on the load flow in a network, and we require to make several further definitions. By power flow $f_{kl} \geq 0$ in arc $(k,l) \in A$ we denote a non-negative value of current flowing in this arc. According to our assumption, $f_{kl}$ and $f_{lk}$ cannot be both positive. A flow profile $\overline{f} = \{f_{kl},\ (k,l) \in A\}$ is a set of flows in all arcs. We assume that each arc $(k,l) \in A$ charges a transmission fee $\beta_{kl}(f_{kl})$ that must be shared between consumers using this arc.

In order to determine the transmission cost of consumer $i$ similarly to (4), we need to answer two questions: how does flow profile $\overline{f}$ depend on contract profile $\overline{e}$? And how should the transmission costs be shared between consumers?

The first question can be answered by applying Kirchhoff's laws to this power grid. Let us denote all neighbours of node $k$ by $W_k = \{l \in V | \ (k, l) \in A\}$. Then the first Kirchhoff's law takes the following form:

$$\sum_{l \in W_k} f_{kl} - \sum_{l \in W_k} f_{lk} = d_k, \ \forall k \in V. \tag{5}$$

If $\Theta_{kl}(f_{kl})$ is a voltage change function for an arc $(k, l) \in A$, and $\pi_k$ is a potential in node $k \in V$, we can formulate the second Kirchhoff's law for our model:

$$\pi_k - \pi_l = \Theta_{kl}(f_{kl}), \ \forall (k, l) \in A. \tag{6}$$

It is well-known that the flow profile can be found as a solution of a non-linear optimization problem (cf. Chap. 2.6 in [10]):

$$\min_{\bar{f}} \sum_{(k,l) \in A} \int_0^{f_{kl}} \Theta_{kl}(s) ds, \tag{7}$$

subject to

$$\sum_{l \in W_k} f_{kl} - \sum_{l \in W_k} f_{lk} = d_k, \quad \forall k \in V, \tag{8}$$

$$f_{lk} \geq 0, \quad \forall (k, l) \in A, \tag{9}$$

where contract profile $\bar{e}$ defines coefficients $\{d_k\}$ in (8).

By $\bar{f}(\bar{e})$ we denote the solution of problem (7)–(9) for a given $\bar{e}$. Mathematical programming formulation of the problem of finding equilibrium currents and voltages in electrical networks is an electrical network analogy of traffic equilibrium problem [10–12].

The second question is more complicated, since electricity is a homogeneous product, and unlike transportation problems it is not possible to track what portion of energy belongs to which consumer-producer contract. There are different methods used to determine transmission cost pricing, e.g. [13]. In general case, we define a set of functions $\{\delta_i^{kl}(\bar{e})\}$ that impose the cost sharing rule:

$$\begin{aligned} &\sum_{i \in V_Q} \delta_i^{kl}(\bar{e}) = 1, \quad \forall (k, l) \in A, \\ &\delta_i^{kl}(\bar{e}) \geq 0, \ \forall (k, l) \in A, \ \forall i \in V_Q. \end{aligned} \tag{10}$$

According to this rule, one can find transmission cost of consumer $i \in V_Q$ charged by arc $(k,l) \in A$ as follows:

$$t_i^{kl}(\overline{e}) = \delta_i^{kl}(\overline{e}) \cdot \beta_{kl}(f_{kl}(\overline{e})). \tag{11}$$

Thus, total transmission cost paid by consumer $i$ is

$$t_i(\overline{e}) = \sum_{(k,l)\in A} t_i^{kl}(\overline{e}) = \sum_{(k,l)\in A} \delta_i^{kl}(\overline{e}) \cdot \beta_{kl}(f_{kl}(\overline{e})) \tag{12}$$

We are ready to formulate a network optimization problem for a system of cooperative consumers.

## 3 Integrated Smart Energy System

Usually, the participants of energy market are assumed to be selfish and independent, that leads us to a system formulated in terms of competitive games. However, if energy resources are scarce and/or uncontrollable (such as renewable energy generation) it may be more profitable for consumers to cooperate. This section formulates a total cost minimization problem for a power grid with multiple suppliers.

### 3.1 Total Cost Minimization Problem

In order to reduce total costs for energy production and transmission, we need to solve a total cost minimization problem. Let us consider the total cost of all consumers, according to (4), (12):

$$\begin{aligned} C(\overline{e}) &= \sum_{i\in V_Q} (c_i(\overline{e}) + t_i(\overline{e})) \\ &= \sum_{i\in V_Q}\sum_{j\in V_P} e_{ij} \cdot \alpha_j\left(\sum_{i=1}^{m} e_{ij}\right) + \sum_{i\in V_Q}\sum_{(k,l)\in A} \delta_i^{kl}(\overline{e}) \cdot \beta_{kl}(f_{kl}(\overline{e})) \\ &= \sum_{j\in V_P} d_j(\overline{e}) \cdot \alpha_j\big(d_j(\overline{e})\big) + \sum_{(k,l)\in A} \beta_{kl}(f_{kl}(\overline{e})). \end{aligned} \tag{13}$$

The minimization problem can be formulated in the following form:

$$\min_{\overline{e}} \sum_{j\in V_P} d_j(\overline{e}) \cdot \alpha_j\big(d_j(\overline{e})\big) + \sum_{(k,l)\in A} \beta_{kl}(f_{kl}(\overline{e})) \tag{14}$$

$$\text{subject to } (1), (2), \tag{15}$$

where flow profile $\overline{f}(\overline{e})$ is found as a solution of optimization problem (7)–(9).

Note that the solution of constrained optimization problem (14), (15) does exist. However, it is not a trivial task to define the uniqueness of the solution in general case. Nevertheless, the properties of a solution could be established via differentiating Lagrangian with respect to $e_{ij}$ and using Kuhn-Tucker conditions. The only thing to be state is that Kuhn-Tucker conditions are necessary and sufficient when product $d_j(\overline{e}) \cdot \alpha_j\left(d_j(\overline{e})\right)$ and $\beta_{kl}(f_{kl}(\overline{e}))$ are convex functions. Generally, one could use most appropriate powerful numerical method for constrained nonlinear optimization [14].

### 3.2 Consumer Cooperation

There are different ways of fair cost sharing among consumers. Ideas of the cooperative game theory may be applied, such as the concept of Shapley value for a coalition of consumers (see, e.g., [15]). Another simplified policy to encourage the cooperation between consumers is to share the total cost proportionally to the demand of each consumer:

$$C_i(\overline{e}) = d_i \cdot C(\overline{e}), \quad \forall i \in V_Q. \tag{16}$$

In this case each consumer $i$ minimizes the same function (up to a constant coefficient $\{d_i\}$.

In the following example we do not consider any specific rule of cost sharing, but rather the possible total cost reduction in the network without cycles.

## 4 Example and Simulation Results

This section shows how our methods can be applied to a specific example of a network with tree structure.

### 4.1 Example

Consider a network with 7 nodes that is depicted in Fig. 1. There are 3 consumers (red nodes), 3 producers (green nodes) and one intermediate node. Therefore, $V_Q = \{1,2,3\}$, $V_P = \{4,5,6\}$, and $V_O = \{7\}$.

All nodes are located in the same local area except for node 4 that depicts a conventional energy generator, e.g. a power plant. Hence, arc (4, 2) is longer than all other arcs, and transmission costs are higher for this arc.

Since there are no cycles in the network, we only need to check the first Kirchhoff's law (5). A flow on each arc is a linear combination of $\left\{e_{ij}\right\}$:

$$\hat{f}_{25} = e_{34} + e_{36} - e_{15} - e_{25}, \quad \hat{f}_{53} = e_{34} + e_{35} + e_{36}, \quad \hat{f}_{42} = e_{14} + e_{24} + e_{34},$$
$$\hat{f}_{27} = e_{14} + e_{15} - e_{26} - e_{36}, \quad \hat{f}_{71} = e_{14} + e_{15} + e_{16}, \quad \hat{f}_{67} = e_{16} + e_{26} + e_{36}.$$

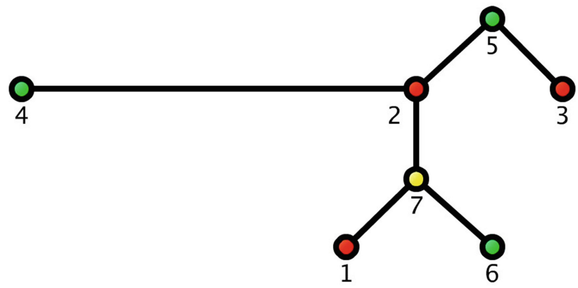


**Fig. 1.** 7-node network with no cycles (Color figure online)

The direction of flow in arcs (2, 5) and (2, 7) may differ depending on the values $\{e_{ij}\}$. If $\hat{f}_{25} < 0$, we assign $\hat{f}_{25} = 0$ and $\hat{f}_{52} = -\hat{f}_{25}$. The same is true for $\hat{f}_{27}$.

Let us assume that functions $\{\alpha_j(\cdot)\}$ and $\{\alpha_j(\cdot)\}$ have the following form:

$$\begin{aligned} \alpha_j(x) &= \lambda_j \cdot x^{1+\varepsilon} + \mu_j, \quad \forall j \in V_P \\ \beta_{kl}(x) &= \lambda_{kl} \cdot x^{1+\zeta}, \quad \forall (k,l) \in A, \end{aligned} \tag{17}$$

where all coefficients are non-negative. We now solve the problem (14), (15) with specific values of demands $\{d_i,\ i \in V_Q\}$ and coefficients in (17), and evaluate the total cost reduction.

### 4.2 Simulation

Let us assume that consumers have similar demand values, and that the transmission cost in arc (4, 2) is higher due to longer distance, while other five arcs have identical coefficients of transmission function (i.e., $\lambda_{42} > \lambda_a,\ \forall a \neq (4,2)$). We also assign $\varepsilon = \zeta = 0.2$.

We compute and compare minimal total costs of consumers in two different cases: $C_{\sin}$ if each consumer buys energy from a single supplier, and $C_{opt}$ for a case, when consumers distribute their contracts between several producers (Table 1).

**Table 1.** Some of simulation results are given.

| $d_1$ | $d_2$ | $d_3$ | $\lambda_{42}$ | $\lambda_a$ | $\lambda_4,\ \mu_4$ | $\lambda_5,\ \mu_5$ | $\lambda_6,\ \mu_6$ | $C_{opt}$ | $C_{\sin}$ | Gain |
|---|---|---|---|---|---|---|---|---|---|---|
| −20 | −22 | −23 | 0.005 | 0.001 | (0.002, 0.2) | (0.001, 0.12) | (0.005, 0.1) | 15.81 | 17.21 | 8.1 % |
| −10 | −12 | −9 | 0.005 | 0.001 | (0.002, 0.2) | (0.001, 0.12) | (0.005, 0.1) | 5.43 | 5.5 | 1.2 % |
| −10 | −12 | −9 | 0.005 | 0.001 | (0.002, 0.2) | (0, 0.12) | (0.01, 0) | 3.98 | 4.33 | 7.9 % |

It is clear that the cooperative contract distribution reduces the total cost significantly (up to 8 %). However, this reduction strongly depends on the demand values of consumers and cost functions of producers.

## 5 Conclusion and Future Work

In this work, we described and studied a collaborative model of local area energy consumption in the presence of multiple producers. We showed that the distribution of power flow can be determined by solving an optimization problem based on Kirchhoff's laws, and we formulated the total cost minimization problem considering the congestion in the network arcs. We also considered an example of a network without cycles and presented promising simulation results. There are several possible extensions of this model. First of all, it is crucial to consider a dynamic setting over several periods of time (e.g., 24 h of a day). Second direction is the investigation of cost sharing techniques.

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