# Advances in Computer Vision and Pattern Recognition

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# Guide to 3D Vision Computation

## Geometric Analysis and Implementation

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# Preface

Today, computer vision techniques are used for various purposes, and there exist many textbooks and references that describe principles of programming and system organization. At the same time, ever new research is going on all over the world, and the achievements are offered in the form of open source code on the Web. It appears, therefore, that sufficient environments already exist for students and researchers for embarking on computer vision research.

However, although executing a public source code may be easy, improving or modifying it for other applications is rather difficult, because the intent of the code author is difficult to discern simply by reading the code. On the other hand, many computer vision textbooks focus on theoretical principles coupled with application demos. As a result, one is often at a loss as to how to write a program oneself. Actual implementation of algorithms requires many small details that must be carefully taken into consideration, which a ready-to-use code does not provide. This book intends to fill that gap, describing in detail the computational procedures for programming 3D geometric tasks. The algorithms presented in this book are based on today's state of the art, yet arranged in a form simple and easy enough to understand. The authors also believe that they are the most appropriate form for practical use in real situations.

In this book, the mathematical background of the presented algorithms is mostly omitted for the ease of reading, but for theoretically minded readers detailed derivations and justifications are given in the form of Problems in each chapter; their Solutions are given at the end of the volume. Also, historical notes and related references are discussed in the Supplemental Note at the end of each chapter. In this sense, this book can also serve as a theoretical reference of computer vision research. To help readers implement the algorithms in this book, sample codes of typical procedures are placed on the publisher's Web page.[1]

This book is based on the teaching materials that the authors used for student projects at Okayama University and Toyohashi University of Technology, Japan. Every year, new students with little background knowledge come to our labs to do computer vision work. According to our experience, the most effective way for them to learn is to let them implement basic algorithms such as those given here.

[1]http://www.springer.com/book/9783319484921

Through this process, they learn the basic know-how of programming and at the same time gradually understand the theoretical background as their interest deepens. Thus we are hoping that this book can serve not only as a reference of the latest computer vision techniques but also as useful material for introductory courses of computer vision.

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[2]http://www.morganclaypool.com.

# Contents

**1 Introduction** . . . 1
1.1 Background. . . . 1
1.2 Organization . . . 2
1.3 Features . . . 7

**Part I Fundamental Algorithms for Computer Vision**

**2 Ellipse Fitting** . . . 11
2.1 Representation of Ellipses. . . . 11
2.2 Least-Squares Approach . . . 12
2.3 Noise and Covariance Matrices. . . . 13
2.4 Algebraic Methods . . . 15
2.4.1 Iterative Reweight . . . 15
2.4.2 Renormalization and the Taubin Method. . . . 16
2.4.3 Hyper-Renormalization and HyperLS . . . 17
2.4.4 Summary of Algebraic Methods . . . 18
2.5 Geometric Methods. . . . 19
2.5.1 Geometric Distance and Sampson Error . . . 19
2.5.2 FNS. . . . 19
2.5.3 Geometric Distance Minimization . . . 20
2.5.4 Hyper-Accurate Correction . . . 22
2.6 Ellipse-Specific Methods. . . . 23
2.6.1 Ellipse Condition. . . . 23
2.6.2 Method of Fitzgibbon et al. . . . 23
2.6.3 Method of Random Sampling . . . 24
2.7 Outlier Removal . . . 25
2.8 Examples . . . 26
2.9 Supplemental Note . . . 27
Problems. . . . 29
References. . . . 31

**3 Fundamental Matrix Computation**. . . . 33
3.1 Fundamental Matrices. . . . 33
3.2 Covariance Matrices and Algebraic Methods . . . 34

3.3 Geometric Distance and Sampson Error . . . 37
3.4 Rank Constraint . . . 38
3.5 A Posteriori Correction . . . 39
3.6 Hidden Variables Approach . . . 41
3.7 Extended FNS . . . 45
3.8 Geometric Distance Minimization . . . 46
3.9 Outlier Removal . . . 48
3.10 Examples . . . 49
3.11 Supplemental Note . . . 50
Problems . . . 53
References . . . 56

**4 Triangulation** . . . 59
4.1 Perspective Projection . . . 59
4.2 Camera Matrix and Triangulation . . . 60
4.3 Triangulation from Noisy Correspondence . . . 62
4.4 Optimal Correction of Correspondences . . . 63
4.5 Examples . . . 65
4.6 Supplemental Note . . . 66
Problems . . . 66
References . . . 68

**5 3D Reconstruction from Two Views** . . . 69
5.1 Camera Modeling and Self-calibration . . . 69
5.2 Expression of the Fundamental Matrix . . . 72
5.3 Focal Length Computation . . . 73
5.4 Motion Parameter Computation . . . 75
5.5 3D Shape Computation . . . 76
5.6 Examples . . . 77
5.7 Supplemental Note . . . 78
Problems . . . 79
References . . . 80

**6 Homography Computation** . . . 81
6.1 Homographies . . . 81
6.2 Noise and Covariance Matrices . . . 83
6.3 Algebraic Methods . . . 85
6.4 Geometric Distance and Sampson Error . . . 88
6.5 FNS . . . 89
6.6 Geometric Distance Minimization . . . 90
6.7 Hyperaccurate Correction . . . 92
6.8 Outlier Removal . . . 93
6.9 Examples . . . 94
6.10 Supplemental Note . . . 95
Problems . . . 96
References . . . 97

**7 Planar Triangulation** . . . . . 99
7.1 Perspective Projection of a Plane . . . . . 99
7.2 Planar Triangulation . . . . . 101
7.3 Procedure of Planar Triangulation . . . . . 101
7.4 Examples . . . . . 103
7.5 Supplemental Note . . . . . 104
Problems . . . . . 105
References . . . . . 105

**8 3D Reconstruction of a Plane** . . . . . 107
8.1 Self-calibration with a Plane . . . . . 107
8.2 Computation of Surface Parameters and Motion Parameters . . . . . 108
8.3 Selection of the Solution . . . . . 109
8.4 Examples . . . . . 112
8.5 Supplemental Note . . . . . 113
Problems . . . . . 113
References . . . . . 115

**9 Ellipse Analysis and 3D Computation of Circles** . . . . . 117
9.1 Intersections of Ellipses . . . . . 117
9.2 Ellipse Centers, Tangents, and Perpendiculars . . . . . 119
9.3 Projection of Circles and 3D Reconstruction . . . . . 120
9.4 Center of Circle . . . . . 123
9.5 Front Image of the Circle . . . . . 124
9.6 Examples . . . . . 125
9.7 Supplemental Note . . . . . 126
Problems . . . . . 128
References . . . . . 129

**Part II Multiview 3D Reconstruction Techniques**

**10 Multiview Triangulation** . . . . . 133
10.1 Trilinear Constraint . . . . . 133
10.2 Triangulation from Three Views . . . . . 134
10.2.1 Optimal Correspondence Correction . . . . . 134
10.2.2 Solving Linear Equations . . . . . 136
10.2.3 Efficiency of Computation . . . . . 138
10.2.4 3D Position Computation . . . . . 138
10.3 Triangulation from Multiple Views . . . . . 140
10.4 Examples . . . . . 144
10.5 Supplemental Note . . . . . 144
Problems . . . . . 146
References . . . . . 147

**11 Bundle Adjustment** . . . . . 149
11.1 Principle of Bundle Adjustment . . . . . 149
11.2 Bundle Adjustment Algorithm . . . . . 151
11.3 Derivative Computation . . . . . 153
11.3.1 Gauss-Newton Approximation . . . . . 153
11.3.2 Derivatives with Respect to 3D Positions . . . . . 154
11.3.3 Derivatives with Respect to Focal Lengths . . . . . 154
11.3.4 Derivatives with Respect to Principal Points . . . . . 154
11.3.5 Derivatives with Respect to Translations . . . . . 154
11.3.6 Derivatives with Respect to Rotations . . . . . 155
11.3.7 Efficient Computation and Memory Use . . . . . 155
11.4 Efficient Linear Equation Solving . . . . . 156
11.5 Examples . . . . . 158
11.6 Supplemental Note . . . . . 160
Problems . . . . . 160
References . . . . . 161

**12 Self-calibration of Affine Cameras** . . . . . 163
12.1 Affine Cameras . . . . . 163
12.2 Factorization and Affine Reconstruction . . . . . 164
12.3 Metric Condition for Affine Cameras . . . . . 167
12.4 Description in the Camera Coordinate System . . . . . 168
12.5 Symmetric Affine Camera . . . . . 169
12.6 Self-calibration of Symmetric Affine Cameras . . . . . 172
12.7 Self-calibration of Simplified Affine Cameras . . . . . 175
12.7.1 Paraperspective Projection Model . . . . . 175
12.7.2 Weak Perspective Projection Model . . . . . 177
12.7.3 Orthographic Projection Model . . . . . 178
12.8 Examples . . . . . 179
12.9 Supplemental Note . . . . . 181
Problems . . . . . 182
References . . . . . 182

**13 Self-calibration of Perspective Cameras** . . . . . 183
13.1 Homogeneous Coordinates and Projective Reconstruction . . . . . 183
13.2 Projective Reconstruction by Factorization . . . . . 185
13.2.1 Principle of Factorization . . . . . 185
13.2.2 Primary Method . . . . . 187
13.2.3 Dual Method . . . . . 190
13.3 Euclidean Upgrading . . . . . 192
13.3.1 Principle of Euclidean Upgrading . . . . . 192
13.3.2 Computation of $\boldsymbol{\Omega}$ . . . . . 194
13.3.3 Modification of $\mathsf{K}_\kappa$ . . . . . 196
13.3.4 Computation of $\mathsf{H}$ . . . . . 198
13.3.5 Procedure for Euclidean Upgrading . . . . . 198

13.4 3D Reconstruction Computation . . . 199
13.5 Examples . . . 201
13.6 Supplemental Notes . . . 202
Problems . . . 207
References . . . 210

**Part III Mathematical Foundation of Geometric Estimation**

**14 Accuracy of Geometric Estimation** . . . 213
14.1 Constraint of the Problem . . . 213
14.2 Noise and Covariance Matrices . . . 214
14.3 Error Analysis . . . 214
14.4 Covariance and Bias . . . 216
14.5 Bias Elimination and Hyper-Renormalization . . . 218
14.6 Derivations . . . 219
14.7 Supplemental Note . . . 228
Problems . . . 228
References . . . 229

**15 Maximum Likelihood of Geometric Estimation** . . . 231
15.1 Maximum Likelihood . . . 231
15.2 Sampson Error . . . 232
15.3 Error Analysis . . . 233
15.4 Bias Analysis and Hyper-Accurate Correction . . . 235
15.5 Derivations . . . 236
15.6 Supplemental Note . . . 241
Problems . . . 242
References . . . 242

**16 Theoretical Accuracy Limit** . . . 243
16.1 Kanatani-Cramer-Rao (KCR) Lower Bound . . . 243
16.2 Structure of Constraints . . . 244
16.3 Derivation of the KCR Lower Bound . . . 246
16.4 Expression of the KCR Lower Bound . . . 249
16.5 Supplemental Note . . . 251
Problems . . . 253
References . . . 254

**Solutions** . . . 255

**Index** . . . 319