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Krassimir T. Atanassov

# Intuitionistic Fuzzy Logics



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### Preface

The work over the present book started in 1993 in a discussion with George Gargov. Ten years earlier, he had proposed the name of the extension of fuzzy sets, introduced by me in [1]. In the next years, first I and later together with G. Gargov gave some of the basic definitions of intuitionistic fuzzy propositional calculus, intuitionistic fuzzy predicate logic and intuitionistic fuzzy modal logic. Unfortunately, in 1996, Gargov died. During next 20 years, I was gathering enthusiasm to write this book. Meantime, I published about 30 books in the areas of the generalized nets (extensions of the Petri nets), intuitionistic fuzzy sets and number theory, but the work on the present book was progressing very slowly. The readers will see that there are a lot of open problems in the book, which I cannot solve at the moment, but I hope that the readers will be more successful than me in this enterprise.

I will repeat a paragraph of my book [2], that I think will be valid for the present book, too: "throughout the years, many colleagues of mine have shown me or published communications about inaccuracies, found in [3]. I myself have also found many mistakes, most of which were due to the "copy-paste" effect. I do not cherish any illusions that the present book is perfect. However, I do believe that the present book holds some interesting ideas that open opportunities for future research. I would encourage researchers to pay more attention in this direction."

During last years, in the area of fuzzy logic a lot of books have been published, which had an impact on my research. An incomprehensive list includes [4–17]. To the best of my knowledge, this book is the first one, exclusively devoted to intuitionistic fuzzy logics.

Now, I have recorded more than 5000 papers on intuitionistic fuzzy sets, written by researchers from more than 50 countries. Most of these papers discuss important issues related to the theory or applications of the intuitionistic fuzzy sets. For this reason, I realized that it would be virtually impossible to make a survey of all the others' results and contented myself with the presentation of my own works only. I am convinced that in recent future many of the colleagues in the field will likewise collect and publish their own ideas as papers and monographs, and I would gladly subserve anyone in that. I am very thankful to my colleagues and students who motivated me to prepare the present book. The list of their names is long, but I must note (in the alphabetical order of their first names) at least those of my coauthors whose research I have used and included in this book: Beloslav Riečan, Boyan Kolev, Dimiter Dimitrov, Eulalia Szmidt, Evgeniy Marinov, Ivan Georgiev, Janusz Kacprzyk, Lilija Atanassova, Nora Angelova, Peter Vassilev, Radoslav Tsvetkov, Trifon Trifonov, Vassia Atanassova.

Ivan Georgiev, Peter Vassilev and my daughter Vassia Atanassova read and corrected the text of the manuscript and deserve my deepest thanks and appreciation.

Special thanks are due to one of the most active contributors and promoters of intuitionistic fuzzy sets worldwide, an active researcher, coauthor and friend–Janusz Kacprzyk, the Editor-in-chief of this book series, who urged me to finalize my 30-year-long work on intuitionistic fuzzy logics.

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Last but not least, I would like to thank the three most important ladies in my life: my mother, my wife, and my daughter for encouraging and stimulating all my scientific research.

Sofia, Bulgaria July 2016 Krassimir T. Atanassov

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