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An Evolutionary Optimization Approach for Path Planning of Arrival Aircraft for Optimal Sequencing

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Abstract. In this paper, we present an evolutionary optimization based path planning algorithm at Terminal Airspace (TAS) that provides a near optimal aircraft arrival sequence at Final Approach Fix (FAF). The sequence obtained minimizes the inter-arrival time as well as provides conflict free path planning to an Air Traffic Controller (ATC). A classic Genetic Algorithm (GA) based optimization technique with conflict detection and resolution is developed. Conflict between any two aircraft is detected based on their future arrival time at the waypoint and resolved by stretching the gap between those two aircraft. The proposed algorithm is compared with the traditional GA. Results indicate that the proposed approach obtains a near optimal solution compared to the traditional GA based algorithm which does not consider TAS constraints.

Keywords: Terminal Airspace; Way-point Manoeuvring; Optimal Path Planning; Optimal Aircraft Sequence; Conflict Detection and Resolution;

1 Introduction

Terminal Area Airspace (TAS) is the airspace surrounding a controlled aerodrome where aircraft transition from the descend phase to the approach phase. TAS is also one of the most resource-constrained elements of an air transportation network as all air traffic converges in TAS and is sequenced for landing [1]. Aircraft sequencing in TAS is a highly challenging task due to complex manoeuvring constraints (i.e., restricted speed, altitude and movement). During the busiest times, operation on this safety critical environment reduces ATC efficiency especially sequencing and manoeuvring the aircraft and eventually degrades the efficiency of the system [2]. One study shows that inefficient sequencing techniques in the TAS area resulted in, on average, 18 minutes of delay for 19% of European flights [3].

The increasing demand of air traffic is stressing the capacity of the current Air Traffic Management System (ATMS). This is likely to cause both safety and performance degradation in the near future. It is believed that by increasing the level of automation, the efficiency of the ATMS can be enhanced. This may assist ATC to handle the increased traffic demand in a more reliable way.

Therefore, increasing the automation of ATM components, an automatic decision support techniques, is imperative to meet future needs and might increase the overall system performance.

In recent years, many optimization based algorithms and technique were proposed for aircraft sequencing as an automatic decision support tool [4, 5]. Most of the approaches available in the literature have goals that were simplistic to obtain the best sequence and hence provide high throughput. However, in practice, obtaining those optimal sequences might be a very challenging task due to the frequently changing environment, complex network structure and cost consideration (i.e., shifting position, vectoring a long way, holding a long time etc). A survey of the literature has failed to discover any other works that use a path planning based approach for solving the aircraft sequencing problem. However, a path-planning approach may deliver near optimal results while addressing and dealing with real-world complexities such as limited capacity to shift sequence. Fig.1 presents the algorithm based optimization sequence that is prevalent in the literature and the real world scenario that is expected to be achieved.

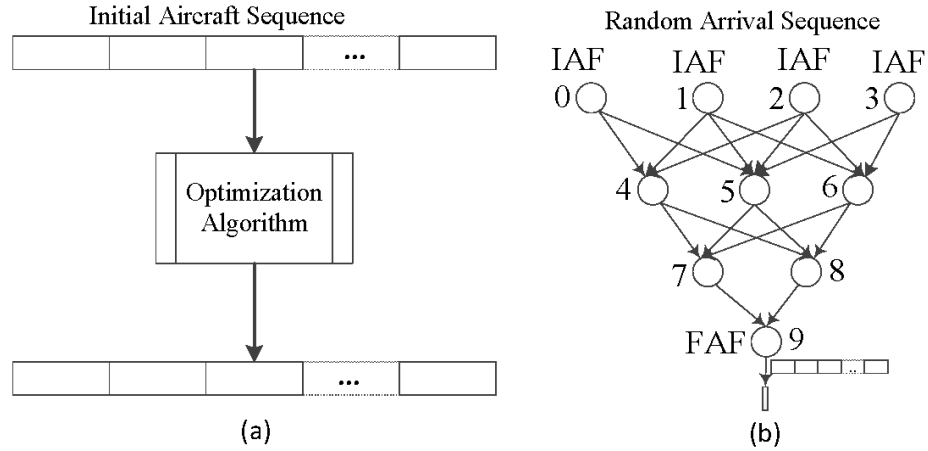


Fig. 1: (a) Optimal Sequence (traditional optimization technique) (b) Expected Optimal Sequence

To achieve the expected arrival sequence, ATCs (the Approach controller) have to juggle the arrivals through the Standard Terminal Arrival Routes (STAR) along with the desired safe separation. A STAR is a flight route defined and published by the Air Navigation Service Provider (ANSP) that usually covers the phase of a flight that lies between the last point of the route filled in the flight plan and the first point of the approach to the airport, normally the Initial Approach Fix (IAF) [6]. Hence, a STAR connects the en route phase with the approach phase of the flight. Having an optimal sequence and implementing it in a TAS provides two different outcomes, given the complexity of the environment.

First, increased probability of potential conflict at different waypoint. Second, it decreases the ATC efficiency. In this paper, we have been motivated to obtain the expected near optimal sequence at FAF by traversing through waypoint, given all the TAS constraints (i.e., speed limit, altitude limit and separation etc). We develop an evolutionary optimization based path planning algorithm that provides an optimal aircraft sequence at the FAF. Our algorithm also predicts potential conflict at future waypoints and uses a path stretch technique to resolve the conflict. We call this algorithm Terminal-Airspace Traversing Algorithm (TATA). This algorithm finds the near optimal path for each aircraft, resolves potential conflict and maintains safe separation. The proposed algorithm also provides a detailed manoeuvring guidance to the ATC.

We compare and analyse our approach (i.e., TATA VS traditional GA) on several random arrival sequences in terms of the inter-arrival time of the sequences. The remainder of the paper is organized as follows: Section 2 explains the problem formulation. Section 3 outlines a methodology and proposed algorithm. Section 4 presents the experimental design. Results and analysis are discussed in section 5. In Section 6, we present our conclusions and future work.

2 Problem Formulation

We subdivide the problem formulation into two stages:

- The optimization model for arrival sequencing using traditional GA
- The optimization model for arrival sequence using path planning (TATA)

2.1 Optimization model for arrival sequencing using traditional GA

The aircraft-sequencing problem is to minimize the inter-arrival time between two consecutive arrivals and hence the total inter-arrival time. The optimization model for the arrival sequence is formulated as follows:

A : set of all aircraft in a sequence

S_{ij} : minimum safe separation between two aircraft i and j

L : length of the final approach path

O_i : runway occupancy time of aircraft i

V_i : approach speed of aircraft i

V_j : approach speed of aircraft j

T_{ij} : inter-arrival time between aircraft i, j

The inter-arrival time between aircraft i and j can be determined by Eq.(1) and Eq.(2) [7]. The runway occupancy time of the aircraft are presented in table 1(b).

$$T_{ij} = \text{Max}[\frac{L + S_{ij}}{V_j} - \frac{L}{V_i}, O_i] \text{ when } V_i > V_j \quad (1)$$

$$T_{ij} = \text{Max}[\frac{S_{ij}}{V_j}, O_i] \text{ when } V_i \leq V_j \quad (2)$$

Objective Function: The objective is to minimize the inter-arrival time of each sequence. The objective function and the corresponding fitness function of the traditional GA based optimization problem is as follows:

$$\Psi = \sum_{i,j=1}^k \sum_{i,j=1}^k P_{ij}.T_{ij} \quad (3)$$

$$\text{Min} \sum_{i=1}^n \Psi \quad (4)$$

$$\text{Fitness} = \text{Max}(1/\Psi) \quad (5)$$

Where k is the number of aircraft classes (i.e., 3 for Heavy, Medium and Light classification), P_{ij} is the probability of the arrival of aircraft i followed by j and n is the total aircraft in the sequence. Eq.(6, 4 and 5) determines the optimal sequence based on traditional GA.

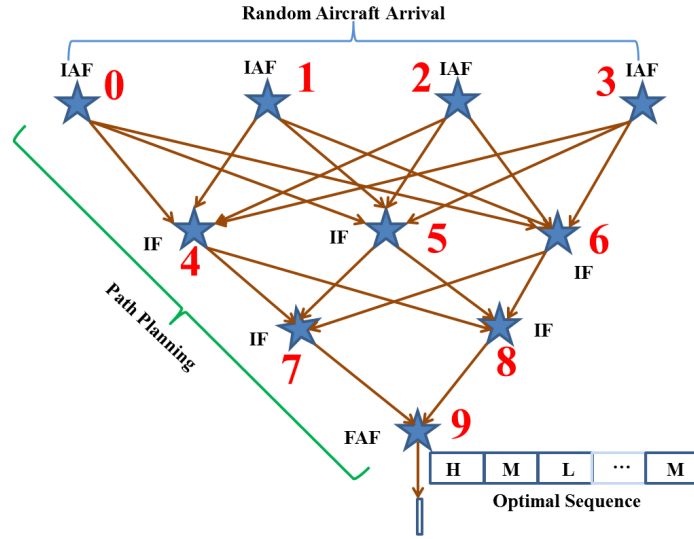


Fig. 2: Conceptual diagram of the methodology

2.2 Optimization model for arrival sequencing using path planning (TATA)

Aircraft are sequenced by traversing through the terminal airspace and the final sequence is obtained at the FAF. Let the arrival time of aircraft i and j at waypoint p be $_p t_i$ and $_p t_j$ respectively. Each aircraft needs to maintain a safe separation at each waypoint.

Objective function: The objective function of the optimization problem is as follows:

$$\Psi = \sum_{i,j=1}^k \sum_{i,j=1}^k P_{ij}.T_{ij} \quad (6)$$

$$Min \sum_{i=1}^n \Psi \quad (7)$$

$$Fitness = Max(1/\Psi) \quad (8)$$

Subject to the constraints that,

$$|_p t_i - _p t_j| \geq S_{ij} \quad (9)$$

TATA achieve the near optimal sequence through Eq.(6,7,8 and 9).

3 Methodology

The proposed methodology consists of two major phases. Fig.2 shows the conceptual diagram of the methodology.

- A method for generating the optimal arrival sequence using GA
- TATA for optimal arrival sequence

3.1 A method for generating the optimal arrival sequence using GA

Fig.3 shows the flowchart of the traditional genetic algorithm implementation for optimal sequence derivation. The initial aircraft arrival sequence is generated using a mixed distribution of the aircraft as presented table.1(a). This distribution represents the aircraft mix for a typical spoke airport (in a Hub-Spoke network) [8]. The random arrival sequence is modelled as a chromosome where each gene of the chromosome represents the aircraft wake category (heavy, medium or light).

Heavy (H)	Medium (M)	Light (L)	Heavy (H)	Medium (M)	Light (L)
20%	40%	40%	60	55	50
(a)			(b)		

Table 1: (a) Aircraft Mix Distribution (%) (b) Runway Occupancy Time (Sec)

GA based optimization considers inter-arrival time as the sole optimization criteria without involving the manoeuvring complexity of TAS. The evolutionary process attempts to maximize the fitness function through genetic operations. In this implementation, according to the problem characteristics and the chromosome size the initial population size is considered 50 [9].

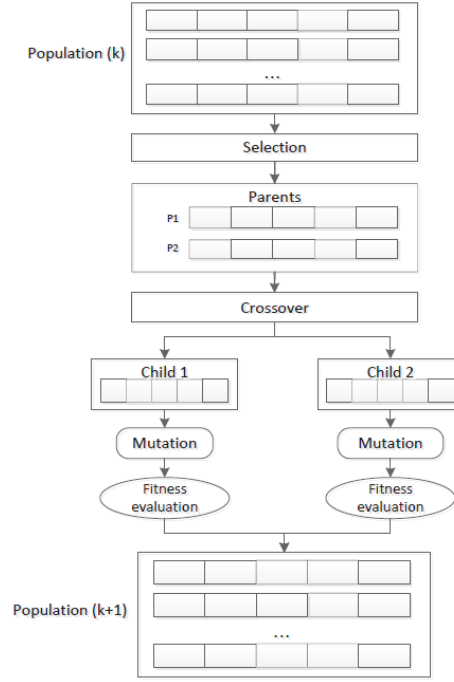


Fig. 3: Flowchart of the genetic algorithm implementation.

Two parent chromosomes (out of a population of K parent chromosomes) are selected (using tournament selection) to undergo a genetic operation (i.e., crossover and mutation). The elitism algorithm is used to determine the survival of parent and offspring in the new generation [10]. The fitness of the offspring is evaluated using a fitness function. The fitness generation curve shows in Fig. 8(a) that the fitness value does not further improve after 200 generation. This is because, after 200 generations the produced sequence's total inter-arrival time remained unchanged. However, we continued our evolution up to 400 generations as an evidence of convergence of the solution.

- Selection

Two individuals are chosen from the population using a selection operator. The preference is given to fitter individuals, allowing them to pass on their genes to the next generation. Fitness is determined by Eq.(5). The lower the inter-arrival time of a particular sequence the higher the fitness value.

- Crossover

Fig.4 shows the crossover procedure. In this crossover method a subset of the gene is selected from the first parent and then that subset is added to the offspring. The missing genes are then added to the offspring from the second parent by ensuring that the total number and types of genes (i.e., aircraft) remain equivalent to the parents'. To make this explanation a little

clearer, consider the example in Fig.4. Note here that a subset of the genes (i.e., M, L, H) of the offspring is taken from the parent 1 chromosome. Next, the remaining genes are taken from parent 2 sequentially.

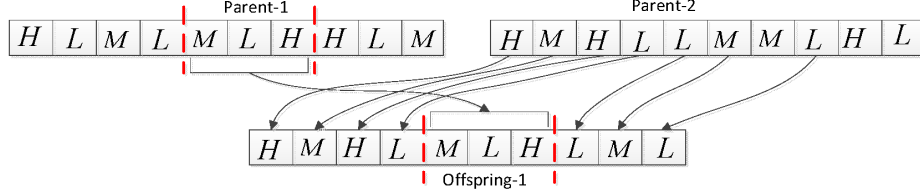


Fig. 4: Crossover procedure of GA based optimization

- Mutation

Mutation is used to maintain genetic diversity from one generation of the population to the next generation. In this implementation, swap mutation is used. With swap mutation two gene's positions in the chromosome are selected in a random fashion. Swap mutation is only swapping of pre-existing genes, it never creates a new gene. Eventually, once the population is not producing offspring that are noticeably different from the previous generation, it is assumed that the population converges to a set of solutions to the problem.

Determining the population size and mutation rate for GA is problem specific [11]. Too high a mutation rate increases the probability of searching more areas in the search space, however, it prevents the population from converging to an optimum solution. On the other hand, too small a mutation rate may results in premature convergence. To prevent both premature convergence and local optima, a small mutation rate of 0.015 is used [12].

3.2 TATA for optimal arrival sequence

The flowchart of the TATA approach is shown in Fig.5. The proposed method utilizes the evolutionary algorithm technique to obtain the near optimal sequence that ensures a conflict free path at TAS.

A. Initial Aircraft Sequence and Activation Time

The population of possible solution sequences are generated randomly according to a poisson arrival rate λ . Aircraft arrival events occur at IAF continuously and independently of one another. Each flight is activated at IAF by following a poisson arrival process. The initial aircraft sequence is randomly generated by a mix of three classes of aircraft (Heavy, Medium, and Light) using the distribution as shown in table 1(a). The probability distribution of the number of homogeneous poisson arrival events in a fixed interval gives the cumulative function of

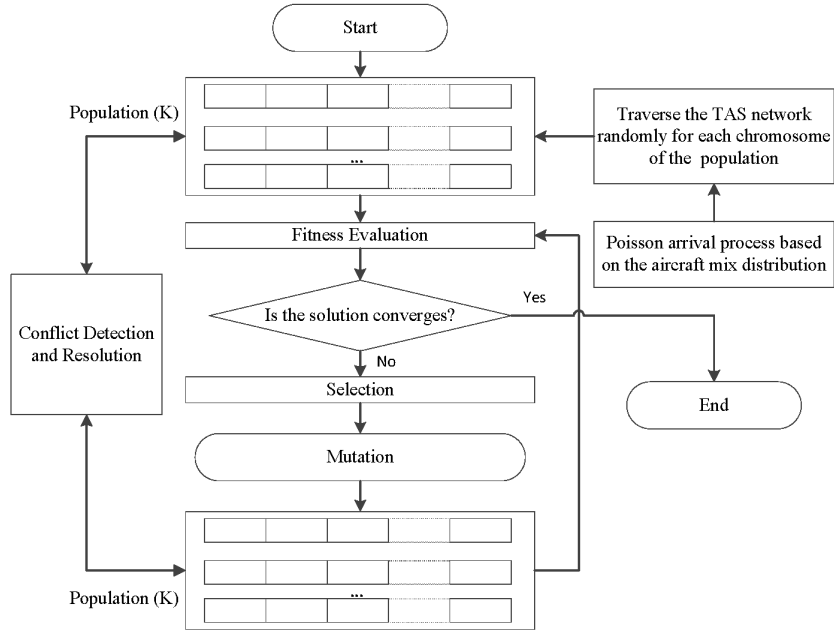


Fig. 5: Flowchart of the TATA approach.

an exponential distribution as,

$$F(t) = \begin{cases} 1 - e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad (10)$$

Given the inverse of the exponential equation $y = 1 - e^{-\lambda t}$, we can write for the next arrival time t in terms of $y \in (0, 1)$,

$$t = F^{-1}(y) = -\frac{1}{\lambda} \ln(1 - y) \quad (11)$$

Eq.(11) gives the continuous activation time at IAF of each random aircraft expressed in terms of the arrival rate.

B. TATA based Path Planning Algorithm

The TAS is considered as a network of waypoints. We assume that all the waypoints are static and that the distance between all waypoint pairs is known. Equations of motion are used for calculating the position of the aircraft at any time t . After activating at the IAF, each flight finds the next possible waypoint towards the destination (FAF). The next waypoint is selected randomly from the next available connected waypoint. At each waypoint the traversing aircraft calculates the distance and the arrival time between the current and next waypoint. The path distance and the arrival time are estimated as follows,

$$a = \sin^2(\Delta\varphi/2) + \cos\varphi_1\cos\varphi_2\sin^2(\Delta\phi/2) \quad (12)$$

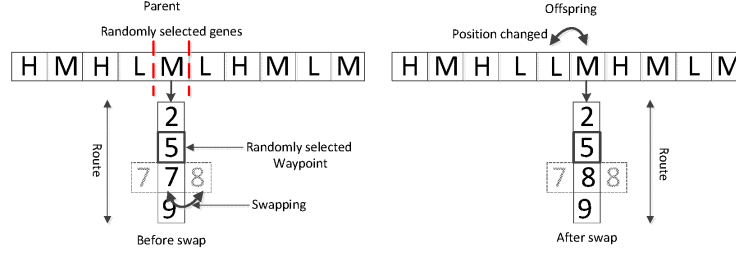


Fig. 6: Swap mutation operator

$$c = 2 \cdot \text{atan2}(\sqrt{a}, \sqrt{(1-a)}) \quad (13)$$

$$d_p = R \cdot c \quad (14)$$

$${}_p t_i = \frac{d_p}{V_i} \quad (15)$$

where $\Delta\varphi$ and $\Delta\phi$ are the difference between latitude and longitude of two waypoints respectively. a is the chord length, C is the great circle distance, d_p is the distance up to waypoint p and R is the earth's radius. From Eq.(15), we can estimate the future arrival time (${}_p t_i$) of aircraft i at waypoint p while V_i is the approach speed. This procedure is repeated until the aircraft arrives at the goal waypoint. Each aircraft of the population obtains a traversal path with an approach speed at a different waypoint.

However, there may be conflict at each waypoint with another aircraft. To resolve the potential conflict at the waypoint, a conflict detection and resolution technique is explained in subsection C and D. Two genetic operators are used in an evolutionary process: selection and mutation.

- **Selection**

The popular tournament selection mechanism is used due to its efficiency and simple implementation. In tournament selection, 5 individuals are selected randomly from the population [13]. The individual with the highest fitness wins and is included as one of the next generation's population. This is repeated. Tournament selection also gives a chance for all individuals to be selected and thus it preserves diversity.

- **Mutation**

To maintain the genetic diversity from one generation of a population to another generation swap mutation is used as described above for traditional GA. However, the swapping content and strategy is different. The swap scheme selects one gene within a chromosome at random and then selects a waypoint from the traversed path of that gene (aircraft) randomly as well. If there is another available path instead of the selected waypoint, swap these contents. An example of the swap mutation procedure is shown in Fig.6. Note here that from the parent chromosome, a gene is chosen randomly as indicated by the dotted lines. The selected gene (i.e., aircraft) has four waypoints in its chosen path ($2 \rightarrow 5 \rightarrow 7 \rightarrow 9$). The selected random waypoint

is 5 which has two available next waypoints i.e., 7 and 8 as indicated in the dotted box. After swapping the waypoints, an offspring is produced. Note that the offspring's gene sequence has also changed as an outcome of the swap.

C. Conflict Detection Technique

In this paper, a prediction based conflict detection model is introduced. Conflict is a situation where two aircraft come closer than a certain prescribed distance to one another. The safety distance is determined by means of a minimum allowed horizontal separation and a minimum vertical separation. In this model, we consider the horizontal separation as conflict detection metrics and the potential conflicts are predicted at different waypoint based on an aircraft's future arrival time.

We assume that each aircraft follows its flight plan moving along the straight line joining successive waypoints p_{k-1} and p_k with the prescribed speed V_i . The nominal arrival time ${}_k t_i$ of an aircraft i at waypoint k is

$${}_k t_i = \frac{\|p_{k-1} - p_k\|}{V_i} \quad (16)$$

where $\|p_{k-1} - p_k\|$ is the distance between two waypoint. A potential conflict between aircraft i and j at waypoint k is predicted if,

$$|{}_k t_i - {}_k t_j| \leq \delta \quad (17)$$

where ${}_k t_j$ is the arrival time of aircraft j and δ is the minimum separation.

D. Conflict Resolution

A path stretching technique is used to resolve the potential conflict. The objective of path stretching is to maintain a smooth motion along the trajectories. Two approaches are used. i.e., speeding the aircraft that will arrive first or slowing the aircraft that will arrive second. Suppose the arrival time of two conflicting aircraft at waypoint p_k is ${}_i t_k$ and ${}_j t_k$ respectively. The resolution advisories(i.e., required adjustment of speed) is estimated by following equation.

$$dt = |{}_i t_k - {}_j t_k| \quad (18)$$

$$\Delta t = \delta_k - dt \quad (19)$$

$$v_{exp} = \frac{d_i}{{}_i t_k + \Delta t} \quad (20)$$

$$\Delta v = |v_{exp} - {}_{ap} v_i| \quad (21)$$

$${}_{ap} v_i = \begin{cases} {}_{ap} v_i - \Delta v, & \text{if } {}_i t_k \geq {}_j t_k \\ {}_{ap} v_i + \Delta v, & \text{if } {}_i t_k < {}_j t_k \end{cases} \quad (22)$$

Here, dt is time gap, Δt is the required time adjustment, δ_k is the minimum separation. Expected speed v_{exp} can be estimated by Eq.(20) where d_i is the distance between aircraft i and next connecting waypoint. Therefore, we can estimate the required approach speed by Eq.(22) where Δv is the required speed adjustment.

4 Experimental Design

The optimization model is evaluated through simulation. The performance of the model is evaluated using a mixture of arrivals by taking into consideration that all the ATC separation rules are satisfied. Table 2 shows the summary of the experimental parameters used. The simulation is conducted 30 times to observe closeness of the obtained sequence with traditional GA.

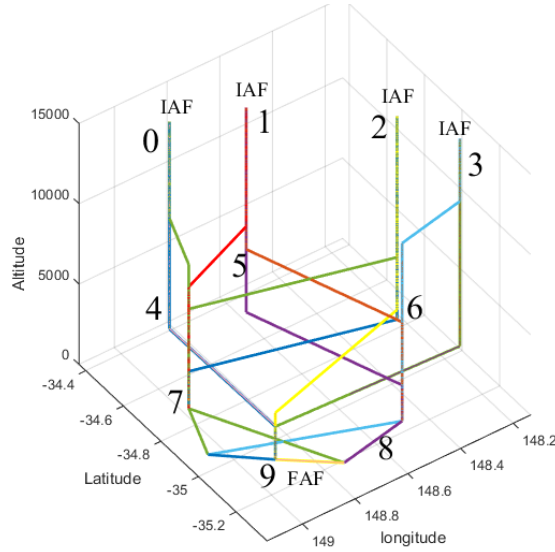


Fig. 7: 3D traversed network

Parameter	Values
Mutation rate	0.015
Chromosome Size	20
Population Size	50
Tournament Size	5
Generation	400
No. of Simulation	30

Table 2: Experimental Set-up

Table 3(a) shows the pair-wise ATC separations from arrival to arrival in seconds. In practice arrival separation is measured as nautical miles (NM) and departure separation is measured in seconds. In this implementation, we convert the separation distance into time (seconds) for simplicity of computation. Table 3(b) presents the probability matrix of two consecutive aircraft based on their wake-type.

		Trail		
		H	M	L
Lead	H	90	120	120
	M	60	60	60
	L	60	60	60

(a) Separation

		Trail		
		H(0.2)	M(0.4)	L(0.4)
Lead	H(0.2)	0.04	0.08	0.08
	M(0.4)	0.08	0.16	0.16
	L(0.4)	0.08	0.16	0.16

(b) Probability

Table 3: (a) Arrival-Arrival (sec) (b) Probability (P_{ij})

IAF Seq	GA Seq	FAF Seq	IAF time	FAF time	Optimal Path
L	L	H	6.83	23.03	3 → 6 → 7 → 9
M	M	M	5.35	23.66	0 → 4 → 7 → 9
L	H	L	7.15	30.69	0 → 5 → 8 → 9
M	L	M	14.56	32.89	3 → 6 → 8 → 9
L	M	M	9.61	34.92	1 → 4 → 8 → 9
M	H	L	5.26	37.79	1 → 5 → 8 → 9
L	M	H	17.28	40.04	1 → 6 → 7 → 9
H	H	L	17.64	41.21	3 → 6 → 7 → 9
L	L	M	24.16	42.47	0 → 4 → 8 → 9
L	L	L	10.31	42.82	2 → 6 → 8 → 9
M	M	M	15.74	43.65	2 → 6 → 8 → 9
H	L	L	20.52	44.09	3 → 5 → 8 → 9
L	L	M	19.89	45.17	2 → 5 → 8 → 9
M	M	L	15.62	48.12	1 → 4 → 8 → 9
L	L	L	25.14	48.71	3 → 5 → 7 → 9
M	L	L	26.62	50.16	0 → 5 → 8 → 9
L	M	H	28.44	51.2	1 → 5 → 8 → 9
M	M	M	31.65	51.65	2 → 5 → 8 → 9
H	L	M	26.4	53.07	3 → 6 → 7 → 9
M	M	L	28.92	54.64	2 → 5 → 8 → 9

Table 4: Near optimal path and obtained sequence at FAF

5 Result Analysis and Discussion

In this section, we present an illustration of the TATA model and a demonstration of the aircraft simulation. Fig. 8 (a) presents the convergence curve of the optimization problem. The visualisation shows that the fittest individual had not improved further after 200 generations.

The simulation result presented in this section is the optimal sequence of GA based optimization and path planning based optimal sequence of the TATA algorithm. An optimal solution of TATA and GA based arrival sequence is shown in table 4. Notice that the arrival sequence of the GA and arrival sequence of the path planning algorithm (TATA) are very close. The TATA algorithm also provides the estimated arrival time at FAF. Separation is maintained between all aircraft at all waypoints at all times. A significant contribution of the TATA algorithm is to provide detailed guidance to the ATC i.e., the path planning, maximization of runway capacity and the estimated arrival time at FAF as shown in table 4.

To observe the mutual closeness of the GA based optimal sequence and the TATA sequence, the simulation is conducted 30 times. The average inter-arrival time is shown in Fig.8(b). Note that the path planning based optimal sequence took slightly greater time, however it is only 0.51%. Finally, we analyze the time-space diagram of the GA based optimal sequence and the TATA based optimal sequence. Fig. 9 shows the comparison of the obtained optimal sequence

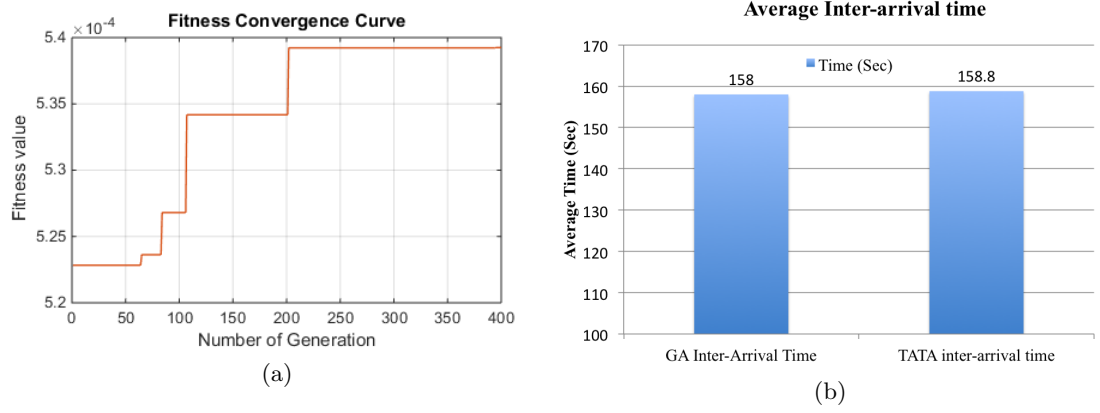


Fig. 8: Simulation result (a) Fitness Generation Curve (b) Average inter-arrival time/window (Seconds)

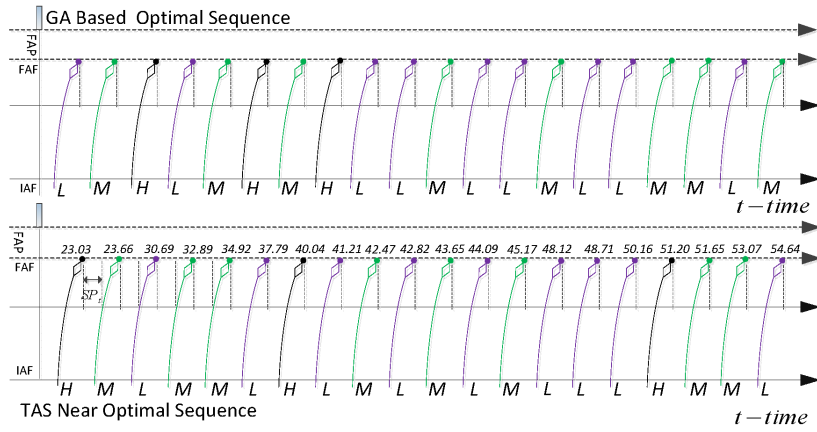


Fig. 9: Time-space diagram of GA optimal sequence VS TATA near optimal sequence

from the traditional GA approach and the TATA based approach. Note here, an interesting result for the best possible sequences in both the cases is that similar wake category aircraft are positioned side by side to reduce the inter-arrival time. A 3D trajectory network is depicted based on the traversed path of the TATA based optimal sequence as presented in Fig.7.

6 Conclusion

In this paper, we addressed one of the common challenges faced by ATC in TAS. How to plan the arrival path of aircraft in transition airspace such that they are

conflict free and their inter-arrival time is minimized. State-of-the-art methods provide the optimal sequence which minimizes the inter-arrival time given an arrival sequence of aircraft, with population based search methods being highly effective. We proposed a GA based path planning technique which can not only achieve an optimal sequence but also address conflict between arriving aircraft and resolve them. The proposed algorithm fills an important gap in advising ATC on arrival aircraft path planning and sequencing to achieve a conflict free optimal sequence which reduces the inter-arrival time which in turn increases the runway capacity. However, this approach comes at the cost of some airborne delay which stems from aircraft speed manoeuvres for conflict resolution.

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