A Matheuristic Approach for the p-Cable Trench Problem

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Abstract. The p-Cable Trench Problem is a telecommunications network design problem, which jointly considers cable and trench installation costs and addresses the optimal location of p facilities. In this work, a matheuristic approach based on the POPMUSIC (Partial Optimization Metaheuristic under Special Intensification Conditions) framework is developed. The inspected neighborhoods for building sub-problems include lexicographic as well as nearest neighbor measures. Using benchmark data available from literature it is shown that existing results can be outperformed.

1 Introduction

The Cable Trench Problem (CTP) reflects a scenario that appears in the installation of information technology infrastructure. In particular, it joins two cost types that appear in the construction of wire-based networks, namely cost for installation of cables and cost for preparing trenches. A trench may contain more than one cable such that a solution has to balance lengths of the cables on the one hand and the distance covered by the trenches on the other hand. As a result, the CTP combines the problems of finding a shortest path tree and of finding a minimum spanning tree. The CTP was proposed by [4] for the problem of connecting buildings to a central facility on a campus. Recent publications suggest further applications. In [5] a problem from bioinformatics, the representation of vascular network connectivity in medical image analysis is addressed by solving a Generalized CTP (GCTP). Moreover, [6] models the setup of a low-frequency radioastronomy station by applying a GCTP. An extension to the CTP is the p-CTP proposed by [1] and now introduced in more detail.

Let $G = (V, E)$ be a connected graph with nodes $i \in V$ and directed edges $e \in E$. For each edge (i, j) in E, the cost of installing one cable is given by $D_{ij} > 0$ and the cost of preparing a trench is denoted by $C_{ij} > D_{ij}$. In contrast to the CTP, where each node has to be connected to a given, single source node, the p-CTP requires to open exactly p facilities. The goal is to choose p of the $n = |V|$ nodes to act as facilities and to assign the remaining nodes to these p facilities, such that the total cost for cable and trenches is minimized.

A small example for a p-CTP is presented in Figs. 1 and 2. We consider a graph with $n = 11$ nodes and fix $p = 2$, i.e., two facilities shall be opened.

P. Festa et al. (Eds.): LION 2016, LNCS 10079, pp. 247–252, 2016.
DOI: 10.1007/978-3-319-50349-3_19

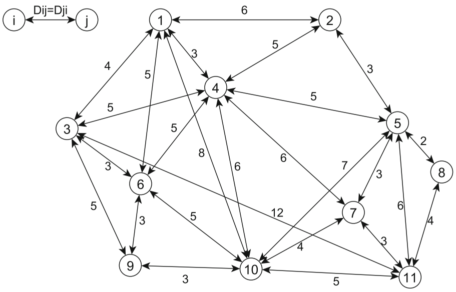

Fig. 1. Instance, $n = 11, p = 2$.

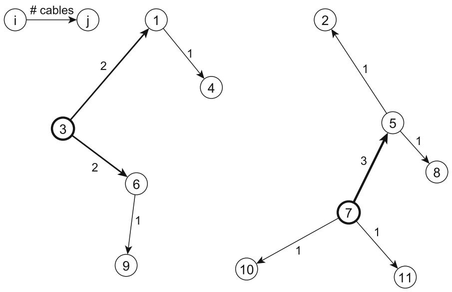

Fig. 2. Optimal solution: cost 97.

Moreover, Fig. 1 illustrates the graph G of the instance together with cable costs D_{ij} for each edge $(i, j) \in E$. The cost for preparing a trench on any edge (i, j) is fixed to $C_{ij} = 2D_{ij}$. An optimal solution is presented in Fig. 2. Nodes 3 and 7 are facility nodes and are indicated by bold circles. Moreover, each edge (i, j) is labeled with the number of installed cables (# cables). The cost of this solution is 97 and divides into trenching cost of 56 and cable cost of 41.

In [1], a mixed-integer programming formulation is proposed for the p-CTP and used to solve instances with up to 200 nodes. Nevertheless, when the dimensions of the instances increase, the solver runs out of memory. Furthermore, two heuristics based on Lagrangean relaxation are provided and tested for instances of up to 300 nodes. In order to improve the solution quality and avoid the memory-fault status, we propose in this work a matheuristic approach for the p-CTP. In particular, the Partial Optimization Metaheuristic under Special Intensification Conditions (POPMUSIC) [3] is applied. POPMUSIC addresses large instances by decomposing them into a set of parts. Subsets of parts are bundled and then used to form sub-problems for subsequently solving them.

The adaption of the POPMUSIC for the p-CTP is described in more detail in Sect. 2. Afterwards, numerical experiments are provided in Sect. 3 and the paper closes with concluding remarks in Sect. 4.

2 A POPMUSIC Approach for the p-CTP

The basic idea of POPMUSIC, proposed in [3], is to split an available solution S of the problem into t parts $part_1, part_2, \ldots, part_t$ and joining some of them to build a sub-problem R. To construct R, first a particular part, namely $part_{seed}$, is selected. Afterwards, r parts closest to $part_{seed}$ are merged with $part_{seed}$ to produce the sub-problem R. In order to determine the closeness of the parts, a distance measure is defined. Once a sub-problem R is constructed, it is solved by using an approximate or an exact solution approach. If parts and sub-problems are defined in an appropriate way, every improvement of a sub-problem corresponds to an improvement of the solution S. This process is repeated until the solution contains no sub-problem that can be improved.

Algorithm 1 depicts the POPMUSIC framework. An initial solution S is generated (line 1). Once it is build, the next step is to divide the solution into t parts (line 2). Then, a seed, $part_{seed}$, is selected (line 5). The sub-problem R is constructed by considering its r nearest parts according to a distance measure (line 6). In this regard, the unique parameter of this framework, r, is used for delimiting the size of the sub-problems. The sub-problem R is then solved by an approximate or exact procedure (line 7). In this framework, the set O gives the seed parts that correspond to sub-problems that have been unsuccessfully optimized. Once O contains all the parts of the complete solution (line 4), the process stops as all sub-problems have been examined without success.

Algorithm 1. POPMUSIC framework

```
1  Generate an initial solution S at random
2  Decompose S in t parts, H = {part_1, ..., part_t}
3  Set O = ∅
4  while O ≠ {part_1, ..., part_t} do
5      Select a seed part_seed ∉ O
6      Build sub-problem R composed of r parts of S closest to part_seed
7      Optimize R by using an approximate or exact solution approach
8      if R has been improved then
9          Update solution S
10         O ← ∅
11     end
12     else
13         O ∪ {part_seed}
14     end
15 end
16 return the improved solution S
```

In order to develop a POPMUSIC approach for the p-CTP, an initial solution is decomposed by considering those p nodes selected to be the facilities. All trenches and cables departing from each facility including the assigned nodes can be seen as a sub-network. Thus, in the context of POPMUSIC, the size of a solution is $t = p$ and each part is a sub-network induced by a facility. In the example provided in Fig. 2, the size of the solution structure is $t = 2$ and it is composed by nodes $part_1 = 3$ and $part_2 = 7$.

The sub-problems are built by means of the sub-networks represented by their starting nodes and the associated edges and nodes. Therefore, in the example provided in Fig. 2, when building a sub-problem of size $r = 2$ with the seed part $part_1 = 3$ all the nodes belonging to the corresponding sub-networks form the sub-problem. In the case of the aforementioned example, the new sub-problem may consider all the nodes from the network starting with $part_1 = 3$ and $part_2 = 7$. Moreover, for building the sub-problems, different measures or strategies can be used to indicate the closeness of the parts among themselves:

– Lexicographic: The sub-problems are grouped according to the indexes of the parts. That is, all the nodes belonging to $part_1$ are grouped to those belonging to $part_2$ if $r = 1$, also to those of $part_3$ if $r = 2$, and so on. For instance, for a solution divided into 4 parts and $r = 2$, we can have the following sub-problems, $R = \{part_1, part_2\}$, $R = \{part_2, part_3\}$, $R = \{part_3, part_4\}$, and $R = \{part_4, part_1\}$.
– Distance: This strategy takes into account the minimum distance between the facilities. For any $part_i \in H$, let i^* be the route node of $part_i$. Then the distance between $part_i$ and $part_j$ is given as $\bar{D}_{ij} = D_{i^*j^*}$, i.e., by the cable costs assigned to edge (i^*, j^*). If (i^*, j^*) does not exist, a high-enough value is assigned. The construction of the sub-problem is then performed in a greedy way. Once the seed $part_i \in H$ has been selected, that part with the minimum arc distance is assigned. That is, one $part_j = argmin(\bar{D}_{ij})_{part_j \in H, j \neq i}$ is chosen. For $r > 2$, the following parts are added taking into account the minimum average arc distance to the already assigned parts such that the next part to be added is calculated by means of $argmin(\sum_{part_i \in R} \bar{D}_{ij})_{part_j \in H \setminus R}$.

Once the sub-problem has been formed, it has to be solved by an approximate or exact method. In this work, we investigate the approach of applying a branch and cut method provided by a general-purpose solver such as CPLEX. The rationale behind this is (i) to provide flexibility in terms of not requiring to develop specific solvers for the problem itself, (ii) investigate the advantage of decomposing the problem for large-sized problem instances, and (iii) provide a competitive solution approach for addressing this problem in terms of solution quality. At this point, we may stress that dividing the problem into parts allows to address memory problems as the one indicated by Marianov et al. [1] for large-sized instances, where directly managing them may require high-amounts of computational memory.

3 Numerical Results

The computational experiments were conducted on a computer with an Intel i7 CPU 3.50 GHz and 6 GB of RAM, restricted to use one CPU. The model was implemented in CPLEX 12.6. The instances used in this work are those large-sized ones from [1] for the p-CTP.

Table 1 shows the results provided by the best approach reported in the literature based on a Lagrangean relaxation [1] and the results of our POPMUSIC approach with $t = p$ and $r = 0.5p$, for both measures distance (dist) and lexicographic (lex). Moreover, with the aim of reducing the computational time, a modified stopping criterion is realized in rPOPMUSIC. In this version, the algorithm stops if the set O, see Algorithm 1, line 4, contains r elements. That is, in the experiments, rPOPMUSIC stops if $0.5p$ parts have been unsuccessfully examined. The relative error of each approach is calculated by means of the lower bound provided by the Lagrangean approach. Moreover, it should be mentioned that the Lagrangean approach is executed until the step size is lower than 0.0001. Therefore, due to the fact that both approaches reach their respective stopping

Table 1. Numerical results for the large-sized instances provided in [1]

		Lagrangean relax. heur. [1]				POPMUSIC - dist			POPMUSIC - lex			rPOPMUSIC - dist			rPOPMUSIC - lex		
Instance	p	UB	LB	Gap (%)	Time (s)	UB	Gap (%)	Time (s)	UB	Gap (%)	Time (s)	UB	Gap (%)	Time (s)	UB	Gap (%)	Time (s)
Pmed11	30	4566	4375.9	4.3	503	**4532**	3.57	3914.65	4533	3.59	2346.04	4533	3.59	336.31	4544	3.84	378.11
Pmed11	60	3295	2907.7	13.3	493.2	**3277**	12.70	4885.59	**3277**	12.70	2688.89	3283	12.91	314.15	3290	13.15	314.85
Pmed11	90	2403	2169.8	10.7	499.1	2372	9.32	6125.81	**2366**	9.04	3915.45	2369	9.18	327.67	2374	9.41	357.65
Pmed12	30	4650	4451	4.5	612.6	4641	4.27	1780.31	**4636**	4.16	1730.32	4648	4.43	361.73	4646	4.38	303.98
Pmed12	60	3432	2924.5	17.4	762.1	**3385**	15.75	4891.89	**3385**	15.75	2211.76	3391	15.95	322.93	3402	16.33	347.29
Pmed12	90	2498	2228.2	12.1	738.7	2481	11.35	8807.69	**2478**	11.21	3935.17	2486	11.57	347.43	2487	11.61	315.09
Pmed13	30	4584	4288.1	6.9	883.2	4575	6.69	2054.39	**4574**	6.67	1604.44	4594	7.13	349.53	4636	8.11	318.38
Pmed13	60	3364	2873.3	17.1	524.3	3346	16.45	5820.94	**3344**	16.38	2426.28	3357	16.83	333.71	3357	16.83	334.94
Pmed13	90	2475	2223.8	11.3	703.8	**2466**	10.89	9492.88	**2466**	10.89	3002.48	2468	10.98	355.67	2469	11.03	302.50
Pmed14	30	4712	4438.7	6.2	347.4	**4676**	5.35	4812.77	**4676**	5.35	2825.34	4720	6.34	317.73	4716	6.25	332.67
Pmed14	60	3416	2752.8	24.1	918.9	**3392**	23.22	3820.63	**3392**	23.22	3076.78	**3392**	23.22	360.93	3410	23.87	340.96
Pmed14	90	2483	2210.5	12.3	880.1	2455	11.06	5514.69	**2453**	10.97	3325.85	2459	11.24	333.69	2454	11.02	342.00
Pmed15	30	4469	4293.9	4.1	577.5	4448	3.59	5771.30	**4437**	3.33	1622.57	4451	3.66	351.36	4450	3.64	356.95
Pmed15	60	3281	2807.9	16.8	748.4	**3266**	16.31	3626.79	**3266**	16.31	2366.72	3302	17.60	342.03	3282	16.88	304.09
Pmed15	90	2456	2236.6	9.8	542.9	**2427**	8.51	10996.87	**2427**	8.51	3472.56	2430	8.65	346.38	**2427**	8.51	346.62
		3472.27	3145.51	11.39	649.01	3449.27	10.60	5487.81	3447.33	10.54	2703.38	3458.87	10.89	340.08	3462.93	10.99	333.07

criteria without running out of memory, the quality of the solutions provided by them is analyzed.

Independent from the measurement used to determine the closeness among the parts, the results are similar in terms of average gap, see Table 1. In terms of computational time, however, differences are observed. In particular, rPOP-MUSIC allows to provide high-quality solutions in less computational time than required for the Lagrangean heuristic. Moreover, new best values for the inspected instances are reported in bold font. It can be highlighted that for all instances, POPMUSIC and its variants are able to provide new best values.

4 Conclusion

In this work, a matheuristic approach for the p-CTP based on the POPMUSIC template is introduced. Two different ways for building the sub-problems and stopping criteria are proposed and assessed. Moreover, solving the sub-problems is done by means of an available mathematical programming formulation and using the standard solver CPLEX. Under this approach, the complete problem is decomposed and can be treated by the solver, while for the full problem, depending on computer performance, it can reach an out-of-memory status. Thus, the POPMUSIC-based approach provides new best values for all the large-sized problem instances considered for this problem.

For future research, we are going to perform an extensive analysis of different configurations for the POPMUSIC parameters (including various options for choosing seed parts) as well as study other stopping criteria and neighborhood measures.

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