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George A. Anastassiou

Intelligent Comparisons II: Operator Inequalities and Approximations

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To My Family

Preface

This is a supplementary, complementary and companion brief monograph to the recently published monograph, by the same author, titled: “Intelligent Comparisons: Analytic Inequalities”, Studies in Computational Intelligence 609, Springer Heidelberg/New York, 2016. It is the analog of the last one, regarding self-adjoint operator well-known inequalities and approximation theory of Korovkin type both in a Hilbert space environment. These are studied for the first time in the literature, and chapters are self-contained and can be read independently. This concise monograph is suitable to be used in related graduate classes and research projects.

The list of presented topics follows:

Self-adjoint operator Korovkin-type quantitative approximations.

Self-adjoint operator Korovkin type and polynomial direct approximations with rates.

Quantitative self-adjoint operator other direct approximations.

Fractional self-adjoint operator Poincaré- and Sobolev-type inequalities.

Self-adjoint operator Ostrowski-type inequalities.

Integer and fractional self-adjoint operator Opial-type inequalities.

Self-adjoint operator Chebyshev–Grüss-type inequalities.

Most general fractional self-adjoint operator representation formulae and operator Poincaré and Sobolev types and other basic inequalities.

Self-adjoint operator harmonic Chebyshev–Grüss inequalities.

Most general self-adjoint operator Chebyshev–Grüss inequalities.

A fractional means inequality.

An extensive list of references is given per chapter.

This book’s results are expected to find applications in many areas of pure and applied mathematics. As such this monograph is suitable for researchers, graduate students, and seminars of the above disciplines, also to be in all science and engineering libraries.

The preparation of this book took place during 2016 in Memphis.

The author likes to thank Prof. Alina Lupas of University of Oradea, Romania, for checking and reading the manuscript.

Memphis, TN, USA
November 2016

George A. Anastassiou

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