Algorithms and Combinatorics

Volume 30

Editorial Board

William J. Cook Ronald Graham Bernhard Korte László Lovász Avi Wigderson Günter M. Ziegler More information about this series at http://www.springer.com/series/13

Alexander Barvinok

Combinatorics and Complexity of Partition Functions



Alexander Barvinok Department of Mathematics University of Michigan Ann Arbor, MI USA

ISSN 0937-5511 Algorithms and Combinatorics ISBN 978-3-319-51828-2 DOI 10.1007/978-3-319-51829-9

ISSN 2197-6783 (electronic)

ISBN 978-3-319-51829-9 (eBook)

Library of Congress Control Number: 2016963427

Mathematics Subject Classification (2010): 05A05, 05A16, 05C31, 05C50, 05C65, 05C70, 15A15, 15A51, 15A57, 15A69, 30A06, 30A08, 30A82, 30C15, 30E10, 37A60, 37E05, 41A05, 41A10, 60C05, 68A10, 68A20, 68E10, 68R05, 68W25, 82A25, 82A67, 82A68, 90C25, 90C27

© Springer International Publishing AG 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Contents

1	Intr	oduction	1	
2	Preliminaries			
	2.1	Convexity	9	
	2.2	Polynomial Approximations	23	
	2.3	Polynomials with Real Roots	28	
	2.4	II-Stable Polynomials III-Stable Polynomials	31	
	2.5	D-Stable Polynomials	38	
3	Peri	nanents	47	
	3.1	Permanents	47	
	3.2	Permanents of Non-negative Matrices		
		and H-Stable Polynomials	52	
	3.3	The van der Waerden Inequality and Its Extensions	55	
	3.4	The Bregman–Minc Inequality and Its Corollaries	58	
	3.5	Matrix Scaling	64	
	3.6	Permanents of Complex Matrices	72	
	3.7	Approximating Permanents of Positive Matrices	78	
	3.8	Permanents of α-Conditioned Matrices and Permutations		
		with Few Cycles	85	
	3.9	Concluding Remarks	88	
4	Haf	nians and Multidimensional Permanents	93	
	4.1	Hafnians	93	
	4.2	Concentration of Hafnians of α-Conditioned Doubly Stochastic		
		Matrices	101	
	4.3	Hafnians and Pfaffians	107	
	4.4	Multidimensional Permanents	116	
	4.5	Mixed Discriminants	129	
	4.6	A Version of Bregman-Minc Inequalities for Mixed		
		Discriminants	135	

vi Contents

5	The	Matching Polynomial	145	
J	5.1	Matching Polynomial	145	
	5.2	Correlation Decay for the Matching Polynomial	150	
	5.3	Matching Polynomials of Bipartite Graphs	158	
	5.4	The Bethe-Entropy Lower Bound	166	
	5.5	Hypergraph Matching Polynomial	174	
6	The	Independence Polynomial	181	
	6.1	The Independence Polynomial of a Graph	181	
	6.2	The Independence Polynomial of Regular Graphs	189	
	6.3	Correlation Decay for Regular Trees	196	
	6.4	Correlation Decay for General Graphs	205	
	6.5	The Roots on and Near the Real Axis	216	
	6.6	On the Local Nature of Independent Sets	224	
7	The	Graph Homomorphism Partition Function	229	
	7.1	The Graph Homomorphism Partition Function	229	
	7.2	Sharpening in the Case of a Positive Real Matrix	238	
	7.3	Graph Homomorphisms with Multiplicities	244	
	7.4	The Lee-Yang Circle Theorem and the Ising Model	258	
8	Partition Functions of Integer Flows		269	
	8.1	The Partition Function of 0-1 Flows	269	
	8.2	The Partition Function of Integer Flows	273	
	8.3	Approximate Log-Concavity	278	
	8.4	Bounds for the Partition Function	284	
	8.5	Concluding Remarks: Partition Functions for Integer Points		
		in Polyhedra	286	
Re	References			
Index				