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Enric Trillas

On the Logos: A Naïve View on Ordinary Reasoning and Fuzzy Logic



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Foreword

-In the beginning was the Logos (John's Gospel)-

Just looking at the title and observing that formulae constellate the pages without the overwhelming presence that has become customary in scientific writing, provides a refreshing sensation of going back to the Greek serenity hinted at by the title.

The title is, usually, an indication of what the author had in mind when he began to conceive of the book. So, let us start from it: *On the Logos*. Oh it is truly demanding! In the Prelude Enric explains that he refers to the “capabilities of reasoning and expressing in some plain language” that are “shared by all mentally sane people.” We (and the reader should) go back to it after finishing reading the book.

Now we start from the subtitle and the concepts he presents in the first pages. Well, these concepts look more attainable. Ordinary reasoning is a very difficult and challenging theme and I know that it is a comprehensive argument for attainment for which Enric in the last years has used all the technical, formal, and conceptual instruments of his toolbox. It is what he has already done not only in many scholarly papers but also in two recent books, one in Catalan, the other in Spanish. But the present one is something different: it is neither a translation nor a reshuffling of them. It is really a new “synthesis”. Synthesis of what? Of all his research, a summary of the path Enric has followed over the years, but also of the questions, the crucial questions that have accompanied the scientific revolutions of the twentieth century—in physics, in mathematics, in logic—and, among them, also in the newcomer to the club of scientific disciplines that can generically identify with “information sciences” but still lacks a stable name after having changed many over the years (from cybernetics to AI, cognitive sciences, etc.).

These were paths that Enric naturally crossed during his studies and his career. However, he did more: he searched and looked for unusual aspects, pushed by his intellectual curiosity and open-minded approach to these problems. The book spans many chapters dealing with or touching upon such different topics as “meaning”, “thinking”, “analogy”, “(ordinary) reasoning”, or “reasoning in quantum physics”. Different topics but united and unified by the common aim of contributing to

understanding them by using a scientific approach to natural (better, ordinary, or plain) language and reasoning, which is the pervasive thread of the book.

“Scientific” for Enric means—among many other things—“measurable”. Also meaning should be measured. This measure, however, needs to come out of a correct adequate modeling of what we are studying, not out of a mechanical routine application of already available results, not the routine work of “applied mathematics” that superimposes known techniques and results, in a sort of Procrustean bed.

We must be aware, as Enric writes, that “plain languages are not the creation of a single person, but are slowly generated by linguistic interactions between groups of people; plain languages are socially constructed over the course of time, and words acquire meaning along such interactions.” Clear! Is it also simple doing a model of this? Rhetorical question.

I am inclined to think that Enric does not believe in “applied” mathematics as a separate category, but in a “new math” for “new problems” (a new math *adequate* to the complexity and novelty of the problems). In a sense this is not customary today (many people use not only the expression “applied mathematics” but also the one of “applied mathematician”); it is natural, anyway, and above all, is not new at all. (Isn’t that what Newton did when he invented the calculus for describing Nature?) Newton was not an applied mathematician.

The division of the book into two parts, one dedicated to “sowing” ideas, the other to “gathering” questions can also be seen as typical of Enric’s style (human before the scientific, and—for this same reason—strongly scientific). We must propose and defend new ideas trying to convince everyone that these can be good points of departure for further investigations (fallible, of course, as is everything in science) but, at the same time, we must ask new questions, trying to discuss them openly involving in the project as many people and attitudes as possible in order to reach a shared common point from which to start (and later divide). Let me write that I saw with great interest that the problem of “(non) monotony in reasoning” has been put in this second part. We really should share a few common conceptual points before trying to confront the question of constructing quantitative models of this feature of reasoning, simple and clear models, not cumbersome machineries of which the scientific literature is full and which remind us of Ptolemy’s epicycles.

So, after all, Enric was right in calling his book *On the Logos* inasmuch as not only, “In the beginning was the logos,” but, as John goes on to write, “All things were made through the same, and without the same nothing was made that was made.” The “same”, as I read it, is the Logos. In addition to the Greek use of the term, we perhaps should also take into account the use by John in his Gospel. Are we dwelling outside our proper territory? I do not think so. Let me be clear at this point. Enric took the term seriously and the reader should take it seriously too. Maybe Enric has in mind only, or mainly, the Greek interpretation and would consider the present extension improper. I think, however, that every reader should take into account all the possible interpretations of this as well as all informal notions we encounter reading the book (the book is doing exactly this regarding ordinary reasoning).

What John did was to try to connect Greek concepts and philosophy with the “needs” of the new, innovative, religion. Today science and scientific knowledge and tradition play a crucial role in our society. We should be able to do similar difficult and profound conceptual operations when trying to scientifically understand and address difficult questions. The problem of the way in which the human being reasons, in general, not when he is concerned with specific and limited chapters of his action, is one such big question.

This is what Frege attempted to do, and Boole before him, the latter successfully but, seen from today’s perspective, in a very limited range, and the former constructing a wonderful cathedral that, unfortunately, was based on very unstable ground. His project, his program, however, was so grandiose and magnificent that a lot of today’s philosophy of language is still full of his ideas.

In this perspective, in our reading, the Logos is at the same time both the “Informal,” the “Tao,” the “Chaos,” and what helps in putting order in it, the “Reason,” the “Measure.” In this second role we have here something very similar to what is alluded to in the title (and the ideas) of a book by Nelson Goodman, *Ways of Worldmaking*. In fact, trying to understand (pieces) of Nature, we are constructing “worlds,” or, as scientists and mathematicians say more modestly, “models.”

As Enric writes, “A model should come from some knowledge of the subject, and a good model allows finding novelties. In the case of ordinary reasoning, models should come from the current *scarce* knowledge of it” (my *Italics*).

A model of such a general thing not only is, obviously, very difficult to obtain, address, and study but presents additional problems (apparently unsurmountable) also of methodological and epistemological type, because we are anyway obliged to delimit what we want to study without losing essential features of what we want to understand. This theme periodically comes up in the book.

We cannot but congratulate Enric for this book that presents the first and basic bricks of what appears to be a very interesting and ambitious research project whose aim is one of attaining and solving one of the most challenging and difficult open problems, a project based on a few already assessed basic results and which starts from and encapsulates many findings of the research done by himself. An open project, however, that is open to the finding of nearby disciplines, mainly neurosciences, and usefully open to suggest crucial questions also to these cognate disciplines.

Without doubt many characters will appear on the stage and many of those already present will change their roles as soon as neurosciences provide us new relevant information. However, we dare to say that the overall structure of the book is general and robust enough to sustain and support many changes.

As it stands, the book seems to conclude the parable of the scientific quest done by Enric, not in the sense that he will not provide, in the future, other challenging contributions, but in the sense that in this book both the philosophical questions (which have always been at the core of his intellectual interests and curiosity) and his mathematical findings (that can give support, flesh, and substance to the

disembodied ideas which are the most fantastic products of human creativity and imagination) are here fused together in an admirable way.

However, I am sure that the present synthesis on the Logos, on his thoughts on “Thought”, are not his final word (his final logos). And we, while appreciating the way in which he has addressed—*now*—these difficult questions, remain waiting for the unexpected suggestions and comments that Enric will provide as soon as new findings ask for new paths to be followed.

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Prelude: Guessing, Telling, and Computing

A new frontier seems to be opened for natural science thanks to the advances in neuroscience's knowledge concerning the human brain's functioning and, in particular, its derived natural phenomenon called thinking. Before such borders will be actually crossed, it cannot be foreseen how the essentially person-driven manifestation of thinking called ordinary, everyday, or commonsense reasoning, will be scientifically understood.

Thinking comprises much more than reasoning, and it is beyond doubt that ordinary reasoning, the spring of rationality, is mainly expressed in natural language's words and statements which, by consequence, can be seen as carriers of information; reasoning and language are in a deep semantic interlinking. Consequently, it seems reasonable to study ordinary reasoning pragmatically by not keeping it too far from its usual expression in natural language; natural language is acquired and fueled by ordinary reasoning, and reciprocally.

It should be pointed out that the expression "natural language" only has sense for distinguishing the plain everyday languages from the artificial ones built up in the formal sciences, but that, actually, a plain language is not properly something naturally fixed, but continuously evolving, influenced, and "loaded" by, for instance, the culture surrounding its speakers.

At the end, the capabilities of reasoning and of expressing themselves in some plain language, the old Greek's *Logos*, are two essential and intermingled characteristics of rationality shared by all mentally sane people on the Earth; at least, it is thanks to plain language that a person can know the reasoning made by other people. *Logos* was translated into Latin by *ratio*, not referring to a ratio of numbers but to a balance between what is pro and what is con in respect to some statements submitted to scrutiny. For this reason, it cannot seem too odd to try to accomplish a symbolic study of ordinary reasoning as it is done by people, through plain words and statements, just excluding contradictions but neither with the strong constraints imposed by "logical laws" (not always presumable in language), nor abusing any artificial language, but keeping the study as close as possible to the ways in which ordinary people seem to reason. That is, done in a mathematically naïve style.

For instance, the classical representation of the linguistic “and” by a lattice conjunction is abusive with respect to, at least, the presumptions of its commutative property, of being the greatest element below the statements the “and” joins, and always identifying “ p and p ” with “ p ”. Formal representations such as a lattice’s, impose on plain language and ordinary reasoning “laws” that often cannot be presumed and which, in any case, should be carefully tested before introducing them.

Because understanding what words actually denote is essential for reasoning, the study of ordinary reasoning requires scientifically domesticating the (dark) philosophical concept of meaning. As shown here, such domestication can be done in a form similar to how probability was domesticated, even if it were in the limited conceptual and strongly structured frames of Boolean algebras and orthomodular lattices. Inasmuch as meaning is attributed to words in any setting, such domestication should be done in the general and, in principle, mathematically unstructured frame of plain language, by giving extension to meaning, that is, measuring it; measuring is but a necessity and a characteristic of science. After doing this, it should be analyzed as to what ordinary reasoning consists in, the role analogy plays as a first engine of it, and the difference between deduction and induction clarified, that is, between the extremes of deductively deploying what is just hidden in the departure’s information, or evidence, and reaching something new, or creative reasoning. Deduction and creation are the pearls of reasoning, gracing both the refuting of arguments and the abducting of hypotheses, as well as facilitating the proof of “theorems” in the deductive own formalism of mathematics. Because deducing in ordinary reasoning often leads to contradiction, it should be differentiated from deducing in a formal frame where reaching contradictions is never accepted.

In this respect there is a recent development remembering, up to some point, how the old scholastic logic did confront reasoning, but was concerned with a symbolic form of representing ordinary reasoning. A development that, based on language and very few and soft formal constraints, came directly from the conceptual skeleton of some fuzzy logic views, indirectly from the neopositivistic philosophical reflections made by the Vienna Circle’s members, and also from some mathematical studies.

It is on these antecedents that what follows is based by following, in its argumentation, William of Occam’s razor methodological rule, advising us not to introduce more entities than those strictly necessary, but also taking into account Karl Menger’s balancing addenda of not introducing fewer entities than those allowing us to capture something new, previously unseen or unknown. It should be remarked that such rules refer here to the considered linguistic, or reasoning’s operations, and the “laws” to which they are supposedly submitted for obtaining both a symbolic representation and a calculus able, at the end, to translate them into a formal framework.

At least and perhaps, such developments can be suggestive for those aiming at the Menger’s “exact thinking,” and wishing to reflect on the flexible, neither formal, nor always deductive, ordinary reasoning. It is done in what can be possible,

by pragmatically proceeding in ways simpler than those currently covered under the label of “mathematical logic” that, usually, go on under an artificial language, rigid and strictly fixed by some finite number of axioms. A novelty in the developments is the classification of conjectures in consequences, hypotheses, and speculations. Once introduced, for the first time, this last particular and deductively wild third class of conjectures, formerly not previously differentiated in the literature, seems to produce, in association with analogy, the creative reasoning that, without previous knowledge, analogy, and speculation, seems to be impossible. A good enough example is the story of how the German chemist August Kekulé discovered the structure of the benzene molecule after an 1861 fantasy dream of a snake eating its own tail (the old *ouroboros* of alchemy), a discovery that later on, and thanks to the forthcoming theory of the chemical molecular structure, showed its great fertility by leading to the success of the German industry of colorants, a discovery that seems to be impossible without the previous knowledge Kekulé held on benzene’s properties.

It is very difficult to arrive at something new without seeing a neighborhood of previous knowledge of the subject, and constituted by hypotheses, refutations, consequences, speculations, and analogies. Continuing Karl Popper’s ideas, if scientific reasoning always consists in conjectures and refutations, it is also important to know how they are reached or abandoned; at the end, and at least in the scientific context of searching, analogy with former knowledge is basic.

This book just pretends to deal with a pragmatic view of common sense reasoning shared by both specialized and ordinary people, and in such a form that, at the end, can admit to being particularized (by adding more suppositions) into specialized modes of reasoning, such as they are classical and already known, models for several types of formal reasoning, namely the models for precise Boolean, quantum orthomodular, and the imprecise De Morgan or Zadeh, each one living in different rooms of the common house of ordinary reasoning.

At the end of each section and as recommended reading, a list of references appears. All of them influenced what is in the book, and are only suggestions to the reader that can help her or him to go deeper concerning the corresponding subjects. After the final section, a complementary list of books is added; without reading at least some of them, no good enough perspective on the ground subject of this book may be grasped.

Ordinary reasoning is neither, in the large, of a formal deductive character, nor is always done in a previously formalized setting. For its part, natural language is not deeply studied here in itself but taken, ideally, as it appears to be; apart from the essential domestication of the meaning of words and statements, only some basic topics of language leading to building statements with words, such as conditionality, connectives, and modifiers, are analyzed and, as much as possible, try to be mathematically modeled. In the same vein, the new model of the meaning of words allows us to clarify what a fuzzy set and its membership functions are and can, eventually, help to shed some light on the debate among the two basic interpretations of probability, the objective and the subjective, and for what concerns, at least, plain language’s use of “probable”, “possible”, and “uncertain”.

Even if, further on, the progress neuroscientists are making regarding the knowledge of thinking can force us to modify the views sustained here, the presented mathematical model (enclosing them) can be of some help towards a future and, possibly just partial, undoing what is known as the Gordian knot of artificial intelligence, that is, the symbolic mastering of commonsense reasoning in view of its mechanizing through computer algorithms and heuristic methods.

What follows is presented with the aim of trying to describe symbolically, without presuming too strong algebraic structures, what seems to surround the naïve expression, “*Questioning, Guessing, Telling, and Computing*,” a scant but not totally wrong résumé for the concept of rationality, the old *Logos*.

This book could even be considered a trial towards a theoretical, but naïve, approach to rationality that, provided someday were transformed in a new experimental science, a physics of language and reasoning intermingling controlled experimentation and theory, with each reinforcing the other, could mean a paramount success towards a deepest knowledge of *Logos*.

It should still be added that this is not a book on certainties, but on questions and doubts. It is not full of technicalities and theorems, but consists in some reflections that, often supported by mathematical developments can, eventually, lead to new views of how to study ordinary reasoning and also fuzzy logic. In general, to know “why” something is what it is, it is better benefiting from previous analysis on “how” it can be managed; our “something” is ordinary reasoning, and our “analysis” concerns how it can be represented in a naïve symbolic form able to reveal some of its characteristics.

No final doctrine is tried to be imparted, and the core of what follows can be seen rooted in the conceptual troubles that can come from always presuming a strict mathematical framework, as is remarked in what follows. The agendas of this book and of mathematical logic, either crisp or fuzzy, are different; actually, at most they facilitate only a “view from the ground” of ordinary reasoning, and of classical and fuzzy logics. What is mainly offered is questioning of ordinary reasoning.

Concerning fuzzy logic, more than 50 years after Lotfi A. Zadeh introduced fuzzy sets, and also more than 20 after he introduced his first and brilliant ideas on *Computing with Words*, the moment seems to have arrived of theoretically rethinking fuzzy sets from their very grounds, plain language and ordinary reasoning, in short, looking at it from down to top, but not from top to down; that is, with a true scientific seriousness, and not only as either a kind of abstract playing, or as a complex of recipes just devoted to immediate applications, even if both existing approaches should not be disdained in the measure that they could help to see, down to top, fuzzy logic and computing with words.