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Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

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Andrés Jiménez-Losada

Models for Cooperative Games with Fuzzy Relations among the Agents

Fuzzy Communication, Proximity Relation and Fuzzy Permission

Springer

Andrés Jiménez-Losada
Departamento de Matemática Aplicada II
Universidad de Sevilla
Seville
Spain

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A book, as a journey, begins with concern
and it ends with melancholy

José Vasconcelos

To my wife and my daughters, thanks
for being capable of putting up with me.

Foreword

Cooperative game theory can be seen as an axiomatic approach to analyze situations in which the players can make coalitions with worth. In a cooperative game with transferable utility, the worth of a coalition is the maximal profit or the minimal cost for the players in their own coalition. A solution concept is a distribution of the profit (cost) of the great coalition among the players. Stable sets, core, kernel, and several types of values are solution concepts, but I believe that the Shapley value is adapted to very different situations and it enables a great variety of extensions through their combinatorial properties.

This book analyzes values for cooperative games with fuzzy information among the players. Each model uses a different relation: a particular coalition, a coalition structure, a communication structure, a priori union system, a permission structure, or a coercive structure, among other combinatorial structures. It is very remarkable that the work of the author on the value of Shapley has extended the classic models of Aumann and Dreze, Myerson, Owen, Gilles, and van den Brink to the situations of fuzzy information between the players.

Mathematics is learned by doing, exploring new models with new tools, and applying the knowledge obtained. Still remains much work to make, especially in the field of applications of the models to propose new methods of conflict resolution in economic, social, cultural, and political situations that can be analyzed from the new perspective of fuzzy approximation.

When I started my research on the Shapley value in games constrained or defined on combinatorial structures, I could not imagine the level of development achieved by this field in which the Prof. Jiménez-Losada has obtained new and interesting characterizations of the Shapley value in the context of fuzzy information.

I hope that this book will be found useful as a text to start working in a new and exciting field of research defined in its title "Models for Cooperative Games with Fuzzy Relations among the Agents."

Seville, Spain J. Mario Bilbao
January 2017

Preface

Game theory is a mathematical discipline which studies situations of competition and cooperation among several agents or players. This is a consistent definition with its large number of applications. These applications come from economy, sociology, engineering, policy, computation, psychology, or biology. I focus this book in the cooperative branch. This branch analyzes only the outcomes that result in situations of cooperation, those cases where players are grouped in coalitions. In the classical model, there are several basic suppositions. Players are symmetric, and in each coalition, any player is as important as the rest and they cooperate at the same level. Coalitions are all feasible, in the sense that the worth of each coalition does not depend on any particular relation among the players. The use of techniques from determined mathematical structures and fuzzy sets has allowed us to describe better real problems by new models in cooperative games. Numerous studies have been introduced describing certain additional information about the players or the feasibility of the coalitions which modify the cooperation behavior. Fuzzy coalitions defined different levels of participation in a continuous model of cooperation. My work for several years closely with my colleagues in the research group have been focussed in the analysis of games with restricted cooperation, first by certain classical mathematical structures and currently fuzzy cooperation structures.

This book has a double vocation, one to be a treatise and one to be a practical manual. It is treatise on games with a bilateral fuzzy relation among the players. The idea is presenting the difference in the models, and then, I focus on one particular classical solution for games, the Shapley value. It is self-contained, in the sense that all the mean contents about the topic are included. I present several fuzzy models that we have studied particularly in their respective papers but, in the same context, given certain degree of generality. Each model is analyzed first in crisp case and later in the fuzzy option. All the models contain certain nuances which allow the reader to see the results as newness about the subject. But this book is also a good manual for students and researchers, in the sense that all the proofs are included showing different ways of analysis in these situations. So, I opt in each model for an axiomatization in different way. Usually, I present axioms and properties in the most feasible general way. But also I study the last one in particular cases because the refinement of the model permits to use specific properties. Any person, only with a common knowledge base about maths, can follow the book. There are also a lot of examples which show the application of the different proposed formulas and concepts, in numbers: 104 definitions, 39 theorems, and 96 propositions with their proofs, 110 examples, 37 tables, and 53 figures. I hope that the reader will find useful this work.

Seville, Spain
January 2017

Andrés Jiménez-Losada

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Thank you very much.

Contents

Symbols

N	Players
i	Player
n	Number of players
2^N	Coalitions
S	Coalition
$\mathscr{G}^N$	Games
$\wedge, \vee$	Minimum, maximum
u_T	Unanimity game
Δ_T^v	Dividend
v^{svg}	Saving game
v^{dual}	Dual game
$\mathscr{G}_{sa}^N$	Superadditive games
$\mathscr{G}_c^N$	Convex games
$\mathscr{G}_m^N$	Monotone games
$\mathscr{G}_s^N$	Simple games
$v\|_T$	Subgame
f	Value
Θ^N	Permutations
$<_T$	T-ordering
$<_i$	i-ordering
$m^\theta(v)$	Marginal vector
$\phi(v)$	Shapley value
$\int_c$	Choquet integral
$[0,1]^N$	Fuzzy coalitions
τ	Fuzzy coalition
e^S	Canonical vector
$[\tau]_t$	t-cut
$im(\tau)$	Images of fuzzy set
$supp(\tau)$	Support of fuzzy set

v^{cr}	Crisp version of game
$\phi^{cr}(v)$	Crisp Shapley value
$\phi^{d}(v)$	Diagonal value
pl	Partition function
v^{pl}	Fuzziness of game
ml	Multilinear extension
pr	Proportional extension
ch	Choquet extension
$R(N)$	Binary relations
r	Binary relation
N^r, N^ρ	Domain
$L(r), L(\rho)$	Links
$N/r, N/\rho$	Components
$[0,1]^{N\times N}$	Fuzzy binary relations
ρ	Fuzzy binary relation
τ^ρ	Fuzzy domain
r^S, ρ^τ	Relations of coalitions
v_T	Restricted game
$\phi(v, T)$	Extended Shapley value
$v^{T\,dual}$	T-dual game
v_τ^{pl}	pl-restricted game
τ^{pl}	pl-game
$\phi^{pl}(v,\tau)$	pl-extended Shapley value
P_0^N	Coalition structures
$\mathscr{B}$	Coalition structure
$v_{\mathscr{B}}$	Coalitional game
$\mu(v, \mathscr{B})$	Coalitional value
$v^{\mathscr{B}\,dual}$	$\mathscr{B}$-dual game
G^N	Communication structures
$g(v, r)$	Myerson graph worth
v/r	Vertex game
r_{-ij}	Deleting a link in a graph
r_{-i}	Isolating a vertex in a graph
$\mu(v, r)$	Myerson value
v^{rdual}	r-dual game
FG^N	Fuzzy communication structure
ρ_{-ij}^t	Decreasing the level of a link
$\gamma(v,\rho)$	Fuzzy graph worth
pg	Proportional by graphs extension
cg	Choquet by graphs extension
$\gamma^{pl}(v,\rho)$	pl-worth
CV	Choquet by vertices extension
$(v/\rho)^{pl}$	pl-vertex game

$(v/\rho)^{\gamma}$	γ-vertex game
pl^{*}	Induced partition
pb	Probabilistic extension
P^{N}	A priori union systems
$w^{v,\mathscr{B}}$	Quotient game
$v_{p}^{\mathscr{B}}$	p-union game
$\omega(v,\mathscr{B})$	Owen value
C^{N}	Cooperation structures
$\omega(v,r)$	Myerson-Owen value
$null^{v}(r)$	Null components
FC^{N}	Proximity relations
pp	Prox-proportional extension
$[N/\rho]_{pl}$	pl-groups
ρ_{pl}^{K}	K-scaling
$\omega^{pl}(v,\rho)$	pl-Myerson-Owen value, pl-Owen value
FP^{N}	Similarity relations
A^{N}	Permission structures
σ^{r}	Sovereign part
r°	Quasi-reflexive interior
$\hat{r}$	Transitive closure
v^{r}	Permission game
$\bar{\sigma}^{r}$	Coercive part
$\bar{v}^{r}$	Coercive game
$\delta(v,r)$	Local permission value
$\hat{\delta}(v,r)$	Permission value
$\bar{\delta}(v,r)$	Coercive value
σ^{ρ}	Fuzzy sovereign part
ρ°	Weakly reflexive interior
v_{pl}^{ρ}	pl-permission game
$\bar{\sigma}^{\rho}$	Fuzzy coercive part
$\bar{v}_{pl}^{\rho}$	pl-coercive game
$\delta^{pl}(v,r)$	Local pl-permission value
$\hat{\rho}$	Fuzzy transitive closure
$\hat{\delta}^{pl}(v,r)$	pl-permission value
$\bar{\delta}^{pl}(v,r)$	pl-coercive value
η^{ρ}	Probabilistic part