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An Introduction to Computational Origami

Springer

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ISSN 0943-853X ISSN 2197-8409 (electronic)
Texts & Monographs in Symbolic Computation
ISBN 978-3-319-59188-9 ISBN 978-3-319-59189-6 (eBook)
https://doi.org/10.1007/978-3-319-59189-6

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This monograph is written for students and researchers who love origami and take interest in computational and geometric aspects of origami. The term *computational origami* in the title is relatively new. Computational origami is the field of study on origami with extensive use of mathematics and computer science and engineering, in the same vein that computational geometry is related to geometry. This observation brings back to the question of what discipline is origami. In this monograph, we treat origami as a subject of art, a science and technology of shapes. It is an art and science of shapes since a variety of shapes constructed from a piece of paper are studied. A system of faces of a piece of paper connected, superposed and folded indeed produces interesting shapes that deserve full exploration.

With the advancement of computing technology, many disciplines have gained one more attribute of "computational" and found their ways to yet another level of depth and sophistication and the industrial applications. Origami is an excellent example of such discipline. Gains in computational origami by applying computational power are not restricted to the speed and scale. Computational origami also attains rigor, abstraction, and generality that are the characteristics of the main ingredients. They build up the field of study a full-fledged science and engineering. This introductory monograph on origami is a glimpse of how the above transformation has happened and is still evolving.

We hope this monograph is used as a textbook in a course on special topics in applied computing for third-year or fourth-year undergraduate in computer science and applied mathematics, or interdisciplinary master course. We also hope that it is read as a research monograph by researchers interested in origami, geometry, and the applications of algebraic computation. To help the readers which part of the monograph they focus on at the first reading, we outline the monograph below.

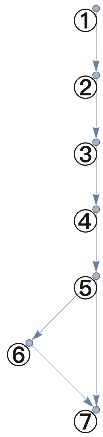
As a monograph in the series of textbooks in symbolic computation, we proceed as follows. We collect some of our research results, glued together with the works of the pioneers in this field, and make the collection into a short coherent story of computational origami. In compiling the materials, we have to assume some maturity level of logic, algebraic, and symbolic manipulation. Since the scope of computational origami is broad, we need the above prerequisite knowledge. However, the examples are mostly from high school mathematics. We hope that if the readers have some difficulty in following some part of this monograph, they refer to supplemental materials. We list them in References to Textbook at the end of the monograph.

After briefly discussing the history of origami as used in daily life and the object of research, we begin with elementary geometry a là Euclid and formalize a simple origami geometry. Next, extend the origami geometry by one more fold rule and then point out the remarkable difference between the Euclidean geometry and the (extended) origami geometry.

Then, we formalize the origami geometry in more generality. Algebraic treatise of the origami geometry realizes the precise construction of lines and points involved in folds. It further enables us to tackle the automated theorem proving in the origami geometry. The logical formulation opens the way to the mathematical interpretation of knot folds. Finally, abstract origami and abstract rewrites of origami enable us to formalize origami construction; and more sophisticated folds that the readers find in origami recipe books.

In all, we have seven chapters. We expect that at the first reading, the readers may choose

1. very beginning of origami viewed computationally - Chapters 1 and 2,
2. until the origami geometry - Chapters 1 - 3,
3. until the logical and algebraic foundation of the origami geometry - Chapters 1 - 5,
4. until Chapters 6 and 7 for readers with strong research motivation.



We have in mind the chapter dependency shown to the right, although we tried to make each chapter self-contained as much as possible. In Appendix B, we give a link to the web site of EOS (E-Origami-System) project. This site is essential for us to communicate with readers. It also provides links to supplementary materials of this monograph and the EOS packages for download together with the tutorial of origami programming.

Tsukuba Japan
May 2020

Acknowledgements

This monograph is one of the outcomes of 30 years of fruitful cooperation between Symbolic Computation Research Group (SCORE), University of Tsukuba, Japan, and Research Institute for Symbolic Computation (RISC), Johannes Kepler University, Austria. I want to thank the colleagues of RISC, in particulars Bruno Buchberger, Franz Winkler, and Peter Paule, who have made the long-term cooperation possible.

All the examples of origami construction in the monograph have been tested and reformulated, when necessary, by EOS. EOS has been developed by the team of SCORE: Tetsuo Ida, Hidekazu Takahashi, Fadoua Ghourabi, Asem Kasem, and Mircea Marin. Yasuhiko Minamide and Cezary Kaliszyk provided us timely suggestions and helped on computer-assisted theorem proving. I am grateful to Bruno Buchberger, Masahiko Sato, Stephen Watt, and Fairouz Kamareddine for stimulating communications on origami, symbolic computation, logic, and automated theorem proving.

The plan of having a monograph collecting, coherently, materials spread in various forms of publications was with us for some time. The Great East Japan Earthquake in 2011 and the subsequent hardships hampered our work. I am grateful for all the people concerned in Springer International and RISC at Johannes Kepler University, who patiently waited for me to complete the monograph and encouraged me all the time.

While writing the monograph, the author has been supported by JSPS research grant Kakenhi 16K00008.

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