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Aggregation Functions in Theory and in Practice

Springer

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Preface

Aggregation functions are usually defined as those functions that are monotonic and that satisfy some boundary conditions. In particular settings, these conditions are relaxed. Aggregation functions are used for data fusion and decision-making. Examples of these functions include means, t-norms and t-conorms, uninorms and nullnorms, copulas and fuzzy integrals (e.g. the Choquet and Sugeno integrals). Besides the aggregation of real inputs, aggregation functions on general/particular lattices are also considered.

This volume collects the final revised manuscripts of 26 accepted contributions of participants to the 9th International Summer School on Aggregation Functions that took place in Skövde (Sweden) on 19–22 June 2017. Note that AGOP conferences are biannually organized by the working group AGOP of the EUSFLAT association, and it is the ninth in a series of AGOP summer schools, including AGOP 2001 (Oviedo, Spain), AGOP 2003 (Alcalá de Henares, Spain), AGOP 2005 (Lugano, Switzerland), AGOP 2007 (Gent, Belgium), AGOP 2009 (Palma de Mallorca, Spain), AGOP 2011 (Benevento, Italy), AGOP 2013 (Pamplona, Spain) and AGOP 2015 (Katowice, Poland). The volume also includes the abstracts of the invited talks and tutorials given in the School. All included contributions were reviewed by PC members and several external reviewers, and they include works from theory and fundamentals of aggregation functions to their use in applications. Together, they provide a good overview of recent trends in research on aggregation functions.

March 2017

Vicenç Torra
Radko Mesiar
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9th International Summer School on Aggregation Functions – AGOP 2017

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Tutorials and Invited Talks

The Role of Aggregation Functions on Auctions

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Auctions are mechanisms for allocating resources (tasks or goods) among self-interested agents [3]. An auction consists in the following four steps:

1. Call for proposals: the auctioneer announces the resources to be committed
2. Bidding: the bidders express their preferences on the resources
3. Winner determination problem (WDP): the auctioneer decides which agents will have the resources
4. Payment: the winner bidders pay to the auctioneer.

This basic mechanism could have several instantiations depending on the role of the participants (forward if the auctioneer sells; reverse if the auctioneer buys), number of bidding sides (one-side when an agent can be either auctioneer or bidder; double-side when the agent can have both roles) bid composition (single attribute or multi-attribute), number of different resources involved (single item versus combinatorial), number of items considered (single-unit versus multi-unit) [1, 6]. Moreover, the strategic decision made in each step depends on the kind of resource being auctioned: static or dynamic (consumable, perishable), divisible or indivisible, controlled or uncontrolled (e.g. public goods) [6].

All of the agents make decisions in order to maximize their utility regarding the selling (auctioneer) or buying (bidders) of the resources, u(R),

- Auctioneer: $u(R) = p - V(R)$
- Bidder: $u(R) = V(R) - p$

where p is the payment made for the resources and $V(R)$ is a valuation function that measures the value of R for the agent. When a single resource is being sold, characterized by a single attribute as, for example, the price, the WDP is simple: take the maximum value. However, when there is a set of resources to be allocated or the number of attributes that characterizes the resources has some dimensionality, then the WDP requires a more complex $V(\cdot)$ function.
Mechanism design [3] is the study concerning on the definition of auction components, as $V(\cdot)$. Other issues include social welfare measures, and dealing with cheaters, among others. Social welfare assesses the quality of the allocation in a global perspective. In that regard, a social welfare measure aggregates either the utility, benefits, satisfaction, or other gratifications of the agents. In recurrent scenarios, in which auctions are repeated over time, an auctioneer could learn trust

models regarding the cheating behaviour of agents that later on conditions the $V(\cdot)$ function too. In this talk, we analyse the use of aggregation functions in all of these issues [2, 8, 12].

First, from a design perspective, the set of requirements that the aggregation function should fulfil to be $V(\cdot)$ will be reviewed, so that the properties desired for the mechanism are guaranteed [8]. There are some frameworks that could help, as for example constraints, in order to verify the properties [11]. Two cases will be studied in detail: the application of aggregation functions on multi-attribute auctions [7, 10] and on combinatorial auctions [14] will be described. Second, the role of the aggregation function in social welfare measures will be presented [9]. Third, an example how the parameters of aggregation functions can be tuned thanks to trust methods will be provided [13].

Cases study will be provided in several application domains, including different types of resources: workflow resource allocation with energy constraints [14], wastewater management [4, 5] and e-services [6].

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References

1. Chevaleyre, Y., Dunne, P.E., Endriss, U., Lang, J., Lemaître, M., Maudet, N., Padget, J., Phelps, S., Rodriguez-Aguilar, J.A., Sousa, P.: Issues in multiagent resource allocation. Informatica **30**(1) (2006)
2. Grabisch, M., Marichal, J.-L., Mesiar, R., Pap, E., Aggregation functions: means. Inf. Sci. **181**(1) (2011)
3. Leyton-Brown, K., Shoham, Y.: Mechanism design and auctions. In: Weiss, G. (ed.) Multiagent Systems, Chap. 7, 2nd edn., pp. 285–32. The MIT Press (2013)
4. Murillo, J., Busquets, D., Dalmau, J., López, B., Muñoz, V., Rodríguez-Roda, I.: Improving urban wastewater management through an auction-based management of discharges. Environ. Model. Softw. **26**(6), 689–696 (2011)
5. Murillo, J., Muñoz, V., Busquets, D., López, B.: Schedule coordination through egalitarian recurrent multi-unit combinatorial auctions. Appl. Intell. **34**(1), 47–63 (2011)
6. Murillo Espinar, J.: Egalitarian behaviour in multi unit combinatorial auctions, Ph.D. thesis, University of Girona (2010)
7. Pla, A.: Multi-attribute auctions: application to workflow management systems, Ph.D. thesis, University of Girona (2014)
8. Pla, A., Lopez, B., Murillo, J.: Multi criteria operators for multi-attribute auctions. In: Torra, V., Narukawa, Y., López, B., Villaret M. (eds,) MDAI 2012. LNCS, vol 7647. Springer, Heidelberg (2012)
9. Pla, A., López, B., Murillo, J.: Multi-dimensional fairness for auction-based resource allocation. Knowl.-Based Syst. **73**, 134–148 (2015)

10. Pla, A., López, B., Murillo, J., Maudet, N.: Multi-attribute auctions with different types of attributes: enacting properties in multi-attribute auctions. Expert Syst. Appl. **41**(10), 4829–4843 (2014)
11. Pla, B., López, A. Murillo, J.: How to demonstrate incentive compatibility in multi-attribute auctions. In: Gibert, K., Botti, V., Reig-Bolaño, R. (eds.) Artificial Intelligence Research and Development (CCIA). Frontiers in Artificial Intelligence and Applications, vol. 256, pp. 303–306 (2013)
12. Torra, V., Narukawa, Y.: Modeling Decisions: Information Fusion and Aggregation Operators. Springer, Heidelberg (2007)
13. Torrent-Fontbona, F., Pla, A., López, B.: New perspective of trust through multi-attribute auctions. In: Papers from the AAAI Workshop in Incentive, Trust in E-Communities, pp. 25–31 (2015)
14. Torrent-Fontbona, F., Pla, B., López, A.: Using multi-attribute combinatorial auctions for resource allocation. In: Müller, J.P., Weyrich, M., Bazzan, A.L.C. (eds.) MATES 2014. LNCS, vol. 8732, pp. 57–71. Springer, Heidelberg (2014)

Aggregation Operators in Information Retrieval

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In the context of Information Retrieval, the issue of employing aggregation operators in various phases of the retrieval process has been extensively investigated in the literature. In particular, these approaches rely on the interpretation of Information Retrieval as a Multi-Criteria Decision Making (MCDM) problem, from various perspectives. The first, more straightforward perspective, is to interpret the overall IR process as a MCDM process aimed at selecting the best alternatives (documents) based on the assessment of the performance of multiple criteria (the keywords specified in a user's query). Another and strongly related perspective is to see the assessment of the overall relevance estimate of a document (still an alternative) to a query as the process of evaluating the performance of several relevance dimensions (e.g. topicality, novelty, recency), which in this case represent the criteria to be aggregated. Another process that may require the application of appropriate aggregation operators is the indexing process, when applied to structured documents. Metasearch constitutes another interesting task that can be seen as an instance of a Multi-Expert Decision Making (MEDM) problem, also strongly relying on the appropriate choice of an aggregation operator. By this task, a user query is separately evaluated by different search engines, each one providing its own relevance assessment of the considered documents. Metasearch aims to merge the ranked lists generated by the various search engines (experts) in response to a query, to the aim of providing a unique, consensual ranked list of results. A quite interesting aspect implied by the above interpretations of various phases of the IR process is that the choice of different aggregation operators can produce different results. In other words, the semantics of aggregation implies an interpretation of the affected process. For example, if considering the aggregation of different relevance assessments for a same query and the same documents, distinct rankings can be obtained by applying distinct aggregation strategies. Despite the potential impact of aggregation on the whole IR process, this aspect has not received the proper attention in the literature. Only recently, some approaches have appeared demonstrating the importance of this issue and its potential impact on the searching process. This lecture aims to shortly review the main contributions that in the literature have made use of aggregation operators in Information Retrieval.

Geometric Analysis on Cantor Sets and Trees

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This is a joint work with A. Björn, J.T. Gill and N. Shanmugalingam [1]. We consider an infinite network represented by a weighted rooted tree which we equip with a metric and measure structure enabling first-order Sobolev spaces and harmonic and p-harmonic functions. This is a special case of a procedure called uniformization, see Bonk, Heinonen and Koskela [2]. The visual boundary of the tree at infinity is an ultrametric space and can be regarded as a Cantor type set, see Semmes [4, 5].

In this setting, we show that the trace of the Sobolev space is exactly a Besov space with an explicit smoothness exponent, cf. Bourdon and Pajot [3]. This, in particular, means that such Besov boundary data have harmonic extensions to the whole tree and it is possible to solve the Dirichlet and obstacle problems with such boundary data. These harmonic extensions can be seen as potentials or stationary flows in the network.

We also consider mappings between pairs of such trees and between their boundaries. It turns out that quasi-symmetries between two Cantor sets exactly extend to rough quasi-isometries between their generating trees, and vice versa.

References

1. Björn, A., Björn, J., Gill, J.T., Shanmugalingam, N.: Geometric analysis on Cantor sets and trees. J. Reine Angew. Math. (to apper). doi:10.1515/crelle-2014-0099, arXiv:1304.0566
2. Bonk, M., Heinonen, J., Koskela, P.: Uniformizing Gromov hyperbolic spaces, Astérisque **270** (2001)
3. Bourdon, M., Pajot, H.: Cohomologie l_p et espaces de Besov. J. Reine Angew. Math. **558**, 85–108 (2003)
4. Semmes, S.: An introduction to the geometry of ultrametric spaces (2007, preprint). arXiv:0711.0709
5. Semmes, S.: Cellular structures, quasisymmetric mappings, and spaces of homogeneous type (2007, preprint). arXiv:0711.1333

A Monometric-Based Approach to Data Aggregation

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Abstract. Data aggregation is a common problem in many fields of application and is historically understood as a process of combining several real values into a single one. However, the aggregation of other types of structured data is lately receiving increasing attention [4]. Some examples are the aggregation of multidimensional data [5], the aggregation of rankings [6] and the aggregation of mappings [3].

As Yager described in his "general theory of information aggregation" [11], a natural approach to data aggregation is based on the search for the element minimizing some notion of "penalty". In the framework of real numbers, this penalty is usually provided with a well-founded semantic basis, for instance, with the requirement of the property of quasi-convexity in the second argument [10]. Some other examples can be found in [2, 12] or in the recent survey on the definition of penalty functions in data aggregation [1]. Unfortunately, outside the framework of real numbers, this well-founded semantic basis is usually disregarded.

This penalty-based approach to data aggregation is similar to that considered in social choice theory for the aggregation of rankings. The monometric rationalization of ranking rules [8] is the branch of social choice theory, where the process of aggregating several rankings is characterized as the minimization of the distance to a consensus state for some appropriate monometric. Formally, a monometric is a function satisfying the axioms of nonnegativity and coincidence of a distance function, while requiring the compatibility with a given betweenness relation [7, 9]. This monometric can be understood as a natural extension of a penalty function outside the framework of real values, where the well-founded semantic basis is provided by the compatibility with the chosen betweenness relation. In this contribution, monometrics and betweenness relations will be considered for different types of structured data (multidimensional data, maps, strings, compositional data, among others), leading to a natural expansion of the definition of a penalty function beyond its current confinement to real values.

References

1. Bustince, H., Beliakov, G., Dimuro, G.P., Bedregal, B., Mesiar, R.: On the definition of penalty functions in data aggregation. Fuzzy Sets, Syst. (accepted). http://dx.doi.org/10.1016/j.fss.2016.09.011
2. Calvo, T., Mesiar, R., Yager, R.R.: Quantitative weights and aggregation. IEEE Trans. Fuzzy Syst. **12**, 62–69 (2004)
3. De Miguel, L., Campóon, M.J., Candeal, J.C., Induráin, E., Paternain, D.: Pointwise aggregation of maps: Its structural functional equation and some applications to social choice theory. Fuzzy Sets Syst. (accepted). http://dx.doi.org/10.1016/j.fss.2016.05.010
4. Gagolewski, M.: Data Fusion. Theory, Methods and Applications, Institute of Computer Science. Polish Academy of Sciences, Warsaw (2015)
5. Gagolewski, M.: Penalty-based aggregation of multidimensional data. Fuzzy Sets, Syst. (accepted). http://dx.doi.org/10.1016/j.fss.2016.12.009
6. Kemeny, J.G.: Mathematics without numbers. Daedalus **88**(4), 577–591 (1959)
7. Pasch, M.: Vorlesungen über neuere Geometrie, vol. 23. Teubner, Leipzig, Berlin (1882)
8. Pérez-Fernández, R., Rademaker, M., Baets, B.: Monometrics and their role in the rationalisation of ranking rules. Inf. Fusion **34**, 16–27 (2017)
9. Pitcher, E., Smiley, M.F.: Transitivities of betweenness. Trans. Am. Math. Soc. **52**(1), 95–114 (1942)
10. Wilkin, T., Beliakov, G.: Weakly monotonic averaging functions. Int. J. Intell. Syst. **30**(2), 144–169 (2015)
11. Yager, R.R.: Toward a general theory of information aggregation. Inf. Sci. **68**, 191–206 (1993)
12. Yager, R.R., Rybalov, A.: Understanding the median as a fusion operator. Int. J. Gen. Syst. **26**, 239–263 (1997)

The Fusion of Uncertain Information: Principles and Examples of Merging Rules Across Uncertainty Theories

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Abstract. We present basic principles for the fusion of incomplete or uncertain information items that should apply regardless of the formalism adopted for representing pieces of information coming from several sources. This formalism can be based on sets, logic, partial orders, possibility theory, belief functions or imprecise probabilities. The presented tutorial is based on past work performed especially with Henri Prade, Weiru Liu and Jianbing Ma, Ronald Yager.

Outline of the Presentation

Information fusion deals with extracting accurate knowledge from possibly conflicting pieces of information stemming from a set of sources, without introducing arbitrary precision [5]. It differs from belief revision [11] and preference aggregation [7]. Information fusion is useful in many areas ranging from databases [6] to image processing [4] and expert opinion aggregation [8]. The main reference for this presentation is the paper [12]. We propose a general notion of information item representing incomplete or uncertain information about the value of an entity of interest. Any kind of uncertain information is supposed to rank possible values in terms of relative plausibility, and explicitly point out impossible ones. Important issues affecting the results of the fusion process, such as the comparison of information items by their relative information content, the consistency of information items, as well as their mutual consistency, are discussed. For each representation setting, we write a version of the fusion postulates, present known fusion rules that obey them, and compare our postulates to existing ones proposed in the past and specific to the representation setting. In the crudest (Boolean) setting (where an information item is just defined as a set of possible values), we show that the understanding of a set in terms of most plausible values, or in terms of non-impossible ones matters for choosing a relevant fusion rule. In particular, in the latter case, our principles justify the old method of maximal consistent subsets [23], while the former is related to the fusion of logical bases [16, 17] that merges sets of preferred values. Then, we consider several formal settings for incomplete or uncertain information items, where our postulates are also instantiated: plausibility orderings [19], qualitative and quantitative possibility distributions [10, 14] and possibilistic knowledge bases [3, 18], the merging of probability distributions [8, 15, 20, 26], of belief functions [9, 13, 24, 25, 28] and of convex sets of probabilities [21, 27]. The aim of this work is to provide a unified picture of fusion rules across such various uncertainty representation settings. Finally, we discuss the connection with the Belnap approach [2] to inference under source-based inconsistent information, and discuss the possibility of non-destructive fusion methods that preserve the original information provided by the sources [1].

References

1. Assaghir, Z., Napoli, A., Kaytoue, M., Dubois, D., Numerical, P.H., Fusion, I.: Lattice of answers with supporting arguments. In: IEEE International Conference on Tools with Artificial Intelligence (ICTAI 2011), Boca Raton, pp. 621–628 (2011)
2. Belnap, N.D.: A useful four-valued logic. In: Dunn, J.M., Epstein, G. (eds.) Modern Uses of Multiple-Valued Logic, pp. 8–37. D. Reidel Publishing Company (1977)
3. Benferhat, S., Dubois, D., Kaci, S., Prade, H.: Possibilistic merging and distance-based fusion of propositional information. Ann. Math. Artif. Intell. **34**(1–3), 217–252 (2002)
4. loch, I.: Fusion of image information under imprecision, uncertainty: numerical methods. In: Della Riccia, G., Lenz, H.-J., Kruse, R. (eds.) Data Fusion and Perception, CISM Courses and Lectures, vol. 431, pp. 135–168 (2001)
5. Bloch, I., Hunter, A. (eds.): Fusion: general concepts and characteristics. Int. J. Intell. Syst. **16**(10–11), 1107–1136 (2001). Special Issue on Data and Knowledge Fusion
6. Cholvy, L., Moral, S.: Merging databases: problems and examples. Int. J. Intell. Syst. **16**(10), 1193–1221 (2001)
7. Chopra, S., Ghose, A.K., Meyer, T.A.: Social choice theory, belief merging, and strategy-proofness. Inf. Fusion **7**(1), 61–79 (2006)
8. Cooke, R.: Experts in Uncertainty. Oxford University Press, Oxford (1991)
9. Destercke, S., Dubois, D.: Idempotent conjunctive combination of belief functions: extending the minimum rule of possibility theory. Inf. Sci. **181**(18), 3925–3945 (2011)
10. Destercke, S., Dubois, D., Chojnacki, E.: Possibilistic information fusion using maximal coherent subsets. IEEE Trans. Fuzzy Syst. **17**(1), 79–92 (2009)
11. Dubois D.: Information fusion and revision in qualitative and quantitative settings. In: Liu, W. (eds.) ECSQARU 2011. LNCS, vol. 6717. Springer, Heidelberg (2011). doi:10.1007/978-3-642-22152-1_1
12. Dubois, D., Liu, W., Ma, J., Prade, H.: The basic principles of uncertain information fusion. An organised review of merging rules in different representation frameworks. Inf. Fusion **32**, 12–39 (2016)
13. Dubois, D., Prade, H.: Representation and combination of uncertainty with belief functions and possibility measures. Comput. Intell. **4**, 244–264 (1988)
14. Dubois, D., Prade, H., Yager, R.R. Merging fuzzy information. In: Bezdek, J.C., Dubois, D., Prade, H. (eds.) Fuzzy Sets in Approximate Reasoning and Information Systems. The Handbooks of Fuzzy Sets Series, pp. 335–401. Kluwer Academic Publishers, Dordrecht (1999)
15. Genest, C., Zidek, J.V.: Combining probability distributions: a critique and an annoted bibliography. Stat. Sci. **1**(1), 114–135 (1986)

16. Konieczny, S., Pino-Pérez, R.: Logic based merging. J. Philos. Logic **40**(2), 239–270 (2011)
17. Liberatore, P., Schaerf, M.: Arbitration (or how to merge knowledge bases). IEEE Trans. Knowl. Data Eng. **10**(1), 76–90 (1998)
18. Liu, W., Qi, G., Bell, D.A.: Adaptive merging of prioritized knowledge bases. Fundamenta Informaticae **73**(3), 389–407 (2006)
19. Maynard-Reid-II, P., Shoham, Y.: Belief fusion: aggregating pedigreed belief states. J. Logic, Lang. Inf. **10**, 183–209 (2001)
20. McConway, K.J.: Marginalization and linear opinion pools. J. Am. Stat. Assoc. **76**(374), 410–414 (June 1981)
21. Moral, S., Sagrado, J.: Aggregation of Imprecise Probabilities. Aggregation and Fusion of Imperfect Information, pp. 162–188. Physica-Verlag, Heidelberg (1997)
22. Oussalah, M.: Study of some algebrical properties of adaptive combination rules. Fuzzy Sets Syst. **114**(3), 391–409 (2000)
23. Rescher, N., Manor, R.: On inference from inconsistent premises. Theor. Decis. **1**, 179–219 (1970)
24. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press (1976)
25. Smets, P.: Analyzing the combination of conflicting belief functions. Inf. Fusion **8**(4), 387–412 (2007)
26. Wagner, C., Lehrer, K.: Rational Consensus in Science and Society. D. Reidel, (1981)
27. Walley, P. The elicitation and aggregation of beliefs. Technical report, University of Warwick (1982)
28. Yager, R.: On the Dempster-Shafer framework and new combination rules. Inf. Sci. **41**, 93–138 (1987)

Aggregation of Multidimensional Data: A Review

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Abstract. Aggregation theory classically deals with functions to summarize a sequence of numeric values, e.g. in the unit interval, see [6, 7]. Since the notion of componentwise monotonicity plays a key role in many situations, there is an increasingly growing interest in methods that act on diverse ordered structures.

However, as far as the definition of a mean or an averaging function is concerned, see, e.g., [1, 2], the internality (or at least idempotence) property seems to be of a relatively higher importance than the monotonicity condition. In particular, the Bajraktarević means or the mode are among some well-known non-monotone means.

The concept of a penalty-based function was first investigated by Yager in [8] and then extended in numerous works, see, e.g., for a recent summary and a critical overview [3]. In such a framework, we are interested in minimizing the amount of "disagreement" between the inputs and the output being computed; the corresponding aggregation functions are at least idempotent and express many existing means in an intuitive and attractive way.

In this talk, I focus on the notion of penalty-based aggregation of sequences of points in $\mathbb{R}$, this time for some $d \geq 1$, see [4, 5]. I review three noteworthy subclasses of penalty functions: componentwise extensions of unidimensional ones, those constructed upon pairwise distances between observations, and those defined by measuring the so-called data depth. Then, I discuss their formal properties, which are particularly useful from the perspective of data analysis, e.g. different possible generalizations of internality or equivariances to various geometric transforms. I also point out the difficulties with extending some notions that are key in classical aggregation theory, like the monotonicity property.

References

1. Beliakov, G., Bustince, H., Calvo, T.: A Practical Guide to Averaging Functions. Springer, Heidelberg (2016)
2. Bullen, P.: Handbook of Means and Their Inequalities. Springer Science+Business Media, Dordrecht (2003)
3. Bustince, H., Beliakov, G., Dimuro, G.P., Bedregal, B., Mesiar, R.: On the definition of penalty functions in data aggregation. Fuzzy Sets Syst. (2016, in press). doi:10.1016/j.fss.2016.09.011
4. Gagolewski, M.: Data Fusion: Theory, Methods, and Applications. Institute of Computer Science, Polish Academy of Sciences, Warsaw, Poland (2015)
5. Gagolewski, M.: Penalty-based aggregation of multidimensional data. Fuzzy Sets Syst. (2016, in press). doi:10.1016/j.fss.2016.12.009
6. Grabisch, M., Marichal, J.L., Mesiar, R., Pap, E.: Aggregation Functions. Cambridge University Press (2009)
7. Torra, V., Narukawa, Y., Modeling Decisions: Information Fusion and Aggregation Operators. Springer-Verlag, Heidelberg (2007)
8. Yager, R.R.: Toward a general theory of information aggregation. Inf. Sci. **68**(3), 191–206 (1993)

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