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# Evidential $k$ -NN for Link Prediction

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**Abstract.** Social networks play a major role in today's society, they have shaped the unfolding of social relationships. To analyze networks dynamics, link prediction i.e., predicting potential new links between actors, is concerned with inspecting networks topology evolution over time. A key issue to be addressed is the imperfection of real world social network data which are usually missing, noisy, or partially observed. This uncertainty is perfectly handled under the general framework of the belief function theory. Here, link prediction is addressed from a supervised learning perspective by extending the evidential  $k$ -nearest neighbors approach. Each nearest neighbor represents a source of information concerning new links existence. Overall evidence is pooled via the belief function theory fusion scheme. Experiments are conducted on real social network data where performance is evaluated along with a comparative study. Experiment results confirm the effectiveness of the proposed framework, especially when handling skewness in data.

**Keywords:** Link prediction, social network, belief function theory, information fusion, evidential  $k$ -nearest neighbor, supervised learning

## 1 Introduction

Link prediction (LP) is an important task in social network analysis and graph mining that plays a major role in the understanding of network evolution. It is a powerful tool with a wide variety of applications such as prediction of protein-protein interactions in bioinformatics [4], construction of recommendation systems for e-commerce [12], detection of criminals or terrorist cells for security applications [24] or users aid to form new connections in social networks [15]. The main goal is to accurately predict the existence of new links between unlinked entities given a state of the network.

Supervised machine learning techniques have been intensively applied to LP. Many classification models successfully addressed link prediction [5, 10] (for details, see [11]). Indeed, LP can be easily transformed into a two-class classification problem. Given a social network graph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$  where  $\mathcal{V}$  is the set nodes and  $\mathcal{E}$  is the set of edges, one can partition the graph into two states by considering the edges that occurred at the time interval  $[t_0, t_1]$  as the training set and those belonging to the time interval  $[t_1, t_2]$  as the test set. To this end, LP is reformulated into a binary classification problem by

assigning class labels to all pairs of nodes such that:

$$class(u, v|\mathcal{G}) = \begin{cases} 1 & \text{if } uv \in \mathcal{E} \\ 0 & \text{if } uv \notin \mathcal{E} \end{cases}$$

The most straightforward perception for LP is that similar nodes are likely to connect. That is, the challenge is how to evaluate this similarity accurately. From this point of view, we propose, in this paper, a framework that extends the  $k$  nearest neighbor ( $k$ -NN) classification approach to LP. We draw on the assumption that query links that are similar to links present in the network are likely to exist. Yet, the challenge is to evaluate the degree of support regarding the class membership of the new links. Actually, information given by the nearest neighbors cannot be considered completely trustworthy as a result of uncertainty and imperfection in the data. As pointed out in [3], real-world networks especially, the large scale ones, are characterized by shifting degrees of uncertainty. Besides, data from real world applications are inherently uncertain, they are frequently incomplete, noisy and sensitive to observation errors. In order to overcome data imperfection issues,  $k$ -NN extensions under uncertainty theories have been proposed. For example, fuzzy  $k$ -NN algorithm [14] is an extension of  $k$ -NN based on fuzzy set theory [30]. It applies a fuzzy editing that alters the membership of each training sample according to its  $k$  nearest neighbors. However, it does not allow to model and deal with imprecise or incomplete information effectively. Another example would be the evidential  $k$ -NN (EKNN) [8] based on the belief function theory (BFT) [7, 25], a formal framework for reasoning under uncertainty that permits to manage and model imprecise information accurately.

Obviously, dealing with uncertainty is relatively correlated to the definition of fusion. It is important to determine the properties of objects and the relations among multiple ones. Yet, one has to quantify the uncertainty regarding certain characteristics of an object and the likelihood with which we can say that some elements are related. The BFT allows to carry out such fusion procedures. Furthermore, it enables to pool evidence while being cautious to the sources' reliability. The usage of the BFT to handle uncertainty in networks has been strongly recommended in the literature [1, 6, 29]. In that regard, we extend, in this paper, evidential  $k$ -NN proposed by [8] under the belief function framework [7, 25] to address link prediction. The proposed framework combines topological properties and the intuition of the nearest neighbors approach. The similarity is evaluated using structural metrics as features. Each nearest neighbor is considered as a distinct item of evidence supporting the class membership of the query link. Finally, the overall evidence given by the  $k$ -nearest neighbors is fused using the belief function theory combination tools.

This paper is organized as follows: in the next two sections, we recall related work on link prediction and a brief background on the belief function theory. In Section 4, our proposals for LP based on supervised learning under the belief function theory framework are presented. In Section 5, we report the conducted experiments to test the novel framework. Lastly, conclusions and possible future work are drawn in Section 6.

## 2 Related Work on Link Prediction

According to [17], LP approaches can be roughly classified into three groups: probabilistic models, maximum likelihood algorithms and similarity based methods [11]. The probabilistic models estimate the likelihood of links existence by building a joint probability distribution representing the graph and applying inference techniques. They are generally based on Markov Networks or Bayesian Networks [9]. On the other hand, maximum likelihood approaches concentrate on a given structure across the network (i.e. hierarchical structure, community structure, etc.) and try to fit the most likely structure through maximum likelihood algorithms. Finally, similarity-based algorithms compute similarity scores between the nodes based on some topological properties of the graph. These scores can be easily employed under a supervised learning. They are generic as they do not depend on the network domain and do not require an overall model. Additionally, each similarity score is independent from the others which allows to compute several ones separately and at the same time. They are the simplest of LP algorithms in terms of computational cost. Actually, LP applies to networks which are continuously evolving in size. In many cases, maximum likelihood and probabilistic models cannot even be checked due the large structure of the networks. On that point, similarity-based methods are more convenient since they do not only perform to large graphs but their performance is also impressive. We presented, in previous works [18–21], LP approaches inspired from similarity-based methods. However, the latter works are applicable to uncertain social networks i.e., edges are attached by uncertainty degrees regarding their existence. They operate merely using the BFT tools. Our proposed framework, in this paper, tackles LP under supervised learning. Furthermore, it applies to social networks without encapsulated uncertainty in their structure.

The similarity-based methods use topological information of the networks. This information is usually grouped into two types: local and global information. The first group of methods computes scores according to node-neighborhoods. An example would be the common neighbors of two nodes [23]. The intuition is that the more two nodes  $u$  and  $v$  share many common neighbors the more likely they tend to connect. This makes sense in many real world networks such as friendship networks, as two persons who have many mutual friends are very likely to become friends. In contrast, global information methods employ proximity in the network where two nodes are likely to connect if they are close in the network in distance terms. An example of such algorithms would be the shortest path between two nodes. Yet, these algorithms have higher computational complexity since they require all the topological information which is frequently not completely available. In this paper, we consider the node neighborhood based metrics as they are simple and not costly in computational terms.

For a node  $u$ , let  $\tau(u)$  be the set of its neighbors in the network, called first level neighbors or direct neighbors. The second level neighbors of  $u$ , denoted  $\tau(u)^2$ , are the nodes connected to the direct neighbors of  $u$ . We recall here the most popular local similarity scores that proved their efficiency in many works from literature [15, 23, 31]:

- Common Neighbors (CN) [23] computes the common neighbors between a pair of nodes  $(u, v)$ .
- Jaccard Coefficient (JC) [13] measures the ratio of the common neighbors of  $u$  and  $v$  and all their neighbors.

- Adamic Adar measure (AA) [2] weights all common neighbors of the pair  $(u, v)$  and penalizes the ones with high degrees.
- Resource Allocation (RA) [31] is inspired from the resource allocation process of networks. For an unlinked pair of nodes  $(u, v)$ , each common neighbor plays the role of a transmitter of a single resource unit. As such, the similarity between  $u$  and  $v$  is the amount of resource  $v$  collected from  $u$ .
- Preferential Attachment (PA) [23] assumes that the probability that a new edge relate to  $u$  is proportional to  $|\tau(u)|$ . Thus, the score of  $uv$  is correlated to the number of neighbors of  $u$  and  $v$ .

Equations of the presented metrics are given in Table 1.

**Table 1.** Structural similarity measures based on local topological information between the pair of nodes  $(u, v)$  where  $\tau(u)$  and  $\tau(v)$  are respectively their sets of neighbors in the graph.

Common Neighbors (CN)	$ \tau(u) \cap \tau(v) $
Adamic Adar (AA)	$\sum_{z \in (\tau(u) \cap \tau(v))} \frac{1}{\log \tau(z) }$
Jaccard Coefficient (JC)	$\frac{ \tau(u) \cap \tau(v) }{ \tau(u) \cup \tau(v) }$
Resource Allocation (RA)	$\sum_{z \in (\tau(u) \cap \tau(v))} \frac{1}{ \tau(z) }$
Preferential Attachment (PA)	$ \tau(u)  \cdot  \tau(v) $

In this paper, local topological metrics are combined with EKNN to evaluate similarities for LP. A feature set is constructed using structural metrics to determine the nearest neighbors according to a distance measure. The assets of the belief function theory for information fusion are subsequently exploited to pool the information gathered from the nearest neighbors. We present, in the next section, some fundamental basic concepts of the BFT.

### 3 Background on the Belief Function Theory

In the belief function theory [7, 25], a problem is represented by a frame of discernment  $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ , an exhaustive and finite set of mutually exclusive events. A basic belief assignment (*bba*), denoted by  $m$ , represents the knowledge committed to the elements of  $2^\Omega$  given a source of information. It is a mapping function  $m : 2^\Omega \rightarrow [0, 1]$ , such that:

$$\sum_{A \subseteq \Omega} m(A) = 1 \quad (1)$$

An element  $A$  is called a focal element of the *bba*  $m$  if  $m(A) > 0$ . The belief committed to  $\Omega$  represents the degree of ignorance. A state of total ignorance is defined by  $m(\Omega) = 1$ . When the *bba* has at most one focal element  $A$  different from  $\Omega$ , it said to be a simple support function (*ssf*) and has the following form [26]:

$$\begin{cases} m(A) &= 1 - \omega \\ m(\Omega) &= \omega \end{cases} \quad (2)$$

for some  $A \subset \Omega$  and  $\omega \in [0, 1]$ .

Combining two basic assignments induced from two distinct sources of information over the same frame of discernment into one may be ensured using the conjunctive rule of combination denoted by  $\odot$ . It is defined as [27]:

$$m_1 \odot m_2(A) = \sum_{B, C \subseteq \Omega: B \cap C = A} m_1(B) \cdot m_2(C) \quad (3)$$

The combination rule permits to aggregate evidence by meaningfully outlining a corpus of data and making it simpler. Furthermore, it allows to fuse information induced from single and multiple sources.

#### 4 Evidential $k$ -nearest neighbors for Link Prediction

The goal is to predict the existence of new links in a network graph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$  where  $\mathcal{V}$  is the set of nodes and  $\mathcal{E}$  is the set of edges. The set of classes is  $\Omega = \{E, \neg E\}$ , where  $E$  points up the existence of a link in  $\mathcal{G}$  and  $\neg E$  its absence. The class of each link in  $\mathcal{E}$  is assumed to be known with certainty. The available information consists in a training set  $\mathcal{T} = \{(e^1, \omega^1), \dots, (e^{|\mathcal{E}|}, \omega^{|\mathcal{E}|})\}$  of single labeled links, where  $e^i \in \mathcal{E}$ ,  $i \in \{0, \dots, |\mathcal{E}|\}$  and its corresponding class label is  $\omega^i \in \Omega$ .

Let  $e$  be a new link to be classified, where  $e$  may connect the pair of nodes  $(u, v)$ . Let  $\mathcal{L}^1$  be the set of links shared with the direct ( $1^{st}$  level) neighbors of  $u$  and  $v$  in  $\mathcal{G}$ , where  $|\mathcal{L}^1| = |\tau(u)| + |\tau(v)|$ . Additionally, let  $\mathcal{L}^2$  be the set of links unshared with the  $2^{nd}$  level neighbors of  $u$  and  $v$  in  $\mathcal{G}$ , where  $|\mathcal{L}^2| = |\tau(u)^2| + |\tau(v)^2|$ . The set  $\mathcal{L}$  incorporates  $\mathcal{L}^1$  and  $\mathcal{L}^2$ .

It is obvious that the class of the links in the set  $\mathcal{L}^1$  is  $E$ . In contrast, the class of the links in  $\mathcal{L}^2$  is  $\neg E$  since it includes the links that are not shared between  $u$  and  $v$  and their respective  $2^{nd}$  level neighbors. As follows, the nearest neighbors of each unseen link are uncovered thought-out the neighborhood of its end points. Accordingly, instead of comparing each unseen link with all the possible edges in the network, which is computationally not feasible since there are  $\frac{(|\mathcal{V}| \times (|\mathcal{V}| - 1))}{2}$  possible links, it is compared to the neighboring ones. As such, the search space is reduced. Besides, the intuition of node neighborhood approaches is inherently employed as they are exactly intended to find similar nodes. In this context, the authors in [28], proposed a framework for LP using  $k$ -NN by considering the local similarity indexes to evaluate the similarity with the neighbors. The prediction of an edge  $uv$  is made by comparing the neighbors of  $u$  to  $v$  and vice versa. However, the proposed approach considers the nearest neighbors equally trustworthy. Besides, only one similarity index is considered in  $k$ -NN at a time.

In our proposed framework, the evidential  $k$ -NN classifier [8] operates in two stages. First, it computes the distances between a test link  $e$  and its neighborhood in  $\mathcal{L}$  and retain the smallest  $k$  distances. Subsequently, the evidence given by the  $k$  nearest neighbors is combined to get an overview about the global belief regarding the existence of  $e$ . The steps are detailed in the following.

At first, one has to determine the  $k$  nearest neighbors. For that, we need to define a distance to evaluate the similarity between the edges in the test set and those in the train set. We propose to use the Euclidean distance  $d(e, e^i)$  between the link  $e$  and its

nearest neighbor  $e^i \in \mathcal{L}$  by computing the similarities between their connecting nodes as follows:

$$d(e, e^i) = \sqrt{\sum_{j=1}^n (s_e^j - s_{e^i}^j)^2} \quad (4)$$

where  $j$  is the index of a local similarity metric (e.g, CN, AA, JC, RA, PA),  $s_e$  and  $s_{e^i}$  are respectively its values for  $e$  and  $e^i$  and  $n$  is the number of local similarities considered.

Each link  $e^i$  in  $\mathcal{L}$  represents a piece of evidence that increases our belief about  $e$  also belonging to  $\omega^i$ . Yet, this information solely does not provide certain knowledge about the class of  $e$ . This situation is modeled in the BFT by simple support functions where only some part of the belief is committed to  $\omega^i$  and the rest is affected to  $\Omega$ . Therefore, we get the following *bba*:

$$\begin{cases} m_i(\{\omega^i\}) = \alpha\phi(d_i) \\ m_i(\Omega) = 1 - \alpha\phi(d_i). \end{cases} \quad (5)$$

where  $d_i = d(e, e^i)$ ,  $\alpha$  is a parameter such that  $0 < \alpha < 1$  and  $\phi$  is a decreasing function. The closer  $e$  is to  $e^i$  according to the distance  $d$ , the more likely for  $e$  to have same class as  $e^i$ . In contrast, when  $e$  is far from  $e^i$ , in distance terms, then  $e^i$  would provide little information regarding the class of  $e$ . On that point, the function  $\phi$  must verify  $\phi(0) = 1$  and  $\lim_{d \rightarrow \infty} \phi(d) = 0$ . Authors in [8] suggest to use the following decreasing function:

$$\phi(d_i) = e^{(-\gamma d_i^\beta)} \quad (6)$$

where  $\gamma > 0$  and  $\beta \in \{1, 2, \dots\}$ .  $\beta$  can be arbitrarily fixed to a small value (1 or 2).

As a result of considering each nearest neighbor in  $\mathcal{L}$  as an independent source of evidence regarding the class of the  $e$ , we obtain  $k$  *bba*'s that can be combined using the conjunctive rule of combination. Thus, a global *bba*  $m$  that synthesizes the belief regarding the existence of  $e$  is produced as follows:

$$m = m_1 \oplus \dots \oplus m_k \quad (7)$$

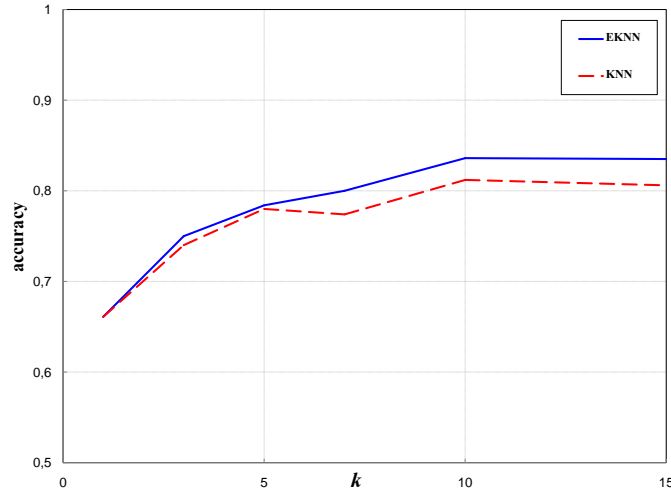
Finally, decision about the membership of  $e$  to one of the classes in  $\Omega$  is made by comparing  $m(\{E\})$  and  $m(\{\neg E\})$ . If  $m(\{E\}) > m(\{\neg E\})$  then  $e$  exists, it is absent otherwise.

## 5 Experiments

Experiments are conducted on a real social network component of 1K nodes and 10K edges of the Facebook dataset from [22]. Since network data with time information are not usually available, we must settle for the more drastic technique by randomly removing a partition of the edges from the network in order to use them as test set. That is, we remove a random 10% of the edges which we try to predict the existence along with randomly generated false links of the same size using the graph as a source composed by the remaining 90%. The results are obtained by averaging over 10 implementations

with independently random divisions of testing set and training set. In order to reduce the computational time, a preprocessing phase is first conducted in which the local similarity scores of all the links from the train and test sets are computed. Evaluation is made according to accuracy which computes the number of correct predictions among all predictions and the precision which takes the fraction of predicted links that are relevant.

Required parameters are  $\alpha$ ,  $\beta$ ,  $\gamma$  for the induced  $bba$ 's and the number of nearest neighbors  $k$ . As discussed in [8], the parameters  $\alpha$  and  $\beta$  do not have a great influence on the approach performance. Thus, as in [8],  $\alpha$  is fixed to 0.95 and  $\beta$  to 1. We tested values of  $k$  ranging from 1 to 15. Tests for the optimization of the  $\gamma$  parameter allowed us to set it to the value of 0.12. A comparison with the standard  $k$ -NN method (KNN) is carried out, where the class of a link is predicted according to the majority classes of its  $k$  nearest neighbors. The results are reported in Fig. 1.



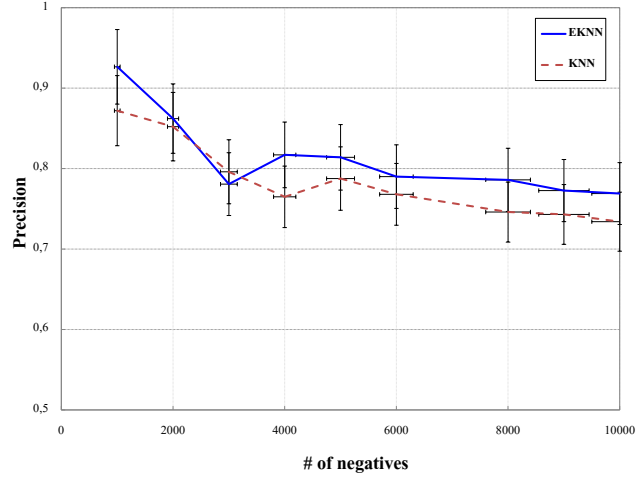
**Fig. 1.** Results according to the values of  $k$

Fig. 1 reports the results in terms of accuracy for different values of  $k$ . It can be seen that both methods have better performances as  $k$  increases. Indeed, as we boost the number of nearest neighbors we get more sources of evidence regarding the class membership of the links. Yet, as shown in Fig. 1, performance stops upon reaching a certain threshold. Rather, by increasing excessively the number of nearest neighbors, we get more distant ones (less similar). Consequently, the associated mass functions are close to the state of total ignorance. Thus, they have no impact in the combination and therefore in the prediction. The EKNN based framework has better classification performance than the standard KNN based LP method. It stands to reason that, EKNN performs better for some values of  $k$  as a results of taking implicitly the relevance of the information given by the sources into account unlike KNN as it considers all the



nearest neighbors equally trustworthy. Although, the results are low for small values of  $k$ , it still gives acceptable results i.e., 75% accuracy for  $k = 3$ .

In a second stage, we conduct implementations by increasing the number of negative instances (non existing links) at each time to evaluate the behavior of our algorithms to class imbalance scenarios. Actually, LP is a very imbalanced class problem where the number of non existing links is much larger than the existing ones. The same parameters specifications are considered except that  $k$  is set to 15. Precision results are presented in Fig. 2 for different negative links number. Measuring precision is very important for evaluating LP since, in many cases, the main goal is to accurately predict the real existing edges. For example, in Facebook, it is more important to not miss actual friends and it does not really matter when unknown friends are recommended. As shown in Fig. 2, EKNN outperforms KNN for most values. Furthermore, the precision plot decreases as more non existing edges are predicted. However, the curve does not fall dramatically but rather slowly reaching 77% for 10K false edges. These good performances are also obtained thanks to the advantages of supervised learning which permits to center on class boundaries and balance data. As opposed to unsupervised methods which cannot address this imbalance well because they are agnostic to class distributions by nature [16]. We conjecture that our framework is capable of dealing with the class imbalance problem. Furthermore, it allows us to consider network topology and deal with uncertainty at the same time.



**Fig. 2.** Precision results for different negative links number

## 6 Conclusion

Local similarity measures naturally operate to detect similar nodes which make it simple to extend them to  $k$ -NN. In this paper, we propose a framework for link prediction that combines structural local topological properties and the evidential  $k$ -NN approach. Based on the direct and second level neighbors of two unlinked nodes  $u$  and  $v$ , local similarity indexes are computed and used as features to find their  $k$ -nearest neighbors. These latter are considered as items of evidence regarding the class membership of the link  $uv$ . Global evidence is pooled using the conjunctive rule of combination from the belief function theory to get an overall information about new links existence. Tests on real world social network data proved the efficiency of the proposed framework. It is interesting to note that the novel framework handles skewness in social network data. It is capable of combating class imbalance that characterizes the link prediction task. Furthermore, it shows performance improvement over the baseline algorithm KNN which does not take into account uncertainty in the analysis.

A straightforward direction for future research is to take supplementary information into account such as node attributes. Obviously, nodes with similar attribute values are likely to share social relationships i.e., two authors who have the same affiliation and the same research fields. Besides, handling node attributes brings semantics to social connections. Therefore, it is an important source of information that may enhance the link prediction task.

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