# Generation of Near-Optimal Solutions Using ILP-Guided Sampling

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#### **Abstract**

Our interest in this paper is in optimisation problems that are intractable to solve by direct numerical optimisation, but nevertheless have significant amounts of relevant domain-specific knowledge. The category of heuristic search techniques known as estimation of distribution algorithms (EDAs) seek to incrementally sample from probability distributions in which optimal (or near-optimal) solutions have increasingly higher probabilities. Can we use domain knowledge to assist the estimation of these distributions? To answer this in the affirmative, we need: (a) a general-purpose technique for the incorporation of domain knowledge when constructing models for optimal values; and (b) a way of using these models to generate new data samples. Here we investigate a combination of the use of Inductive Logic Programming (ILP) for (a), and standard logic-programming machinery to generate new samples for (b). Specifically, on each iteration of distribution estimation, an ILP engine is used to construct a model for good solutions. The resulting theory is then used to guide the generation of new data instances, which are now restricted to those derivable using the ILP model in conjunction with the background knowledge). We demonstrate the approach on two optimisation problems (predicting optimal depth-of-win for the KRK endgame, and job-shop scheduling). Our results are promising: (a) On each iteration of distribution estimation, samples obtained with an ILP theory have a substantially greater proportion of good solutions than samples without a theory; and (b) On termination of distribution estimation, samples obtained with an ILP theory contain more near-optimal samples than samples without a theory. Taken together, these results suggest that the use of ILP-constructed theories could be a useful technique for incorporating complex domain-knowledge into estimation distribution procedures.

### l Introduction

There are many real-world planning problems for which domain knowledge is qualitative, and not easily encoded in a form suitable for numerical optimisation. Here, for instance, are some guiding principles that are followed by the Australian Rail Track Corporation when scheduling trains: (1) If a "healthy" Train is running late, it should be given equal preference to other healthy Trains; (2) A higher priority train should be given preference to a lower priority train, provided the delay to the lower priority train is kept to a minimum; and so on. It is evident from this that train-scheduling may benefit from knowing if a train is "healthy", what a train's priority is, and so on. But are priorities and train-health fixed, irrespective of the context? What values constitute acceptable delays to a low-priority train? Generating good train-schedules will require a combination of quantitative knowledge of a train's running times and qualitative knowledge about the train in isolation, and in relation to other trains. In this paper, we propose a heuristic search method, that comes under the broad category of an estimation distribution algorithm (EDA). EDAs iteratively generates better solutions to the optimisation problem using machine-constructed models. Usually EDA's have used generative probabilistic models, such as Bayesian Networks, where domain-knowledge needs to be translated into prior distributions and/or network topology. In this paper, we are concerned with problems for such a translation is not evident. Our interest in ILP is that it presents perhaps one the most flexible ways to use domain-knowledge when constructing models.

What can this form of optimisation do differently? First, there is the straightforward difference to standard optimisation, arising from the use of domain-knowledge in first-order logic. Traditionally, optimisation methods have required domain knowledge to be in the form of linear inequalities. This quickly becomes complicated. For example,  $y = x_1 \oplus x_2$  requires the inequalities  $y \le x_1 + x_2 \land y \le 2 - x_1 - x_2 \land y \ge x_1 - x_2 \land y \ge x_2 - x_1 \land y \ge 0 \land y \le 1$ . As a statement in logic, the relation is clearly trivial: so, we would expect to do better on problems for which domain-knowledge is far easier to express in logical form than as linear constraints (of course, one could consider non-linear constraints, but then the optimisation problem becomes much harder). Secondly, there is the difference arising from constructing models in first-order logic. Most probabilistic models used in EDA only allow use models that involve statements about propositions. This restricts the expressivity of the models, or requires large numbers of propositions representing pre-defined

relations. We would therefore expect to to do better on problems that require models that involve relationships amongst background predicates that are not easy to know beforehand.

The rest of the paper is organised as follows. Section 2 provides a brief description of the EDA method we use for optimisation problems. Section 2.1 describes how ILP can be used within the iterative loop of an EDA, including a procedure for sampling data instances entailed by the ILP-theory. Section 3 describes an empirical evaluation followed by conclusions in Section 4.

## 2 EDA for Optimisation

The basic EDA approach we use is the one proposed by the MIMIC algorithm [5]. Assuming that we are looking to minimise an objective function  $F(\mathbf{x})$ , where  $\mathbf{x}$  is an instance from some instance-space  $\mathcal{X}$ , the approach first constructs an appropriate machine-learning model to discriminate between samples of lower and higher value, i.e.,  $F(\mathbf{x}) \leq \theta$  and  $F(\mathbf{x}) > \theta$ , and then sampling from this model to generate a population for the next iteration, while also lowering  $\theta$ . This is described by the procedure in Fig. 1.

Procedure EOMS: Evolutionary Optimisation using Model-Assisted Sampling

- 1. Initialize population  $P := \{\mathbf{x}_i\}; \theta := \theta_0$
- 2. while not converged do
  - (a) for all  $\mathbf{x}_i$  in P label( $\mathbf{x}_i$ ) := 1 if  $F(\mathbf{x}_i) < \theta$  else label( $\mathbf{x}_i$ ) := 0
  - (b) train model M to discriminate between 1 and 0 labels i.e.,  $P(\mathbf{x} : label(\mathbf{x}) = 1|M) > P(\mathbf{x} : label(\mathbf{x}) = 0|M)$
  - (c) regenerate P by repeated sampling using model M
  - (d) reduce threshold  $\theta$
- 3. return P

Figure 1: Evolutionary optimisation using machine-learning models to guide sampling.

## 2.1 ILP-assisted EDA for Optimisation

We propose the use ILP as the model construction technique in the EOMS procedure, since it provides an extremely flexible way to construct models using domain-knowledge. On the face of it, this would seem to pose a difficulty for the sampling step: how are we to generate new instances that are entailed by an ILP-constructed model? There are two straightforward options. First, if we have an enumerator of the instance space  $\mathcal{X}$ , then we could resort to a form of rejection-sampling. Second, we can restrict ILP-theories for any predicate to generative clauses, which allows the theories to be used generatively. using standard logic-programming inference machinery to generate instances of the success-set of each predicate. Instances obtained in this manner are selected with some probability to achieve a non-uniform sampling of the success-set. Both these methods are viable for the purposes of this paper, but in general, we expect that more sophisticated ways of sampling would be needed: see for example [4]. The procedure EOIS in Fig. 2 is a refinement of the EOMS procedure above.

In EOIS,  $ilp(B, E^+, E^-)$  is an ILP algorithm that returns a theory M s.t.  $B \wedge M \models E^+$ ;  $B \wedge M$  is inconsistent with the  $E^-$  only to the extent allowed by constraints in B; and sample(n, M, B) returns a set of at most n instances entailed by  $B \wedge M$ , if  $M \neq \emptyset$ . If  $M = \emptyset$ , it returns a random selection of n instances from the instance-space. In general, we require sample to be draw instances from the success-set of the ILP-constructed theory, since these are the "good" solutions entailed by the model on each iteration (see [4] for techniques for doing this). Here, we make do by providing an initial sample to EOIS as input when  $M = \emptyset$  (to prevent biasing future iterations: this sample is obtained by uniform random selection from the instance space). For subsequent steps, we assume the availability of a generator that returns a sample of the success-set of target-predicate. That is, when  $M \neq \emptyset$ , sample returns the set  $S = \{e_i : 1 \leq i \leq n \ e_i \in \mathcal{X} \ \text{and} \ B \wedge M \vdash e \ \text{and} \ Pr(e_i) \geq \delta\}$ , where  $\delta$  is some probability threshold (correctly therefore sample(n, M, B) should be  $sample(n, \delta, M, B)$ ). Thus if  $\delta = 1$ , the first n instances derived (or fewer if there are less) using SLD-resolution with  $B \wedge M$  will be selected. Finally, we note that on iterations  $k \geq 1$ , we can use the data  $P_0 \cup \cdots \cup P_{k-1}$  to obtain training examples  $E_k^+$  and  $E_k^-$  since the actual costs for the P's have have already been computed. For clarity, this detail has been omitted.

<sup>&</sup>lt;sup>1</sup>A syntactic way to do this is by adding constraints to the body of the clause that impose range (that is, type) restrictions on the variables in the head: see [6].

#### **Procedure EOIS:** Evolutionary Optimisation using ILP-Assisted Sampling

**Given:** (a) Background knowledge B; (b) an upper-bound  $\theta^*$  on the cost of acceptable solutions; (c) a decreasing sequence of cost-values  $\theta_1, \theta_2, \dots, \theta_n$  s.t.  $\theta_1 \ge \theta^* \ge \theta_n$ ; and (d) an upper-bound on the sample size n

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1. Let M_0 := \emptyset and P_0 := sample(n, M_0, B)
2. Let k = 1
3. while (\theta_k \ge \theta^*) do

(a) E_k^+ := \{ \mathbf{x}_i : \mathbf{x}_i \in P \text{ and } F(\mathbf{x}_i) \le \theta_k \} and E_k^- := \{ \mathbf{x}_i : \mathbf{x}_i \in P \text{ and } F(\mathbf{x}_i) > \theta_k \}

(b) M_k := ilp(B, E_k^+, E_k^-)

(c) P_k := sample(n, M_k, B)

(d) increment k
4. return P_{k-1}
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Figure 2: Evolutionary optimisation using ILP models to guide sampling.

## 3 Empirical Evaluation

### **3.1** Aims

Our aims in the empirical evaluation are to investigate the following conjectures:

- (1) On each iteration, the EOIS procedure will yield better samples than simple random sampling of the instance-space; and
- (2) On termination, the EOIS procedure will yield more near-optimal instances than simple random sampling of the same number of instances as used for constructing the model.

It is relevant here to clarify what the comparisons are intended in the statements above. Conjecture (1) is essentially a statement about the gain in precision obtained by using the model. Let us denote  $Pr(F(\mathbf{x}) \leq \theta)$  the probability of generating an instance  $\mathbf{x}$  with cost at most  $\theta$  without a model to guide sampling (that is, using simple random sampling of the instance space), and by  $Pr(F(\mathbf{x}) \leq \theta|M_{k,B})$  the probability of obtaining such an instance with an ILP-constructed model  $M_{k,B}$  obtained on iteration k of the EOIS procedure using some domain-knowledge B. (note if  $M_{k,B} = \emptyset$ , then we will mean  $Pr(F(\mathbf{x}) \leq \theta|M_{k,B}) = Pr(F(\mathbf{x}) \leq \theta)$ ). Then for (1) to hold, we would require  $Pr(F(\mathbf{x}) \leq \theta_k|M_{k,B}) > Pr(F(\mathbf{x}) \leq \theta_k)$ . given some relevant B. We will estimate the probability on the lhs from the sample generated using the model, and the probability on the rhs from the datasets provided.

Conjecture (2) is related to the gain in recall obtained by using the model, although it is more practical to examine actual numbers of near-optimal instances (true-positives in the usual terminology). We will compare the numbers of near-optimal in the sample generated by the model to those obtained using random sampling.

### 3.2 Materials

#### 3.2.1 Data

We use two synthetic datasets, one arising from the KRK chess endgame (an endgame with just White King, White Rook and Black King on the board), and the other a restricted, but nevertheless hard  $5 \times 5$  job-shop scheduling (scheduling 5 jobs taking varying lengths of time onto 5 machines, each capable of processing just one task at a time).

The optimisation problem we examine for the KRK endgame is to predict the depth-of-win with optimal play [1]. Although aspect of the endgame has not been as popular in ILP as task of predicting "White-to-move position is illegal" [2, 8], it offers a number of advantages as a *Drosophila* for optimisation problems of the kind we are interested. First, as with other chess endgames, KRK-win is a complex, enumerable domain for which there is complete, noise-free data. Second, optimal "costs" are known for all data instances. Third, the problem has been studied by chess-experts at least since Torres y Quevado built a machine, in 1910, capable of playing the KRK endgame. This has resulted in a substantial amount of domain-specific knowledge. We direct the reader to [3] for the history of automated methods for the KRK-endgame. For us, it suffices to treat the problem as a form of optimisation, with the cost being the depth-of-win with Black-to-move, assuming minimax-optimal play. In principle, there are  $64^3 \approx 260,000$  possible positions for the KRK endgame, not all legal. Removing illegal positions, and redundancies arising from symmetries of the board reduces the size of the

Cost	Instances	Cost	Instances
0	27 (0.001)	9	1712 (0.196)
1	78 (0.004)	10	1985 (0.267)
2	246 (0.012)	11	2854 (0.368)
3	81 (0.152)	12	3597 (0.497)
4	198 (0.022)	13	4194 (0.646)
5	471 (0.039)	14	4553 (0.808)
6	592 (0.060)	15	2166 (0.886)
7	683 (0.084)	16	390 (0.899)
8	1433 (0.136)	draw	2796 (1.0)

Total Instances: 28056

Cost	Instances	Cost	Instances
400-500	10 (0.0001)	1000-1100	24067 (0.748)
500-600	294 (0.003)	1100-1200	15913 (0.907)
600-700	2186 (0.025)	1200-1300	7025 (0.978)
700-800	7744 (0.102)	1300-1400	1818 (0.996)
800–900	16398 (0.266)	1400-1500	345 (0.999)
900-1000	24135 (0.508)	1500-1700	66 (1.0)

Total Instances: 100000

(a) Chess (b) Job-Shop

Figure 3: Distribution of cost values. Numbers in parentheses are cumulative frequencies.

instance space to about 28,000 and the distribution shown in Fig. 3(a). The sampling task here is to generate instances with depth-of-win equal to 0. Simple random sampling has a probability of about 1/1000 of generating such an instance once redundancies are removed.

The job-shop scheduling problem is less controlled than the chess endgame, but is nevertheless representative of many real-life applications (like scheduling trains), which are, in general, known to be computationally hard. We use a job-shop problem with five jobs, each consisting of five tasks that need to be executed in order. These 25 tasks are to be performed using 5 machines, each capable of performing a particular task, albeit for any of the jobs. A  $5 \times 5$  matrix defines how long task j of job i takes to execute on machine j.

Data instances for Chess are in the form of 6-tuples, representing the rank and file (X and Y values) of the 3 pieces involved. At each iteration k of the EOIS procedure, some instances with depth-of-win  $\leq \theta_k$  and the rest with depth-of-win  $> \theta_k$  are used to construct a model.<sup>2</sup>

Data instances for Job-Shop are in the form of schedules defining the sequence in which tasks of different jobs are performed on each machine, along with the total cost (i.e., time duration) implied by the schedule. On iteration i of the EOIS procedure, models are to be constructed to predict if the cost of schedule will be  $\leq \theta_i$  or otherwise.<sup>3</sup>

#### 3.2.2 Background Knowledge

For Chess, background predicates encode the following (WK denotes, WR the White Rook, and BK the Black King): (a) Distance between pieces WK-BK, WK-BK, WK-WR; (b) File and distance patterns: WR-BK, WK-WR, WK-BK; (c) "Alignment distance": WR-BK; (d) Adjacency patterns: WK-WR, WK-BK, WR-BK; (e) "Between" patterns: WR between WK and BK, WK between WR and BK, BK between WK and WR; (f) Distance to closest edge: BK; (g) Distance to closest corner: BK; (h) Distance to centre: WK; and (i) Inter-piece patterns: Kings in opposition, Kings almost-in-opposition, L-shaped pattern. We direct the reader to [3] for the history of using these concepts, and their definitions.

For Job-Shop, background predicates encode: (a) schedule job J "early" on machine M (early means first or second); (b) schedule job J "late" on machine M (late means last or second-last); (c) job J has the fastest task for machine M; (d) job J has the slowest task for machine M; (e) job J has a fast task for machine M (fast means the fastest or second-fastest); (f) Job J has a slow task for machine M (slow means slowest or second-slowest); (g) Waiting time for machine M; (h) Total waiting time; (i) Time taken before executing a task on a machine. Correctly, the predicates for (g)–(i) encode upper and lower bounds on times, using the standard inequality predicates  $\leq$  and  $\geq$ .

### 3.2.3 Algorithms and Machines

The ILP-engine we use is Aleph (Version 6, available from A.S. on request). All ILP theories were constructed on an Intel Core i7 laptop computer, using VMware virtual machine running Fedora 13, with an allocation of 2GB for the virtual machine. The Prolog compiler used was Yap, version 6.1.3<sup>4</sup>.

#### 3.3 Method

Our method is straightforward:

For each optimisation problem, and domain-knowledge B:

 $<sup>^2</sup>$ The  $\theta_k$  values are pre-computed assuming optimum play. We note that when constructing a model on iteration k, it is permissible to use all instances used on iterations  $1, 2, \ldots, (k-1)$  to obtain data for model-construction.

 $<sup>^{3}</sup>$ The total cost of a schedule includes any idle-time, since for each job, a task before the next one can be started for that job. Again, on iteration i, it is permissible to use data from previous iterations.

<sup>4-</sup>http://www.dcc.fc.up.pt/ vsc/Yap/

Using a sequence of threshold values  $\langle \theta_1, \theta_2, \dots, \theta_n \rangle$  on iteration k  $(1 \le k \le n)$  for the EOIS procedure:

- 1. Obtain an estimate of  $Pr(F(\mathbf{x}) \leq \theta_k)$  using a simple random sample from the instance space;
- 2. Obtain an estimate of  $Pr(F(\mathbf{x}) \leq \theta_k | M_{k,B})$  by constructing an ILP model for discriminating between  $F(\mathbf{x}) \leq \theta_k$  and  $F(\mathbf{x}) > \theta_k$
- 3. Compute the ratio of  $Pr(F(\mathbf{x}) \leq \theta_k | M_{k,B})$  to  $P(F(\mathbf{x}) \leq \theta_k)$

The following details are relevant:

- The sequence of thresholds for Chess are  $\langle 8, 4, 0 \rangle$ . For Job-Shop, this sequence is  $\langle 1000, 750, 600 \rangle$ ; Thus,  $\theta^* = 0$  for Chess and 600 for Job-Shop, which means we require exactly optimal solutions for Chess.
- Experience with the use of ILP engine used here (Aleph) suggests that the most sensitive parameter is the one defining a lower-bound on the precision of acceptable clauses (the *minacc* setting in Aleph). We report experimental results obtained with minacc = 0.7, which has been used in previous experiments with the KRK dataset. The background knowledge for Job-Shop does not appear to be sufficiently powerful to allow the identification of good theories with short clauses. That is, the usual Aleph setting of upto 4 literals per clause leaves most of the training data ungeneralised. We therefore allow an upper-bound of upto 10 literals for Job-Shop, with a corresponding increase in the number of search nodes to 10000 (Chess uses the default setting of 4 and 5000 for these parameters).
- In the EOIS procedure, the bound on sample size n is 1000. The initial sample is obtained using a uniform distribution over all instances. Let us call this  $P_0$ . On the first iteration of EOIS (k=1), the datasets  $E_1^+$  and  $E_1^-$  are obtained by computing the (actual) costs for instances in  $P_0$ , and an ILP model  $M_{1,B}$ , or simply  $M_1$ , constructed. To obtain a sample of instances entailed by the model  $M_{k,B}$  we use the sample function with a  $\delta$  value of 1.0. That is, the first n unique instances (or fewer, if less) obtained by employing SLD-resolution on  $B \wedge M_{k,B}$  are taken as the sample  $P_k$ . For Chess, it has been possible to ensure that the logic-programs involved are generative. Thus, we are able to use  $M_{k,B}$  directly as a generator of instances entailed by the  $B \wedge M_{k,B}$ . For Job-Shop, we employ rejection-sampling instead. That is, we randomly draw from the instance-space, and then check to see if it is entailed by  $B \wedge M_{k,B}$ . Both approaches have not proved to be especially inefficient, probably because the instance-spaces are small. On each iteration k, an estimate of  $Pr(F(\mathbf{x}) \leq \theta_k)$  can be obtained from the empirical frequency distribution of instances with values  $\leq \theta_k$  and  $> \theta_k$ . For the synthetic problems here, these estimates are in Fig. 3. For  $Pr(F(\mathbf{x}) \leq \theta_k | M_{k,B})$ , we use obtain the frequency of  $F(\mathbf{x}) \leq \theta_k$  in  $P_k$
- Readers will recognise that the ratio of  $Pr(F(\mathbf{x}) \leq \theta_k | M_{k,B})$  to  $P(F(\mathbf{x}) \leq \theta_k)$  is equivalent to computing the gain in precision obtained by using an ILP model over random selection. Specifically, if this ratio is approximately 1, then there is no value in using the ILP model. The probabilities computed also provide one way of estimating sampling efficiency of the models (the higher the probability, the fewer samples will be needed to obtain an instance  $\mathbf{x}$  with  $F(\mathbf{x}) \leq \theta_k$ ).

#### 3.4 Results

Results relevant to conjectures (1) and (2) are tabulated in Fig. 4 and Fig. 5. The principal conclusions that can drawn from the results are these:

- (1) For both problems, and every threshold value  $\theta_k$ , the probability of obtaining instances with cost at most  $\theta_k$  with model-guided sampling is substantially higher than without a model. This provides evidence that model-guided sampling results in better samples than simple random sampling (Conjecture 1);
- (2) For both problems and every threshold value  $\theta_k$ , samples obtained with model-guided sampling contain a substantially number of near-optimal instances than samples obtained with a model (Conjecture 2)

We note also that all results have been obtained by sampling a small portion of the instance space (about 10 % for Chess, and about 3 % for Job-Shop).

We now examine the result in more detail. It is evident that the performance on the Job-Shop domain is not as good as on Chess. The natural question that arises is: Why is this so? We conjecture that this is a consequence of the background knowledge for Job-Shop not being as relevant to low cost values, as was the case for Chess. Some evidence for this was already apparent when we had to increase the lengths of clauses allowed for the ILP engine (this is usually a sign that the background knowledge is somewhat low-level). In contrast, with Chess, some of the concepts refer specifically to "cornering" the Black King, with a view of ending the game as soon as possible. We would expect these predicates to be especially useful for positions at depths-of-win near 0. Evidence of the unreliable performance of the EOIS procedure in Chess, with irrelevant background knowledge is in Fig. 6. These results suggest a refinement to the conclusions we can draw from the use of EOIS, namely: we expect the EOIS procedure to be less effective if the background knowledge is not very relevant to low-cost solutions.

Model	$Pr(F(\mathbf{x}) \le \theta_k   M_k)$			
	k = 1  k = 2  k = 3			
None	0.136	0.022	0.001	
ILP	0.816	0.462	0.409	
	(6.0) (21.0) (409.0)			

(a) Chess

Model	$Pr(F(\mathbf{x}) \le \theta_k   M_k)$			
	k = 1	k = 2	k = 3	
None	0.507	0.025	0.003	
ILP	0.647	0.171	0.080	
	(1.3)	(6.8)	(26.7)	

(b) Job-Shop

Figure 4: Probabilities of obtaining good instances  $\mathbf{x}$  for each iteration k of the EOIS procedure. That is, the column k=1 denotes  $P(F(\mathbf{x}) \leq \theta_1$  after iteration 1; the column k=2 denotes  $P(F(\mathbf{x}) \leq \theta_2$  after iteration 2 and so on. In effect, this is an estimate of the precision when predicting  $F(\mathbf{x}) \leq \theta_k$ . "None" in the model column stands for probabilities of the instances, corresponding to simple random sampling  $(M_k=\emptyset)$ . The number in parentheses below each ILP entry denotes the ratio of that entry against the corresponding entry for "None". This represents the gain in precision of using the ILP model over simple random sampling.

Model	Near-Optimal Instances		
	k = 1	k = 3	
None	1/27	2/27	3/27
ILP	11/27	22/27	27/27
	(1000) (1964) (2549)		

(a) Chess

Model	Near-Optimal Instances		
	k = 1	k = 3	
None	3/304	6/304	9/304
ILP	6/304	28/304	36/304
	(1000)	(2895)	

(b) Job-Shop

Figure 5: Fraction of near-optimal instances  $(F(\mathbf{x}) \leq \theta^*)$  generated on each iteration of EOIS. In effect, this is an estimate of the recall (true-positive rate, or sensitivity) when predicting  $F(\mathbf{x}) \leq \theta^*$ . The fraction a/b denotes that a instances of b are generated. The numbers in parentheses denote the number of training instances used by the ILP engine. The values with "None" are the numbers expected by sampling the same number of training instances used for training the ILP engine.

Finally, we note that the experiments with synthetic data have ignored an important aspect of the optimisation problem, namely the time taken to obtain the value of the objective function for each data instance. Clearly, there is a trade-off between the time taken to construct a model, and the time taken to simply draw instances without a model. To address this trade-off we would have estimate the number of instances that need to be randomly sampled to obtain the same numbers of near-optimal instances as with model-assisted EDA; and to compare: (a) the time to obtain values of the objective function for randomly-sampled instances; and (b) the time taken to obtain the corresponding values for the training data used to construct models and the total time taken for model construction. For model-construction to be beneficial, clearly the time in (b) has to be less than (a). For the problems here, random sampling we require approximately 4 (JobShop) to 10 (Chess) times as many samples to obtain the same numbers of near-optimal instances. The times for theory-construction are small enough to expect that (b) is less than (a) for these ratios.

# 4 Concluding Remarks

It is uncommon to see ILP applied to optimisation problems. The use of ILP-constructed theories within an evolutionary optimisation procedure is one answer to the question of how to use ILP for optimisation (but not the only answer: recent work in [7], for example, suggests using ILP-constructed clauses as soft-constraints to a constraint solver). This requires the ILP model to be used generatively, which is not common practice in ILP. In this paper we have been able to do this by a combination of careful definition of the background knowledge, and adding range-restrictions to clauses constructed by ILP. Our results provide evidence that this combination of ILP and logic-programming provides one way of incorporating complex, but relevant domain-knowledge into evolutionary optimisation.

Concerning the specific problems examined here, it is possible that we could have directly constructed a model discriminating near-optimal instances from the rest using ILP alone. The focus of this paper however is on a different question, namely, whether evolutionary optimisation methods can benefit from the use of ILP. The results here should therefore be seen as evidence of improvements possible in an EDA technique when it includes ILP-assisted models. In turn, this evidence could be of relevance for problems where ILP models alone would be insufficient, and we would have to resort to sampling-based methods.

There are several ways in which the work here could be extended. The most immediate is to examine ways of sampling by using techniques developed in probabilistic ILP. Indeed, the principal conjecture of the paper is that the use of models constructed by any form of learning that allows the inclusion domain-knowledge can greatly improve the sampling efficiency of EDA methods. We have

Background	$Pr(F(\mathbf{x}) \le \theta_k   M_{k,B})$			
B	k = 1	k=2	k = 3	
$B_{low}$	0.658	0.417	0.0	
$B_{high}$	0.816	0.462	0.409	
(a)				

Background	Near-Optimal Instances			
B	k = 1	k=2	k = 3	
$B_{low}$	17/27	4/27	0/27	
	(1000)	(1959)	(2581)	
$B_{high}$	11/27	17/27	27/27	
	(1000)	(1964)	(2549)	
(b)				

Figure 6: Precision (a), and recall of near-optimal instances (b) of the EOIS procedure with background knowledge of low relevance to near-optimal solutions. The results are for Chess, with  $B_{low}$  denoting background predicates that simply define the geometry of the board, using the predicates  $less\_than$  and adjacent. These predicates form the background knowledge for most ILP applications to the problem of detecting illegal positions in the KRK endgame.  $B_{high}$  denotes background used previously, with high relevance to low depths-of-win. The numbers in parentheses in (b) are the number of training instances as before.

provided evidence for this conjecture using a classical ILP method. Given these results, we can expect first-order learning that is capable of using domain-knowledge and constructing rules that can allow a non-uniform sampling of ground instances (for example, through the incorporation of probabilities with the rules) will provide even better. We would recommend this as the next step in this line of work.

It is of interest also to consider whether there are any gains to be made by re-use of theories (currently, we re-use data, but re-learn theories from scratch on each new iteration of the EOIS procedure). There is the straightforward approach to this, of simply providing theories constructed earlier as background knowledge for subsequent iterations. A more ambitious variant would retain some portions of the earlier theory (those clauses that entail the current set of positive instances and and none, or only few, of the negative instances, for example), thus reducing the model-construction effort.

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